THE MECHANICAL ENGINEER'S REFERENCE BOOK

A HAND-BOOK OF
TABLES, FORMULAS, AND METHODS
FOR ENGINEERS, STUDENTS, AND DRAFTSMEN

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PREFACE

IN preparing a hand-book for engineering reference it is necessary to select from among a great mass of detailed information the matter which shall be most generally available. Naturally, the differentiation which has taken place in the science of engineering makes it desirable that some one department of work shall predominate, and, as indicated in the title, this book is devoted principally to the presentation of tables, formulas, and reference data for mechanical engineers. It is, therefore, purposely full in the portions relating to machine design and to such information as will render it useful in the drawing room and in the designing department, the intention being to render it available broadly in furnishing a record of general principles, as well as of detailed methods.

The many and varying rules and formulas existing in this connection have been carefully examined, and only those which in the judgment of the author are most generally applicable have been given, since the presentation of a mass of data, much of it contradictory, throws the burden of selection upon the user. In this portion of the work the author has sought to relieve the user of the necessity of selecting from among a mass of contradictory information the matter of the most general value, leaving special work to be conducted—as it should be—under the control of special investigation.

In view of the fact that the metric system has been under active discussion of late, a number of the tables have been presented in both British and metric units, so that those engineers who are desirous of using the latter system may do so. Among these tables may be mentioned the metric steam tables, which render it convenient for steam computations to be made in the metric system.

This work is intended to be a successor to the well-known pocket-book written many years ago by the late John W. Nystrom, and published by Messrs. J. B. Lippincott Company. The plates and stock of that valuable work having been destroyed by fire in 1899, certain of the information therein contained has been utilized, with such modifications as are necessary to meet engineering problems and needs of the present.

Among the valuable works to which acknowledgments are due in the preparation of this hand-book may be mentioned Reuleaux's "Constructor," Unwin's "Machine Design," Weisbach's "Ingenieur," "Des Ingenieurs Taschenbuch Hütte," the Smithsonian Physical Tables, and the hand-books of the Pencoyd Iron Works and the Passaic Steel Company, as well as the various authorities mentioned in the text.

HENRY HARRISON SUPLEE.

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The Mechanical Engineer's Reference Book

MATHEMATICS.

THE engineer should use mechanical appliances for mathematical computations whenever possible, including the slide-rule in some of its various modifications, but the following tables will also be found useful:

MULTIPLICATION TABLE.

By the use of the following table products of numbers from 1 to 10 by numbers from 1 to 100 may be obtained directly, and of larger numbers by successive operations, as follows:

$$67 \times 489 = 67 \times 400 + 67 \times 80 + 67 \times 9 = \left\{\begin{array}{r}26800\\5360\\603\end{array}\right\} = 32763.$$

If both factors consist of more than three figures, one of the factors may be modified and the operation performed as follows:

$$854 \times 279 = 850 \times 279 + 4 \times 279.$$

Here we subtract 4 from 854 and then get the product of 850 by 279 from the table, and add to this the product of 4 by 279, also readily taken from the table; thus:

$$850 \times 279 + 4 \times 279 = \left\{\begin{array}{r}170000\\59500\\7650\end{array}\right\} + \left\{\begin{array}{r}800\\280\\36\end{array}\right\}$$

$$= \quad 237150 \quad + \quad 1116 = 238266.$$

1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0
1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
11	22	33	44	55	66	77	88	99
12	24	36	48	60	72	84	96	108
13	26	39	52	65	78	91	104	117
14	28	42	56	70	84	98	112	126
15	30	45	60	75	90	105	120	135
16	32	48	64	80	96	112	128	144
17	34	51	68	85	102	119	136	153
18	36	54	72	90	108	126	144	162
19	38	57	76	95	114	133	152	171
20	40	60	80	100	120	140	160	180
21	42	63	84	105	126	147	168	189
22	44	66	88	110	132	154	176	198
23	46	69	92	115	138	161	184	207
24	48	72	96	120	144	168	192	216
25	50	75	100	125	150	175	200	225
26	52	78	104	130	156	182	208	234
27	54	81	108	135	162	189	216	243
28	56	84	112	140	168	196	224	252
29	58	87	116	145	174	203	232	261
30	60	90	120	150	180	210	240	270
31	62	93	124	155	186	217	248	279
32	64	96	128	160	192	224	256	288
33	66	99	132	165	198	231	264	297
34	68	102	136	170	204	238	272	306
35	70	105	140	175	210	245	280	315
36	72	108	144	180	216	252	288	324
37	74	111	148	185	222	259	296	333
38	76	114	152	190	228	266	304	342
39	78	117	156	195	234	273	312	351
40	80	120	160	200	240	280	320	360
41	82	123	164	205	246	287	328	369
42	84	126	168	210	252	294	336	378
43	86	129	172	215	258	301	344	387
44	88	132	176	220	264	308	352	396
45	90	135	180	225	270	315	360	405
46	92	138	184	230	276	322	368	414
47	94	141	188	235	282	329	376	423
48	96	144	192	240	288	336	384	432
49	98	147	196	245	294	343	392	441
50	100	150	200	250	300	350	400	450
51	102	153	204	255	306	357	408	459
52	104	156	208	260	312	364	416	468
53	106	159	212	265	318	371	424	477
54	108	162	216	270	324	378	432	486
55	110	165	220	275	330	385	440	495
56	112	168	224	280	336	392	448	504
57	114	171	228	285	342	399	456	513
58	116	174	232	290	348	406	464	522
59	118	177	236	295	354	413	472	531
60	120	180	240	300	360	420	480	540
61	122	183	244	305	366	427	488	549
62	124	186	248	310	372	434	496	558
63	126	189	252	315	378	441	504	567
64	128	192	256	320	384	448	512	576

1	2	3	4	5	6	7	8	9
65	130	195	260	325	390	455	520	585
66	132	198	264	330	396	462	528	594
67	134	201	268	335	402	469	536	603
68	136	204	272	340	408	476	544	612
69	138	207	276	345	414	483	552	621
70	140	210	280	350	420	490	560	630
71	142	213	284	355	426	497	568	639
72	144	216	288	360	432	504	576	648
73	146	219	292	365	438	511	584	657
74	148	222	296	370	444	518	592	666
75	150	225	300	375	450	525	600	675
76	152	228	304	380	456	532	608	684
77	154	231	308	385	462	539	616	693
78	156	234	312	390	468	546	624	702
79	158	237	316	395	474	553	632	711
80	160	240	320	400	480	560	640	720
81	162	243	324	405	486	567	648	729
82	164	246	328	410	492	574	656	738
83	166	249	332	415	498	581	664	747
84	168	252	336	420	504	588	672	756
85	170	255	340	425	510	595	680	765
86	172	258	344	430	516	602	688	774
87	174	261	348	435	522	609	696	783
88	176	264	352	440	528	616	704	792
89	178	267	356	445	534	623	712	801
90	180	270	360	450	540	630	720	810
91	182	273	364	455	546	637	728	819
92	184	276	368	460	552	644	736	828
93	186	279	372	465	558	651	744	837
94	188	282	376	470	564	658	752	846
95	190	285	380	475	570	665	760	855
96	192	288	384	480	576	672	768	864
97	194	291	388	485	582	679	776	873
98	196	294	392	490	588	686	784	882
99	198	297	396	495	594	693	792	891

FACTOR TABLE.

It is often desirable to know whether a number is a prime number or a product of two or more factors. The following table gives the factors of all numbers not divisible by 2, 3, or 5 up to 9599, and shows all prime numbers up to 9595.

If the last figure of a number is divisible by 2, the whole number is divisible by 2. Thus 26154 is divisible by 2.

If the sum of the digits of which a number is composed is divisible by 3, the number is divisible by 3. Thus the sum of the digits of 26154 is equal to 18, which is divisible by 3; hence the whole number is divisible by 3.

Any number ending with 0 or 5 is divisible by 5.

It is therefore possible to discover by inspection whether a number is divisible by 2, 3, or 5, and such a division will bring most large numbers—not prime numbers—within the compass of the table.

To use the table, look along the top lines of the successive sections for the hundreds, and in the vertical columns at the left for the units and tens. The factors will be found at the intersection. If no factors are given, the number is a prime.

Thus given the number 5203, which is not divisible by 2, 3, or 5, according to the above rules, we find under 5200, and opposite 3, the factors $11 \times 11 \times 43 = 5203$. In like manner we see that 5233 is a prime number, and so on for any other number.

N	0	300	600	900
1	.	7 . 43	.	17 . 53
7	.	.	.	.
11	.	.	13 . 47	.
13	.	.	.	11 . 83
17	.	.	.	7 . 131
19	.	11 . 29	.	.
23	.	17 . 19	7 . 89	13 . 71
29	.	7 . 47	17 . 37	.
31	.	.	.	7 . 7 . 19
37	.	.	7 . 7 . 13	.
41	.	11 . 31	.	.
43	.	7 . 7 . 7	.	23 . 41
47	.	.	.	.
49	7 . 7	.	11 . 59	13 . 73
53	.	.	.	.
59	.	.	.	7 . 137
61	.	19 . 19	.	31 . 31
67	.	.	23 . 29	.
71	.	7 . 53	11 . 61	.
73	.	.	.	7 . 139
77	7 . 11	13 . 29	.	.
79	.	.	7 . 97	11 . 89
83	.	.	.	.
89	.	.	13 . 53	23 . 43
91	7 . 13	17 . 23	.	.
97	.	.	17 . 41	.

N	100	400	700	1000
1	.	.	.	7 . 11 . 13
3	.	13 . 31	19 . 37	17 . 59
7	.	11 . 37	7 . 101	19 . 53
9	.	.	.	.
13	.	7 . 59	23 . 31	.
19	7 . 17	.	.	.
21	11 . 11	.	7 . 103	.
27	.	7 . 61	.	13 . 79
31	.	.	17 . 43	.
33	7 . 19	.	.	.
37	.	19 . 23	11 . 67	17 . 61
39	.	.	.	.
43	11 . 13	.	.	7 . 149
49	.	.	7 . 107	.
51	.	11 . 41	.	.

N	100	400	700	1000
57	.	.	.	7 . 151
61	7 . 23	.	.	.
63	.	.	7 . 109	.
67	.	.	13 . 59	11 . 97
69	13 . 13	7 . 67	.	.
73	.	11 . 43	.	29 . 37
79	.	.	19 . 41	13 . 83
81	.	13 . 37	11 . 71	23 . 47
87	11 . 17	.	.	.
91	.	.	7 . 113	.
93	.	17 . 29	13 . 61	.
97	.	7 . 71	.	.
99	.	.	17 . 47	7 . 157

N	200	500	800	1100
3	7 . 29	.	11 . 73	.
9	11 . 19	.	.	.
11	.	7 . 73	.	11 . 101
17	7 . 31	11 . 47	19 . 43	.
21	13 . 17	.	.	19 . 59
23	.	.	.	.
27	.	17 . 31	.	7 . 7 . 23
29	.	23 . 23	.	.
33	.	13 . 41	7 . 7 . 17	11 . 103
39	.	7 . 7 . 11	.	17 . 67
41	.	.	29 . 29	7 . 163
47	13 . 19	.	7 . 11 . 11	31 . 37
51	.	19 . 29	23 . 37	.
53	11 . 23	7 . 79	.	.
57	.	.	.	13 . 89
59	7 . 37	13 . 43	.	19 . 61
63	.	.	.	.
69	.	.	11 . 79	7 . 167
71	.	.	13 . 67	.
77	.	.	.	11 . 107
81	.	7 . 83	.	.
83	.	11 . 53	.	7 . 13 . 13
87	7 . 41	.	.	.
89	17 . 17	19 . 31	7 . 127	29 . 41
93	.	.	19 . 47	.
99	13 . 23	.	29 . 31	11 . 109

N	1200	1500	1800	2100
1	.	19 . 79	.	11 . 191
7	17 . 71	11 . 137	13 . 139	7 . 7 . 43
11	7 . 173	.	.	.
13	.	17 . 89	7 . 7 . 37	.
17	.	37 . 41	23 . 79	29 . 73
19	23 . 53	7 . 7 . 31	17 . 107	13 . 163
23	.	.	.	11 . 193
29	.	11 . 139	31 . 59	.
31	.	.	.	.
37	.	29 . 53	11 . 167	.
41	17 . 73	23 . 67	7 . 263	.
43	11 . 113	.	19 . 97	.
47	29 . 43	7 . 13 . 17	.	19 . 113
49	.	.	43 . 43	7 . 307
53	7 . 179	.	17 . 109	.
59	.	.	11 . 13 . 13	17 . 127
61	13 . 97	7 . 223	.	.
67	7 . 181	.	.	11 . 197
71	31 . 41	.	.	13 . 167
73	19 . 67	11 . 11 . 13	.	41 . 53
77	.	19 . 83	.	7 . 311
79	.	.	.	.
83	.	.	7 . 269	37 . 59
89	.	7 . 227	.	11 . 199
91	.	37 . 43	31 . 61	7 . 313
97	.	.	7 . 271	13 . 13 . 13

N	1300	1600	1900	2200
1	.	.	.	31 . 71
3	.	7 . 229	11 . 173	.
7	.	.	.	.
9	7 . 11 . 17	.	23 . 83	47 . 47
13	13 . 101	.	.	.
19	.	.	19 . 101	7 . 317
21	.	.	17 . 113	.
27	.	.	41 . 47	17 . 131
31	11 . 11 . 11	7 . 233	.	23 . 97
33	31 . 43	23 . 71	.	7 . 11 . 29
37	7 . 191	.	13 . 149	.
39	13 . 103	11 . 149	7 . 277	.
43	17 . 79	31 . 53	29 . 67	.
49	19 . 71	17 . 97	.	13 . 173
51	7 . 193	13 . 127	.	.

N	1300	1600	1900	2200
57	23 . 59	.	19 . 103	37 . 61
61	.	11 . 151	37 . 53	7 . 17 . 19
63	29 . 47	.	13 . 151	31 . 73
67	.	.	7 . 281	.
69	37 . 37	.	11 . 179	.
73	.	7 . 239	.	.
79	7 . 197	23 . 73	.	43 . 53
81	.	41 . 41	7 . 283	.
87	19 . 73	7 . 241	.	.
91	13 . 107	19 . 89	11 . 181	29 . 79
93	7 . 199	.	.	.
97	11 . 127	.	.	.
99	.	.	.	11 . 11 . 19

N	1400	1700	2000	2300
3	23 . 61	13 . 131	.	7 . 7 . 47
9	.	.	7 . 7 . 41	.
11	17 . 83	29 . 59	.	.
17	13 . 109	17 . 101	.	7 . 331
21	7 . 7 . 29	.	43 . 47	11 . 211
23	.	.	7 . 17 . 17	23 . 101
27	.	11 . 157	.	13 . 179
29	.	7 . 13 . 19	.	17 . 137
33	.	.	19 . 107	.
39	.	37 . 47	.	.
41	11 . 131	.	13 . 157	.
47	.	.	23 . 89	.
51	.	17 . 103	7 . 293	.
53	.	.	.	13 . 181
57	31 . 47	7 . 251	11 . 11 . 17	.
59	.	.	29 . 71	7 . 337
63	7 . 11 . 19	41 . 43	.	17 . 139
69	13 . 113	29 . 61	.	23 . 103
71	.	7 . 11 . 23	19 . 109	.
77	7 . 211	.	31 . 67	.
81	.	13 . 137	.	.
83	.	.	.	.
87	.	.	.	7 . 11 . 31
89	.	.	.	.
93	.	11 . 163	7 . 13 . 23	.
99	.	7 . 257	.	.

N	2400	2700	3000	3300
1	7 . 7 . 7 . 7	37 . 73	.	.
7	29 . 83	.	31 . 97	.
11	.	.	.	7 . 11 . 43
13	19 . 127	.	23 . 131	.
17	.	11 . 13 . 19	7 . 431	31 . 107
19	41 . 59	.	.	.
23	.	7 . 389	.	.
29	7 . 347	.	13 . 233	.
31	11 . 13 . 17	.	7 . 433	.
37	.	7 . 17 . 23	.	47 . 71
41	.	.	.	13 . 257
43	7 . 349	13 . 211	17 . 179	.
47	.	41 . 67	11 . 277	.
49	31 . 79	.	.	17 . 197
53	11 . 223	.	43 . 71	7 . 479
59	.	31 . 89	7 . 19 . 23	.
61	23 . 107	11 . 251	.	.
67	.	.	.	7 . 13 . 37
71	7 . 353	17 . 163	37 . 83	.
73	.	47 . 59	7 . 439	.
77	.	.	17 . 181	11 . 307
79	37 . 67	7 . 397	.	31 . 109
83	13 . 191	11 . 11 . 23	.	17 . 199
89	19 . 131	.	.	.
91	47 . 53	.	11 . 281	.
97	11 . 227	.	19 . 163	43 . 79

N	2500	2800	3100	3400
1	41 . 61	.	7 . 443	19 . 179
3	.	.	29 . 107	41 . 83
7	23 . 109	7 . 401	13 . 239	.
9	13 . 193	53 . 53	.	7 . 487
13	13 . 359	29 . 97	11 . 283	.
19	11 . 229	.	.	13 . 263
21	.	7 . 13 . 31	.	11 . 311
27	7 . 19 . 19	11 . 257	53 . 59	23 . 149
31	.	19 . 149	31 . 101	47 . 73
33	17 . 149	.	13 . 241	.
37	43 . 59	.	.	7 . 491
39	.	17 . 167	43 . 73	19 . 181
43	.	.	7 . 449	11 . 313
49	.	7 . 11 . 37	47 . 67	.
51	.	.	23 . 137	7 . 17 . 29

N	2500	2800	3100	3400
57	.	.	7 . 11 . 41	.
61	13 . 197	.	29 . 109	.
63	11 . 233	7 . 409	.	.
67	17 . 151	47 . 61	.	.
69	7 . 367	19 . 151	.	.
73	31 . 83	13 . 13 . 17	19 . 167	23 . 151
79	.	.	11 . 17 . 17	7 . 7 . 71
81	29 . 89	43 . 67	.	59 . 59
87	13 . 199	.	.	11 . 317
91	.	7 . 7 . 59	.	.
93	.	11 . 263	31 . 103	7 . 499
97	7 . 7 . 53	.	23 . 139	13 . 269
99	23 . 113	13 . 223	7 . 457	.

N	2600	2900	3200	3500
3	19 . 137	.	.	31 . 113
9	.	.	.	11 . 11 . 29
11	7 . 373	41 . 71	13 . 13 . 19	.
17	.	.	.	.
21	.	23 . 127	.	7 . 503
23	43 . 61	37 . 79	11 . 293	13 . 271
27	37 . 71	.	7 . 461	.
29	11 . 239	29 . 101	.	.
33	.	7 . 419	53 . 61	.
39	7 . 13 . 29	.	41 . 79	.
41	19 . 139	17 . 173	7 . 463	.
47	.	7 . 421	17 . 191	.
51	11 . 241	13 . 227	.	53 . 67
53	7 . 379	.	.	11 . 17 . 19
57	.	.	.	.
59	.	11 . 269	.	.
63	.	.	13 . 251	7 . 509
69	17 . 157	.	7 . 467	43 . 83
71	.	.	.	.
77	.	13 . 229	29 . 113	7 . 7 . 73
81	7 . 383	11 . 271	17 . 193	.
83	.	19 . 157	7 . 7 . 67	.
87	.	29 . 103	19 . 173	17 . 211
89	.	7 . 7 . 61	11 . 13 . 23	37 . 97
93	.	41 . 73	37 . 89	.
99	.	.	.	59 . 61

N	3600	3900	4200	4500
1	13 . 277	47 . 83	.	7 . 643
7	.	.	7 . 601	.
11	23 . 157	.	.	13 . 347
13	.	7 . 13 . 43	11 . 383	.
17	.	.	.	.
19	7 . 11 . 47	.	.	.
23	.	.	41 . 103	.
29	19 . 191	.	.	7 . 647
31	.	.	.	23 . 197
37	.	31 . 127	19 . 223	13 . 349
41	11 . 331	7 . 563	.	19 . 239
43	.	.	.	7 . 11 . 59
47	7 . 521	.	31 . 137	.
49	41 . 89	11 . 359	7 . 607	.
53	13 . 281	59 . 67	.	29 . 157
59	.	37 . 107	.	47 . 97
61	7 . 523	17 . 233	.	.
67	19 . 193	.	17 . 251	.
71	.	11 . 19 . 19	.	7 . 653
73	.	29 . 137	.	17 . 269
77	.	41 . 97	7 . 13 . 47	23 . 199
79	13 . 283	23 . 173	11 . 389	19 . 241
83	29 . 127	7 . 569	.	.
89	7 . 17 . 31	.	.	13 . 353
91	.	13 . 307	7 . 613	.
97	.	7 . 571	.	.

N	3700	4000	4300	4600
1	.	.	11 . 17 . 23	43 . 107
3	7 . 23 . 23	.	13 . 331	.
7	11 . 337	.	59 . 73	17 . 271
9	.	19 . 211	31 . 139	11 . 419
13	47 . 79	.	19 . 227	7 . 659
19	.	.	7 . 617	31 . 149
21	61 . 61	.	29 . 149	.
27	.	.	.	7 . 661
31	7 . 13 . 41	29 . 139	61 . 71	11 . 421
33	.	37 . 109	7 . 619	41 . 113
37	37 . 101	11 . 367	.	.
39	.	7 . 577	.	.
43	19 . 197	13 . 311	43 . 101	.
49	23 . 163	.	.	.
51	11 . 11 . 31	.	19 . 229	.

N	3700	4000	4300	4600
57	13 . 17 . 17	.	.	.
61	.	31 . 131	7 . 7 . 89	59 . 79
63	53 . 71	17 . 239	.	.
67	.	7 . 7 . 83	11 . 397	13 . 359
69	.	13 . 313	17 . 257	7 . 23 . 29
73	7 . 7 . 7 . 11	.	.	.
79	.	.	29 . 151	.
81	19 . 199	7 . 11 . 53	13 . 337	31 . 151
87	7 . 541	61 . 67	41 . 107	43 . 109
91	17 . 223	.	.	.
93	.	.	23 . 191	13 . 19 . 19
97	.	17 . 241	.	7 . 11 . 61
99	29 . 131	.	53 . 83	37 . 127

N	3800	4100	4400	4700
3	.	11 . 373	7 . 17 . 37	.
9	13 . 293	7 . 587	.	17 . 277
11	37 . 103	.	11 . 401	7 . 673
17	11 . 347	23 . 179	7 . 631	53 . 89
21	.	13 . 317	.	.
23	.	7 . 19 . 31	.	.
27	43 . 89	.	19 . 233	29 . 163
29	7 . 547	.	43 . 103	.
33	.	.	11 . 13 . 31	.
39	11 . 349	.	23 . 193	7 . 677
41	23 . 167	41 . 101	.	11 . 431
47	.	11 . 13 . 29	.	47 . 101
51	.	7 . 593	.	.
53	.	.	61 . 73	7 . 7 . 97
57	7 . 19 . 29	.	.	67 . 71
59	17 . 227	.	7 . 7 . 7 . 13	.
63	.	23 . 181	.	11 . 433
69	53 . 73	11 . 379	41 . 109	19 . 251
71	7 . 7 . 79	43 . 97	17 . 263	13 . 367
77	.	.	11 . 11 . 37	17 . 281
81	.	37 . 113	.	7 . 683
83	11 . 353	47 . 89	.	.
87	13 . 13 . 23	53 . 79	7 . 641	.
89	.	59 . 71	67 . 67	.
93	17 . 229	7 . 599	.	.
99	7 . 557	13 . 17 . 19	11 . 409	.

N	4800	5100	5400	5700
1	.	.	11 . 491	.
7	11 . 19 . 23	.	.	13 . 439
11	17 . 283	19 . 269	7 . 773	.
13	.	.	.	29 . 197
17	.	7 . 17 . 43	.	.
19	61 . 79	.	.	7 . 19 . 43
23	7 . 13 . 53	47 . 109	11 . 17 . 29	59 . 97
29	11 . 439	23 . 223	61 . 89	17 . 337
31	.	7 . 733	.	11 . 521
37	7 . 691	11 . 467	.	.
41	47 . 103	53 . 97	.	.
43	29 . 167	37 . 139	.	.
47	37 . 131	.	13 . 419	7 . 821
49	13 . 373	19 . 271	.	.
53	23 . 211	.	7 . 19 . 41	11 . 523
59	43 . 113	7 . 11 . 67	53 . 103	13 . 443
61	.	13 . 397	43 . 127	7 . 823
67	31 . 157	.	7 . 11 . 71	73 . 79
71	.	.	.	29 . 199
73	11 . 443	7 . 739	13 . 421	23 . 251
77	.	31 . 167	.	53 . 109
79	7 . 17 . 41	.	.	.
83	19 . 257	71 . 73	.	.
89	.	.	11 . 499	7 . 827
91	67 . 73	29 . 179	17 . 17 . 19	.
97	59 . 83	.	23 . 239	11 . 17 . 31

N	4900	5200	5500	5800
1	13 . 13 . 29	7 . 743	.	.
3	.	11 . 11 . 43	.	7 . 829
7	7 . 701	41 . 127	.	.
9	.	.	7 . 787	37 . 157
13	17 . 17 . 17	13 . 401	37 . 149	.
19	.	17 . 307	.	11 . 23 . 23
21	7 . 19 . 37	23 . 227	.	.
27	13 . 379	.	.	.
31	.	.	.	7 . 7 . 7 . 17
33	.	.	11 . 503	19 . 307
37	.	.	7 . 7 . 113	13 . 449
39	11 . 449	13 . 13 . 31	29 . 191	.
43	.	7 . 7 . 107	23 . 241	.
49	7 . 7 . 101	29 . 181	31 . 179	.
51	.	59 . 89	7 . 13 . 61	.

N	4900	5200	5500	5800
57	.	7 . 751	.	.
61	11 . 11 . 41	.	67 . 83	.
63	7 . 709	19 . 277	.	11 . 13 . 41
67	.	23 . 229	19 . 293	.
69	.	11 . 479	.	.
73	.	.	.	7 . 839
79	13 . 383	.	7 . 797	.
81	17 . 293	.	.	.
87	.	17 . 311	37 . 151	7 . 29 . 29
91	7 . 23 . 31	11 . 13 . 37	.	43 . 137
93	.	67 . 79	7 . 17 . 47	71 . 83
97	19 . 263	.	29 . 193	.
99	.	7 . 757	11 . 509	17 . 347

N	5000	5300	5600	5900
3	.	.	13 . 431	.
9	.	.	71 . 79	19 . 311
11	.	47 . 113	31 . 181	23 . 257
17	29 . 173	13 . 409	41 . 137	61 . 97
21	.	17 .. 313	7 . 11 . 73	31 . 191
23	.	.	.	.
27	11 . 457	7 . 761	17 . 331	.
29	47 . 107	73 . 73	13 . 433	7 . 7 . 11 . 11
33	7 . 719	.	43 . 131	17 . 349
39	.	19 . 281	.	.
41	71 . 71	7 . 7 . 109	.	13 . 457
47	7 . 7 . 103	.	.	19 . 313
51	.	.	.	11 . 541
53	31 . 163	53 . 101	.	.
57	13 . 389	11 . 487	..	7 . 23 . 37
59	.	23 . 233	.	59 . 101
63	61 . 83	31 . 173	7 . 809	67 . 89
69	37 . 137	7 . 13 . 59	.	47 . 127
71	11 . 461	41 . 131	53 . 107	7 . 853
77	.	19 . 283	7 . 811	43 . 139
81	.	.	13 . 19 . 23	.
83	13 . 17 . 23	7 . 769	.	31 . 193
87	.	.	11 . 11 . 47	.
89	7 . 727	17 . 317	.	53 . 113
93	11 . 463	.	.	13 . 461
99	.	.	41 . 139	7 . 857

N	6000	6300	6600	6900
1	17 . 353	.	7 . 23 . 41	67 . 103
7	.	7 . 17 . 53	.	.
11	.	.	11 . 601	.
13	7 . 859	59 . 107	17 . 389	31 . 223
17	11 . 547	.	13 . 509	.
19	13 . 463	71 . 89	.	11 . 17 . 37
23	19 . 317	.	37 . 179	7 . 23 . 43
29	.	.	7 . 947	13 . 13 . 41
31	37 . 163	13 . 487	19 . 349	29 . 239
37	.	.	.	7 . 991
41	7 . 863	17 . 373	29 . 229	11 . 631
43	.	.	7 . 13 . 73	53 . 131
47	.	11 . 577	17 . 17 . 23	.
49	23 . 263	7 . 907	61 . 109	.
53	.	.	.	17 . 409
59	73 . 83	.	.	.
61	11 . 19 . 29	.	.	.
67	.	.	59 . 113	.
71	13 . 467	23 . 277	7 . 953	.
73	.	.	.	19 . 367
77	59 . 103	7 . 911	11 . 607	.
79	.	.	.	7 . 997
83	7 . 11 . 79	13 . 491	41 . 163	.
89	.	.	.	29 . 241
91	.	7 . 11 . 83	.	.
97	7 . 13 . 67	.	37 . 181	.

N	6100	6400	6700	7000
1	.	37 . 173	.	.
3	17 . 359	19 . 337	.	47 . 149
7	31 . 197	43 . 149	19 . 353	7 . 7 . 11 . 13
9	41 . 149	13 . 17 . 29	.	43 . 163
13	.	11 . 11 . 53	7 . 7 . 137	.
19	29 . 211	7 . 7 . 131	.	.
21	.	.	11 . 13 . 47	7 . 17 . 59
27	11 . 557	.	7 . 31 . 31	.
31	.	59 . 109	53 . 127	79 . 89
33	.	7 . 919	.	13 . 541
37	17 . 19 . 19	41 . 157	.	31 . 227
39	7 . 877	47 . 137	23 . 293	.
43	.	17 . 379	11 . 613	.
49	11 . 13 . 43	.	17 . 397	7 . 19 . 53
51	.	.	43 . 157	11 . 641

N	6100	6400	6700	7000
57	47 . 131	11 . 587	29 . 233	.
61	61 . 101	7 . 13 . 71	.	23 . 307
63	.	23 . 281	.	7 . 1009
67	7 . 881	29 . 223	67 . 101	37 . 191
69	31 . 199	.	7 . 967	.
73	.	.	13 . 521	11 . 643
79	37 . 167	11 . 19 . 31	.	.
81	7 . 883	.	.	73 . 97
87	23 . 269	13 . 499	11 . 617	19 . 373
91	41 . 151	.	.	7 . 1013
93	11 . 563	43 . 151	.	41 . 173
97	.	73 . 89	7 . 971	47 . 151
99	.	67 . 97	13 . 523	31 . 229

N	6200	6500	6800	7100
3	.	7 . 929	.	.
9	7 . 887	23 . 283	11 . 619	.
11	.	17 . 383	7 . 7 . 139	13 . 547
17	.	7 . 7 . 7 . 19	17 . 401	11 . 647
21	.	.	19 . 359	.
23	7 . 7 . 127	11 . 593	.	17 . 419
27	13 . 479	61 . 107	.	.
29	.	.	.	.
33	23 . 271	47 . 139	.	7 . 1019
39	17 . 367	13 . 503	7 . 977	11 . 11 . 59
41	79 . 79	31 . 211	.	37 . 193
47	.	.	41 . 167	7 . 1021
51	7 . 19 . 47	.	13 . 17 . 31	.
53	13 . 13 . 37	.	7 . 11 . 89	23 . 311
57	.	79 . 83	.	17 . 421
59	11 . 569	7 . 937	19 . 19 . 19	.
63	.	.	.	13 . 19 . 29
69	.	.	.	67 . 107
71	.	.	.	71 . 101
77	.	.	13 . 23 . 23	.
81	11 . 571	.	7 . 983	43 . 167
83	61 . 103	29 . 227	.	11 . 653
87	.	7 . 941	71 . 97	.
89	19 . 331	11 . 599	83 . 83	7 . 13 . 79
93	7 . 29 . 31	19 . 347	61 . 113	.
99	.	.	.	23 . 313

N	7200	7500	7800	8100
1	19 . 379	13 . 577	29 . 269	.
7	.	.	37 . 211	11 . 11 . 67
11	.	7 . 29 . 37	73 . 107	.
13	.	11 . 683	13 . 601	7 . 19 . 61
17	7 . 1031	.	.	.
19	.	73 . 103	7 . 1117	23 . 353
23	31 . 233	.	.	.
29	.	.	.	11 . 739
31	7 . 1033	17 . 443	41 . 191	47 . 173
37	.	.	17 . 461	79 . 103
41	13 . 557	.	.	7 . 1163
43	.	19 . 397	11 . 23 . 31	17 . 479
47	.	.	7 . 19 . 59	.
49	11 . 659	.	47 . 167	29 . 281
53	.	7 . 13 . 83	.	31 . 263
59	7 . 17 . 61	.	29 . 271	41 . 199
61	53 . 137	.	7 . 1123	.
67	13 . 13 . 43	7 . 23 . 47	.	.
71	11 . 661	67 . 113	17 . 463	.
73	7 . 1039	.	.	11 . 743
77	19 . 383	.	.	13 . 17 . 37
79	29 . 251	11 . 13 . 53	.	.
83	.	.	.	7 . 7 . 167
89	37 . 197	.	7 . 7 . 7 . 23	19 . 431
91	23 . 317	.	13 . 607	.
97	.	71 . 107	53 . 149	7 . 1171

N	7300	7600	7900	8200
1	7 . 7 . 149	11 . 691	.	59 . 139
3	67 . 109	.	7 . 1129	13 . 631
7	.	.	.	29 . 283
9	.	7 . 1087	11 . 719	.
13	71 . 103	23 . 331	41 . 193	43 . 191
19	13 . 563	19 . 401	.	.
21	.	.	89 . 89	.
27	17 . 431	29 . 263	.	19 . 433
31	.	13 . 587	7 . 11 . 103	.
33	.	17 . 449	.	.
37	11 . 23 . 29	7 . 1091	.	.
39	41 . 179	.	17 . 467	7 . 11 . 107
43	7 . 1049	.	13 . 13 . 47	.
49	.	.	.	73 . 113
51	.	7 . 1093	.	37 . 223

N	7300	7600	7900	8200
57	7 . 1051	13 . 19 . 31	73 . 109	23 . 359
61	17 . 433	47 . 163	19 . 419	11 . 751
63	37 . 199	79 . 97	.	.
67	53 . 139	11 . 17 . 41	31 . 257	7 . 1181
69	.	.	13 . 613	.
73	73 . 101	.	7 . 17 . 67	.
79	47 . 157	7 . 1097	79 . 101	17 . 487
81	11 . 11 . 61	.	23 . 347	7 . 7 . 13 . 13
87	83 . 89	.	7 . 7 . 163	.
91	19 . 389	.	61 . 131	.
93	.	7 . 7 . 157	.	.
97	13 . 569	43 . 179	11 . 727	.
99	7 . 7 . 151	.	19 . 421	43 . 193

N	7400	7700	8000	8300
3	11 . 673	.	53 . 151	19 . 19 . 23
9	31 . 239	13 . 593	.	7 . 1187
11	.	11 . 701	.	.
17	.	.	.	.
21	41 . 181	7 . 1103	13 . 617	53 . 157
23	13 . 571	.	71 . 113	7 . 29 . 41
27	7 . 1061	.	23 . 349	11 . 757
29	17 . 19 . 23	59 . 131	7 . 31 . 37	.
33	.	11 . 19 . 37	29 . 277	13 . 641
39	43 . 173	71 . 109	.	31 . 269
41	7 . 1063	.	11 . 17 . 43	19 . 439
47	11 . 677	61 . 127	13 . 619	17 . 491
51	.	23 . 337	83 . 97	7 . 1193
53	29 . 257	.	.	.
57	.	.	7 . 1151	61 . 137
59	.	.	.	13 . 643
63	17 . 439	7 . 1109	11 . 733	.
69	7 . 11 . 97	17 . 457	.	.
71	31 . 241	19 . 409	7 . 1153	11 . 761
77	.	7 . 11 . 101	41 . 197	.
81	.	31 . 251	.	17 . 17 . 29
83	7 . 1069	43 . 181	59 . 137	83 . 101
87	.	13 . 599	.	.
89	.	.	.	.
93	59 . 127	.	.	7 . 11 . 109
99	.	11 . 709	7 . 13 . 89	37 . 227

N	8400	8700	9000	9300
1	31 . 271	7 . 11 . 113	.	71 . 131
7	7 . 1201	.	.	41 . 227
11	13 . 647	31 . 281	.	.
13	47 . 179	.	.	67 . 139
17	19 . 443	23 . 379	71 . 127	7 . 11 . 11 . 11
19	.	.	29 . 311	.
23	.	11 . 13 . 61	7 . 1289	.
29	.	7 . 29 . 43	.	19 . 491
31	.	.	11 . 821	7 . 31 . 43
37	11 . 13 . 59	.	7 . 1291	.
41	23 . 367	.	.	.
43	.	7 . 1249	.	.
47	.	.	83 . 109	13 . 719
49	7 . 17 . 71	13 . 673	.	.
53	79 . 107	.	11 . 823	47 . 199
59	11 . 769	19 . 461	.	7 . 7 . 191
61	.	.	13 . 17 . 41	11 . 23 . 37
67	.	11 . 797	.	17 . 19 . 29
71	43 . 197	7 . 7 . 179	47 . 193	.
73	37 . 229	31 . 283	43 . 211	7 . 13 . 103
77	7 . 7 . 173	67 . 131	29 . 313	.
79	61 . 139	.	7 . 1297	83 . 113
83	17 . 499	.	31 . 293	11 . 853
89	13 . 653	11 . 17 . 47	61 . 149	41 . 229
91	7 . 1213	59 . 149	.	.
97	29 . 293	19 . 463	11 . 827	.

N	8500	8800	9100	9400
1	.	13 . 677	19 . 479	7 . 17 . 79
3	11 . 773	.	.	.
7	47 . 181	.	7 . 1301	23 . 409
9	67 . 127	23 . 383	.	97 . 97
13	.	7 . 1259	13 . 701	.
19	7 . 1217	.	11 . 829	.
21	.	.	7 . 1303	.
27	.	7 . 13 . 97	.	11 . 857
31	19 . 449	.	23 . 397	.
33	7 . 23 . 53	11 . 11 . 73	.	.
37	.	.	.	.
39	.	.	13 . 19 . 37	.
43	.	37 . 239	41 . 223	7 . 19 . 71
49	83 . 103	.	7 . 1307	11 . 859
51	17 . 503	53 . 167	.	13 . 727

N	8500	8800	9100	9400
57	43 . 199	17 . 521	.	7 . 7 . 193
61	7 . 1223	.	.	.
63	.	.	7 . 7 . 11 . 17	.
67	13 . 659	.	89 . 103	.
69	11 . 19 . 41	7 . 7 . 181	53 . 173	17 . 557
73	.	19 . 467	.	.
79	23 . 373	13 . 683	67 . 137	.
81	.	83 . 107	.	19 . 499
87	31 . 277	.	.	53 . 179
91	11 . 11 . 71	17 . 523	7 . 13 . 101	.
93	13 . 661	.	29 . 317	11 . 863
97	.	7 . 31 . 41	17 . 541	.
99	.	11 . 809	.	7 . 23 . 59

N	8600	8900	9200	9500
3	7 . 1229	29 . 307	.	13 . 17 . 43
9	.	59 . 151	.	37 . 257
11	79 . 109	7 . 19 . 67	61 . 151	.
17	7 . 1231	37 . 241	13 . 709	31 . 307
21	37 . 233	11 . 811	.	.
23	.	.	23 . 401	89 . 107
27	.	79 . 113	.	7 . 1361
29	.	.	11 . 839	13 . 733
33	89 . 97	.	7 . 1319	.
39	53 . 163	7 . 1277	.	.
41	.	.	.	7 . 29 . 47
47	.	23 . 389	7 . 1321	.
51	41 . 211	.	11 . 29 . 29	.
53	17 . 509	7 . 1279	19 . 487	41 . 233
57	11 . 787	13 . 13 . 53	.	19 . 503
59	7 . 1237	17 . 17 . 31	47 . 197	11 . 11 . 79
63	.	.	59 . 157	73 . 131
69	.	.	13 . 23 . 31	7 . 1367
71	13 . 23 . 29	.	73 . 127	17 . 563
77	.	47 . 191	.	61 . 157
81	.	7 . 1283	.	11 . 13 . 67
83	19 . 457	13 . 691	.	7 . 37 . 37
87	7 . 17 . 73	11 . 19 . 43	37 . 251	.
89	.	89 . 101	7 . 1327	43 . 223
93	.	17 . 23 . 23	.	53 . 181
99	.	.	17 . 547	29 . 331

FRACTIONS.

There are two methods of indicating subdivisions in general use,—one by continual bisection, as on the common foot-rule, in which the inch is divided into halves, quarters, eighths, sixteenths, etc., the other by division into tenths, hundredths, thousandths, etc. Since the latter is based on the same principle as our system of numeration, it is desirable for general use, and the following conversion table will enable the common fractions to be converted into their equivalent decimals.

Fractions Reduced to Equivalent Decimals.

$\frac{1}{64}$	.015625	$\frac{17}{64}$	.265625	$\frac{33}{64}$	.515625	$\frac{49}{64}$	.765625
$\frac{1}{32}$	.03125	$\frac{9}{32}$	.28125	$\frac{17}{32}$	.53125	$\frac{25}{32}$	.78125
$\frac{3}{64}$	.046875	$\frac{19}{64}$	.296875	$\frac{35}{64}$	.546875	$\frac{51}{64}$	.796875
$\frac{1}{16}$	.0625	$\frac{5}{16}$	.3125	$\frac{9}{16}$	.5625	$\frac{13}{16}$	.8125
$\frac{5}{64}$	.078125	$\frac{21}{64}$	.328125	$\frac{37}{64}$	.578125	$\frac{53}{64}$	.828125
$\frac{3}{32}$	.09375	$\frac{11}{32}$	.34375	$\frac{19}{32}$	.59375	$\frac{27}{32}$	.84375
$\frac{7}{64}$	.109375	$\frac{23}{64}$	.359375	$\frac{39}{64}$	.609375	$\frac{55}{64}$	.859375
$\frac{1}{8}$	.125	$\frac{3}{8}$	.375	$\frac{5}{8}$	.625	$\frac{7}{8}$	.875
$\frac{9}{64}$	.140625	$\frac{25}{64}$	.390625	$\frac{41}{64}$	.640625	$\frac{57}{64}$	.890625
$\frac{5}{32}$	.15625	$\frac{13}{32}$	.40625	$\frac{21}{32}$	.65625	$\frac{29}{32}$	.90625
$\frac{11}{64}$	.171875	$\frac{27}{64}$	.421875	$\frac{43}{64}$	.671875	$\frac{59}{64}$	.921875
$\frac{3}{16}$	.1875	$\frac{7}{16}$	.4375	$\frac{11}{16}$	.6875	$\frac{15}{16}$	.9375
$\frac{13}{64}$	.203125	$\frac{29}{64}$	.453125	$\frac{45}{64}$	.703125	$\frac{61}{64}$	.953125
$\frac{7}{32}$	.21875	$\frac{15}{32}$	.46875	$\frac{23}{32}$	.71875	$\frac{31}{32}$	.96875
$\frac{15}{64}$	.234375	$\frac{31}{64}$	.484375	$\frac{47}{64}$	.734375	$\frac{63}{64}$	.984375
$\frac{1}{4}$	.25	$\frac{1}{2}$	.5	$\frac{3}{4}$	.75	1	1.

Any common fraction may be converted into its equivalent decimal by dividing the numerator by the denominator, a fraction really being merely a form of indicating division, and the decimal being the result of the performance of the division thus indicated.

POWERS AND ROOTS.

Any number multiplied by itself is said to be raised to its second power, or squared; any number multiplied by itself twice is said to be raised to its third power, or cubed, etc. It is clear from this that every squared number, or second power, is composed of two equal factors, and either one of these equal factors is called the square root of the number. In like manner every cubed number is composed of the product of three equal factors, and any one of these equal factors is called the cube root of the number.

Since squares, cubes, square roots, and cube roots are much used, the following table is given for all numbers up to 1600. If much work is to be done in this line, reference may be made to Barlow's Tables (Spon), which give the squares, cubes, square roots, and cube roots of all numbers up to 10,000.

In the right-hand column of the following table the reciprocals of the numbers in the first column are given, these being the quotients resulting from the division of unity by the given numbers.

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
1	1	1	1.000 0000	1.000 0000	1.000 000 000
2	4	8	1.414 2136	1.259 9210	.500 000 000
3	9	27	1.732 0508	1.442 2496	.333 333 333
4	16	64	2.000 0000	1.587 4011	.250 000 000
5	25	125	2.236 0680	1.709 9759	.200 000 000
6	36	216	2.449 4897	1.817 1206	.166 666 667
7	49	343	2.645 7513	1.912 9312	.142 857 143
8	64	512	2.828 4271	2.000 0000	.125 000 000
9	81	729	3.000 0000	2.080 0837	.111 111 111
10	100	1 000	3.162 2777	2.154 4347	.100 000 000
11	121	1 331	3.316 6248	2.223 9801	.090 909 091
12	144	1 728	3.464 1016	2.289 4286	.083 333 333
13	169	2 197	3.605 5513	2.351 3347	.076 923 077
14	196	2 744	3.741 6574	2.410 1422	.071 428 571
15	225	3 375	3.872 9833	2.466 2121	.066 666 667
16	256	4 096	4.000 0000	2.519 8421	.062 500 000
17	289	4 913	4.123 1056	2.571 2816	.058 823 529
18	324	5 832	4.242 6407	2.620 7414	.055 555 556
19	361	6 859	4.358 8989	2.668 4016	.052 631 579
20	400	8 000	4.472 1360	2.714 4177	.050 000 000
21	441	9 261	4.582 5757	2.758 9243	.047 619 048
22	484	10 648	4.690 4158	2.802 0393	.045 454 545
23	529	12 167	4.795 8315	2.843 8670	.043 478 261
24	576	13 824	4.898 9795	2.884 4991	.041 666 667
25	625	15 625	5.000 0000	2.924 0177	.040 000 000
26	676	17 576	5.099 0195	2.962 4960	.038 461 538
27	729	19 683	5.196 1524	3.000 0000	.037 037 037
28	784	21 952	5.291 5026	3.036 5889	.035 714 286
29	841	24 389	5.385 1648	3.072 3168	.034 482 759
30	900	27 000	5.477 2256	3.107 2325	.033 333 333
31	961	29 791	5.567 7644	3.141 3806	.032 258 065
32	1 024	32 768	5.656 8542	3.174 8021	.031 250 000
33	1 089	35 937	5.744 5626	3.207 5343	.030 303 030
34	1 156	39 304	5.830 9519	3.239 6118	.029 411 765
35	1 225	42 875	5.916 0798	3.271 0663	.028 571 429
36	1 296	46 656	6.000 0000	3.301 9272	.027 777 778
37	1 369	50 653	6.082 7625	3.332 2218	.027 027 027
38	1 444	54 872	6.164 4140	3.361 9754	.026 315 789
39	1 521	59 319	6.244 9980	3.391 2114	.025 641 026
40	1 600	64 000	6.324 5553	3.419 9519	.025 000 000
41	1 681	68 921	6.403 1242	3.448 2172	.024 390 244
42	1 764	74 088	6.480 7407	3.476 0266	.023 809 524
43	1 849	79 507	6.557 4385	3.503 3981	.023 255 814
44	1 936	85 184	6.633 2496	3.530 3483	.022 727 273
45	2 025	91 125	6.708 2039	3.556 8933	.022 222 222
46	2 116	97 336	6.782 3300	3.583 0479	.021 739 130
47	2 209	103 823	6.855 6546	3.608 8261	.021 276 600
48	2 304	110 592	6.928 2032	3.634 2411	.020 833 333
49	2 401	117 649	7.000 0000	3.659 3057	.020 408 163
50	2 500	125 000	7.071 0678	3.684 0314	.020 000 000
51	2 601	132 651	7.141 4284	3.708 4298	.019 607 843
52	2 704	140 608	7.211 1026	3.732 5111	.019 230 769

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
53	2 809	148 877	7.280 1099	3.756 2858	.018 867 925
54	2 916	157 464	7.348 4692	3.779 7631	.018 518 519
55	3 025	166 375	7.416 1985	3.802 9525	.018 181 818
56	3 136	175 616	7.483 3148	3.825 8624	.017 857 143
57	3 249	185 193	7.549 8344	3.848 5011	.017 543 860
58	3 364	195 112	7.615 7731	3.870 8766	.017 241 379
59	3 481	205 379	7.681 1457	3.892 9965	.016 949 153
60	3 600	216 000	7.745 9667	3.914 8676	.016 666 667
61	3 721	226 981	7.810 2497	3.930 4972	.016 393 443
62	3 844	238 328	7.874 0079	3.957 8915	.016 129 032
63	3 969	250 047	7.937 2539	3.979 0571	.015 873 016
64	4 096	262 144	8.000 0000	4.000 0000	.015 625 000
65	4 225	274 625	8.062 2577	4.020 7256	.015 384 615
66	4 356	287 496	8.124 0384	4.041 2401	.015 151 515
67	4 489	300 763	8.185 3528	4.061 5480	.014 925 373
68	4 624	314 432	8.246 2113	4.081 6551	.014 705 882
69	4 761	328 509	8.306 6239	4.101 5661	.014 492 754
70	4 900	343 000	8.366 6003	4.121 2853	.014 285 714
71	5 041	357 911	8.426 1498	4.140 8178	.014 084 517
72	5 184	373 248	8.485 2814	4.160 1676	.013 888 889
73	5 329	389 017	8.544 0037	4.179 3390	.013 698 630
74	5 476	405 224	8.602 3253	4.198 3364	.013 513 514
75	5 625	421 875	8.660 2540	4.217 1633	.013 333 333
76	5 776	438 976	8.717 7979	4.235 8236	.013 157 895
77	5 929	456 533	8.774 9644	4.254 3210	.012 987 013
78	6 084	474 552	8.831 7609	4.272 6586	.012 820 513
79	6 241	493 039	8.888 1944	4.290 8404	.012 658 228
80	6 400	512 000	8.944 2719	4.308 8695	.012 500 000
81	6 561	531 441	9.000 0000	4.326 7487	.012 345 679
82	6 724	551 368	9.055 3851	4.344 4815	.012 195 122
83	6 889	571 787	9.110 4336	4.362 0707	.012 048 193
84	7 056	592 704	9.165 1514	4.379 5191	.011 904 762
85	7 225	614 125	9.219 5445	4.396 8296	.011 764 706
86	7 396	636 056	9.273 6185	4.414 0049	.011 627 907
87	7 569	658 503	9.327 3791	4.431 0476	.011 494 253
88	7 744	681 472	9.380 8315	4.447 9692	.011 363 636
89	7 921	704 969	9.433 9811	4.464 7451	.011 235 955
90	8 100	729 000	9.486 8330	4.481 4047	.011 111 111
91	8 281	753 571	9.539 3920	4.497 9414	.010 989 011
92	8 464	778 688	9.591 6630	4.514 3574	.010 869 565
93	8 649	804 357	9.643 6508	4.530 6549	.010 752 688
94	8 836	830 584	9.695 3597	4.546 8359	.010 638 298
95	9 025	857 375	9.746 7943	4.562 9026	.010 526 316
96	9 216	884 736	9.797 9590	4.578 8570	.010 416 667
97	9 409	912 673	9.848 8578	4.594 7009	.010 309 278
98	9 604	941 192	9.899 4949	4.610 4363	.010 204 082
99	9 801	970 299	9.949 8744	4.626 0650	.010 101 010
100	10 000	1 000 000	10.000 0000	4.641 5888	.010 000 000
101	10 201	1 030 301	10.049 8756	4.657 0095	.009 900 990
102	10 404	1 061 208	10.099 5049	4.672 3287	.009 803 922
103	10 609	1 092 727	10.148 8916	4.687 5482	.009 708 738
104	10 816	1 124 864	10.198 0390	4.702 6694	.009 615 385

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
105	11 025	1 157 625	10.246 9508	4.717 6940	.009 523 810
106	11 236	1 191 016	10.295 6301	4.732 6235	.009 433 962
107	11 449	1 225 043	10.344 0804	4.747 4594	.009 345 794
108	11 664	1 259 712	10.392 3048	4.762 2032	.009 259 259
109	11 881	1 295 029	10.440 3065	4.776 8562	.009 174 312
110	12 100	1 331 000	10.488 0885	4.791 4199	.009 090 909
111	12 321	1 367 631	10.535 6538	4.805 8995	.009 009 009
112	12 544	1 404 928	10.583 0052	4.820 2845	.008 928 571
113	12 769	1 442 897	10.630 1458	4.834 5881	.008 849 558
114	12 996	1 481 544	10.677 0783	4.848 8076	.008 771 930
115	13 225	1 520 875	10.723 8053	4.862 9442	.008 695 652
116	13 456	1 560 896	10.770 3296	4.876 9990	.008 620 690
117	13 689	1 601 613	10.816 6538	4.890 9732	.008 547 009
118	13 924	1 643 032	10.862 7805	4.904 8681	.008 474 576
119	14 161	1 685 159	10.908 7121	4.918 6847	.008 403 361
120	14 400	1 728 000	10.954 4512	4.932 4242	.008 333 333
121	14 641	1 771 561	11.000 0000	4.946 0874	.008 264 463
122	14 884	1 815 848	11.045 3610	4.959 6757	.008 196 721
123	15 129	1 860 867	11.090 5365	4.973 1898	.008 130 081
124	15 376	1 906 624	11.135 5287	4.986 6310	.008 064 516
125	15 625	1 953 125	11.180 3399	5.000 0000	.008 000 000
126	15 876	2 000 376	11.224 9722	5.013 2979	.007 936 508
127	16 129	2 048 383	11.269 4277	5.026 5257	.007 874 016
128	16 384	2 097 152	11.313 7085	5.039 6842	.007 812 500
129	16 641	2 146 689	11.357 8167	5.052 7743	.007 751 938
130	16 900	2 197 000	11.401 7543	5.065 7970	.007 692 308
131	17 161	2 248 091	11.445 5231	5.078 7531	.007 633 588
132	17 424	2 299 968	11.489 1253	5.091 6434	.007 575 758
133	17 689	2 352 637	11.532 5626	5.104 4687	.007 518 797
134	17 956	2 406 104	11.575 8369	5.117 2299	.007 462 687
135	18 225	2 460 375	11.618 9500	5.129 9278	.007 407 407
136	18 496	2 515 456	11.661 9038	5.142 5632	.007 352 941
137	18 769	2 571 353	11.704 6999	5.155 1367	.007 299 270
138	19 044	2 628 072	11.747 3401	5.167 6493	.007 246 377
139	19 321	2 685 619	11.789 8261	5.180 1015	.007 194 245
140	19 600	2 744 000	11.832 1596	5.192 4941	.007 142 857
141	19 881	2 803 221	11.874 3421	5.204 8279	.007 092 199
142	20 164	2 863 288	11.916 3753	5.217 1034	.007 042 254
143	20 449	2 924 207	11.958 2607	5.229 3215	.006 993 007
144	20 736	2 985 984	12.000 0000	5.241 4828	.006 944 444
145	21 025	3 048 625	12.041 5946	5.253 5879	.006 896 552
146	21 316	3 112 136	12.083 0460	5.265 6374	.006 849 315
147	21 609	3 176 523	12.124 3557	5.277 6321	.006 802 721
148	21 904	3 241 792	12.165 5251	5.289 5725	.006 756 757
149	22 201	3 307 949	12.206 5556	5.301 4592	.006 711 409
150	22 500	3 375 000	12.247 4487	5.313 2928	.006 666 667
151	22 801	3 442 951	12.288 2057	5.325 0740	.006 622 517
152	23 104	3 511 008	12.328 8280	5.336 8033	.006 578 947
153	23 409	3 581 577	12.369 3169	5.348 4812	.006 535 948
154	23 716	3 652 264	12.409 6736	5.360 1084	.006 493 506
155	24 025	3 723 875	12.449 8996	5.371 6854	.006 451 613
156	24 336	3 796 416	12.489 9960	5.383 2126	.006 410 256

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
157	24 649	3 869 893	12.529 9641	5.394 6907	.006 369 427
158	24 964	3 944 312	12.569 8051	5.406 1202	.006 329 114
159	25 281	4 019 679	12.609 5202	5.417 5015	.006 289 308
160	25 600	4 096 000	12.649 1106	5.428 8352	.006 250 000
161	25 921	4 173 281	12.688 5775	5.440 1218	.006 211 180
162	26 244	4 251 528	12.727 9221	5.451 3618	.006 172 840
163	26 569	4 330 747	12.767 1453	5.462 5556	.006 134 969
164	26 896	4 410 944	12.806 2485	5.473 7037	.006 097 561
165	27 225	4 492 125	12.845 2326	5.484 8066	.006 060 606
166	27 556	4 574 296	12.884 0987	5.495 8647	.006 024 096
167	27 889	4 657 463	12.922 8480	5.506 8784	.005 988 024
168	28 224	4 741 632	12.961 4814	5.517 8484	.005 952 381
169	28 561	4 826 809	13.000 0000	5.528 7748	.005 917 160
170	28 900	4 913 000	13.038 4048	5.539 6583	.005 882 353
171	29 241	5 000 211	13.076 6968	5.550 4991	.005 847 953
172	29 584	5 088 448	13.114 8770	5.561 2978	.005 813 953
173	29 929	5 177 717	13.152 9464	5.572 0546	.005 780 347
174	30 276	5 268 024	13.190 9060	5.582 7702	.005 747 126
175	30 625	5 359 375	13.228 7566	5.593 4447	.005 714 286
176	30 976	5 451 776	13.266 4992	5.604 0787	.005 681 818
177	31 329	5 545 233	13.304 1347	5.614 6724	.005 649 718
178	31 684	5 639 752	13.341 6641	5.625 2263	.005 617 978
179	32 041	5 735 339	13.379 0882	5.635 7408	.005 586 592
180	32 400	5 832 000	13.416 4079	5.646 2162	.005 555 556
181	32 761	5 929 741	13.453 6240	5.656 6528	.005 524 862
182	33 124	6 028 568	13.490 7376	5.667 0511	.005 494 505
183	33 489	6 128 487	13.527 7493	5.677 4114	.005 464 481
184	33 856	6 229 504	13.564 6600	5.687 7340	.005 434 783
185	34 225	6 331 625	13.601 4705	5.698 0192	.005 405 405
186	34 596	6 434 856	13.638 1817	5.708 2675	.005 376 344
187	34 969	6 539 203	13.674 7943	5.718 4791	.005 347 594
188	35 344	6 644 672	13.711 3092	5.728 6543	.005 319 149
189	35 721	6 751 269	13.747 7271	5.738 7936	.005 291 005
190	36 100	6 859 000	13.784 0488	5.748 8971	.005 263 158
191	36 481	6 967 871	13.820 2750	5.758 9652	.005 235 602
192	36 864	7 077 888	13.856 4065	5.768 9982	.005 208 333
193	37 249	7 189 517	13.892 4400	5.778 9966	.005 181 347
194	37 636	7 301 384	13.928 3883	5.788 9604	.005 154 639
195	38 025	7 414 875	13.964 2400	5.798 8900	.005 128 205
196	38 416	7 529 536	14.000 0000	5.808 7857	.005 102 041
197	38 809	7 645 373	14.035 6688	5.818 6479	.005 076 142
198	39 204	7 762 392	14.071 2473	5.828 4867	.005 050 505
199	39 601	7 880 599	14.106 7360	5.838 2725	.005 025 126
200	40 000	8 000 000	14.142 1356	5.848 0355	.005 000 000
201	40 401	8 120 601	14.177 4469	5.857 7660	.004 975 124
202	40 804	8 242 408	14.212 6704	5.867 4673	.004 950 495
203	41 209	8 365 427	14.247 8068	5.877 1307	.004 926 108
204	41 616	8 489 664	14.282 8569	5.886 7653	.004 901 961
205	42 025	8 615 125	14.317 8211	5.896 3685	.004 878 049
206	42 436	8 741 816	14.352 7001	5.905 9406	.004 854 369
207	42 849	8 869 743	14.387 4946	5.915 4817	.004 830 918
208	43 264	8 998 912	14.422 2051	5.924 9921	.004 807 692

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
209	43 681	9 129 329	14.456 8323	5.934 4721	.004 784 689
210	44 100	9 261 000	14.491 3767	5.943 9220	.004 761 905
211	44 521	9 393 931	14.525 8390	5.953 3418	.004 739 336
212	44 944	9 528 128	14 560 2198	5.962 7320	.004 716 981
213	45 369	9 663 597	14.594 5195	5.972 0926	.004 694 836
214	45 796	9 800 344	14.628 7388	5.981 4240	.004 672 897
215	46 225	9 938 375	14.662 8783	5.990 7264	.004 651 163
216	46 656	10 077 696	14.696 9385	6.000 0000	.004 629 630
217	47 089	10 218 313	14.730 9199	6.009 2450	.004 608 295
218	47 524	10 360 232	14.764 8231	6.018 4617	.004 587 156
219	47 961	10 503 459	14.798 6486	6.027 6502	.004 566 210
220	48 400	10 648 000	14.832 3970	6.036 8107	.004 545 455
221	48 841	10 793 861	14.866 0687	6.045 9435	.004 524 887
222	49 284	10 941 048	14.899 6644	6.055 0489	.004 504 505
223	49 729	11 089 567	14.933 1845	6.064 1270	.004 484 305
224	50 176	11 239 424	14.966 6295	6.073 1779	.004 464 286
225	50 625	11 390 625	15.000 0000	6.082 4020	.004 444 444
226	51 076	11 543 176	15.033 2964	6.099 1994	.004 424 779
227	51 529	11 697 083	15.066 5192	6.100 1702	.004 405 286
228	51 984	11 852 352	15.099 6689	6.109 1147	.004 385 965
229	52 441	12 008 989	15.132 7460	6.118 0332	.004 366 812
230	52 900	12 167 000	15.165 7509	6.126 9257	.004 347 826
231	53 361	12 326 391	15.198 6842	6.135 7924	.004 329 004
232	53 824	12 487 168	15.231 5462	6.144 6337	.004 310 345
233	54 289	12 649 337	15.264 3375	6.153 4495	.004 291 845
234	54 756	12 812 904	15.297 0585	6.162 2401	.004 273 504
235	55 225	12 977 875	15.329 7097	6.171 0058	.004 255 319
236	55 696	13 144 256	15.362 2915	6.179 7466	.004 237 288
237	56 169	13 312 053	15.394 8043	6.188 4628	.004 219 409
238	56 644	13 481 272	15.427 2486	6.197 1544	.004 201 681
239	57 121	13 651 919	15.459 6248	6.205 8218	.004 184 100
240	57 600	13 824 000	15.491 9334	6.214 4650	.004 166 667
241	58 081	13 997 521	15.524 1747	6.223 0843	.004 149 378
242	58 564	14 172 488	15.556 3492	6.231 6797	.004 132 231
243	59 049	14 348 907	15.588 4573	6.240 2515	.004 115 226
244	59 536	14 526 784	15.620 4994	6.248 7998	.004 098 361
245	60 025	14 706 125	15.652 4758	6.257 3248	.004 081 633
246	60 516	14 886 936	15.684 3871	6.265 8266	.004 065 041
247	61 009	15 069 223	15.716 2336	6.274 3054	.004 048 583
248	61 504	15 252 992	15.748 0157	6.282 7613	.004 032 258
249	62 001	15 438 249	15.779 7338	6.291 1946	.004 016 064
250	62 500	15 625 000	15.811 3883	6.299 6053	.004 000 000
251	63 001	15 813 251	15.842 9795	6.307 9935	.003 984 064
252	63 504	16 003 008	15.874 5079	6.316 3596	.003 968 254
253	64 009	16 194 277	15.905 9737	6.324 7035	.003 952 569
254	64 516	16 387 064	15.937 3775	6.333 0256	.003 937 008
255	65 025	16 581 375	15.968 7194	6.341 3257	.003 921 569
256	65 536	16 777 216	16.000 0000	6.349 6042	.003 906 250
257	66 049	16 974 593	16.031 2195	6.357 8611	.003 891 051
258	66 564	17 173 512	16.062 3784	6.366 0968	.003 875 969
259	67 081	17 373 979	16.093 4769	6.374 3111	.003 861 004
260	67 600	17 576 000	16.124 5155	6.382 5043	.003 846 154

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
261	68 121	17 779 581	16.155 4944	6.390 6765	.003 831 418
262	68 644	17 984 728	16.186 4141	6.398 8279	.003 816 794
263	69 169	18 191 447	16.217 2747	6.406 9585	.003 802 281
264	69 696	18 399 744	16.248 0768	6.415 0687	.003 787 879
265	70 225	18 609 625	16.278 8206	6.423 1583	.003 773 585
266	70 756	18 821 096	16.309 5064	6.431 2276	.003 759 398
267	71 289	19 034 163	16.340 1346	6.439 2767	.003 745 318
268	71 824	19 248 832	16.370 7055	6.447 3057	.003 731 343
269	72 361	19 465 109	16.401 2195	6.455 3148	.003 717 472
270	72 900	19 683 000	16.431 6767	6.463 3041	.003 703 704
271	73 441	19 902 511	16.462 0776	6.471 2736	.003 690 037
272	73 984	20 123 643	16.492 4225	6.479 2236	.003 676 471
273	74 529	20 346 417	16.522 7116	6.487 1541	.003 663 004
274	75 076	20 570 824	16.552 9454	6.495 0653	.003 649 635
275	75 625	20 796 875	16.583 1240	6.502 9572	.003 636 364
276	76 176	21 024 576	16.613 2477	6.510 8300	.003 623 188
277	76 729	21 253 933	16.643 3170	6.518 6839	.003 610 108
278	77 284	21 484 952	16.673 3320	6.526 5189	.003 597 122
279	77 841	21 717 639	16.703 2931	6.534 3351	.003 584 229
280	78 400	21 952 000	16.733 2005	6.542 1326	.003 571 429
281	78 961	22 188 041	16.763 0546	6.549 9116	.003 558 719
282	79 524	22 425 768	16.792 8556	6.557 6722	.003 546 099
283	80 089	22 665 187	16.822 6038	6 565 4144	.003 533 569
284	80 656	22 906 304	16.852 2995	6.573 1385	.003 521 127
285	81 225	23 149 125	16.881 9430	6.580 8443	.003 508 772
286	81 796	23 393 656	16.911 5345	6.588 5323	.003 496 503
287	82 369	23 639 903	16.941 0743	6.596 2023	.003 484 321
288	82 944	23 887 872	16.970 5627	6.603 8545	.003 472 222
289	83 521	24 137 569	17.000 0000	6.611 4890	.003 460 208
290	84 100	24 389 000	17.029 3864	6.619 1060	.003 448 276
291	84 681	24 642 171	17.058 7221	6.626 7054	.003 436 426
292	85 264	24 897 088	17.088 0075	6.634 2874	.003 424 658
293	85 849	25 153 757	17.117 2428	6.641 8522	.003 412 969
294	86 436	25 412 184	17.146 4282	6.649 3998	.003 401 361
295	87 025	25 672 375	17.175 5640	6.656 9302	.003 389 831
296	87 616	25 934 836	17.204 6505	6.664 4437	.003 378 378
297	88 209	26 198 073	17.233 6879	6.671 9403	.003 367 003
298	88 804	26 463 592	17.262 6765	6.679 4200	.003 355 705
299	89 401	26 730 899	17.291 6165	6.686 8831	.003 344 482
300	90 000	27 000 000	17.320 5081	6.694 3295	.003 333 333
301	90 601	27 270 901	17.349 3516	6.701 7593	.003 322 259
302	91 204	27 543 608	17.378 1472	6.709 1729	.003 311 258
303	91 809	27 818 127	17.406 8952	6.716 5700	.003 301 330
304	92 416	28 094 464	17.435 5958	6.723 9508	.003 289 474
305	93 025	28 372 625	17.464 2492	6.731 3155	.003 278 689
306	93 636	28 652 616	17.492 8557	6.738 6641	.003 267 974
307	94 249	28 934 443	17.521 4155	6.745 9967	.003 257 329
308	94 864	29 218 112	17.549 9288	6.753 3134	.003 246 753
309	95 481	29 503 609	17.578 3958	6.760 6143	.003 236 246
310	96 100	29 791 000	17.606 8169	6.767 8995	.003 225 806
311	96 721	30 080 231	17.635 1921	6.775 1690	.003 215 434
312	97 344	30 371 328	17.663 5217	6.782 4229	.003 205 128

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
313	97 969	30 664 297	17.691 8060	6.789 6613	.003 194 888
314	98 596	30 959 144	17.720 0451	6.796 8844	.003 184 713
315	99 225	31 255 875	17.748 2393	6.804 0921	.003 174 603
316	99 856	31 554 496	17.776 3888	6.811 2847	.003 164 557
317	100 489	31 855 013	17.804 4938	6.818 4620	.003 154 574
318	101 124	32 157 432	17.832 5545	6.825 6242	.003 144 654
319	101 761	32 461 759	17.860 5711	6.832 7714	.003 134 796
320	102 400	32 768 000	17.888 5438	6.839 9037	.003 125 000
321	103 041	33 076 161	17.916 4729	6.847 0213	.003 115 265
322	103 684	33 386 248	17.944 3584	6.854 1240	.003 105 590
323	104 329	33 698 267	17.972 2008	6.861 2120	.003 095 975
324	104 976	34 012 224	18.000 0000	6.868 2855	.003 086 420
325	105 625	34 328 125	18.027 7564	6.875 3433	.003 076 923
326	106 276	34 645 976	18.055 4701	6.882 3888	.003 067 485
327	106 929	34 965 783	18.083 1413	6.889 4188	.003 048 104
328	107 584	35 287 552	18.110 7703	6.896 4345	.003 048 780
329	108 241	35 611 289	18.138 3571	6.903 4359	.003 039 514
330	108 900	35 937 000	18.165 9021	6.910 4232	.003 030 303
331	109 561	36 264 691	18.193 4054	6.917 3964	.003 021 148
332	110 224	36 594 368	18.220 8672	6.924 3556	.003 012 048
333	110 889	36 926 037	18.248 2876	6.931 3088	.003 003 003
334	111 556	37 259 704	18.275 6669	6.938 2321	.002 994 012
335	112 225	37 595 375	18.303 0052	6.945 1496	.002 985 075
336	112 896	37 933 056	18.330 3028	6.952 0533	.002 976 190
337	113 569	38 272 753	18.357 5598	6.958 9434	.002 967 359
338	114 244	38 614 472	18.384 7763	6.965 8198	.002 958 580
339	114 921	38 958 219	18.411 9526	6.972 6826	.002 949 853
340	115 600	39 304 000	18.439 0889	6 979 5321	.002 941 176
341	116 281	39 651 821	18.466 1853	6.986 3681	.002 932 551
342	116 964	40 001 688	18.493 2420	6.993 1906	.002 923 977
343	117 649	40 353 607	18.520 2592	7.000 0000	.002 915 452
344	118 336	40 707 584	18.547 2370	7.006 7962	.002 906 977
345	119 025	41 063 625	18.574 1756	7.013 5791	.002 898 551
346	119 716	41 421 736	18.601 0752	7.020 3490	.002 890 173
347	120 409	41 781 923	18.627 9360	7.027 1058	.002 881 844
348	121 104	42 144 192	18.654 7581	7.033 8497	.002 873 563
349	121 801	42 508 549	18.681 5417	7.040 5860	.002 865 330
350	122 500	42 875 000	18.708 2869	7.047 2987	.002 857 143
351	123 201	43 243 551	18.734 9940	7.054 0041	.002 849 003
352	123 904	43 614 208	18.761 6630	7.060 6967	.002 840 909
353	124 609	43 986 977	18.788 2942	7.067 3767	.002 832 861
354	125 316	44 361 864	18.814 8877	7.074 0440	.002 824 859
355	126 025	44 738 875	18.841 4437	7.080 6988	.002 816 901
356	126 736	45 118 016	18.867 9623	7.087 3411	.002 808 989
357	127 449	45 499 293	18.894 4436	7.093 9709	.002 801 120
358	128 164	45 882 712	18.920 8879	7.100 5885	.002 793 296
359	128 881	46 268 279	18.947 2953	7.107 1937	.002 785 515
360	129 600	46 656 000	18.973 6660	7.113 7866	.002 777 778
361	130 321	47 045 831	19.000 0000	7.120 3674	.002 770 083
362	131 044	47 437 928	19.026 2976	7.126 9360	.002 762 431
363	131 769	47 832 147	19.052 5589	7.133 4925	.002 754 821
364	132 496	48 228 544	19.078 7840	7.140 0370	.002 747 253

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
365	133 225	48 627 125	19.104 9732	7.146 5695	.002 739 726
366	133 956	49 027 896	19.131 1265	7.153 0901	.002 732 240
367	134 689	49 430 863	19.157 2441	7.159 5988	.002 724 796
368	135 424	49 836 032	19.183 3261	7.166 0957	.002 717 391
369	136 161	50 243 409	19.209 3727	7.172 5809	.002 710 027
370	136 900	50 653 000	19.235 3841	7.179 0544	.002 702 703
371	137 641	51 064 811	19.261 3603	7.185 5162	.002 695 418
372	138 384	51 478 848	19.287 3015	7.191 9663	.002 688 172
373	139 129	51 895 117	19.313 2079	7.198 4050	.002 680 965
374	139 876	52 313 624	19.339 0796	7.204 8322	.002 673 797
375	140 625	52 734 375	19.364 9167	7.211 2479	.002 666 667
376	141 376	53 157 376	19.390 7194	7.217 6522	.002 659 574
377	142 129	53 582 633	19.416 4878	7.224 0450	.002 652 520
378	142 884	54 010 152	19.442 2221	7.230 4268	.002 645 503
379	143 641	54 439 939	19.467 9223	7.236 7972	.002 638 521
380	144 400	54 872 000	19.493 5887	7.243 1565	.002 631 579
381	145 161	55 306 341	19.519 2213	7.249 5045	.002 624 672
382	145 924	55 742 968	19.544 8203	7.255 8415	.002 617 801
383	146 689	56 181 887	19.570 3858	7.262 1675	.002 610 966
384	147 456	56 623 104	19.595 9179	7.268 4824	.002 604 167
385	148 225	57 066 625	19.621 4169	7.274 7864	.002 597 403
386	148 996	57 512 456	19.646 8827	7.281 0794	.002 590 674
387	149 769	57 960 603	19.672 3156	7.287 3617	.002 583 979
388	150 544	58 411 072	19.697 7156	7.293 6330	.002 577 320
389	151 321	58 863 869	19.723 0829	7.299 8936	.002 570 694
390	152 100	59 319 000	19.748 4177	7.306 1436	.002 564 103
391	152 881	59 776 471	19.773 7199	7.312 3828	.002 557 545
392	153 664	60 236 288	19.798 9899	7.318 6114	.002 551 020
393	154 449	60 698 457	19.824 2276	7.324 8295	.002 544 529
394	155 236	61 162 984	19.849 4332	7.331 0369	.002 538 071
395	156 025	61 629 875	19.874 6069	7.337 2339	.002 531 646
396	156 816	62 099 136	19.899 7487	7.343 4205	.002 525 253
397	157 609	62 570 773	19.924 8588	7.349 5966	.002 518 892
398	158 404	63 044 792	19.949 9373	7.355 7624	.002 512 563
399	159 201	63 521 199	19.974 9844	7.361 9178	.002 506 266
400	160 000	64 000 000	20.000 0000	7.368 0630	.002 500 000
401	160 801	64 481 201	20.024 9844	7.374 1979	.002 493 766
402	161 604	64 964 808	20.049 9377	7.380 3227	.002 487 562
403	162 409	65 450 827	20.074 8599	7.386 4373	.002 481 390
404	163 216	65 939 264	20.099 7512	7.392 5418	.002 475 248
405	164 025	66 430 125	20.124 6118	7.398 6363	.002 469 136
406	164 836	66 923 416	20.149 4417	7.404 7206	.002 463 054
407	165 649	67 419 143	20.174 2410	7.410 7950	.002 457 002
408	166 464	67 917 312	20.199 0099	7.416 8595	.002 450 980
409	167 281	68 417 929	20.223 7484	7.422 9142	.002 444 988
410	168 100	68 921 000	20.248 4567	7.428 9589	.002 439 024
411	168 921	69 426 531	20.273 1349	7.434 9938	.002 433 090
412	169 744	69 934 528	20.297 7831	7.441 0189	.002 427 184
413	170 569	70 444 997	20.322 4014	7.447 0343	.002 421 308
414	171 396	70 957 944	20.346 9899	7.453 0399	.002 415 459
415	172 225	71 473 375	20.371 5488	7.459 0359	.002 409 639
416	173 056	71 991 296	20.396 0781	7.465 0223	.002 403 846

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
521	271 441	141 420 761	22.825 4244	8.046 6030	.001 919 386
522	272 484	142 236 648	22.847 3193	8.051 7479	.001 915 709
523	273 529	143 055 667	22.869 1933	8.056 8862	.001 912 046
524	274 576	143 877 824	22.891 0463	8.062 0180	.001 908 397
525	275 625	144 703 125	22.912 8785	8.067 1432	.001 904 762
526	276 676	145 531 576	22.934 6899	8.072 2620	.001 901 141
527	277 729	146 363 183	22.956 4806	8.077 3743	.001 897 533
528	278 784	147 197 952	22.978 2506	8.082 4800	.001 893 939
529	279 841	148 035 889	23.000 0000	8.087 5794	.001 890 359
530	280 900	148 877 001	23.021 7289	8.092 6723	.001 886 792
531	281 961	149 721 291	23.043 4372	8.097 7589	.001 883 239
532	283 024	150 568 768	23.065 1252	8.102 8390	.001 879 699
533	284 089	151 419 437	23.086 7928	8.107 9128	.001 876 173
534	285 156	152 273 304	23.108 4400	8.112 9803	.001 872 659
535	286 225	153 130 375	23.130 0670	8.118 0414	.001 869 159
536	287 296	153 990 656	23.151 6738	8.123 0962	.001 865 672
537	288 369	154 854 153	23.173 2605	8.128 1447	.001 862 197
538	289 444	155 720 872	23.194 8270	8.133 1870	.001 858 736
539	290 521	156 590 819	23.216 3735	8.138 2230	.001 855 288
540	291 600	157 464 000	23.237 9001	8.143 2529	.001 851 852
541	292 681	158 340 421	23.259 4067	8.148 2765	.001 848 429
542	293 764	159 220 088	23.280 8935	8.153 2939	.001 845 018
543	294 849	160 103 007	23.302 3604	8.158 3051	.001 841 621
544	295 936	160 989 184	23.323 8076	8.163 3102	.001 838 235
545	297 025	161 878 625	23.345 2351	8.168 3092	.001 834 862
546	298 116	162 771 336	23.366 6429	8.173 3020	.001 831 502
547	299 209	163 667 323	23.388 0311	8.178 2888	.001 828 154
548	300 304	164 566 592	23.409 3998	8.183 2695	.001 824 818
549	301 401	165 469 149	23.430 7490	8.188 2441	.001 821 494
550	302 500	166 375 000	23.452 0788	8.193 2127	.001 818 182
551	303 601	167 284 151	23.473 3892	8.198 1753	.001 814 882
552	304 704	168 196 608	23.494 6802	8.203 1319	.001 811 594
553	305 809	169 112 377	23.515 9520	8.208 0825	.001 808 318
554	306 916	170 031 464	23.537 2046	8.213 0271	.001 805 054
555	308 025	170 953 875	23.558 4380	8.217 9657	.001 801 802
556	309 136	171 879 616	23.579 6522	8.222 8985	.001 798 561
557	310 249	172 808 693	23.600 8474	8.227 8254	.001 795 332
558	311 364	173 741 112	23.622 0236	8.232 7463	.001 792 115
559	312 481	174 676 879	23.643 1808	8.237 6614	.001 788 909
560	313 600	175 616 000	23.664 3191	8.242 5706	.001 785 714
561	314 721	176 558 481	23.685 4386	8.247 4740	.001 782 531
562	315 844	177 504 328	23.706 5392	8.252 3715	.001 779 359
563	316 969	178 453 547	23.727 6210	8.257 2635	.001 776 199
564	318 096	179 406 144	23.748 6842	8.262 1492	.001 773 050
565	319 225	180 362 125	23.769 7286	8.267 0294	.001 769 912
566	320 356	181 321 496	23.790 7545	8.271 9039	.001 766 784
567	321 489	182 284 263	23.811 7618	8.276 7726	.001 763 668
568	322 624	183 250 432	23.832 7506	8.281 6255	.001 760 563
569	323 761	184 220 009	23.853 7209	8.286 4928	.001 757 469
570	324 900	185 193 000	23.874 6728	8.291 3444	.001 754 386
571	326 041	186 169 411	23.895 6063	8.296 1903	.001 751 313
572	327 184	187 149 248	23.916 5215	8.301 0304	.001 748 252

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
573	328 329	188 132 517	23.937 4184	8.305 8651	.001 745 201
574	329 476	189 119 224	23.958 2971	8.310 6941	.001 742 160
575	330 625	190 109 375	23.979 1576	8.315 5175	.001 739 130
576	331 776	191 102 976	24.000 0000	8.320 3353	.001 736 111
577	332 927	192 100 033	24.020 8243	8.325 1475	.001 733 102
578	334 084	193 100 552	24.041 6306	8.329 9542	.001 730 104
579	335 241	194 104 539	24.062 4188	8.334 7553	.001 727 116
580	336 400	195 112 000	24.083 1891	8.339 5509	.001 724 138
581	337 561	196 122 941	24.103 9416	8.344 3410	.001 721 170
582	338 724	197 137 368	24.124 6762	8.349 1256	.001 718 213
583	339 889	198 155 287	24.145 3929	8.353 9047	.001 715 266
584	341 056	199 176 704	24.166 0919	8.358 6784	.001 712 329
585	342 225	200 201 625	24.186 7732	8.363 4466	.001 709 402
586	343 396	201 230 056	24.207 4369	8.368 2095	.001 706 485
587	344 569	202 262 003	24.228 0829	8.372 9668	.001 703 578
588	345 744	203 297 472	24.248 7113	8.377 7188	.001 700 680
589	346 921	204 336 469	24.269 3222	8.382 4653	.001 697 793
590	348 100	205 379 000	24.289 9156	8.387 2065	.001 694 915
591	349 281	206 425 071	24.310 4996	8.391 9428	.001 692 047
592	350 464	207 474 688	24.331 0501	8.396 6729	.001 689 189
593	351 649	208 527 857	24.351 5913	8.401 3981	.001 686 341
594	352 836	209 584 584	24.372 1152	8.406 1180	.001 683 502
595	354 025	210 644 875	24.392 6218	8.410 8326	.001 680 672
596	355 216	211 708 736	24.413 1112	8.415 5419	.001 677 852
597	356 409	212 776 173	24.433 5834	8.420 2460	.001 675 042
598	357 604	213 847 192	24.454 0385	8.424 9448	.001 672 241
599	358 801	214 921 799	24.474 4765	8.429 6383	.001 669 449
600	360 000	216 000 000	24.494 8974	8.434 3267	.001 666 667
601	361 201	217 081 801	24.515 3013	8.439 0098	.001 663 894
602	362 404	218 167 208	24.535 6883	8.443 6877	.001 661 130
603	363 609	219 256 227	24.556 0583	8.448 3605	.001 658 375
604	364 816	220 348 864	24.576 4115	8.453 0281	.001 655 629
605	366 025	221 445 125	24.596 7478	8.457 6906	.001 652 893
606	367 236	222 545 016	24.617 0673	8.462 3479	.001 650 165
607	368 449	223 648 543	24.637 3700	8.467 0001	.001 647 446
608	369 664	224 755 712	24.657 6560	8.471 6471	.001 644 737
609	370 881	225 866 529	24.677 9254	8.476 2892	.001 642 036
610	372 100	226 981 000	24.698 1781	8.480 9261	.001 639 344
611	373 321	228 099 131	24.718 4142	8.485 5579	.001 636 661
612	374 544	229 220 928	24.738 6338	8.490 1848	.001 633 987
613	375 769	230 346 397	24.758 8368	8.494 8065	.001 631 321
614	376 996	231 475 544	24.779 0234	8.499 4233	.001 628 664
615	378 225	232 608 375	24.799 1935	8.504 0350	.001 626 016
616	379 456	233 744 896	24.819 3473	8.508 6417	.001 623 377
617	380 689	234 885 113	24.839 4847	8.513 2435	.001 620 746
618	381 924	236 029 032	24.859 6058	8.517 8403	.001 618 123
619	383 161	237 176 659	24.879 7106	8.522 4331	.001 615 509
620	384 400	238 328 000	24.899 7992	8.527 0189	.001 612 903
621	385 641	239 483 061	24.919 8716	8.531 6009	.001 610 306
622	386 884	240 641 848	24 939 9278	8.536 1780	.001 607 717
623	388 129	241 804 367	24.959 9679	8.540 7501	.001 605 136
624	389 376	242 970 624	24.979 9920	8.545 3173	.001 602 564

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
625	390 625	244 140 625	25.000 0000	8.549 8797	.001 600 000
626	391 876	245 134 376	25.019 9920	8.554 4372	.001 597 444
627	393 129	246 491 883	25.039 9681	8.558 9899	.001 594 896
628	394 384	247 673 152	25.059 9282	8.563 5377	.001 592 357
629	395 641	248 858 189	25.079 8724	8.568 0807	.001 589 825
630	396 900	250 047 000	25.099 8008	8.572 6189	.001 587 302
631	398 161	251 239 591	25.119 7134	8.577 1523	.001 584 786
632	399 424	252 435 968	25.139 6102	8.581 6809	.001 582 278
633	400 689	253 636 137	25.159 4913	8.586 2247	.001 579 779
634	401 956	254 840 104	25.179 3566	8.590 7238	.001 577 287
635	403 225	256 047 875	25.199 2063	8.595 2380	.001 574 803
636	404 496	257 259 456	25.219 0404	8.599 7476	.001 572 327
637	405 769	258 474 853	25.238 8589	8.604 2525	.001 569 859
638	407 044	259 694 072	25.258 6619	8.608 7526	.001 567 398
639	408 321	260 917 119	25.278 4493	8.613 2480	.001 564 945
640	409 600	262 144 000	25.298 2213	8.617 7388	.001 562 500
641	410 881	263 374 721	25.317 9778	8.622 2248	.001 560 062
642	412 164	264 609 288	25.337 7189	8.626 7063	.001 557 632
643	413 449	265 847 707	25.357 4447	8.631 1830	.001 555 210
644	414 736	267 089 984	25.377 1551	8.635 6551	.001 552 795
645	416 025	268 336 125	25.396 8502	8.640 1226	.001 550 388
646	417 316	269 585 136	25.416 5302	8.644 5855	.001 547 988
647	418 609	270 840 023	25.436 1947	8.649 0437	.001 545 595
648	419 904	272 097 792	25.455 8441	8.653 4974	.001 543 210
649	421 201	273 359 449	25.475 4784	8.657 9465	.001 540 832
650	422 500	274 625 000	25.495 0976	8.662 3911	.001 538 462
651	423 801	275 894 451	25.514 7013	8.666 8310	.001 536 098
652	425 104	277 167 808	25.534 2907	8.671 2665	.001 533 742
653	426 409	278 445 077	25.553 8647	8.675 6974	.001 531 394
654	427 716	279 726 264	25.573 4237	8.680 1237	.001 529 052
655	429 025	281 011 375	25.592 9678	8.684 5456	.001 526 718
656	430 336	282 300 416	25.612 4969	8.688 9630	.001 524 390
657	431 649	283 593 393	25.632 0112	8.693 3759	.001 522 070
658	432 964	284 890 312	25.651 5107	8.697 7843	.001 519 757
659	434 281	286 191 179	25.670 9953	8.702 1882	.001 517 451
660	435 600	287 496 000	25.690 4652	8.706 5877	.001 515 152
661	436 921	288 804 781	25.709 9203	8.710 9827	.001 512 859
662	438 244	290 117 528	25.729 3607	8.715 3734	.001 510 574
663	439 569	291 434 247	25.748 7864	8.719 7596	.001 508 296
664	440 896	292 754 944	25.768 1975	8.724 1414	.001 506 024
665	442 225	294 079 625	25.787 5939	8.728 5187	.001 503 759
666	443 556	295 408 296	25.806 9758	8.732 8918	.001 501 502
667	444 889	296 740 963	25.826 3431	8.737 2604	.001 499 250
668	446 224	298 077 632	25.845 6960	8.741 6246	.001 497 006
669	447 561	299 418 309	25.865 0343	8.745 9846	.001 494 768
670	448 900	300 763 000	25.884 3582	8.750 3401	.001 492 537
671	450 241	302 111 711	25.903 6677	8.754 6913	.001 490 313
672	451 584	303 464 448	25.922 9628	8.759 0383	.001 488 095
673	452 929	304 821 217	25.942 2435	8.763 3809	.001 485 884
674	454 276	306 182 024	25.961 5100	8.767 7192	.001 483 680
675	455 625	307 546 875	25.980 7621	8.772 0532	.001 481 481
676	456 976	308 915 776	26.000 0000	8.776 3830	.001 479 290

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
677	458 329	310 288 733	26.019 2237	8.780 7084	.001 477 105
678	459 684	311 665 752	26.038 4331	8.785 0296	.001 474 926
679	461 041	313 046 839	26.057 6284	8.789 3466	.001 472 754
680	462 400	314 432 000	26.076 8096	8.793 6593	.001 470 588
681	463 761	315 821 241	26.095 9767	8.797 9679	.001 468 429
682	465 124	317 214 568	26.115 1297	8.802 2721	.001 466 276
683	466 489	318 611 987	26.134 2687	8.806 5722	.001 464 129
684	467 856	320 013 504	26.153 3937	8.810 8681	.001 461 988
685	469 225	321 419 125	26.172 5047	8.815 1598	.001 459 854
686	470 596	322 828 856	26.191 6017	8.819 4474	.001 457 726
687	471 969	324 242 703	26.210 6848	8.823 7307	.001 455 604
688	473 344	325 660 672	26.229 7541	8.828 0099	.001 453 488
689	474 721	327 082 769	26.248 8095	8.832 2850	.001 451 379
690	476 100	328 509 000	26.267 8511	8.836 5559	.001 449 275
691	477 481	329 939 371	26.286 8789	8.840 8227	.001 447 178
692	478 864	331 373 888	26.305 8929	8.845 0854	.001 445 087
693	480 249	332 812 557	26.324 8932	8.849 3440	.001 443 001
694	481 636	334 255 384	26.343 8797	8.853 5985	.001 440 922
695	483 025	335 702 375	26.362 8527	8.857 8489	.001 438 849
696	484 416	337 153 536	26.381 8119	8.862 0952	.001 436 782
697	485 809	338 608 873	26.400 7576	8.866 3375	.001 434 720
698	487 204	340 068 392	26.419 6896	8.870 5757	.001 432 665
699	488 601	341 532 099	26.438 6081	8.874 8099	.001 430 615
700	490 000	343 000 000	26.457 5131	8.879 0400	.001 428 571
701	491 401	344 472 101	26.476 4046	8.883 2661	.001 426 534
702	492 804	345 948 408	26.495 2826	8.887 4882	.001 424 501
703	494 209	347 428 927	26.514 1472	8.891 7063	.001 422 475
704	495 616	348 913 664	26.532 9983	8.895 9204	.001 420 455
705	497 025	350 402 625	26.551 8361	8.900 1304	.001 418 440
706	498 436	351 895 816	26.570 6605	8.904 3366	.001 416 431
707	499 849	353 393 243	26.589 4716	8.908 5387	.001 414 427
708	501 264	354 894 912	26.608 2694	8.912 7369	.001 412 429
709	502 681	356 400 829	26.627 0539	8.916 9311	.001 410 437
710	504 100	357 911 000	26.645 8252	8.921 1214	.001 408 451
711	505 521	359 425 431	26.664 5833	8.925 3078	.001 406 470
712	506 944	360 944 128	26.683 3281	8.929 4902	.001 404 494
713	508 369	362 467 097	26.702 0598	8.933 6687	.001 402 525
714	509 796	363 994 344	26.720 7784	8.937 8433	.001 400 560
715	511 225	365 525 875	26.739 4839	8.942 0140	.001 398 601
716	512 656	367 061 696	26.758 1763	8.946 1809	.001 396 648
717	514 089	368 601 813	26.776 8557	8.950 3438	.001 394 700
718	515 524	370 146 232	26.795 5220	8.954 5029	.001 392 758
719	516 961	371 694 959	26.814 1754	8.958 6581	.001 390 821
720	518 400	373 248 000	26.832 8157	8.962 8095	.001 388 889
721	519 841	374 805 361	26.851 4432	8.966 9570	.001 386 963
722	521 284	376 367 048	26.870 0577	8.971 1007	.001 385 042
723	522 729	377 933 067	26.888 6593	8.975 2406	.001 383 126
724	524 176	379 503 424	26.907 2481	8.979 3766	.001 381 215
725	525 625	381 078 125	26.925 8240	8.983 5089	.001 379 310
726	527 076	382 657 176	26.944 3872	8.987 6373	.001 377 410
727	528 529	384 240 583	26.962 9375	8.991 7620	.001 375 516
728	529 984	385 828 352	26.981 4751	8.995 8899	.001 373 626

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
729	531 441	387 420 489	27.000 0000	9.000 0000	.001 371 742
730	532 900	389 017 000	27.018 5122	9.004 1134	.001 369 863
731	534 361	390 617 891	27.037 0117	9.008 2229	.001 367 989
732	535 824	392 223 168	27.055 4985	9.012 3288	.001 366 120
733	537 289	393 832 837	27.073 9727	9.016 4309	.001 364 256
734	538 756	395 446 904	27.092 4344	9.020 5293	.001 362 398
735	540 225	397 065 375	27.110 8834	9.024 6239	.001 360 544
736	541 696	398 688 256	27.129 3199	9.028 7149	.001 358 696
737	543 169	400 315 553	27.147 7149	9.032 8021	.001 356 852
738	544 644	401 947 272	27.166 1554	9.036 8857	.001 355 014
739	546 121	403 583 419	27.184 5544	9.040 9655	.001 353 180
740	547 600	405 224 000	27.202 9140	9.045 0419	.001 351 351
741	549 081	406 869 021	27.221 3152	9.049 1142	.001 349 528
742	550 564	408 518 488	27.239 6769	9.053 1831	.001 347 709
743	552 049	410 172 407	27.258 0263	9.057 2482	.001 345 895
744	553 536	411 830 784	27.276 3634	9.061 3098	.001 344 086
745	555 025	413 493 625	27.294 6881	9.065 3677	.001 342 282
746	556 516	415 160 936	27.313 0006	9.069 4220	.001 340 483
747	558 009	416 832 723	27.331 3007	9.073 4726	.001 338 688
748	559 504	418 508 992	27.349 5887	9.077 5197	.001 336 898
749	561 001	420 189 749	27.367 8644	9.081 5631	.001 335 113
750	562 500	421 875 000	27.386 1279	9.085 6030	.001 333 333
751	564 001	423 564 751	27.404 3792	9.089 6352	.001 331 558
752	565 504	425 259 008	27.422 6184	9.093 6719	.001 329 787
753	567 009	426 957 777	27.440 8455	9.097 7010	.001 328 021
754	568 516	428 661 064	27.459 0604	9.101 7265	.001 326 260
755	570 025	430 368 875	27.477 2633	9.105 7485	.001 324 503
756	571 536	432 081 216	27.495 4542	9.109 7669	.001 322 751
757	573 049	433 798 093	27.513 6330	9.113 7818	.001 321 004
758	574 564	435 519 512	27.531 7998	9.117 7931	.001 319 261
759	576 081	437 245 479	27.549 9546	9.121 8010	.001 317 523
760	577 600	438 976 000	27.568 0975	9.125 8053	.001 315 789
761	579 121	440 711 081	27.586 2284	9.129 8061	.001 314 060
762	580 644	442 450 728	27.604 3475	9.133 8034	.001 312 336
763	582 169	444 194 947	27.622 4546	9.137 7971	.001 310 616
764	583 696	445 943 744	27.640 5499	9.141 7874	.001 308 901
765	585 225	447 697 125	27.658 6334	9.145 7742	.001 307 190
766	586 756	449 455 096	27.676 7050	9.149 7576	.001 305 483
767	588 289	451 217 663	27.694 7648	9.153 7375	.001 303 781
768	589 824	452 984 832	27.712 8129	9.157 7139	.001 302 083
769	591 361	454 756 609	27.730 8492	9.161 6869	.001 300 390
770	592 900	456 533 000	27.748 8739	9.165 6565	.001 298 701
771	594 441	458 314 011	27.766 8868	9.169 6225	.001 297 017
772	595 984	460 099 648	27.784 8880	9.173 5852	.001 295 337
773	597 529	461 889 917	27.802 8775	9.177 5445	.001 293 661
774	599 076	463 684 824	27.820 8555	9.181 5003	.001 291 990
775	600 625	465 484 375	27.838 8218	9.185 4527	.001 290 323
776	602 176	467 288 576	27.856 7766	9.189 4018	.001 288 660
777	603 729	469 097 433	27.874 7197	9.193 3474	.001 287 001
778	605 284	470 910 952	27.892 6514	9.197 2897	.001 285 347
779	606 841	472 729 139	27.910 5715	9.201 2286	.001 283 697
780	608 400	474 552 000	27.928 4801	9.205 1641	.001 282 051

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
781	609 961	476 379 541	27.946 3772	9.209 0962	.001 280 410
782	611 524	478 211 768	27.964 2629	9.213 0250	.001 278 772
783	613 089	480 048 687	27.982 1372	9.216 9505	.001 277 139
784	614 656	481 890 304	28.000 0000	9.220 8726	.001 275 510
785	616 225	483 736 625	28.017 8515	9.224 7914	.001 273 885
786	617 796	485 587 656	28.035 6915	9.228 7068	.001 272 265
787	619 369	487 443 403	28.053 5203	9.232 6189	.001 270 648
788	620 944	489 303 872	28.071 3377	9.236 5277	.001 269 036
789	622 521	491 169 069	28.089 1438	9.240 4333	.001 267 427
790	624 100	493 039 000	28.106 9386	9.244 3355	.001 265 823
791	625 681	494 913 671	28.124 7222	9.248 2344	.001 264 223
792	627 264	496 793 088	28.142 4946	9.252 1300	.001 262 626
793	628 849	498 677 257	28.160 2557	9.256 0224	.001 261 034
794	630 436	500 566 184	28.178 0056	9.259 9114	.001 259 446
795	632 025	502 459 875	28.195 7444	9.263 7973	.001 257 862
796	633 616	504 358 336	28.213 4720	9.267 6798	.001 256 281
797	635 209	506 261 573	28.231 1884	9.271 5592	.001 254 705
798	636 804	508 169 592	28.248 8938	9.275 4352	.001 253 133
799	638 401	510 082 399	28.266 5881	9.279 3081	.001 251 564
800	640 000	512 000 000	28.284 2712	9.283 1777	.001 250 000
801	641 601	513 922 401	28.301 9434	9.287 0444	.001 248 439
802	643 204	515 849 608	28.319 6045	9.290 9072	.001 246 883
803	644 809	517 781 627	28.337 2546	9.294 7671	.001 245 330
804	646 416	519 718 464	28.354 8938	9.298 6239	.001 243 781
805	648 025	521 660 125	28.372 5219	9.302 4775	.001 242 236
806	649 636	523 606 616	28.390 1391	9.306 3278	.001 240 695
807	651 249	525 557 943	28.407 7454	9.310 1750	.001 239 157
808	652 864	527 514 112	28.425 3408	9.314 0190	.001 237 624
809	654 481	529 475 129	28.442 9253	9.317 8599	.001 236 094
810	656 100	531 441 000	28.460 4989	9.321 6975	.001 234 568
811	657 721	533 411 731	28.478 0617	9.325 5320	.001 233 046
812	659 344	535 387 328	28.495 6137	9.329 3634	.001 231 527
813	660 969	537 367 797	28.513 1549	9.333 1916	.001 230 012
814	662 596	539 353 144	28.530 6852	9.337 0167	.001 228 501
815	664 225	541 343 375	28.548 2048	9.340 8386	.001 226 994
816	665 856	543 338 496	28.565 7137	9.344 6575	.001 225 499
817	667 489	545 338 513	28.583 2119	9.348 4731	.001 223 990
818	669 124	547 343 432	28.600 6993	9.352 2857	.001 222 494
819	670 761	549 353 259	28.618 1760	9.356 0952	.001 221 001
820	672 400	551 368 000	28.635 6421	9.359 9016	.001 219 512
821	674 041	553 387 661	28.653 0976	9.363 7049	.001 218 027
822	675 684	555 412 248	28.670 5424	9.367 5051	.001 216 545
823	677 329	557 441 767	28.687 9716	9.371 3022	.001 215 067
824	678 976	559 476 224	28.705 4002	9.375 0963	.001 213 592
825	680 625	561 515 625	28.722 8132	9.378 8873	.001 212 121
826	682 276	563 559 976	28.740 2157	9.382 6752	.001 210 654
827	683 929	565 609 283	28.757 6077	9.386 4600	.001 209 190
828	685 584	567 663 552	28.774 9891	9.390 2419	.001 207 729
829	687 241	569 722 789	28.792 3601	9.394 0206	.001 206 273
830	688 900	571 787 000	28.809 7206	9.397 7964	.001 204 819
831	690 561	573 856 191	28.827 0706	9.401 5691	.001 203 369
832	692 224	575 930 368	28.844 4102	9.405 3387	.001 201 923

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
833	693 889	578 009 537	28.861 7394	9.409 1054	.001 200 480
834	695 556	580 093 704	28.879 0582	9.412 8690	.001 199 041
835	697 225	582 182 875	28.896 3666	9.416 6297	.001 197 605
836	698 896	584 277 056	28.913 6646	9.420 3873	.001 196 172
837	700 569	586 376 253	28.930 9523	9.424 1420	.001 194 743
838	702 244	588 480 472	28.948 2297	9.427 8936	.001 193 317
839	703 921	590 589 719	28.965 4967	9.431 6423	.001 191 895
840	705 600	592 704 000	28.982 7535	9.435 3800	.001 190 476
841	707 281	594 823 321	29.000 0000	9.439 1307	.001 189 061
842	708 964	596 947 688	29.017 2363	9.442 8704	.001 187 648
843	710 649	599 077 107	29.034 4623	9.446 6072	.001 186 240
844	712 336	601 211 584	29.051 6781	9.450 3410	.001 184 834
845	714 025	603 351 125	29.068 8837	9.454 0719	.001 183 432
846	715 716	605 495 736	29.086 0791	9.457 7999	.001 182 033
847	717 409	607 645 423	29.103 2644	9.461 5249	.001 180 638
848	719 104	609 800 192	29.120 4396	9.465 2470	.001 179 245
849	720 801	611 960 049	29.137 6046	9.468 9661	.001 177 856
850	722 500	614 125 000	29.154 7595	9.472 6824	.001 176 471
851	724 201	616 295 051	29.171 9043	9.476 3957	.001 175 088
852	725 904	618 470 208	29.189 0390	9.480 1061	.001 173 709
853	727 609	620 650 477	29.206 1637	9.483 8136	.001 172 333
854	729 316	622 835 864	29.223 2784	9.487 5182	.001 170 960
855	731 025	625 026 375	29.240 3830	9.491 2200	.001 169 591
856	732 736	627 222 016	29.257 4777	9.494 9188	.001 168 224
857	734 449	629 422 793	29.274 5623	9.498 6147	.001 166 861
858	736 164	631 628 712	29.291 6370	9.502 3078	.001 165 501
859	737 881	633 839 779	29.308 7018	9.505 9980	.001 164 144
860	739 600	636 056 000	29.325 7566	9.509 6854	.001 162 791
861	741 321	638 277 381	29.342 8015	9.513 3699	.001 161 440
862	743 044	640 503 928	29.359 8365	9.517 0515	.001 160 093
863	744 769	642 735 647	29.376 8616	9.520 7303	.001 158 749
864	746 496	644 972 544	29.393 8769	9.524 4063	.001 157 407
865	748 225	647 214 625	29.410 8823	9.528 0794	.001 156 069
866	749 956	649 461 896	29.427 8779	9.531 7497	.001 154 734
867	751 689	651 714 363	29.444 8637	9.535 4172	.001 153 403
868	753 424	653 972 032	29.461 8397	9.539 0818	.001 152 074
869	755 161	656 234 909	29.478 8059	9.542 7437	.001 150 748
870	756 900	658 503 000	29.495 7624	9.546 4027	.001 149 425
871	758 641	660 776 311	29.512 7091	9.550 0589	.001 148 106
872	760 384	663 054 848	29.529 6461	9.553 7123	.001 146 789
873	762 129	665 338 617	29.546 5734	9.557 3630	.001 145 475
874	763 876	667 627 624	29.563 4910	9.561 0108	.001 144 165
875	765 625	669 921 875	29.580 3989	9.564 6559	.001 142 857
876	767 376	672 221 376	29.597 2972	9.568 2782	.001 141 553
877	769 129	674 526 133	29.614 1858	9.571 9377	.001 140 251
878	770 884	676 836 152	29.631 0648	9.575 5745	.001 138 952
879	772 641	679 151 439	29.647 9342	9.579 2085	.001 137 656
880	774 400	681 472 000	29.664 7939	9.582 8397	.001 136 364
881	776 161	683 797 841	29.681 6442	9.586 4682	.001 135 074
882	777 924	686 128 968	29.698 4848	9.590 0937	.001 133 787
883	779 689	688 465 387	29.715 3159	9.593 7169	.001 132 503
884	781 456	690 807 104	29.732 1375	9.597 3373	.001 131 222

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
885	783 225	693 154 125	29.748 9496	9.600 9548	.001 129 944
886	784 996	695 506 456	29.765 7521	9.604 5696	.001 128 668
887	786 769	697 864 103	29.782 5452	9.608 1817	.001 127 396
888	788 544	700 227 072	29.799 3289	9.611 7911	.001 126 126
889	790 321	702 595 369	29.816 1030	9.615 3977	.001 124 859
890	792 100	704 969 000	29.832 8678	9.619 0017	.001 123 596
891	793 881	707 347 971	29.849 6231	9.622 6030	.001 122 334
892	795 664	707 932 288	29.866 3690	9.626 2016	.001 121 076
893	797 449	712 121 957	29.883 1056	9.629 7975	.001 119 821
894	799 236	714 516 984	29.899 8328	9.633 3907	.001 118 568
895	801 025	716 917 375	29.916 5506	9.636 9812	.001 117 818
896	802 816	719 323 136	29.933 2591	9.640 5690	.001 116 071
897	804 609	721 734 273	29.949 9583	9.644 1542	.001 114 827
898	806 404	724 150 792	29.966 6481	9.647 7367	.001 113 586
899	808 201	726 572 699	29.983 3287	9.651 3166	.001 112 347
900	810 000	729 000 000	30.000 0000	9.654 8938	.001 111 111
901	811 801	731 432 701	30.016 6621	9.658 4684	.001 109 878
902	813 604	733 870 808	30.033 3148	9.662 0403	.001 108 647
903	815 409	736 314 327	30.049 9584	9.665 6096	.001 107 420
904	817 216	738 763 264	30.066 5928	9.669 1762	.001 106 195
905	819 025	741 217 625	30.083 2179	9.672 7403	.001 104 972
906	820 836	743 677 416	30.099 8339	9.676 3017	.001 103 753
907	822 649	746 142 643	30.116 4407	9.679 8604	.001 102 536
908	824 464	748 613 312	30.133 0383	9.683 4166	.001 101 322
909	826 281	751 089 429	30.149 6269	9.686 9701	.001 100 110
910	828 100	753 571 000	30.166 2063	9.690 5211	.001 098 901
911	829 921	756 058 031	30.182 7765	9.694 0694	.001 097 695
912	831 744	758 550 828	30.199 3377	9.697 6151	.001 096 491
913	833 569	761 048 497	30.215 8899	9.701 1583	.001 095 290
914	835 396	763 551 944	30.232 4329	9.704 6989	.001 094 092
915	837 225	766 060 875	30.248 9669	9.708 2369	.001 092 896
916	839 056	768 575 296	30.265 4919	9.711 7723	.001 091 703
917	840 889	771 095 213	30.282 0079	9.715 3051	.001 090 513
918	842 724	773 620 632	30.298 5148	9.718 8354	.001 089 325
919	844 561	776 151 559	30.315 0128	9.722 3631	.001 088 139
920	846 400	778 688 000	30.331 5018	9.725 8883	.001 086 957
921	848 241	781 229 961	30.347 9818	9.729 4109	.001 085 776
922	850 084	783 777 448	30.364 4529	9.732 9309	.001 084 599
923	851 929	786 330 467	30.380 9151	9.736 4484	.001 083 423
924	853 776	788 889 024	30.397 3683	9.739 9634	.001 082 251
925	855 625	791 453 125	30.413 8127	9.743 4758	.001 081 081
926	857 476	794 022 776	30.430 2481	9.746 9857	.001 079 914
927	859 329	796 597 983	30.446 6747	9.750 4930	.001 078 749
928	861 184	799 178 752	30.463 0924	9.753 9979	.001 077 586
929	863 041	801 765 089	30.479 5013	9.757 5002	.001 076 426
930	864 900	804 357 000	30.495 9014	9.761 0001	.001 075 269
931	866 761	806 954 491	30.512 2926	9.764 4974	.001 074 114
932	868 624	809 557 568	30.528 6750	9.767 9922	.001 072 961
933	870 489	812 166 237	30.545 0487	9.771 4845	.001 071 811
934	872 356	814 780 504	30.561 4136	9.774 9743	.001 070 664
935	874 225	817 400 375	30.577 7697	9.778 4616	.001 069 519
936	876 096	820 025 856	30.594 1171	9.781 9466	.001 068 376

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
937	877 969	822 656 953	30.610.4557	9.785 4288	.001 067 236
938	879 844	825 293 672	30.626 7857	9.788 9087	.001 066 098
939	881 721	827 936 019	30.643 1069	9.792 3861	.001 064 963
940	883 600	830 584 000	30.659 4194	9.795 8611	.001 063 830
941	885 481	833 237 621	30.675 7233	9.799 3336	.001 062 699
942	887 364	835 896 888	30.692 0185	9.802 8036	.001 061 571
943	889 249	838 561 807	30.708 3051	9.806 2711	.001 060 445
944	891 136	841 232 384	30.724 5830	9.809 7362	.001 059 322
945	893 025	843 908 625	30.740 8523	9.813 1989	.001 058 201
946	894 916	846 590 536	30.757 1130	9.816 6591	.001 057 082
947	896 809	849 278 123	30.773 3651	9.820 1169	.001 055 966
948	898 704	851 971 392	30.789 6086	9.823 5723	.001 054 852
949	900 601	854 670 349	30.805 8436	9.827 0252	.001 053 741
950	902 500	857 375 000	30.822 0700	9.830 4757	.001 052 632
951	904 401	860 085 351	30.838 2879	9.833 9238	.001 051 525
952	906 304	862 801 408	30.854 4972	9.837 3695	.001 050 420
953	908 209	865 523 177	30.870 6981	9.840 8127	.001 049 318
954	910 116	868 250 664	30.886 8904	9.844 2536	.001 048 218
955	912 025	870 983 875	30.903 0743	9.847 6920	.001 047 120
956	913 936	873 722 816	30.919 2477	9.851 1280	.001 046 025
957	915 849	876 467 493	30.935 4166	9.854 5617	.001 044 932
958	917 764	879 217 912	30.951 5751	9.857 9929	.001 043 841
959	919 681	881 974 079	30.967 7251	9.861 4218	.001 042 753
960	921 600	884 736 000	30.983 8668	9.864 8483	.001 041 667
961	923 521	887 503 681	31.000 0000	9.868 2724	.001 040 583
962	925 444	890 277 128	31.016 1248	9.871 6941	.001 039 501
963	927 369	893 056 347	31.032 2413	9.875 1135	.001 038 422
964	929 296	895 841 344	31.048 3494	9.878 5305	.001 037 344
965	931 225	898 632 125	31.064 4491	9.881 9451	.001 036 269
966	933 156	901 428 696	31.080 5405	9.885 3574	.001 035 197
967	935 089	904 231 063	31.096 6236	9.888 7673	.001 034 126
968	937 024	907 039 232	31.112 6984	9.892 1749	.001 033 058
969	938 961	909 853 209	31.128 7648	9.895 5801	.001 031 992
970	940 900	912 673 000	31.144 8230	9.898 9830	.001 030 928
971	942 841	915 498 611	31.160 8729	9.902 3835	.001 029 866
972	944 784	918 330 048	31.176 9145	9.905 7817	.001 028 807
973	946 729	921 167 317	31.192 9479	9.909 1776	.001 027 749
974	948 676	924 010 424	31.208 9731	9.912 5712	.001 026 694
975	950 625	926 859 375	31.224 9900	9.915 9624	.001 025 641
976	952 576	929 714 176	31.240 9987	9.919 3513	.001 024 590
977	954 529	932 574 833	31.256 9992	9.922 7379	.001 023 541
978	956 484	935 441 352	31.272 9915	9.926 1222	.001 022 495
979	958 441	938 313 739	31.288 9757	9.929 5042	.001 021 450
980	960 400	941 192 000	31.304 9517	9.932 8839	.001 020 408
981	962 361	944 076 141	31.320 9195	9.936 2613	.001 019 168
982	964 324	946 966 168	31.336 8792	9.939 6363	.001 018 330
983	966 289	949 862 087	31.352 8308	9.943 0092	.001 017 294
984	968 256	952 763 904	31.368 7743	9.946 3797	.001 016 260
985	970 225	955 671 625	31.384 7097	9.949 7479	.001 015 228
986	972 196	958 585 256	31.400 6369	9.953 1138	.001 014 199
987	974 169	961 504 803	31.416 5561	9.956 4775	.001 013 171
988	976 144	964 430 272	31.432 4673	9.959 8389	.001 012 146

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
989	978 121	967 361 669	31.448 3704	9.963 1981	.001 011 122
990	980 100	970 299 000	31.464 2654	9.966 5549	.001 010 101
991	982 081	973 242 271	31.480 1525	9.969 9055	.001 009 082
992	984 064	976 191 488	31.496 0315	9.973 2619	.001 008 065
993	986 049	979 146 657	31.511 9025	9.976 6120	.001 007 049
994	988 036	982 107 784	31.527 7655	9.979 9599	.001 006 036
995	990 025	985 074 875	31.543 6206	9.983 3055	.001 005 025
996	992 016	988 047 936	31.559 4677	9.986 6488	.001 004 016
997	994 009	991 026 973	31.575 3068	9.989 9900	.001 003 009
998	996 004	994 011 992	31.591 1380	9.993 3289	.001 002 004
999	998 001	997 002 999	31.606 9613	9.996 6656	.001 001 001
1000	1 000 000	1 000 000 000	31.622 7766	10.000 0000	.001 000 000
1001	1 002 001	1 003 003 001	31.638 5840	10.003 3222	.000 999 0010
1002	1 004 004	1 006 012 008	31.654 3866	10.006 6622	.000 998 0040
1003	1 006 009	1 009 027 027	31.670 1752	10.009 9899	.000 997 0090
1004	1 008 016	1 012 048 064	31.685 9590	10.013 3155	.000 996 0159
1005	1 010 025	1 015 075 125	31.701 7349	10.016 6389	.000 995 0249
1006	1 012 036	1 018 108 216	31.717 5030	10.019 9601	.000 994 0358
1007	1 014 049	1 021 147 343	31.733 2633	10.023 2791	.000 993 0487
1008	1 016 064	1 024 192 512	31.749 0157	10.026 5958	.000 992 0635
1009	1 018 081	1 027 243 729	31.764 7603	10.029 9104	.000 991 0803
1010	1 020 100	1 030 301 000	31.780 4972	10.033 2228	.000 990 0990
1011	1 022 121	1 033 364 331	31.796 2262	10.036 5330	.000 989 1197
1012	1 024 144	1 036 433 728	31.811 9474	10.039 8410	.000 988 1423
1013	1 026 169	1 039 509 197	31.827 6609	10.043 1469	.000 987 1668
1014	1 028 196	1 042 590 744	31.843 3666	10.046 4506	.000 986 1933
1015	1 030 225	1 045 678 375	31.859 0646	10.049 7521	.000 985 2217
1016	1 032 256	1 048 772 096	31.874 7549	10.053 0514	.000 984 2520
1017	1 034 289	1 051 871 913	31.890 4374	10.056 3485	.000 983 2842
1018	1 036 324	1 054 977 832	31.906 1123	10.059 6435	.000 982 3183
1019	1 038 361	1 058 089 859	31.921 7794	10.062 9364	.000 981 3543
1020	1 040 400	1 061 208 000	31.937 4388	10.066 2271	.000 980 3922
1021	1 042 441	1 064 332 261	31.953 0906	10.069 5156	.000 979 4319
1022	1 044 484	1 067 462 648	31.968 7347	10.072 8020	.000 978 4736
1023	1 046 529	1 070 599 167	31.984 3712	10.076 0863	.000 977 5171
1024	1 048 576	1 073 741 824	32.000 0000	10.079 3684	.000 976 5625
1025	1 050 625	1 076 890 625	32.015 6212	10.082 6484	.000 975 6098
1026	1 052 676	1 080 045 576	32.031 2348	10.085 9262	.000 974 6589
1027	1 054 729	1 083 206 683	32.046 8407	10.089 2019	.000 973 7098
1028	1 056 784	1 086 373 952	32.062 4391	10.092 4755	.000 972 7626
1029	1 058 841	1 089 547 389	32.078 0298	10.095 7469	.000 971 8173
1030	1 060 900	1 092 727 000	32.093 6131	10.099 0163	.000 970 8738
1031	1 062 961	1 095 912 791	32.109 1887	10.102 2835	.000 969 9321
1032	1 065 024	1 099 104 768	32.124 7568	10.105 5487	.000 968 9922
1033	1 067 089	1 102 302 937	32.140 3173	10.108 8117	.000 968 0542
1034	1 069 156	1 105 507 304	32.155 8704	10.112 0726	.000 967 1180
1035	1 071 225	1 108 717 875	32.171 4159	10.115 3314	.000 966 1836
1036	1 073 296	1 111 934 656	32.186 9539	10.118 5882	.000 965 2510
1037	1 075 369	1 115 157 653	32.202 4844	10.121 8428	.000 964 3202
1038	1 077 444	1 118 386 872	32.218 0074	10.125 0953	.000 963 3911
1039	1 079 521	1 121 622 319	32.233 5229	10.128 3457	.000 962 4639
1040	1 081 600	1 124 864 000	32.249 0310	10.131 5941	.000 961 5385

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
1041	1 083 681	1 128 111 921	32.264.5316	10.134 8403	.000 960 6148
1042	1 085 764	1 131 366 088	32.280 0248	10.138 0845	.000 959 6929
1043	1 087 849	1 134 626 507	32.295 5105	10.141 3266	.000 958 7728
1044	1 089 936	1 137 893 184	32.310 9888	10.144 5667	.000 957 8544
1045	1 092 025	1 141 166 125	32.326 4598	10.147 8047	.000 956 9378
1046	1 094 116	1 144 445 336	32.341 9233	10.151 0406	.000 956 0229
1047	1 096 209	1 147 730 823	32.357 3794	10.154 2744	.000 955 1098
1048	1 098 304	1 151 022 592	32.372 8281	10.157 5062	.000 954 1985
1049	1 100 401	1 154 320 649	32.388 2695	10.160 7359	.000 953 2888
1050	1 102 500	1 157 625 000	32.403 7035	10.163 9636	.000 952 3810
1051	1 104 601	1 160 935 651	32.419 1301	10.167 1893	.000 951 4748
1052	1 106 704	1 164 252 608	32.434 5495	10.170 4129	.000 950 5703
1053	1 108 809	1 167 575 877	32.449 9615	10.173 6344	.000 949 6676
1054	1 110 916	1 170 905 464	32.465 3662	10.176 8539	.000 948 7666
1055	1 113 025	1 174 241 375	32.480 7635	10.180 0714	.000 947 8673
1056	1 115 136	1 177 583 616	32.496 1536	10.183 2868	.000 946 9697
1057	1 117 249	1 180 932 193	32.511 5364	10.186 5002	.000 946 0738
1058	1 119 364	1 184 287 112	32.526 9119	10.189 7116	.000 945 1796
1059	1 121 481	1 187 648 379	32.542 2802	10.192 9209	.000 944 2871
1060	1 123 600	1 191 016 000	32.557 6412	10.196 1283	.000 943 3962
1061	1 125 721	1 194 389 981	32.572 9949	10.199 3336	.000 942 5071
1062	1 127 844	1 197 770 328	32.588 3415	10.202 5369	.000 941 6196
1063	1 129 969	1 201 157 047	32.603 5807	10.205 7382	.000 940 7338
1064	1 132 096	1 204 550 144	32.619 0129	10.208 9375	.000 939 8496
1065	1 134 225	1 207 949 625	32.634 3377	10.212 1347	.000 938 9671
1066	1 136 356	1 211 355 496	32.649 6554	10.215 3300	.000 938 0863
1067	1 138 489	1 214 767 763	32.664 9659	10.218 5233	.000 937 2071
1068	1 140 624	1 218 186 432	32.680 2693	10.221 7146	.000 936 3296
1069	1 142 761	1 221 611 509	32.695 5654	10.224 9039	.000 935 4537
1070	1 144 900	1 225 043 000	32.710 8544	10.228 0912	.000 934 5794
1071	1 147 041	1 228 480 911	32.726 1363	10.231 2766	.000 933 7068
1072	1 149 184	1 231 925 248	32.741 4111	10.234 4599	.000 932 8358
1073	1 151 329	1 235 376 017	32.756 6787	10.237 6413	.000 931 9664
1074	1 153 476	1 238 833 224	32.771 9392	10.240 8207	.000 931 0987
1075	1 155 625	1 242 296 875	32.787 1926	10.243 9981	.000 930 2326
1076	1 157 776	1 245 766 976	32.802 4398	10.247 1735	.000 929 3680
1077	1 159 929	1 249 243 533	32.817 6782	10.250 3470	.000 928 5051
1078	1 162 084	1 252 726 552	32.832 9103	10.253 5186	.000 927 6438
1079	1 164 241	1 256 216 039	32.848 1354	10.256 6881	.000 926 7841
1080	1 166 400	1 259 712 000	32.863 3535	10.259 8557	.000 925 9259
1081	1 168 561	1 263 214 441	32.878 5644	10.263 0213	.000 925 0694
1082	1 170 724	1 266 723 368	32.893 7684	10.266 1850	.000 924 2144
1083	1 172 889	1 270 238 787	32.908 9653	10.269 3467	.000 923 3610
1084	1 175 056	1 273 760 704	32.924 1553	10.272 5065	.000 922 5092
1085	1 177 225	1 277 289 125	32.939 3382	10.275 6644	.000 921 6590
1086	1 179 396	1 280 824 056	32.954 5141	10.278 8203	.000 920 8103
1087	1 181 569	1 284 365 503	32.969 6830	10.281 9743	.000 919 9632
1088	1 183 744	1 287 913 472	32.984 8450	10.285 1264	.000 919 1176
1089	1 185 921	1 291 467 969	33.000 0000	10.288 2765	.000 918 2736
1090	1 188 100	1 295 029 000	33.015 1480	10.291 4247	.000 917 4312
1091	1 190 281	1 298 596 571	33.030 2891	10.294 5709	.000 916 5903
1092	1 192 464	1 302 170 688	33.045 4233	10.297 7153	.000 915 7509

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
1093	1 194 649	1 305 751 357	33.060 5505	10.300 8577	.000 914 9131
1094	1 196 836	1 309 338 584	33.075 6708	10.303 9982	.000 914 0768
1095	1 199 025	1 312 932 375	33.090 7842	10.307 1368	.000 913 2420
1096	1 201 216	1 316 532 736	33.105 8907	10.310 2735	.000 912 4008
1097	1 203 409	1 320 139 673	33.120 9903	10.313 4083	.000 911 5770
1098	1 205 604	1 323 753 192	33.136 0830	10.316 5411	.000 910 7468
1099	1 207 801	1 327 373 299	33.151 1689	10.319 6721	.000 909 9181
1100	1 210 000	1 331 000 000	33.166 2479	10.322 8012	.000 909 0909
1101	1 212 201	1 334 633 301	33.181 3200	10.325 9284	.000 908 2652
1102	1 214 404	1 338 273 208	33.196 3853	10.329 0537	.000 907 4410
1103	1 216 609	1 341 919 727	33.211 4438	10.332 1770	.000 906 6183
1104	1 218 816	1 345 572 864	33.226 6955	10.335 2985	.000 905 7971
1105	1 221 025	1 349 232 625	33.241 5403	10.338 4181	.000 904 9774
1106	1 223 236	1 352 899 016	33.256 5783	10.341 5358	.000 904 1591
1107	1 225 449	1 356 572 043	33.271 6095	10.344 6517	.000 903 3424
1108	1 227 664	1 360 251 712	33.286 6339	10.347 7657	.000 902 5271
1109	1 229 881	1 363 938 029	33.301 6516	10.350 8778	.000 901 7133
1110	1 232 100	1 367 631 000	33.316 6625	10.353 9880	.000 900 9009
1111	1 234 321	1 371 330 631	33.331 6666	10.357 0964	.000 900 0900
1112	1 236 544	1 375 036 928	33.346 6640	10.360 2029	.000 899 2806
1113	1 238 769	1 378 749 897	33.361 6546	10.363 3076	.000 898 4726
1114	1 240 996	1 382 469 544	33.376 6385	10.366 4103	.000 897 6661
1115	1 243 225	1 386 195 875	33.391 6157	10.369 5113	.000 896 8610
1116	1 245 456	1 389 928 896	33.406 5862	10.372 6103	.000 896 0753
1117	1 247 689	1 393 668 613	33.421 5499	10.375 7076	.000 895 2551
1118	1 249 924	1 397 415 032	33.436 5070	10.378 8030	.000 894 4544
1119	1 252 161	1 401 168 159	33.451 4573	10.381 8965	.000 893 6550
1120	1 254 400	1 404 928 000	33.466 4011	10.384 9882	.000 892 8571
1121	1 256 641	1 408 694 561	33.481 3381	10.388 0781	.000 896 0607
1122	1 258 884	1 412 467 848	33.496 2684	10.391 1661	.000 892 2656
1123	1 261 129	1 416 247 867	33.511 1921	10.394 2527	.000 890 4720
1124	1 263 376	1 420 034 624	33.526 1092	10.397 3366	.000 889 6797
1125	1 265 625	1 423 828 125	33.541 0196	10.400 4192	.000 888 8889
1126	1 267 876	1 427 628 376	33.555 9234	10.403 4999	.000 888 0995
1127	1 270 129	1 431 435 383	33.570 8206	10.406 5787	.000 887 3114
1128	1 272 384	1 435 249 152	33.585 7112	10.409 6557	.000 886 5248
1129	1 274 641	1 439 069 689	33.600 5952	10.412 7310	.000 885 7396
1130	1 276 900	1 442 897 000	33.615 4726	10.415 8044	.000 884 9558
1131	1 279 161	1 446 731 091	33.630 3434	10.418 8760	.000 884 1733
1132	1 281 424	1 450 571 968	33.645 2077	10.421 9458	.000 883 3922
1133	1 283 689	1 454 419 637	33.660 0653	10.425 0138	.000 882 6125
1134	1 285 956	1 458 274 104	33.674 9165	10.428 0800	.000 881 8342
1135	1 288 225	1 462 135 375	33.689 7610	10.431 1443	.000 881 0573
1136	1 290 496	1 466 003 456	33.704 5991	10.434 2069	.000 880 2817
1137	1 292 769	1 469 878 353	33.717 4306	10.437 2677	.000 879 5075
1138	1 295 044	1 473 760 072	33.734 0556	10.440 3677	.000 878 7346
1139	1 297 321	1 477 648 619	33.749 0741	10.443 3839	.000 877 9631
1140	1 299 600	1 481 544 000	33.763 8860	10.446 4393	.000 877 1930
1141	1 301 881	1 485 446 221	33.778 6915	10.449 4929	.000 876 4242
1142	1 304 164	1 489 355 288	33.793 4905	10.452 5448	.000 875 6567
1143	1 306 449	1 493 271 207	33.808 2830	10.455 5948	.000 874 8906
1144	1 308 736	1 497 193 984	33.823 0691	10.458 6431	.000 874 1259

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
1145	1 311 025	1 501 123 625	33.837 8486	10.461 6896	.000 873 3624
1146	1 313 316	1 505 060 136	33.852 6218	10.464 7343	.000 872 6003
1147	1 315 609	1 509 003 523	33.867 3884	10.467 7773	.000 871 8396
1148	1 317 904	1 512 953 792	33.882 1487	10.470 8158	.000 871 0801
1149	1 320 201	1 516 910 949	33.896 9025	10.473 8579	.000 870 3220
1150	1 322 500	1 520 875 000	33.911 6499	10.476 8955	.000 869 5652
1151	1 324 801	1 524 845 951	33.926 3909	10.479 9314	.000 868 8097
1152	1 327 104	1 528 823 808	33.941 1255	10.482 9656	.000 868 0556
1153	1 329 409	1 532 808 577	33.955 8537	10.485 9980	.000 867 3027
1154	1 331 716	1 536 800 264	33.970 5755	10.489 0286	.000 866 5511
1155	1 334 025	1 540 798 875	33.985 2910	10.492 0575	.000 865 8009
1156	1 336 336	1 544 804 416	34.000 0000	10.495 0847	.000 865 0519
1157	1 338 649	1 548 816 893	34.014 7027	10.498 1101	.000 864 3042
1158	1 340 964	1 552 836 312	34.029 3990	10.501 1337	.000 863 5579
1159	1 343 281	1 556 862 679	34.044 0890	10.504 1556	.000 862 8128
1160	1 345 600	1 560 896 000	34.058 7727	10.507 1757	.000 862 0690
1161	1 347 921	1 564 936 281	34.073 4501	10.510 1942	.000 861 3264
1162	1 350 244	1 568 983 528	34.088 1211	10.513 2109	.000 860 5852
1163	1 352 569	1 573 037 747	34.012 7858	10.516 2259	.000 859 8452
1164	1 354 896	1 577 098 944	34.117 4442	10.519 2391	.000 859 1065
1165	1 357 225	1 581 167 125	34.132 0963	10.522 2506	.000 858 3691
1166	1 359 556	1 585 242 296	34.146 7422	10.525 2604	.000 857 6329
1167	1 361 889	1 589 324 463	34.161 3817	10.528 2685	.000 856 8980
1168	1 364 224	1 593 413 632	34.176 0150	10.531 2749	.000 856 1644
1169	1 366 561	1 597 509 809	34.190 6420	10.534 2795	.000 855 4320
1170	1 368 900	1 601 613 000	34.205 2627	10.537 2825	.000 854 7009
1171	1 371 241	1 605 723 211	34.219 8773	10.540 2837	.000 853 9710
1172	1 373 584	1 609 840 448	34.234 4855	10.543 2832	.000 853 2423
1173	1 375 929	1 613 964 717	34.249 0875	10.546 2810	.000 852 5149
1174	1 378 276	1 618 096 024	34.263 6834	10.549 2771	.000 851 7888
1175	1 380 625	1 622 234 375	34.278 2730	10.552 2715	.000 851 0638
1176	1 382 976	1 626 379 776	34.292 8564	10.555 2642	.000 850 3401
1177	1 385 329	1 630 532 233	34.307 4336	10.558 2552	.000 849 6177
1178	1 387 684	1 634 691 752	34.322 0046	10.561 2445	.000 848 8964
1179	1 390 041	1 638 858 339	34.336 5694	10.564 2322	.000 848 1764
1180	1 392 400	1 643 032 000	34.351 1281	10.567 2181	.000 847 1576
1181	1 394 761	1 647 212 741	34.365 6805	10.570 2024	.000 846 7401
1182	1 397 124	1 651 400 568	34.380 2268	10.573 1849	.000 846 0237
1183	1 399 489	1 655 595 487	34.394 7670	10.576 1658	.000 845 3085
1184	1 401 856	1 659 797 504	34.409 3011	10.579 1449	.000 844 5946
1185	1 404 225	1 664 006 625	34.423 8289	10.582 1225	.000 843 8819
1186	1 406 596	1 668 222 856	34.438 3507	10.585 0983	.000 843 1703
1187	1 408 969	1 672 446 203	34.452 8663	10.588 0725	.000 842 4600
1188	1 411 344	1 676 676 672	34.467 3759	10.591 0450	.000 841 7508
1189	1 413 721	1 680 914 629	34.481 8793	10.594 0158	.000 841 0429
1190	1 416 100	1 685 159 000	34.496 3766	10.596 9850	.000 840 3361
1191	1 418 481	1 689 410 871	34.510 8678	10.599 9525	.000 839 6306
1192	1 420 864	1 693 669 888	34.525 3530	10.602 9184	.000 838 9262
1193	1 423 249	1 697 936 057	34.539 8321	10.605 8826	.000 838 2320
1194	1 425 636	1 702 209 384	34.554 3051	10.608 8451	.000 837 5209
1195	1 428 025	1 706 489 875	34.568 7720	10.611 8060	.000 836 8201
1196	1 430 416	1 710 777 536	34.583 2329	10.614 7652	.000 836 1204

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
1197	1 432 809	1 715 072 373	34.597 6879	10.617 7228	.000 835 4219
1198	1 435 204	1 719 374 392	34.612 1366	10.620 6788	.000 834 7245
1199	1 437 601	1 723 683 599	34.626 5794	10.623 6331	.000 834 0284
1200	1 440 000	1 728 000 000	34.641 0162	10.626 5857	.000 833 3333
1201	1 442 401	1 732 323 601	34.655 4469	10.629 5367	.000 832 6395
1202	1 444 804	1 736 654 408	34.669 8716	10.632 4860	.000 831 9468
1203	1 447 209	1 740 992 427	34.684 2904	10.635 4338	.000 831 2552
1204	1 449 616	1 745 337 664	34.698 7031	10.638 3799	.000 830 5648
1205	1 452 025	1 749 690 125	34.713 1099	10.641 3244	.000 829 8755
1206	1 454 436	1 754 049 816	34.727 5107	10.644 2672	.000 829 1874
1207	1 456 849	1 758 416 743	34.741 9055	10.647 2085	.000 828 5004
1208	1 459 264	1 762 790 912	34.756 2944	10.650 1480	.000 827 8146
1209	1 461 681	1 767 172 329	34.770 6773	10.653 0860	.000 827 1299
1210	1 464 100	1 771 561 000	34.785 0543	10.656 0223	.000 826 4463
1211	1 466 521	1 775 956 931	34.799 4253	10.658 9570	.000 825 7638
1212	1 468 944	1 780 360 128	34.813 7904	10.661 8902	.000 825 0825
1213	1 471 369	1 784 770 597	34.828 1495	10.664 8217	.000 824 4023
1214	1 473 796	1 789 188 344	34.842 5028	10.667 7516	.000 823 7232
1215	1 476 225	1 793 613 375	34.856 8501	10.670 6799	.000 823 0453
1216	1 478 656	1 798 045 696	34.871 1915	10.673 6066	.000 822 3684
1217	1 481 089	1 802 485 313	34.885 5271	10.676 5317	.000 821 6927
1218	1 483 524	1 806 932 232	34.899 8567	10.679 4552	.000 821 0181
1219	1 485 961	1 811 386 459	34.914 1805	10.682 3771	.000 820 3445
1220	1 488 400	1 815 848 000	34.928 4984	10.685 2973	.000 819 6721
1221	1 490 841	1 820 316 861	34.942 8104	10.688 2160	.000 819 0008
1222	1 493 284	1 824 793 048	34.957 1166	10.691 1331	.000 818 3306
1223	1 495 729	1 829 276 567	34.971 4169	10.694 0486	.000 817 6615
1224	1 498 176	1 833 764 247	34.985 7114	10.696 9625	.000 816 9935
1225	1 500 625	1 838 265 625	35.000 0000	10.699 8748	.000 816 3265
1226	1 503 276	1 842 771 176	35.014 2828	10.702 7855	.000 815 6607
1227	1 505 529	1 847 284 083	35.028 5598	10.705 6947	000 814 9959
1228	1 507 984	1 851 804 352	35.042 8309	10.708 6023	.000 814 3322
1229	1 510 441	1 856 331 989	35.057 0963	10.711 5083	.000 813 6696
1230	1 512 900	1 860 867 000	35.071 3558	10.714 4127	.000 813 0081
1231	1 515 361	1 865 409 391	35.085 6096	10.717 3155	.000 812 3477
1232	1 517 824	1 869 959 168	35.099 8575	10.720 2168	.000 811 6883
1233	1 520 289	1 874 516 337	35.114 0997	10.723 1165	.000 811 0300
1234	1 522 756	1 879 080 904	35.128 3361	10.726 0146	.000 810 3728
1235	1 525 225	1 883 652 875	35.142 5668	10.728 9112	.000 809 7166
1236	1 527 696	1 888 232 256	35.156 7917	10.731 8062	.000 809 0615
1237	1 530 169	1 892 819 053	35.171 0108	10.734 6997	.000 808 4074
1238	1 532 644	1 897 413 272	35.185 2242	10.737 5916	.000 807 7544
1239	1 535 121	1 902 014 919	35.199 4318	10.740 4819	.000 807 1025
1240	1 537 600	1 906 624 000	35.213 6337	10.743 3707	.000 806 4516
1241	1 540 081	1 911 240 521	35.227 8299	10.746 2579	.000 805 8018
1242	1 542 564	1 915 864 488	35.242 0204	10.749 1436	.000 805 1530
1243	1 545 049	1 920 495 907	35.256 2051	10.752 0277	.000 804 5052
1244	1 547 536	1 925 134 784	35.270 3842	10.754 9103	.000 803 8585
1245	1 550 025	1 929 781 125	35.284 5575	10.757 7913	.000 803 2129
1246	1 552 516	1 934 434 936	35.298 7252	10.760 6708	.000 802 5682
1247	1 555 009	1 939 096 223	35.312 8872	10.763 5488	.000 801 9246
1248	1 557 504	1 943 764 992	35.327 0435	10.766 4252	.000 801 2821

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1249	1 560 001	1 948 441 249	35.341 1941	10.769 3001	.000 800 6405
1250	1 562 500	1 953 125 000	35.355 3391	10.772 1735	.000 800 0000
1251	1 565 001	1 957 816 251	35.369 4784	10.775 0453	.000 799 3605
1252	1 567 504	1 962 515 008	35.383 6120	10.777 9156	.000 798 7220
1253	1 570 009	1 967 221 277	35.397 7400	10.780 7843	.000 798 0846
1254	1 572 516	1 971 935 064	35.411 8624	10.783 6516	.000 797 4482
1255	1 575 025	1 976 656 375	35.425 9792	10.786 5173	.000 796 8127
1256	1 577 536	1 981 385 216	35.440 0903	10.789 3815	.000 796 1783
1257	1 580 049	1 986 121 593	35.454 1958	10.792 2441	.000 795 5449
1258	1 582 564	1 990 865 512	35.468 2957	10.795 1053	.000 794 9126
1259	1 585 081	1 995 616 979	35.482 3900	10.797 9649	.000 794 2812
1260	1 587 600	2 000 376 000	35.496 4787	10.800 8230	.000 793 6508
1261	1 590 121	2 005 142 581	35.510 5618	10.803 6797	.000 793 0214
1262	1 592 644	2 009 916 728	35.524 6393	10.806 5348	.000 792 3930
1263	1 595 169	2 014 698 447	35.538 7113	10.809 3884	.000 791 7656
1264	1 597 696	2 019 487 744	35.552 7777	10.812 2404	.000 791 1392
1265	1 600 225	2 024 284 625	35.566 8385	10.815 0909	.000 790 5138
1266	1 602 756	2 029 089 096	35.580 8937	10.817 9400	.000 789 8894
1267	1 605 289	2 033 901 163	35.594 9434	10.820 7876	.000 789 2660
1268	1 607 824	2 038 720 832	35.608 9876	10.823 6336	.000 788 6435
1269	1 610 361	2 043 548 109	35.623 0262	10.826 4782	.000 788 0221
1270	1 612 900	2 048 383 000	35.637 0593	10.829 3213	.000 787 4016
1271	1 615 441	2 053 225 511	35.651 0869	10.832 1629	.000 786 7821
1272	1 617 984	2 058 075 648	35.665 1090	10.835 0030	.000 786 1635
1273	1 620 529	2 062 933 417	35.679 1255	10.837 8416	.000 785 5460
1274	1 623 076	2 067 798 824	35.693 1366	10.840 6788	.000 784 9294
1275	1 625 625	2 072 671 875	35.707 1421	10.843 5144	.000 784 3137
1276	1 628 176	2 077 552 576	35.721 1422	10.846 3485	.000 783 6991
1277	1 630 729	2 082 440 933	35.735 1367	10.849 1812	.000 783 0854
1278	1 633 284	2 087 336 952	35.749 1258	10.852 0125	.000 782 4726
1279	1 635 841	2 092 240 639	35.763 1095	10.854 8422	.000 781 8608
1280	1 638 400	2 097 152 000	35.777 0876	10.857 6704	.000 781 2500
1281	1 640 961	2 102 071 841	35.791 0603	10.860 4972	.000 780 6401
1282	1 643 524	2 106 997 768	35.805 0276	10.863 3225	.000 780 0312
1283	1 646 089	2 111 932 187	35.818 9894	10.866 1454	.000 779 4232
1284	1 648 656	2 116 874 304	35.832 9457	10.868 9687	.000 778 8162
1285	1 651 225	2 121 824 125	35.846 8966	10.871 7897	.000 778 2101
1286	1 653 796	2 126 781 656	35.860 8421	10.874 6091	.000 777 6050
1287	1 656 369	2 131 746 903	35.874 7822	10.877 4271	.000 777 0008
1288	1 658 944	2 136 719 872	35.888 7169	10.880 2436	.000 776 3975
1289	1 661 521	2 141 700 569	35.902 6461	10.883 0587	.000 775 7952
1290	1 664 100	2 146 689 000	35.916 5699	10.885 8723	.000 775 1938
1291	1 666 681	2 151 685 171	35 930 4884	10.888 6845	.000 774 5933
1292	1 669 264	2 156 689 088	35.944 4015	10.891 4952	.000 773 9938
1293	1 671 849	2 161 700 757	35.958 3092	10.894 3044	.000 773 3952
1294	1 674 436	2 166 720 184	35.972 2115	10.897 1123	.000 772 7975
1295	1 677 025	2 171 747 375	35.986 1084	10.899 9186	.000 772 2008
1296	1 679 616	2 176 782 336	36.000 0000	10.902 7235	.000 771 6049
1297	1 682 209	2 181 825 073	36.013 8862	10.905 5269	.000 771 0100
1298	1 684 804	2 186 875 592	36.027 7671	10.908 3290	.000 770 4160
1299	1 687 401	2 191 933 899	36.041 6426	10.911 1296	.000 769 8229
1300	1 690 000	2 197 000 000	36.055 5128	10.913 9287	.000 769 2308

Number.	Squares.	Cubes.	$\sqrt{}$ Roots.	$\sqrt[3]{}$ Roots.	Reciprocals.
1301	1 692 601	2 202 073 901	36.069 3776	10.916 7265	.000 768 6395
1302	1 695 204	2 207 155 608	36.083 2371	10.919 5228	.000 768 0492
1303	1 697 809	2 212 245 127	36.097 0913	10.922 3177	.000 767 4579
1304	1 700 416	2 217 342 464	36.110 9402	10.925 1111	.000 766 8712
1305	1 703 025	2 222 447 625	36.124 7837	10.927 9031	.000 766 2835
1306	1 705 636	2 227 560 616	36.138 6220	10.930 6937	.000 765 6968
1307	1 708 249	2 232 681 443	36.152 4550	10.933 4829	.000 765 1109
1308	1 710 864	2 237 810 112	36.166 2826	10.936 2706	.000 764 5260
1309	1 713 481	2 242 946 629	36.180 1050	10.939 0569	.000 763 9419
1310	1 716 100	2 248 091 000	36.193 9221	10.941 8418	.000 763 3588
1311	1 718 721	2 253 243 231	36.207 7340	10.944 6253	.000 762 7765
1312	1 721 344	2 258 403 328	36.221 5406	10.947 5074	.000 762 1951
1313	1 723 969	2 263 571 297	36.235 3419	10.950 1880	.000 761 6446
1314	1 726 596	2 268 747 144	36.249 1379	10.952 9673	.000 761 0350
1315	1 729 225	2 273 930 875	36.262 6287	10.955 7451	.000 760 4563
1316	1 731 856	2 279 122 496	36.276 7143	10.958 5215	.000 759 8784
1317	1 734 489	2 284 322 013	36.290 4246	10.961 2965	.000 759 3014
1318	1 737 124	2 289 529 432	36.304 2697	10.964 0701	.000 758 7253
1319	1 739 761	2 294 744 759	36.318 0396	10.966 8423	.000 758 1501
1320	1 742 400	2 299 968 000	36.331 8042	10.969 6131	.000 757 5758
1321	1 745 041	2 305 199 161	36.345 5637	10.972 3825	.000 757 0023
1322	1 747 684	2 310 438 248	36.359 3179	10.975 1505	.000 756 4297
1323	1 750 329	2 315 685 267	36.373 0670	10.977 9171	.000 755 8579
1324	1 752 976	2 320 940 224	36.386 8108	10.980 6823	.000 755 2870
1325	1 755 625	2 326 203 125	36.400 5494	10.983 4462	.000 754 7170
1326	1 758 276	2 331 473 976	36.414 2829	10.986 2086	.000 754 1478
1327	1 760 929	2 336 752 783	36.428 0112	10.988 9696	.000 753 5795
1328	1 763 584	2 342 039 552	36.441 7343	10.991 7293	.000 753 0120
1329	1 766 241	2 347 334 289	36.455 4523	10.994 4876	.000 752 4454
1330	1 768 900	2 352 637 000	36.469 1650	10.997 2445	.000 751 8797
1331	1 771 561	2 357 947 691	36.482 8727	11.000 0000	.000 751 3148
1332	1 774 224	2 363 266 368	36.496 5752	11.002 7541	.000 750 7508
1333	1 776 889	2 368 593 037	36.510 2725	11.005 5069	.000 750 1875
1334	1 779 556	2 373 927 704	36.523 9647	11.008 2583	.000 749 6252
1335	1 782 225	2 379 270 375	36.537 6518	11.011 0082	.000 749 0637
1336	1 784 896	2 384 621 056	36.551 3388	11.013 7569	.000 748 5030
1337	1 787 569	2 389 979 753	36.565 0106	11.016 5041	.000 747 9432
1338	1 790 244	2 395 346 472	36.578 6823	11.019 2500	.000 747 3842
1339	1 792 921	2 400 721 219	36.592 3489	11.021 9945	.000 746 8260
1340	1 795 600	2 406 104 000	36.606 0104	11.024 7377	.000 746 2687
1341	1 798 281	2 411 494 821	36.619 6668	11.027 4795	.000 745 7122
1342	1 800 964	2 416 893 688	36.633 3181	11.030 2199	.000 745 1565
1343	1 803 649	2 422 300 607	36.646 9144	11.032 9590	.000 744 6016
1344	1 806 336	2 427 715 584	36.660 6056	11.035 6967	.000 744 0476
1345	1 809 025	2 433 138 625	36.674 2416	11.038 4330	.000 743 4944
1346	1 811 716	2 438 569 736	36.687 8726	11.041 1680	.000 742 9421
1347	1 814 409	2 444 008 923	36.701 4986	11.043 9017	.000 742 3905
1348	1 817 104	2 449 456 192	36.715 1195	11.046 6339	.000 741 8398
1349	1 819 801	2 454 911 549	36.728 7353	11.049 3649	.000 741 2898
1350	1 822 500	2 460 375 000	36.742 3461	11.052 0945	.000 740 7407
1351	1 825 201	2 465 846 551	36.755 9519	11.054 8227	.000 740 1924
1352	1 827 904	2 471 326 208	36.769 5526	11.057 5497	.000 739 6450

Number.	Squares.	Cubes.	√Roots.	∛Roots.	Reciprocals.
1353	1 830 609	2 476 813 977	36.783 1483	11.060 2752	.000 739 0983
1354	1 833 316	2 482 309 864	36.796 7390	11.062 9994	.000 738 5524
1355	1 836 025	2 487 813 875	36.810 3246	11.065 7222	.000 738 0074
1356	1 838 736	2 493 326 016	36.823 9053	11.068 4437	.000 737 4631
1357	1 841 449	2 498 846 293	36.837 4809	11.071 1639	.000 736 9197
1358	1 844 164	2 504 374 712	36.851 0515	11.073 8828	.000 736 3770
1359	1 846 881	2 509 911 279	36.864 6172	11.076 6003	.000 735 8352
1360	1 849 600	2 515 456 000	36.878 1778	11.079 3165	.000 735 2941
1361	1 852 321	2 521 008 881	36.891 7335	11.082 0314	.000 734 7539
1362	1 855 044	2 526 569 928	36.905 2842	11.084 7449	.000 734 2144
1363	1 857 769	2 532 139 147	36.918 8299	11.087 4571	.000 733 6757
1364	1 860 496	2 537 716 544	36.932 3706	11.090 1679	.000 733 1378
1365	1 863 225	2 543 302 125	36.945 9064	11.092 8775	.000 732 6007
1366	1 865 956	2 548 895 896	36.959 4372	11.095 5857	.000 732 0644
1367	1 868 689	2 554 497 863	36.972 9631	11.098 2926	.000 731 5289
1368	1 871 424	2 560 108 032	36.986 4840	11.100 9982	.000 730 9942
1369	1 874 161	2 565 726 409	37.000 0000	11.103 7025	.000 730 4602
1370	1 876 900	2 571 353 000	37.013 5110	11.106 4054	.000 729 9270
1371	1 879 641	2 576 987 811	37.027 0172	11.109 1070	.000 729 3946
1372	1 882 384	2 582 630 848	37.040 5184	11.111 8073	.000 728 8630
1373	1 885 129	2 588 282 117	37.054 0146	11.114 5064	.000 728 3321
1374	1 887 876	2 593 941 624	37.067 5060	11.117 2041	.000 727 8020
1375	1 890 625	2 599 609 375	37.089 9924	11.119 9004	.000 727 2727
1376	1 893 376	2 605 285 376	37.094 4740	11.122 5955	.000 726 7442
1377	1 896 129	2 610 969 633	37.107 9506	11.125 2893	.000 726 2164
1378	1 898 884	2 616 662 152	37.121 4224	11.127 9817	.000 725 6894
1379	1 901 641	2 622 362 939	37.134 8893	11.130 6729	.000 725 1632
1380	1 904 400	2 628 072 000	37.148 3512	11.133 3628	.000 724 6377
1381	1 907 161	2 633 789 341	37.161 8084	11.136 0514	.000 724 1130
1382	1 909 924	2 639 514 968	37.175 2606	11.138 7386	.000 723 5890
1383	1 912 689	2 645 248 887	37.188 7079	11.141 4246	.000 723 0658
1384	1 915 456	2 650 991 104	37.202 1505	11.144 1093	.000 722 5434
1385	1 918 225	2 656 741 625	37.215 5881	11.146 7926	.000 722 0217
1386	1 920 996	2 662 500 456	37.229 0209	11.149 4747	.000 721 5007
1387	1 923 769	2 668 267 603	37.242 4489	11.152 1555	.000 720 9805
1388	1 926 544	2 674 043 072	37.255 8720	11.154 8350	.000 720 4611
1389	1 929 321	2 679 826 869	37.269 2903	11.157 5133	.000 719 9424
1390	1 932 100	2 685 619 000	37.282 7037	11.160 1903	.000 719 4245
1391	1 934 881	2 691 419 471	37.296 1124	11.162 8659	.000 718 9073
1392	1 937 664	2 697 228 288	37.309 5162	11.165 5403	.000 718 3908
1393	1 940 449	2 703 045 457	37.322 9152	11.168 2134	.000 717 8751
1394	1 943 236	2 708 870 984	37.336 3094	11.170 8852	.000 717 3601
1395	1 946 025	2 714 704 875	37.349 6988	11.173 5558	.000 716 8459
1396	1 948 816	2 720 547 136	37.363 0834	11.176 2250	.000 716 3324
1397	1 951 609	2 726 397 773	37.376 4632	11.178 8930	.000 715 8196
1398	1 954 404	2 732 256 792	37.389 8382	11.181 5598	.000 715 3076
1399	1 957 201	2 738 124 199	37.403 2084	11.184 2252	.000 714 7963
1400	1 960 000	2 744 000 000	37.416 5738	11.186 8894	.000 714 2857
1401	1 962 801	2 749 884 201	37.429 9345	11.189 5523	.000 713 7759
1402	1 965 604	2 755 776 808	37.443 2904	11.192 2139	.000 713 2668
1403	1 968 409	2 761 677 827	37.456 6416	11.194 8743	.000 712 7584
1404	1 971 216	2 767 587 264	37.469 9880	11.197 5334	.000 712 2507

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1405	1 974 025	2 773 505 123	37.483 3296	11.200 1913	.000 711 7438
1406	1 976 836	2 779 431 416	37.496 6665	11.202 8479	.000 711 2376
1407	1 979 649	2 785 366 143	37.509 9987	11.205 5032	.000 710 7321
1408	1 982 464	2 791 309 312	37.523 3261	11.208 1573	.000 710 2273
1409	1 985 281	2 797 260 929	37.536 6487	11.210 8101	.000 709 7232
1410	1 988 100	2 803 221 000	37.549 9667	11.213 4617	.000 709 2199
1411	1 990 921	2 809 189 531	37.563 2799	11.216 1120	.000 708 7172
1412	1 993 744	2 815 166 528	37.576 5885	11.218 7611	.000 708 2153
1413	1 996 569	2 821 151 997	37.589 8922	11.221 4089	.000 707 7141
1414	1 999 396	2 827 145 944	37.603 1913	11.224 0054	.000 707 2136
1415	2 002 225	2 833 148 375	37.616 4857	11.226 7007	.000 706 7138
1416	2 005 056	2 839 159 296	37.629 7754	11.229 3448	.000 706 2147
1417	2 007 889	2 845 178 713	37.643 0604	11.231 9876	.000 705 7163
1418	2 010 724	2 851 206 632	37.656 3407	11.234 6292	.000 705 2186
1419	2 013 561	2 857 243 059	37.669 6164	11.237 2696	.000 704 7216
1420	2 016 400	2 863 288 000	37.682 8874	11.239 9087	.000 704 2254
1421	2 019 241	2 869 341 461	37.696 1536	11.242 5465	.000 703 7298
1422	2 022 084	2 875 403 448	37.709 4153	11.245 1831	.000 703 2349
1423	2 024 929	2 881 473 967	37.722 6722	11.247 8185	.000 702 7407
1424	2 027 776	2 887 553 024	37.735 9245	11.250 4527	.000 702 2472
1425	2 030 625	2 893 640 625	37.749 1722	11.253 0856	.000 701 7544
1426	2 033 476	2 899 736 776	37.762 4152	11.255 7173	.000 701 2623
1427	2 036 329	2 905 841 483	37.775 6535	11.258 3478	.000 700 7708
1428	2 039 184	2 911 954 752	37.788 8873	11.260 9770	.000 700 2801
1429	2 042 041	2 918 076 589	37.802 1163	11.263 6050	.000 699 7901
1430	2 044 900	2 924 207 000	37.815 3408	11.266 2318	.000 699 3007
1431	2 047 761	2 930 345 991	37.828 5606	11.268 8573	.000 698 8120
1432	2 050 624	2 936 493 568	37.841 7759	11.271 4816	.000 698 3240
1433	2 053 489	2 942 649 737	37.854 9864	11.274 1047	.000 697 8367
1434	2 056 356	2 948 814 504	37.868 1924	11.276 7266	.000 697 3501
1435	2 059 225	2 954 987 875	37.881 3938	11.279 3472	.000 696 8641
1436	2 062 096	2 961 169 856	37.894 5906	11.281 9666	.000 696 3788
1437	2 064 969	2 967 360 453	37.907 7828	11.284 5849	.000 695 8942
1438	2 067 844	2 973 559 672	37.920 9704	11.287 2019	.000 695 4103
1439	2 070 721	2 979 767 519	37.934 1535	11.289 8177	.000 694 9270
1440	2 073 600	2 985 984 000	37.947 3319	11.292 4323	.000 694 4444
1441	2 076 481	2 992 209 121	37.960 5058	11.295 0457	.000 693 9625
1442	2 079 364	2 998 442 888	37.973 6751	11.297 6579	.000 693 4813
1443	2 082 249	3 004 685 307	37.986 8398	11.300 2688	.000 693 0007
1444	2 085 136	3 010 936 384	38.000 0000	11.302 8786	.000 692 5208
1445	2 088 025	3 017 196 125	38.013 1556	11.305 4871	.000 692 0415
1446	2 090 916	3 023 464 536	38.026 3067	11.308 0945	.000 691 5629
1447	2 093 809	3 029 741 623	38.039 4532	11.310 7006	.000 691 0850
1448	2 096 704	3 036 027 392	38.052 5952	11.313 3056	.000 690 6078
1449	2 099 601	3 042 321 849	38.065 7326	11.315 9094	.000 690 1312
1450	2 102 500	3 048 625 000	38.078 8655	11.318 5119	.000 689 6552
1451	2 105 401	3 054 936 851	38.091 9939	11.321 1132	.000 689 1799
1452	2 108 304	3 061 257 408	38.105 1178	11.323 7134	.000 688 7052
1453	2 111 209	3 067 586 777	38.118 2371	11.326 3124	.000 688 2312
1454	2 114 116	3 073 924 664	38.131 3519	11.328 9102	.000 687 7579
1455	2 117 025	3 080 271 375	38.144 4622	11.331 5067	.000 687 2852
1456	2 119 936	3 086 626 816	38.157 5681	11.334 1022	.000 686 8132

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1457	2 122 849	3 092 990 993	38.170 6693	11.336 6964	.000 686 3412
1458	2 125 764	3 099 363 912	38.183 7662	11.339 2894	.000 685 8711
1459	2 128 681	3 105 745 579	38.196 8585	11.341 8813	.000 685 4010
1460	2 131 600	3 112 136 000	38.209 9463	11.344 4719	.000 684 9315
1461	2 134 521	3 118 535 181	38.223 0297	11.347 0614	.000 684 4627
1462	2 137 444	3 124 943 128	38.236 1085	11.349 6497	.000 683 9945
1463	2 140 369	3 131 359 847	38.249 1829	11.352 2368	.000 683 5270
1464	2 143 296	3 137 785 344	38.262 2529	11.354 8227	.000 683 0601
1465	2 146 225	3 144 219 625	38.275 3184	11.357 4075	.000 682 5939
1466	2 149 156	3 150 662 696	38.288 3794	11.359 9911	.000 682 1282
1467	2 152 089	3 157 114 563	38.301 4360	11.362 5735	.000 681 6633
1468	2 155 024	3 163 575 232	38.314 4881	11.365 1547	.000 681 1989
1469	2 157 961	3 170 044 709	38.327 5358	11.367 7347	.000 680 7352
1470	2 160 900	3 176 523 000	38.340 5790	11.370 3136	.000 680 2721
1471	2 163 841	3 183 010 111	38.353 6178	11.372 8914	.000 679 8097
1472	2 166 784	3 189 506 048	38.366 6522	11.375 4679	.000 679 3478
1473	2 169 729	3 196 010 817	38.379 6821	11.378 0433	.000 678 8866
1474	2 172 676	3 202 524 424	38.392 7076	11.380 6175	.000 678 4261
1475	2 175 625	3 209 046 875	38.405 7287	11.383 1906	.000 677 9661
1476	2 178 576	3 215 578 176	38.418 7454	11.385 7625	.000 677 5068
1477	2 181 529	3 222 118 333	38.431 7577	11.388 3332	.000 677 0481
1478	2 184 484	3 228 667 352	38.444 7656	11.390 9028	.000 676 5900
1479	2 187 441	3 235 225 239	38.457 7691	11.393 4712	.000 676 1325
1480	2 190 400	3 241 792 000	38.470 7681	11.396 0384	.000 675 6757
1481	2 193 361	3 248 367 641	38.483 7627	11.398 6045	.000 675 2194
1482	2 196 324	3 254 952 168	38.496 7530	11.401 1695	.000 674 7638
1483	2 199 289	3 261 545 587	38.509 7390	11.403 7332	.000 674 3088
1484	2 202 256	3 268 147 904	38.522 7206	11.406 2959	.000 673 8544
1485	2 205 225	3 274 759 125	38.535 6977	11.408 8574	.000 673 4007
1486	2 208 196	3 281 379 256	38.548 6705	11.411 4177	.000 672 9474
1487	2 211 169	3 288 008 303	38.561 6389	11.413 9769	.000 672 4950
1488	2 214 144	3 294 646 272	38.574 6030	11.416 5349	.000 672 0430
1489	2 217 121	3 301 293 169	38.587 5627	11.419 0918	.000 671 5917
1490	2 220 100	3 307 949 000	38.600 5181	11.420 6476	.000 671 1409
1491	2 223 081	3 314 613 771	38.613 4691	11.424 2022	.000 670 6908
1492	2 226 064	3 321 287 488	38.626 4158	11.426 7556	.000 670 2413
1493	2 229 049	3 327 970 157	38.639 3582	11.429 3079	.000 669 7924
1494	2 232 036	3 334 661 784	38.652 2962	11.431 8591	.000 669 3440
1495	2 235 025	3 341 362 375	38.665 2299	11.434 4092	.000 668 8963
1496	2 238 016	3 348 071 936	38.678 1593	11.436 9581	.000 668 4492
1497	2 241 009	3 354 790 473	38.691 0843	11.439 5059	.000 668 0027
1498	2 244 004	3 361 517 992	38.704 0050	11.442 0525	.000 667 5567
1499	2 247 001	3 368 254 499	38.716 9214	11.444 5980	.000 667 1114
1500	2 250 000	3 375 000 000	38.729 8335	11.447 1424	.000 666 6667
1501	2 253 001	3 381 754 501	38.742 7412	11.449 6857	.000 666 2225
1502	2 256 004	3 388 518 008	38.755 6447	11.452 2278	.000 665 7790
1503	2 259 009	3 395 290 527	38.768 5439	11.454 7688	.000 655 3360
1504	2 262 016	3 402 072 064	38.781 4389	11.457 3087	.000 664 8936
1505	2 265 025	3 408 862 625	38.794 3294	11.459 8476	.000 664 4518
1506	2 268 036	3 415 662 216	38.807 2158	11.462 3850	.000 664 0106
1507	2 271 049	3 422 470 843	38.820 0978	11.464 9215	.000 663 5700
1508	2 274 064	3 429 288 512	38.832 9757	11.467 4568	.000 663 1300

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1509	2 277 081	3 436 115 229	38.845 8491	11.469 9911	.000 662 6905
1510	2 280 100	3 442 951 000	38.858 7184	11.472 5242	.000 662 2517
1511	2 283 121	3 449 795 831	38.871 5834	11.475 0562	.000 661 8134
1512	2 286 144	3 456 649 728	38.884 4442	11.477 5871	.000 661 3757
1513	2 289 169	3 463 512 697	38.897 3006	11.480 1169	.000 660 9385
1514	2 292 196	3 470 384 744	38.910 1529	11.482 6455	.000 660 5020
1515	2 295 225	3 477 265 875	38.923 0009	11.485 1731	.000 660 0660
1516	2 298 256	3 484 156 096	38.935 8447	11.487 6995	.000 659 6306
1517	2 301 289	3 491 055 413	38.948 6841	11.490 2249	.000 659 1958
1518	2 304 324	3 597 963 832	38.961 5194	11.492 7491	.000 658 7615
1519	2 307 361	3 504 881 359	38.974 3505	11.495 2722	.000 658 3278
1520	2 310 400	3 511 808 000	38.987 1774	11.497 7942	.000 657 8947
1521	2 313 441	3 518 743 761	39.000 0000	11.500 3151	.000 657 4622
1522	2 316 484	3 525 688 648	39.012 8184	11.502 8348	.000 657 0302
1523	2 319 529	3 532 642 667	39.025 6326	11.505 3535	.000 656 5988
1524	2 322 576	3 539 605 824	39.038 4426	11.507 8711	.000 656 1680
1525	2 325 625	3 546 578 125	39.051 2483	11.510 3876	.000 655 7377
1526	2 328 676	3 553 559 576	39.064 0499	11.512 9030	.000 655 3080
1527	2 331 729	3 560 558 183	39.076 8473	11.515 4173	.000 654 8788
1528	2 334 784	3 567 549 552	39.089 6406	11.517 9305	.000 654 4503
1529	2 337 841	3 574 558 889	39.102 4296	11.520 4425	.000 654 0222
1530	2 340 900	3 581 577 000	39.115 2144	11.522 9535	.000 653 5948
1531	2 343 961	3 588 604 291	39.127 9951	11.525 4634	.000 653 1679
1532	2 347 024	3 595 640 768	39.140 7716	11.527 9722	.000 652 7415
1533	2 350 089	3 602 686 437	39.153 5439	11.530 4799	.000 652 3157
1534	2 353 156	3 609 741 304	39.166 3120	11.532 9865	.000 651 8905
1535	2 356 225	3 616 805 375	39.179 0760	11.535 4920	.000 651 4658
1536	2 359 296	3 623 878 656	39.191 8359	11.537 9965	.000 651 0417
1537	2 362 369	3 630 961 153	39.204 5915	11.540 4998	.000 650 6181
1538	2 365 444	3 638 052 872	39.217 3431	11.543 0021	.000 650 1951
1539	2 368 521	3 645 153 819	39.230 0905	11.545 5033	.000 649 7726
1540	2 371 600	3 652 264 000	39.242 8337	11.548 0034	.000 649 3506
1541	2 374 681	3 657 983 421	39.255 5728	11.550 5025	.000 648 9293
1542	2 377 764	3 666 512 088	39.268 3078	11.553 0004	.000 648 5084
1543	2 380 849	3 673 650 007	39.281 0387	11.555 4972	.000 648 0881
1544	2 383 936	3 680 797 184	39.293 7654	11.557 9931	.000 647 6684
1545	2 387 025	3 687 953 625	39.306 4880	11.560 4878	.000 647 2492
1546	2 390 116	3 695 119 336	39.319 2065	11.562 9815	.000 646 8305
1547	2 393 209	3 702 294 323	39.331 9208	11.565 4740	.000 646 4124
1548	2 396 304	3 709 478 592	39.344 6311	11.567 9655	.000 645 9948
1549	2 399 401	3 716 672 149	39.357 3373	11.570 4559	.000 645 5778
1550	2 402 500	3 723 875 000	39.370 0394	11.572 9453	.000 645 1613
1551	2 405 601	3 731 087 151	39.382 7373	11.575 4336	.000 644 7453
1552	2 408 704	3 738 308 608	39.395 4312	11.577 9208	.000 644 3299
1553	2 411 809	3 745 539 377	39.408 1210	11.580 4069	.000 643 9150
1554	2 414 916	3 752 779 464	39.420 8067	11.582 8919	.000 643 5006
1555	2 418 025	3 760 028 875	39.433 4883	11.585 3759	.000 643 0868
1556	2 421 136	3 767 287 616	39.446 1658	11.587 8588	.000 642 6735
1557	2 424 249	3 774 555 693	39.458 8393	11.590 3407	.000 642 2608
1558	2 427 364	3 781 833 112	39.471 5087	11.592 8215	.000 641 8485
1559	2 430 481	3 789 119 879	39.484 1740	11.595 3013	.000 641 4368
1560	2 433 600	3 796 416 000	39.496 8353	11.597 7799	.000 641 0256

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1561	2 436 721	3 803 721 481	39.509 4925	11.600 2576	.000 640 6150
1562	2 439 844	3 811 036 328	39.522 1457	11.602 7342	.000 640 2049
1563	2 442 969	3 818 360 547	39.534 7948	11.605 2097	.000 639 7953
1564	2 446 096	3 825 641 444	39.547 4399	11.607 6841	.000 639 3862
1565	2 449 225	3 833 037 125	39.560 0809	11.610 1575	.000 638 9776
1566	2 452 356	3 840 389 496	39.572 7179	11.612 6299	.000 638 5696
1567	2 455 489	3 847 751 263	39.585 3508	11.615 1012	.000 638 1621
1568	2 458 624	3 855 123 432	39.597 9797	11.617 5715	.000 637 7551
1569	2 461 761	3 862 503 009	39.610 6046	11.620 0407	.000 637 3486
1570	2 464 900	3 869 883 000	39.623 2255	11.622 5088	.000 636 9427
1571	2 468 041	3 877 292 411	39.635 8424	11.624 9759	.000 636 5372
1572	2 471 184	3 884 701 248	39.648 4552	11.627 4420	.000 636 1323
1573	2 474 329	3 892 119 157	39.661 0640	11.629 9070	.000 635 7279
1574	2 477 476	3 899 547 224	39.673 6688	11.632 3710	.000 635 3240
1575	2 480 625	3 906 984 375	39.686 2696	11.634 8339	.000 634 9206
1576	2 483 776	3 914 430 976	39.698 8665	11.637 2957	.000 634 5178
1577	2 486 929	3 921 887 033	39.711 4593	11.639 7566	.000 634 1154
1578	2 490 084	3 929 352 552	39.724 0481	11.642 2164	.000 633 7136
1579	2 493 241	3 936 827 539	39.736 6329	11.644 6751	.000 633 3122
1580	2 496 400	3 944 312 000	39.749 2138	11.647 1329	.000 632 9114
1581	2 499 561	3 951 805 941	39.761 7907	11.649 5895	.000 632 5111
1582	2 502 724	3 959 309 368	39.774 3636	11.652 0452	.000 632 1113
1583	2 505 889	3 966 822 287	39.786 9325	11.654 4998	.000 631 7119
1584	2 509 056	3 974 344 704	39.799 4976	11.656 9534	.000 631 3131
1585	2 512 225	3 981 876 625	39.812 0585	11.659 4059	.000 630 9148
1586	2 515 396	3 989 418 056	39.824 6155	11.661 8574	.000 630 5170
1587	2 518 569	3 996 969 003	39.837 1686	11.664 3079	.000 630 1197
1588	2 521 744	4 004 529 472	39.849 7177	11.666 7574	.000 629 7229
1589	2 524 921	4 012 099 469	39.862 2628	11.669 2058	.000 629 3266
1590	2 528 100	4 014 679 000	39.874 8040	11.671 6532	.000 628 9308
1591	2 531 281	4 027 268 071	39.887 3413	11.674 0996	.000 628 5355
1592	2 534 464	4 034 866 688	39.899 8747	11.676 5449	.000 628 1407
1593	2 537 649	4 042 474 857	39.912 4041	11.678 9892	.000 627 7464
1594	2 540 836	4 050 092 584	39.924 9295	11.681 4325	.000 627 3526
1595	2 544 025	4 057 719 875	39.937 4511	11.683 8748	.000 626 9592
1596	2 547 216	4 065 356 736	39.949 9687	11.686 3161	.000 626 5664
1597	2 550 409	4 073 003 173	39.962 4824	11.688 7563	.000 626 1741
1598	2 553 604	4 080 659 192	39.974 9922	11.691 1955	.000 625 7822
1599	2 556 801	4 088 324 799	39.987 4980	11.693 6337	.000 625 3909
1600	2 560 000	4 096 000 000	40.000 0000	11.696 0709	.000 625 0000

The use of the table of powers and roots may be extended far beyond its apparent limits by the observance of the following rules:

Remembering that the extraction of the square root of a number is simply the separating it into two equal factors, we have: to extract the square root of any whole number and decimal, when the whole number is within the limits of the table, simply find the square root of the whole number in the table and divide the given number and decimal by this root. The quotient will be another factor, *very nearly* equal to the required root. Add the divisor and the quotient together and divide by *two*, and the result will be the true root to a very close degree of approximation.

Thus, let it be required to find the square root of 346.285.

We find from the table that the square root of 346 is 18.6010752, or, for moderate precision, 18.6011, which is, of course, too small.

We then have 346.285 ÷ 18.6011 = 18.6163, so that we have the number

346.285, composed of the two factors, 18.6011×18.6163, which are very nearly equal. Adding them together and dividing by 2, we get

$$\sqrt{346.285} = \frac{18.6011 + 18.6163}{2} = 18.6087.$$

The true root is 18.60873.

To extract the cube root of a whole number and decimal we proceed in a similar manner, remembering that the cube root is one of three equal factors, so that we divide *twice* by the cube root of the whole number and then take the mean of the two divisors and the final quotient,—*i.e.*, of the three nearly equal factors.

Thus, to find the cube root of 346.285, we find in the table $\sqrt[3]{346} = 7.0203490$, or, for moderate precision, $= 7.02035$.

We then have $346.285 \div 7.02035 = 49.32588$ and $49.32588 \div 7.02035 = 7.02612$, and we have

$$\sqrt[3]{346.285} = \frac{7.02035 + 7.02035 + 7.02612}{3} = 7.02227.$$

The true root is 7.02226.

If the square root or the cube root of a number larger than 1600 is required, look for the nearest number in the column of squares or cubes, as the case may be, and the approximate root will be the corresponding number in the first column. By using this as the divisor the given number may be resolved into two or three nearly equal factors, and their mean will be the required root, very nearly.

Thus, if it is required to find the square root of 569,245, we look in the column of squares and find the nearest number to be 570,025, and the corresponding number in the first column is 755. Taking this as a divisor, we have

$$\frac{569{,}245}{755} = 753.935, \quad \text{and} \quad \frac{755 + 753.935}{2} = 754.476.$$

The true root is 754.483.

INTEREST.

Simple Interest.

Interest is money paid for use of money which is lent for a certain time.

Notation.

$c =$ the amount lent;
$r =$ interest on the amount, c;
$p =$ per cent. in the certain time.

Analogy, $c : r = 100 : p$.

If p is the per cent. on 100 in one year, then $t =$ time in years for the standing capital c and the interest r.

Analogy, $c : r = 100 : pt$.

From this analogy we obtain the equations:

1. Interest,	$r = \frac{cpt}{100}$.	3. Capital,	$c = \frac{100r}{pt}$.
2. Per cent.,	$p = \frac{100r}{ct}$.	4. Time in years,	$t = \frac{100r}{cp}$.

Now for any question in Simple Interest there is one equation which gives the answer. If the time is given in *months*, *weeks*, or *days*, multiply the 100 correspondingly by 12, 52, 365.

Example 1. What is the interest on \$3789.35, for 3 years and 5 months, at 6 per cent. per annum?

$t = 3 \times 12 + 5 = 41$ months; from the Equation 1 we have,

$$\text{Interest,} \qquad r = \frac{3789.35 \times 6 \times 41}{12 \times 100} = 776.81 \text{ dollars.}$$

Example 2. A capital $c = 469.78$ dollars, returned interest $r = 150.72$ dollars in time $t = 4$ years and 7 months. Required the per cent. per annum?

$t = 4 \times 12 + 7 = 55$ months; from the Equation 2 we have,

Per cent., $p = \frac{12 \times 100 \times 150.72}{469.78 \times 55} = 7$ per cent.

Example 3. What amount is required to return interest $r = 345$ dollars in 6 years, at 5 per cent. per annum?

From the Equation 3 we have,

Capital, $c = \frac{100 \times 345}{5 \times 6} = 1150$ dollars.

Example 4. An amount $c = 2365$ dollars is to stand until the interest $r = 550$ dollars, at $p = 6$ per cent. per annum. How long must the amount stand?

From the Equation 4 we have,

Time, $t = \frac{100 \times 550}{2365 \times 6} = 3.876$ years.

$12 \times 0.876 = 10.512$ months, $4 \times 0.512 = 2.048$ weeks; the time $t = 3$ years, 10 months, and 2 weeks.

Compound Interest.

Compound Interest is when the interest is added to the capital for each year, and the sum is the capital for the following year.

1. Amount, $a = c(1+p)^n$.	3. Per cent., $p = \sqrt[n]{\frac{a}{c}} - 1$.
2. Capital, $c = \frac{a}{(1+p)^n}$.	4. Number of years, $n = \frac{\log. a - \log. c}{\log. (1+p)}$.

☞ In these formulas p must be expressed in hundredths.

Example 1. A capital $c = 8650$ standing with compound interest at $p = 5$ per cent. What will it amount to in $n = 9$ years?

Amount $a = 8650\ (1.05)^9 = 13{,}419$ dollars.

Example 2. A man commenced business with $c = 300$ dollars: after $n = 5$ years he had $a = 6875$ dollars. At what rate did his money increase, and how soon will he have a fortune of 50,000 dollars?

The first question, or the percentage, will be answered by the Formula 3.

$$p = \sqrt[5]{\frac{6875}{300}} - 1 = \sqrt[5]{22.9166} - 1 = 0.87, \text{ or } 87 \text{ per cent.}$$

The time from the commencement of business until the fortune is completed will be answered from the Formula 4.

$$n = \frac{\log.\ 50{,}000 - \log.\ 300}{\log.\ 187} = \frac{4.69897 - 2.47712}{0.2720048} = 8.169 \text{ years,}$$

or 8 years and 2 months.

Compound Interest Table, CALCULATED FROM FORMULA 1.

n	COMPOUND INTEREST.			n	COMPOUND INTEREST.		
Years.	5 per cent.	6 per cent.	7 per cent.	Years.	5 per cent.	6 per cent.	7 per cent.
1	1.0500	1.0600	1.0700	17	2.2920	2.6928	3.1588
2	1.1025	1.1236	1.1449	18	2.4066	2.8543	3.3799
3	1.1576	1.1910	1.2250	19	2.5269	3.0256	3.6165
4	1.2155	1.2625	1.3108	20	2.6533	3.2071	3.8697
5	1.2770	1.3382	1.4025	21	2.7859	3.3995	4.1406
6	1.3400	1.4185	1.5007	22	2.9252	3.6035	4.4304
7	1.4071	1.5036	1.6058	23	3.0715	3.8197	4.7405
8	1.4774	1.5938	1.7182	24	3.2251	4.0487	5.0724
9	1.5513	1.6895	1.8385	25	3.3864	4.2919	5.4274
10	1.6289	1.7908	1.9671	30	4.3219	5.7435	7.6123
11	1.7103	1.8983	2.1048	35	5.5166	7.6861	10.6766
12	1.7958	2.0122	2.2522	40	7.0400	10.2858	14.9745
13	1.8856	2.1329	2.4098	45	8.9850	13.7646	21.0025
14	1.9799	2.2609	2.5785	50	11.6792	18.4190	29.4570
15	2.0789	2.3965	2.7599	60	18.6792	32.9878	57.9466
16	2.1829	2.5403	2.9522				

This table shows the value of one unit of money at the rates of 5, 6, and 7 per cent. per annum, compound interest, up to 60 years.

Example 1. What is the amount of 864 pounds sterling for 12 years, at 6 per cent. compound interest?

Table, $2.01219 \times 864 = 1738.53216$, or £1738 10s. 7.7d.

Example 2. What is the amount of 3450 dollars for 18 years, at 5 per cent. compound interest?

Table, $2.40661 \times 3450 = 8302.80$ dollars.

When the interest is compounded in more or less than one year, at the rate of interest per year, and $m =$ the number of months in which the interest is compounded;

then, instead of p in the formulas, put $\frac{mp}{12}$, and instead of n, put $\frac{12n}{m}$.

Example 3. A capital of 500 dollars bears compound interest semi-annually at 5 per cent. per annum; what will it amount to in 10 years?

$$m = 6 \text{ months}, \ p = \frac{mp}{12} = \frac{6 \times 0.05}{12} = 0.025, \text{ and } n = \frac{12 \times 10}{6} = 20;$$

then, $a = c(1 + p)^n = 500(1 + 0.025)^{20} = 8193.11$ dollars, the answer.

	log. (1 + 0.025) =	0.0107239
		20
		0.2144780
	log. 500 =	2.6989700
Amount,	8193.11 =	2.9134480

WEIGHTS AND MEASURES.

There are now but two really important systems of weights and measures in use in civilized countries,—the English and the metric. Many of the older English tables are falling into disuse, volumes of all kinds being expressed in cubic feet, solutions in percentages instead of grains per gallon, and similar simplifications.

The metric system is used everywhere in Europe, except in Great Britain, and it is also extensively used in America, except in the United States and Canada.

The following tables will be found to cover practically all necessary requirements:

Measures of Length—United States and Great Britain.

12 inches = 1 foot.
3 feet = 1 yard = 36 inches.
5½ yards = 1 rod = 16½ feet = 198 inches.
40 rods = 1 furlong = 220 yards = 660 feet.
8 furlongs = 1 mile = 320 rods = 1760 yards = 5280 feet.

Of the above, the inch and the foot are most frequently used by mechanics. The ordinary two-foot rule has the inches subdivided by the system of repeated halving, thus giving ½, ¼, ⅛, and $\frac{1}{16}$ of an inch; and this is sometimes carried as far as to include 32ds and 64ths. This system, however, is now used principally by carpenters, builders, etc., while machinists are generally using scales, calipers, and measuring tools which have the inch subdivided into 10ths, 100ths, and 1000ths.

The yard is much used by shopkeepers for measuring cloth, carpet, and fabrics generally, and is by them also subdivided into halves, quarters, and eighths.

For long distances the mile is universally used, and portions of a mile are given either in furlongs and feet or in halves and quarters.

For engineering measurements steel tapes are much used,—100 feet long, with the feet subdivided into 10ths instead of inches, thus giving 10ths, 100ths, and 1000ths of the length of the tape.

The mile given in the above table is called the *statute* mile, and is always used on land. The *nautical* mile, used only at sea, is equal to 6080 feet, being about 15 per cent. longer than the statute mile.

A knot is not a distance, but a rate of speed, corresponding to 1 nauti-

cal mile per hour. The expression "knots per hour" is incorrect, as the time element is included in the word knot.

The only other system of measures of length which is extensively used is the Metric System.

Metrical Measures of Length—Used generally on the Continent of Europe.

The unit is the Metre = 39.37 inches.

The metre is subdivided decimally and multiplied decimally, as below:

1 millimetre = $\frac{1}{1000}$ metre = 0.03937 inches.
1 centimetre = $\frac{1}{100}$ metre = 0.3937 inches.
1 decimetre = $\frac{1}{10}$ metre = 3.937 inches.
1 metre = 39.37 inches = 3.2808 feet.
1 dekametre = 10 metres = 32.8087 feet.
1 hectometre = 100 metres = 328.0869 feet.
1 kilometre = 1000 metres = 3280.869 feet = 0.621 mile

In using the metric system it is important to *think* of the metre as a main unit and the subdivisions as decimals of it. In mechanical and scientific work the metre and the millimetre are usually employed, and sometimes the centimetre, the decimetre more rarely. In the machine shop, for instance, measurements are usually given directly in millimetres, as 325 mm., not 3 dcm., 2 cm., 5 mm.

For longer distances the kilometre is used exclusively, and should be kept in mind as the unit of out-door measurement, with the metre, its $\frac{1}{1000}$ part, for all subdivisions, the dekametre and hectometre being hardly used at all. It is very desirable that the student should learn the values of these measurements directly from the use of a metric scale, and *not* by transformation into English measures. When such transformations must be roughly made, however, it will be convenient to remember the following:

1 millimetre = $\frac{1}{25}$ inch, approximately.
1 decimetre = 4 inches, approximately.
1 metre = 3 feet and $3\frac{3}{8}$ inches, very closely.
1 kilometre = $\frac{5}{8}$ of a mile, nearly.

An approximate rule to convert metres to feet is to multiply by 3 and add 10 per cent. Thus, 100 metres would be 300 + 30 = 330 feet, while it really is equal to 328 feet, the error being less than 1 per cent.

Measures of Weight—United States and British.

The commercial system is the Avoirdupois; the unit being the pound of 7000 grains.

The system for weighing gold and silver is called Troy Weight, of which the pound contains 5760 grains.

For medicines and drugs the Apothecaries' System is used, the grain and pound being the same as in Troy Weight, but the subdivisions of the pound being different.

Avoirdupois or Commercial Weight.

1 dram = 27.34375 grains.
16 drams = 1 ounce = $437\frac{1}{2}$ grains.
16 ounces = 1 pound = 7000 grains.
14 pounds = 1 stone.
28 pounds = 1 quarter.
4 quarters = 1 hundredweight = 112 pounds.
20 hundredweight = 1 ton = 2240 pounds.

It will be noticed that the "hundredweight" (so called) is 12 pounds more than 100 pounds, this having been the allowance for loss in handling merchandise in old times. The ton of 2240 pounds is sometimes called the long ton in commerce, as distinguished from the short ton of 2000 pounds. When no explanation is made, the long ton of 2240 pounds is the legal value of the ton, but in engineering calculations, such as the load upon a bridge, the pressure of a mass of earthwork, or the lifting capacity of a crane, it is customary to use the word ton to mean 2000 pounds. In prac-

tice a hundredweight (used as one word) means always 112 pounds, while a hundred pounds means 100 pounds exactly.

Troy Weight.

1 pennyweight = 24 grains.
20 pennyweights = 1 ounce = 480 grains.
12 ounces = 1 pound Troy = 5760 grains.

Apothecaries' Weight.

1 scruple = 20 grains.
3 scruples = 1 dram = 60 grains.
8 drams = 1 ounce = 480 grains.
12 ounces = 1 pound = 5760 grains.

Measures of Weight—Metric System.

The metric unit of weight is the Gramme, which is the weight of a cubic centimetre of pure water at a temperature of 4° C., and which is equal to 15.432 grains. The gramme is subdivided and multiplied decimally, as follows:

1 milligram = $\frac{1}{1000}$ gramme = 0.015432 grains.
1 centigram = $\frac{1}{100}$ gramme = 0.15432 grains.
1 decigram = $\frac{1}{10}$ gramme = 1.5432 grains.
1 gramme = 1 gramme = 15.432 grains.
1 dekagram = 10 grammes = 154.32 grains.
1 hectogram = 100 grammes = 1543.2 grains.
1 kilogram = 1000 grammes = 2.2046 pounds.
1 myriagram = 10,000 grammes = 22.046 pounds.
1 metric ton = 1000 kilograms = 2204.6 pounds.

In practice many of these subdivisions and multiples are rarely used. The gramme and the milligram are used by chemists and physicists all over the world. The kilogram is used almost everywhere on the continent of Europe except in Russia, and its subdivisions are generally referred to as $\frac{1}{10}$ kilo, ½ kilo, etc., instead of the tabular names, while the multiples are similarly named at 10 kilos, 100 kilos, etc. It will be noticed that the metric ton, or *tonne*, as it is written in France, is very nearly the same as the English long ton, so nearly that for ordinary commercial purposes they may be considered the same.

Measures of Volume.

Measures of Volume *are not the same* in the United States and in Great Britain, and hence it should always be stated as to which is meant.

In the United States the systems for Liquid and for Dry Measures of volume are also different from each other, while in England both liquid and dry substances are measured by the same system.

Liquid Measure—U. S. A. only.

The unit of volume is the Gallon = 231 cubic inches. The gallon is subdivided and multiplied as follows:

4 gills = 1 pint = 28.875 cubic inches.
2 pints = 1 quart = 57.750 cubic inches.
4 quarts = 1 gallon = 231 cubic inches.
63 gallons = 1 hogshead.
2 hogsheads = 1 pipe or butt.
2 pipes = 1 tun.

Of the above measures the pint and quart are most frequently used. The barrel is not a standard volume, although in the United States and in England a wine barrel is supposed to contain 31½ gallons, but in referring to a barrel in liquid measure the number of gallons it contains should be stated.

A cylinder 7 inches in diameter and 6 inches high contains almost precisely a gallon, and a gallon of pure water at its greatest density weighs 8.33888 pounds. Ordinarily it may be taken at 8.34 pounds. A cubic foot contains 7.48052 United States gallons.

Dry Measure—U. S. A. only.

The unit of dry measure is the Bushel = 2150.42 cubic inches. The bushel is subdivided as follows:

2 pints = 1 quart = 67.2 cubic inches.
4 quarts = 1 gallon = 268.8 cubic inches.
2 gallons = 1 peck = 537.6 cubic inches.
4 pecks = 1 struck bushel = 2150.42 cubic inches.

The barrel is not a legalized unit in dry measure, and its value should always be stated in gallons or in pounds weight of the substance it contains. A barrel of flour is equal to 196 pounds.

British Measures of Volume.

In the British or Imperial system the same measures are used both for liquid and for dry measure. The unit of the system is the Imperial Gallon = 277.274 cubic inches. This is intended to be equal to 10 pounds avoirdupois weight of pure water at a temperature of 62° Fahrenheit.

The imperial gallon is subdivided and multiplied as follows:

4 gills = 1 pint = 1.25 pounds water.
2 pints = 1 quart = 2.50 pounds water.
2 quarts = 1 pottle = 5.00 pounds water.
2 pottles = 1 gallon = 10.00 pounds water.
2 gallons = 1 peck = 20.00 pounds water.
4 pecks = 1 bushel = 80.00 pounds water.
4 bushels = 1 coomb = 320.00 pounds water.
2 coombs = 1 quarter = 640.00 pounds water.

The measures above the gallon are used for dry measures exclusively, and it is customary to state all quantities above the bushel in bushels.

Metric Measures of Volume.

The unit of volume is the Litre = 1 cubic decimetre. This is subdivided and multiplied decimally, as follows:

Liquid.

1 millilitre = $\frac{1}{1000}$ litre.
1 centilitre = $\frac{1}{100}$ litre.
1 decilitre = $\frac{1}{10}$ litre.
1 litre = 1 litre.
1 decalitre = 10 litres.
1 hectolitre = 100 litres.
1 kilolitre = 1000 litres.

The principal measure used is the litre itself, and in trade the $\frac{1}{2}$ litre is often used, this being a little more than a pint ($\frac{1}{2}$ litre = 1.056 pint), and so convenient that the fact of its not being a decimal equivalent is overlooked. For chemical and physical measurements the cubic centimetre is much used, and called by this name, c.c., and not millilitre, which latter it really is.

The unit of dry measure in the metric system is supposed to be the Stere = 1 cubic metre, but in practice the term cubic metre is very generally used, and the subdivisions and multiples so named,—*i.e.*, $\frac{1}{10}$ cubic metre, 100 cubic metres, etc.

MONETARY SYSTEMS.

The various systems used for the money of different countries are too numerous to be described here, but a few of the most important will be given.

United States and Canada.

The unit is the Dollar ($), subdivided and multiplied decimally. The dollar is divided into 100 cents, and the other units are as follows:

1 dime = 10 cents = $\frac{1}{10}$ dollar.
1 dollar = 100 cents.
10 dollars = 1 eagle.

Besides these decimal units there are coins as follows:

$\frac{1}{4}$ dollar = 25 cents.
$\frac{1}{2}$ dollar = 50 cents.
Double eagle = 20 dollars.

These coins are made for convenience, but are not known by their names in reckoning, the quarter- and half-dollar being counted as 25 and 50 cents, and the double eagle, as well as the eagle, as so many dollars.

Great Britain.

The unit is the Pound Sterling, or Sovereign (£), subdivided as follows:

The penny = $\frac{1}{240}$ pound.
1 shilling = 12 pence = about 24 cents.
1 pound = 20 shillings = 240 pence = about $4.86.

Besides these there are the following coins:

Half-penny = $\frac{1}{2}$ penny.
Crown = 5 shillings.
Half-crown = $2\frac{1}{2}$ shillings.
Florin = 2 shillings.

But the calculations are all made in pounds, shillings, and pence. The Guinea, often used in giving prices, is equal to 21 shillings, but it has not been coined for many years.

Latin Monetary Union.

On the Continent of Europe the following countries have formed themselves into the Latin Monetary Union, and use the same system,—*i.e.*, France, Belgium, Switzerland, Italy, and Greece. The unit is the Franc, called Lira in Italy and Drachma in Greece.

The franc is subdivided into 100 centimes,—Centesemi in Italy, Lepta in Greece. There are also gold pieces of 20 francs and silver coins = $\frac{1}{2}$ franc, besides minor coins of nickel, but these have no special names, all the reckoning being done in francs and 100ths. The equivalent value of the franc in United States money is about 19.3 cents.

Germany.

The unit is the Mark = about 24 cents, subdivided into 100 pfennigs. There are gold coins of 20 marks, but all the reckoning is done in marks and 100ths.

Besides the tables and terms already described, there are many other calculations made in trade and commerce which cannot be given here, but which must be learned by actual experience. There are many words, such as net, gross, rebate, tare, tret, etc., etc., for the meanings of which the reader must refer to the dictionary.

There are two ratios, however, which are of sufficient interest to be described here. The "fineness" (so called) of gold or silver is determined by the number of parts of pure gold or silver there are in 1000 parts of the alloy. The metal is, of course, pure only when it contains no alloy whatever, and is then $\frac{1000}{1000}$ fine. The standard alloy for gold for United States coinage is 900 parts of pure gold and 100 parts alloy, and hence is $\frac{900}{1000}$ fine.

Of this alloy the gold dollar contains 25.8 grains, the eagle 258 grains, and the double eagle 516 grains.

The standard "fineness" for silver is also $\frac{900}{1000}$, and the standard dollar contains 412.5 grains of this alloy.

THE METRIC SYSTEM.

The principal advantage of the metric system consists in its use of the decimal subdivisions. The attempt to consider the metre as $\frac{1}{10,000,000}$ of a quadrant of the earth's surface has been abandoned, and it is now held only to be the length of the standard known as the *Mètre des Archives*, copies of which are issued by the *Bureau Internationale des Poids et Mésures*, at Breteuil, near Paris.

The kilogramme was originally intended to be the weight of a cubic decimetre or litre of pure water at the temperature of maximum density, but it is really now considered only as the weight of a platinum standard. At the same time, this relation between the unit of weight and a standard volume of water is sufficiently close for the specific gravity of any substance to be considered as equal to the weight of a cubic decimetre of that substance. In all hydraulic measurements a cubic metre of water is equal in weight to the metric tonne of 1000 kilogrammes, a most convenient fact in the determination of the power developed by a given fall and volume of water.

The French Metrical System.

The French units of weight, measure, and coin are arranged into a perfect decimal system, except those of time and the circle. The division and multiplication of the units are expressed by Latin and Greek names, as follows:

Latin, Division.	*Greek, Multiplication.*
Milli = 1000th of the unit.	Deca = 10 times the unit.
Centi = 100th of the unit.	Hecato = 100 times the unit.
Deci = 10th of the unit.	Kilio = 1000 times the unit.
Metre, litre, stere, are, franc, gramme.	Myrio = 10000 times the unit.

French Measure of Length.

1 millimetre	= 0.03937 inch.	1 metre (unit)	= 3.28083 feet.
1 centimetre	= 0.3937 inch.	1 decametre	= 32.8083 feet.
1 decimetre	= 3.937 inches.	1 hectometre	= 328.083 feet.
1 metre (unit)	= 39.37 inches.	1 kilometre	= 3280.83 ft. = 0.62137 mile.
1 sea mile	= 1853.25 metres.	1 statute mile	= 1.60935 kilomets.
1 kilometre	= 0.53959 sea mile.	1 kilometre	= 49.7096 chains.

French Measure of Surface.

1 square metre	= 10.764 square feet.	1 are	= 1076.4 square feet.
1 are	= 100 square metres.	1 decare	= 107.64 square feet.
1 decare	= 10 ares.	1 hectare	= 2.471 Eng. acres.
1 hectare	= 100 ares.	1 square mile	= 259 hectares.

French Measure of Volume.

1 stere (cubic metre)	= 10 decasteres.	1 stere	= 35.314 Eng. cubic feet.
1 stere	= 1000 litres.	1 litre	= 61.023 Eng. cub. inches.
1 litre	= 1 cubic decimetre.	1 gallon	= 3.7854 litres.
1 decistere	= 3.5314 cubic feet.	1 decistere	= 2.838 bushels (nearly).

French Measure of Weight.

1 ton	= 1 cubic metre distilled water.	1 gramme	= 10 decigrammes.
1 ton	= 1000 kilogrammes.	1 decigramme	= 10 centigrammes.
1 kilogramme	= 1000 grammes.	1 centigramme	= 10 milligrammes.
1 hectogramme	= 100 grammes.	1 kilogramme	= 2.20462 pounds avoirdupois.
1 decagramme	= 10 grammes.	1 Eng. pound	= 0.45359 kilograms.
1 gramme	= 1 cubic centimetre distilled water.	1 gramme	= 15.43 grains troy.
1 French ton	= 0.9842 Eng. ton.	1 English ton	= 1.016 French tons.

Conversion of English Inches into Centimetres.

Inches.	0	1	2	3	4	5	6	7	8	9
	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.
0	0.000	2.540	5.080	7.620	10.16	12.70	15.24	17.78	20.32	22.86
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.26
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.66
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.06
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.46
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.86
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172.72	175.26
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.96
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.06
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.46
100	254.00	256.54	259.08	261.62	264.16	266.70	269.24	271.78	274.32	276.85

Conversion of Centimetres into English Inches.

Cm.	0	1	2	3	4	5	6	7	8	9
	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
0	0.000	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.742	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.662	9.055	9.449	9.843	10.236	10.630	11.024	11.418
30	11.811	12.205	12.599	12.992	13.386	13.780	14.173	14.567	14.961	15.355
40	15.748	16.142	16.536	16.929	17.323	17.717	18.111	18.504	18.898	19.292
50	19.685	20.079	20.473	20.867	21.260	21.654	22.048	22.441	22.835	23.229
60	23.622	24.016	24.410	24.804	25.197	25.591	25.985	26.378	26.772	27.166
70	27.560	27.953	28.347	28.741	29.134	29.528	29.922	30.316	30.709	31.103
80	31.497	31.890	32.284	32.678	33.071	33.465	33.859	34.253	34.646	35.040
90	35.434	35.827	36.221	36.615	37.009	37.402	37.796	38.190	38.583	38.977
100	39.370	39.764	40.158	40.552	40.945	41.339	41.733	42.126	42.520	42.914

Conversion of English Feet into Metres.

Feet.	0	1	2	3	4	5	6	7	8	9
	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.
0	0.000	0.3048	0.6096	0.9144	1.2192	1.5239	1.8287	2.1335	2.4383	2.7431
10	3.0479	3.3527	3.6575	3.9623	4.2671	4.5719	4.8767	5.1815	5.4863	5.7911
20	6.0359	6.4006	6.7055	7.0102	7.3150	7.6198	7.9246	8.2294	8.5342	8.8390
30	9.1438	9.4486	9.7534	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18.287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

Conversion of Metres into English Feet.

Metres.	0	1	2	3	4	5	6	7	8	9
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0	0.000	3.2809	6.5618	9.8427	13.123	16.404	19.685	22.966	26.247	29.528
10	32.809	36.090	39.371	42.651	45.932	49.213	52.494	55.775	59.056	62.337
20	65.618	68.899	72.179	75.461	78.741	82.022	85.303	88.584	91.865	95.146
30	98.427	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.96
40	131.24	134.52	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.33	170.61	173.89	177.17	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.42	206.70	209.98	213.26	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.51	242.79	246.07	249.35	252.63	255.91	259.19
80	262.47	265.75	269.03	272.31	275.60	278.88	282.16	285.44	288.72	292.00
90	295.28	298.56	301.84	305.12	308.40	311.69	314.97	318.25	321.53	324.81
100	328.09	331.37	334.65	337.93	341.21	344.49	347.78	351.06	354.34	357.62

Conversion of English Statute-miles into Kilometres.

Miles.	0	1	2	3	4	5	6	7	8	9
	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.
0	0.0000	1.6093	3.2186	4.8279	6.4372	8.0465	9.6558	11.2652	12.8745	14.4848
10	16.093	17.702	19.312	20.921	22.530	24.139	25.749	27.358	28.967	30.577
20	32.186	33.795	35.405	37.014	38.623	40.232	41.842	43.451	45.060	46.670
30	48.279	49.888	51.498	53.107	54.716	56.325	57.935	59.544	61.153	62.763
40	64.372	65.981	67.591	69.200	70.809	72.418	74.028	75.637	77.246	78.856
50	80.465	82.074	83.684	85.293	86.902	88.511	90.121	91.730	93.339	94.949
60	96.558	98.167	99.777	101.39	102.99	104.60	106.21	107.82	109.43	111.04
70	112.65	114.26	115.87	117.48	119.08	120.69	122.30	123.91	125.52	127.13
80	128.74	130.35	131.96	133.57	135.17	136.78	138.39	140.00	141.61	143.22
90	144.85	146.44	148.05	149.66	151.26	152.87	154.48	156.09	157.70	159.31
100	160.93	162.53	164.14	165.75	167.35	168.96	170.57	172.18	173.79	175.40

Conversion of Kilometres into English Statute-miles.

Kilom.	0	1	2	3	4	5	6	7	8	9
	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.
0	0.0000	0.6214	1.2427	1.8641	2.4855	3.1069	3.7282	4.3497	4.9711	5.5924
10	6.2138	6.8352	7.4565	8.0780	8.6994	9.3208	9.9421	10.562	11.185	11.805
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.776	17.399	18.019
30	18.641	19.263	19.884	20.506	21.127	21.748	22.370	22.990	23.613	24.233
40	24.855	25.477	26.098	26.720	27.341	27.962	28.584	29.204	29.827	30.447
50	31.069	31.690	32.311	32.933	33.554	34.175	34.797	35.417	36.040	36.660
60	37.282	37.904	38.525	39.147	39.768	40.389	41.011	41.631	42.254	42.874
70	43.497	44.118	44.739	45.361	45.982	46.603	47.225	47.845	48.468	49.088
80	49.711	50.332	50.953	51.575	52.196	52.817	53.439	54.059	54.682	55.302
90	55.924	56.545	57.166	57.788	58.409	59.030	59.652	60.272	60.895	61.515
100	62.138	62.759	63.380	64.002	64.623	65.244	65.866	66.486	67.109	67.729

Conversion of Sea-miles into Kilometres.

Sea-miles.	0	1	2	3	4	5	6	7	8	9
	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.
0	0.0000	1.8532	3.7046	5.5596	7.4128	9.2660	11.119	12.972	14.825	16.788
10	18.532	20.386	22.237	24.128	25.945	27.798	29.651	31.504	33.357	35.320
20	37.064	38.918	40.769	42.660	44.477	46.331	48.183	50.036	51.889	53.852
30	55.596	57.450	59.301	61.192	63.009	64.863	66.715	68.568	70.421	72.384
40	74.128	75.982	77.833	79.724	81.541	83.396	85.247	87.100	88.953	90.916
50	92.660	94.514	96.365	98.256	100.07	101.92	103.78	105.63	107.48	109.45
60	111.19	113.05	114.90	116.79	118.61	120.45	122.21	124.16	126.01	127.98
70	129.72	131.58	133.43	135.32	137.14	139.98	140.74	142.69	144.54	146.51
80	148.25	150.11	151.96	153.85	155.67	157.52	159.27	161.22	163.07	165.04
90	166.78	168.64	170.49	172.38	174.20	176.05	177.80	179.75	181.60	183.57
100	185.32	187.18	189.03	190.88	192.73	194.58	196.44	198.28	200.14	201.99

Conversion of Kilometres into Sea-miles.

Kilom.	0	1	2	3	4	5	6	7	8	9
	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.	Sea-m.
0	0.0000	0.5396	1.0792	1.6188	2.1584	2.6880	3.2375	3.7771	4.3167	4.8563
10	5.3959	5.9356	6.4751	7.0147	7.5543	8.0839	8.6334	9.1730	9.7126	10.252
20	10.792	11.331	11.870	12.410	12.950	13.480	14.029	14.568	15.108	15.647
30	16.188	16.727	17.265	17.806	18.345	18.876	19.424	19.965	20.504	21.044
40	21.584	22.123	22.661	23.202	23.740	24.271	24.819	25.360	25.900	26.439
50	26.980	27.519	28.059	28.598	29.135	29.667	30.214	30.757	31.296	31.835
60	32.375	32.915	33.456	33.994	34.530	35.063	35.609	36.151	36.692	37.231
70	37.771	38.310	38.852	39.390	39.925	40.459	41.004	41.574	42.088	42.627
80	43.167	43.705	44.284	44.786	45.320	45.855	46.399	46.943	47.483	48.023
90	48.563	49.103	49.644	50.182	50.715	51.251	51.794	52.339	52.879	53.419
100	53.959	54.498	55.038	55.575	56.117	56.658	57.198	57.737	58.275	58.816

Conversion of Square Inches into Square Centimetres.

Square in.	0	1	2	3	4	5	6	7	8	9
	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².	Cm².
0	0.0000	6.4515	12.903	19.354	25.806	32.257	38.709	45.160	51.612	58.063
10	64.515	70.967	77.418	83.869	90.321	96.772	103.22	109.67	116.12	122.57
20	129.03	135.48	141.93	148.38	154.83	161.29	167.74	174.19	180.64	187.09
30	193.54	199.99	206.44	212.89	219.34	225.80	231.25	238.70	245.15	251.60
40	258.06	264.51	270.96	277.41	283.86	290.32	296.77	303.22	309.67	316.12
50	322.57	329.02	335.47	341.92	348.37	354.83	361.28	367.73	374.18	380.63
60	387.09	393.54	399.99	406.44	412.89	419.35	425.80	432.25	438.70	445.15
70	451.60	458.05	464.50	470.95	477.40	483.86	490.31	496.76	503.21	509.66
80	516.12	522.57	529.02	535.47	541.92	548.38	554.83	561.28	567.73	574.18
90	580.63	587.08	593.53	599.98	606.43	612.89	619.34	625.79	632.24	638.69
100	645.15	651.60	658.05	664.50	670.95	677.41	683.86	690.31	696.76	703.21

Conversion of Square Centimetres into Square Inches.

Square cm.	0	1	2	3	4	5	6	7	8	9
	In².	In².	In².	In².	In².	In².	In².	In².	In².	In².
0	0.0000	0.1550	0.3100	0.4650	0.6200	0.7750	0.9300	1.0850	1.2400	1.3950
10	1.5500	1.7050	1.8600	2.0150	2.1700	2.3250	2.4800	2.6350	2.7900	2.9450
20	3.1000	3.2550	3.4100	3.5650	3.7200	3.8750	4.0300	4.1850	4.3400	4.4950
30	4.6501	4.8051	4.9601	5.1151	5.2701	5.4251	5.5801	5.7351	5.8901	6.0451
40	6.2001	6.3551	6.5101	6.6651	6.8201	6.9751	7.1301	7.2851	7.4401	7.5951
50	7.7501	7.9051	8.0601	8.2151	8.3701	8.5251	8.6801	8.8351	8.9901	9.1451
60	9.3002	9.4552	9.6102	9.7652	9.9202	10.075	10.230	10.385	10.540	10.695
70	10.850	11.040	11.160	11.315	11.470	11.625	11.780	11.935	12.090	12.245
80	12.400	12.555	12.710	12.865	13.020	13.175	13.330	13.485	13.640	13.795
90	13.950	14.105	14.260	14.415	14.570	14.725	14.880	15.035	15.190	15.345
100	15.500	15.655	15.810	15.965	16.120	16.275	16.430	16.585	16.740	16.895

Conversion of Cubic Inches into Cubic Centimetres.

Cubic in.	0	1	2	3	4	5	6	7	8	9
	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.	Cm³.
0	0.0000	16.383	32.773	49.160	65.546	81.933	98.320	114.71	131.01	147.48
10	163.87	180.26	196.64	213.03	229.41	245.80	262.19	278.58	294.88	311.35
20	327.73	344.12	360.50	376.89	393.27	409.66	426.05	442.44	458.74	475.21
30	491.60	507.99	524.37	540.76	557.14	573.53	569.92	606.31	622.61	639.08
40	655.46	671.85	688.23	704.52	721.00	737.39	753.78	770.17	786.47	802.94
50	819.33	835.72	851.10	868.49	884.87	901.26	917.65	934.04	950.34	966.81
60	983.20	999.59	1016.0	1032.4	1048.7	1065.1	1081.5	1097.9	1114.2	1130.7
70	1147.1	1163.5	1179.9	1196.3	1212.6	1229.0	1245.4	1261.8	1278.1	1294.6
80	1310.9	1327.3	1343.7	1360.1	1376.4	1392.8	1409.2	1425.6	1441.9	1458.4
90	1474.8	1491.2	1507.6	1524.0	1540.3	1556.7	1573.1	1589.5	1605.8	1622.3
100	1638.7	1655.1	1671.5	1687.9	1704.2	1720.6	1737.0	1753.4	1769.7	1786.2

Conversion of Cubic Centimetres into Cubic Inches.

Cubic cm.	0	1	2	3	4	5	6	7	8	9
	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.	In³.
0	0.0000	0.0610	0.1221	0.1831	0.2441	0.3051	0.3661	0.4272	0.4882	0.5492
10	0.6102	0.6712	0.7323	0.7933	0.8543	0.9153	0.9763	1.0374	1.0984	1.1594
20	1.2205	1.2815	1.3426	1.4036	1.4646	1.5256	1.5866	1.6477	1.7087	1.7697
30	1.8308	1.8918	1.9529	2.0139	2.0749	2.1359	2.1969	2.2580	2.3190	2.3800
40	2.4410	2.5020	2.5631	2.6241	2.6851	2.7461	2.8071	2.8682	2.9292	2.9902
50	3.0513	3.1123	3.1734	3.2344	3.2954	3.3564	3.4174	3.4785	3.5395	3.6005
60	3.6615	3.7225	3.7836	3.8446	3.9056	3.9666	4.0276	4.0887	4.1497	4.2107
70	4.2718	4.3328	4.3939	4.4549	4.5159	4.5769	4.6379	4.6990	4.7600	4.8210
80	4.8820	4.9430	5.0041	5.0651	5.1261	5.1871	5.2481	5.3092	5.3702	5.4312
90	5.4923	5.5533	5.6144	5.6754	5.7364	5.7974	5.8584	5.9195	5.9805	6.0415
100	6.1025	6.1635	6.2246	6.2856	6.3466	6.4076	6.4686	6.5297	6.5907	6.6517

Conversion of Cubic Yards into Cubic Metres.

Cubic yds.	0	1	2	3	4	5	6	7	8	9
	Met³.	Met³.	Met³.	Met³.	Met³.	Met³.	Met³.	Met³.	Met³.	Met³.
0	0.0000	0.7645	1.5291	2.2936	3.0581	3.8226	4.5872	5.3517	6.1163	6.8808
10	7.6453	8.4098	9.1744	9.9389	10.703	11.468	12.232	12.997	13.761	14.526
20	15.291	16.055	16.820	17.585	18.349	19.114	19.878	20.643	21.407	22.172
30	22.936	23.700	24.455	25.230	25.994	26.759	27.523	28.288	29.052	29.817
40	30.581	31.345	32.110	32.875	33.639	34.404	35.168	35.933	36.797	37.462
50	38.226	38.990	39.755	40.520	41.284	42.049	42.813	43.578	44.342	45.107
60	45.872	46.636	47.401	48.166	48.930	49.695	50.459	51.224	51.988	52.753
70	53.517	54.281	55.046	55.811	56.575	57.340	58.104	58.869	59.633	60.398
80	61.163	61.927	62.692	63.457	64.221	64.986	65.750	66.515	67.279	68.044
90	68.808	69.572	70.337	71.102	71.866	72.631	73.395	74.160	74.924	75.689
100	76.453	77.217	77.982	78.747	79.511	80.276	81.040	81.805	82.569	83.334

Conversion of Cubic Metres into Cubic Yards.

Cubic met.	0	1	2	3	4	5	6	7	8	9
	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.	Yds³.
0	0.0000	1.3080	2.6160	3.9240	5.2329	6.5399	7.8479	9.1559	10.464	11.772
10	13.080	14.388	15.696	17.004	18.313	19.620	20.928	22.236	23.544	24.852
20	26.160	27.468	28.776	30.084	31.393	32.700	34.008	35.316	36.624	37.932
30	39.240	40.548	41.856	43.164	44.473	45.780	47.088	48.396	49.704	51.012
40	52.319	53.627	54.935	56.243	57.552	58.859	60.167	61.475	62.783	63.091
50	65.399	66.707	68.015	69.323	70.632	71.939	73.247	74.545	75.863	77.171
60	78.479	79.787	81.095	82.403	83.712	85.019	86.327	87.535	88.943	90.251
70	91.559	92.867	94.175	95.483	96.792	98.099	99.407	100.71	102.02	103.33
80	104.63	105.94	107.25	108.56	109.87	111.17	112.48	113.79	115.10	116.41
90	117.72	119.03	120.34	121.64	122.95	124.26	125.57	126.88	128.18	129.49
100	130.80	132.11	133.42	134.72	136.03	137.34	138.65	139.96	141.26	142.57

Conversion of U. S. Gallons into Litres.

Gallons.	0	1	2	3	4	5	6	7	8	9
	Litres.	Litres.	Litres.	Litres.	Litres.	Litres.	Litres.	Litres.	Litres.	Litres.
0	0.0000	3.7853	7.5706	11.356	15.141	18.946	22.712	26.497	30.282	34.068
10	37.853	41.638	45.423	49.209	52.994	56.799	60.565	64.350	68.135	71.921
20	75.706	79.491	83.276	87.062	90.847	94.652	98.418	102.20	105.99	109.77
30	113.56	117.34	121.13	124.92	128.66	132.50	136.27	140.06	143.84	147.63
40	151.42	155.22	158.99	162.78	166.56	170.36	174.13	177.92	181.70	185.49
50	189.46	193.24	197.03	200.82	204.60	208.40	212.17	215.96	219.74	223.53
60	227.12	230.90	234.69	238.48	242.26	246.06	249.83	253.62	257.40	261.19
70	264.97	268.75	272.54	276.33	280.11	283.91	286.68	291.47	295.25	299.04
80	302.82	306.60	310.39	314.18	317.96	321.76	324.53	329.32	333.10	336.89
90	440.68	444.46	448.25	452.04	455.82	459.62	463.39	467.18	470.96	474.75
100	478.53	482.31	486.10	789.89	493.67	497.47	501.24	505.03	508.81	512.60

Conversion of Litres into U. S. Gallons.

Litres.	0	1	2	3	4	5	6	7	8	9
	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.	Gal.
0	0.0000	0.2642	0.5284	0.7925	1.0567	1.3209	1.5851	1.8492	2.1134	2.3776
10	2.6418	2.9060	3.1702	3.4343	3.6985	3.9627	4.2269	4.4910	4.7552	5.0194
20	5.2836	5.5478	5.8120	6.0761	6.3403	6.6045	6.8687	7.1328	7.3970	7.6612
30	7.9254	8.1896	8.4538	8.7179	8.9821	9.2463	9.5105	9.8746	10.030	10.303
40	10.567	10.831	11.095	11.360	11.624	11.888	12.152	12.416	12.680	12.945
50	13.209	13.473	13.737	14.002	14.266	14.530	14.794	15.058	15.322	15.587
60	15.851	16.115	16.379	16.644	16.908	17.172	17.436	17.700	17.964	18.229
70	18.492	18.756	19.020	19.284	19.549	19.813	20.077	20.341	20.605	20.870
80	21.134	21.398	21.662	21.926	22.191	22.455	22.719	22.983	23.247	23.512
90	23.776	24.040	24.304	24.568	24.832	25.097	25.361	25.625	25.889	26.154
100	26.418	26.682	26.946	27.210	27.475	27.739	28.003	28.267	28.531	28.796

Conversion of Yards into Metres.

Yards.	0	1	2	3	4	5	6	7	8	9
	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.
0	0.0000	0.9144	1.8288	2.7432	3.6576	4.5719	5.4863	6.4007	7.3151	8.2295
10	9.1439	10.058	10.973	11.887	12.801	13.716	14.630	15.544	16.458	17.373
20	18.288	19.202	20.117	21.031	21.945	22.860	23.774	24.689	25.603	26.518
30	27.432	28.346	29.260	30.174	31.088	32.003	32.917	33.832	34.746	35.661
40	36.576	37.490	38.404	39.318	40.232	41.147	42.061	42.976	43.890	44.805
50	45.719	46.634	47.548	48.462	49.376	50.291	51.205	52.120	53.034	53.949
60	54.863	55.778	56.692	57.606	58.520	59.435	60.349	61.264	62.178	63.093
70	64.007	64.922	65.836	66.750	67.664	68.578	69.493	70.408	71.322	72.237
80	73.151	74.066	74.980	75.894	76.808	77.723	78.637	79.552	80.466	81.381
90	82.295	83.210	84.124	85.038	85.952	86.867	87.781	88.696	89.610	90.525
100	91.439	92.353	93.267	94.181	95.095	96.010	96.924	97.839	98.753	99.668

Conversion of Metres into Yards.

Metres.	0	1	2	3	4	5	6	7	8	9
	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.	Yds.
0	0.0000	1.0936	2.1872	3.2809	4.3745	5.4681	6.5617	7.6553	8.7490	9.8426
10	10.936	12.029	13.122	14.217	15.310	16.404	17.498	18.591	19.685	20.778
20	21.872	22.966	24.059	25.153	26.247	27.340	28.434	29.527	30.621	31.715
30	32.809	33.900	34.993	36.090	37.184	38.277	39.371	40.464	41.558	42.652
40	43.745	44.839	45.932	47.026	48.120	49.213	50.307	51.400	52.544	53.588
50	54.681	55.775	56.868	57.962	59.056	60.149	61.243	62.336	63.430	64.524
60	65.617	66.711	67.804	68.898	69.992	71.085	72.179	73.272	74.366	75.460
70	76.553	77.647	78.740	79.834	80.928	82.021	83.115	84.208	85.302	86.396
80	87.490	88.584	89.677	90.771	91.865	92.958	94.052	95.145	96.239	97.333
90	98.426	99.520	100.61	101.71	102.80	103.89	104.99	106.08	107.17	108.27
100	109.36	110.45	111.55	112.64	113.73	114.83	115.92	117.02	118.11	119.20

Conversion of Square Yards into Square Metres.

Sq. yards.	0	1	2	3	4	5	6	7	8	9
	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.	Met^2.
0	0.0000	0.8361	1.6722	2.5803	3.3444	4.1805	5.0167	5.8528	6.6889	7.5250
10	8.3611	9.1972	10.033	10.941	11.706	12.542	13.378	14.214	15.050	15.886
20	16.722	17.558	18.394	19.102	20.066	20.903	21.739	22.575	23.411	24.247
30	25.083	25.919	26.755	27.663	28.431	29.264	30.100	30.936	31.772	32.608
40	33.444	34.280	35.116	36.024	36.788	37.625	38.461	39.297	40.133	40.969
50	41.805	42.641	43.477	44.385	45.149	45.986	46.822	47.658	48.494	49.330
60	50.167	51.003	51.839	52.747	53.511	54.348	55.184	56.020	56.856	57.692
70	58.528	59.364	60.190	61.108	61.872	62.709	63.545	64.381	65.217	66.053
80	66.889	67.725	68.561	69.469	70.233	71.070	71.906	72.742	73.578	74.414
90	75.250	76.086	76.922	77.830	78.594	79.431	80.267	81.103	81.939	82.775
100	83.611	84.447	85.283	86.191	86.955	87.792	88.628	89.464	90.300	91.136

Conversion of Square Metres into Square Yards.

Sq. metres.	0	1	2	3	4	5	6	7	8	9
	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.	Yds^2.
0	0.0000	1.1960	2.3920	3.5880	4.7840	5.9800	7.1760	8.3720	9.5681	10.764
10	11.960	13.156	14.352	15.548	16.744	17.940	19.136	20.332	21.528	22.724
20	23.920	25.116	26.312	27.508	28.704	29.900	31.096	32.292	33.488	34.684
30	35.880	37.076	38.272	39.468	40.664	41.860	43.056	44.252	45.448	46.644
40	47.840	49.036	50.232	51.428	52.624	53.820	55.016	56.212	57.408	58.604
50	59.800	60.996	62.192	63.388	63.584	65.780	66.976	68.172	69.368	70.564
60	71.760	72.956	74.152	75.348	76.544	77.740	78.936	80.132	81.328	82.524
70	83.721	84.917	86.113	87.309	88.505	89.701	90.897	92.093	93.289	94.485
80	95.681	96.877	98.073	99.269	100.46	101.66	102.86	104.06	105.25	106.44
90	107.64	108.84	110.03	111.24	112.44	113.62	114.81	116.01	117.21	118.40
100	119.60	120.80	121.99	123.19	124.38	125.58	126.77	127.97	129.17	130.36

Conversion of Hectares into Acres.

Hectares.	0	1	2	3	4	5	6	7	8	9
	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.	Acres.
0	0.0000	2.4711	4.9422	7.4133	9.8844	12.355	14.836	17.298	19.769	22.240
10	24.711	27.182	29.653	32.124	34.695	37.046	39.547	42.009	44.480	46.951
20	49.422	51.893	54.364	56.835	59.306	61.757	64.258	66.720	68.191	71.662
30	74.133	76.604	79.075	81.546	84.017	86.468	88.969	91.431	93.902	96.373
40	98.844	101.31	103.79	106.26	108.73	111.18	113.68	116.14	118.61	121.08
50	123.55	126.02	128.49	130.96	133.43	135.88	138.38	140.85	143.32	145.79
60	148.36	150.83	153.30	155.77	158.24	160.69	163.19	165.66	168.13	170.60
70	172.95	175.45	177.92	180.39	182.86	185.31	187.81	190.28	192.75	195.22
80	197.69	200.16	202.63	205.10	207.57	210.02	212.52	214.99	217.46	219.93
90	222.40	224.87	227.34	229.81	232.28	234.73	237.23	239.70	242.17	244.64
100	247.11	249.58	252.05	254.52	256.99	259.44	261.94	264.41	266.88	269.35

Conversion of Acres into Hectares.

Acres.	0	1	2	3	4	5	6	7	8	9
	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.	Hect.
0	0.0000	0.4047	0.8093	1.2140	1.6187	2.0234	2.4280	2.8327	3.2374	3.6420
10	4.0468	4.4515	4.8561	5.2608	5.6655	6.0702	6.4748	6.8795	7.2782	7.6888
20	8.0936	8.4983	8.9029	9.3076	9.7123	10.117	10.521	10.926	11.331	11.735
30	12.140	12.545	12.949	13.354	13.759	14.163	14.568	14.973	15.377	15.782
40	16.187	16.592	16.996	17.401	17.806	18.210	18.615	19.020	19.414	19.829
50	20.234	20.639	21.043	21.448	21.853	22.257	22.662	23.067	23.471	23.876
60	24.280	24.685	25.089	25.494	25.899	26.303	26.708	27.113	27.517	27.922
70	28.327	28.732	29.136	29.541	29.946	30.350	30.755	31.160	31.564	31.969
80	32.374	32.779	33.183	33.588	33.993	34.397	34.802	35.207	35.611	36.016
90	36.420	36.825	37.229	37.634	38.039	38.443	38.848	39.253	39.657	40.062
100	40.468	40.873	41.277	41.682	42.087	42.491	42.896	43.301	43.695	44.110

Conversion of Square Miles into Square Kilometres.

Sq. miles.	0	1	2	3	4	5	6	7	8	9
	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².	Kil².
0	0.0000	2.5899	5.1798	7.7697	10.359	12.929	15.539	18.129	20.718	23.309
10	25.899	28.490	31.079	33.669	36.259	38.829	41.439	44.029	46.619	49.209
20	51.798	54.388	56.978	59.568	62.158	64.728	67.338	69.928	72.518	75.108
30	77.697	80.287	82.877	85.467	88.057	90.627	93.238	96.828	98.417	101.01
40	103.59	106.18	108.77	111.36	113.95	116.52	119.13	121.72	124.31	126.90
50	129.29	131.88	134.47	137.06	139.65	142.22	144.83	147.42	150.01	152.50
60	155.39	157.98	160.57	163.16	165.75	168.32	170.93	173.52	176.11	178.70
70	181.29	183.88	186.47	188.06	191.65	194.22	196.83	199.42	202.01	204.60
80	207.19	209.77	212.36	214.95	217.55	220.11	222.73	225.31	227.91	230.50
90	233.09	235.68	238.27	240.86	243.45	246.02	248.63	251.22	253.81	256.40
100	258.99	261.58	264.17	266.76	269.35	271.92	274.53	277.12	279.71	282.20

Conversion of Square Kilometres into Square Miles.

Sq. kilom.	0	1	2	3	4	5	6	7	8	9
	Sq. m.	Sq. m.	Sq. m.	Sq. m.	Sq. m.	Sq. m.	Sq. m.	Sq. m	Sq. m.	Sq. m.
0	0.0000	0.3861	0.7722	1.1583	1.5445	1.9304	2.3166	2.7028	3.0890	3.4749
10	3.8612	4.2471	4.6334	5.0195	5.4057	5.7916	6.1778	6.5640	6.9502	7.3362
20	7.7224	8.1081	8.4946	8.8807	9.2669	9.6528	10.039	10.425	10.811	11.197
30	11.583	11.969	12.355	12.741	13.127	13.513	13.899	14.286	14.672	15.058
40	15.445	15.830	16.217	16.603	16.989	17.375	17.761	18.146	18.534	18.920
50	19.304	19.691	20.076	20.462	20.848	21.234	21.620	22.007	22.393	22.779
60	23.166	23.552	23.938	24.324	24.710	25.096	25.482	25.869	26.245	26.641
70	27.028	27.413	27.800	28.186	28.572	28.958	29.344	29.731	30.117	30.503
80	30.890	31.274	31.662	32.048	32.434	32.820	33.206	33.593	33.979	34.365
90	34.749	35.135	35.521	35.907	36.293	36.679	37.065	37.452	37.838	38.224
100	38.612	38.996	39.384	39.770	40.156	40.542	40.928	41.315	41.701	42.087

Conversion of Cubic Feet into Cubic Decimetres.

Cubic feet.	0	1	2	3	4	5	6	7	8	9
	Dm^3.	Dm^3.	Dm^3.	Dm^2.	Dm^3.	Dm^3.	Dm^3.	Dm^3.	Dm^3.	Dm^3.
0	0.0000	28.316	56.632	84.948	113.26	141.58	169.90	198.21	226.53	254.84
10	283.16	311.47	339.79	268.11	396.42	424.74	453.06	481.37	509.69	538.00
20	566.32	594.64	622.95	651.27	679.58	707.90	736.22	764.53	792.85	821.16
30	849.48	877.80	906.11	934.43	962.74	991.06	1019.4	1047.7	1076.0	1104.3
40	1132.6	1160.8	1189.2	1217.5	1245.9	1274.2	1302.5	1330.8	1359.1	1387.4
50	1415.8	1444.0	1472.4	1500.7	1529.1	1557.4	1585.7	1614.0	1642.3	1670.6
60	1698.9	1727.2	1755.5	1783.8	1812.2	1840.5	1868.8	1897.1	1925.4	1953.7
70	1982.1	2010.3	2038.7	2067.0	2095.4	2123.7	2152.0	2180.3	2208.6	2236.9
80	2265.3	2293.5	2321.9	2350.2	2378.6	2406.9	2435.2	2463.5	2491.8	2520.1
90	2548.4	2576.6	2605.0	2633.3	2661.6	2690.0	2718.3	2746.6	2774.9	2803.2
100	2831.6	2859.8	2888.2	2916.5	2944.9	2973.2	3001.5	3029.8	3058.1	3086.4

Conversion of Cubic Decimetres into Cubic Feet.

Cubic dm.	0	1	2	3	4	5	6	7	8	9
	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.	Ft^3.
0	0.0000	0.0353	0.0706	0.1059	0.1413	0.1766	0.2119	0.2472	0.2825	0.3178
10	0.3531	0.3884	0.4237	0.4590	0.4944	0.5297	0.5540	0.6003	0.6356	0.6709
20	0.7063	0.7416	0.7766	0.8122	0.8476	0.8829	0.9182	0.9535	0.9888	1.0241
30	1.0594	1.0947	1.1300	1.1653	1.2007	1.2360	1.2713	1.3066	1.3419	1.3772
40	1.4126	1.4479	1.4832	1.5185	1.5539	1.5892	1.6245	1.6608	1.6951	1.7304
50	1.7658	1.8011	1.8364	1.8717	1.9071	1.9424	1.9777	2.0130	2.0483	2.0836
60	2.1189	2.1542	2.1895	2.2248	2.2602	2.2955	2.3308	2.3661	2.4014	2.4367
70	2.4721	2.5074	2.5427	2.5780	2.6134	2.6487	2.6840	2.7193	2.7546	2.7899
80	2.8252	2.8605	2.8958	2.9311	2.9665	3.0018	3.0371	3.0724	3.1077	3.1430
90	3.1784	3.2137	3.2490	3.2843	3.3197	3.3550	3.3903	3.4256	3.4609	3.4962
100	3.5315	3.5668	3.6021	3.6374	3.6728	3.7081	3.7434	3.7787	3.8140	3.8493

Pounds per Square Foot into Kilogrammes per Square Metre.

Lbs. pr ft^2.	0	1	2	3	4	5	6	7	8	9
	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.	K. m^2.
0	0.0000	4.8825	9.7650	14.647	19.530	24.413	29.295	34.177	39.006	43.943
10	48.825	53.707	58.590	63.472	68.355	73.238	78.120	83.002	87.831	92.768
20	97.650	102.53	107.41	112.30	117.18	122.06	126.94	131.83	136.66	141.59
30	146.47	151.35	156.23	161.12	165.90	170.88	175.76	180.65	185.47	190.41
40	195.30	200.13	205.06	209.95	214.83	219.71	224.59	229.48	234.30	239.24
50	244.13	249.01	253.89	258.78	263.66	268.54	273.42	278.31	283.13	288.08
60	292.95	297.83	302.71	307.60	312.48	317.36	322.24	327.13	331.95	336.89
70	341.77	346.65	351.53	356.42	361.20	366.18	371.06	375.95	380.77	385.71
80	390.06	394.94	399.82	404.71	409.59	414.47	419.35	424.24	429.06	434.00
90	439.43	444.31	449.19	454.08	458.96	463.34	468.72	473.61	478.43	483.37
100	488.25	493.13	498.01	502.90	507.78	512.66	517.54	522.43	527.25	532.19

Kilogrammes per Square Metre into Pounds per Square Foot.

K. per m^2.	0	1	2	3	4	5	6	7	8	9
	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2	Lb. ft^2
0	0.0000	0.2048	0.4096	0.6144	0.8192	1.0240	1.2289	1.4337	1.6385	1.8433
10	2.0481	2.2529	2.4577	2.6625	2.8673	3.0721	3.2770	3.4818	3.6866	3.8914
20	4.0962	4.3010	4.5058	4.7106	4.9154	5.1202	5.3251	5.5299	5.7347	5.9395
30	6.1444	6.3492	6.5540	6.7588	6.9636	7.1684	7.3733	7.5781	7.7829	7.9877
40	8.1925	8.3973	8.6021	8.8069	9.0117	9.2165	9.4214	9.6262	9.8310	10.036
50	10.240	10.445	10.649	10.854	11.059	11.264	11.469	11.674	11.878	12.083
60	12.289	12.494	12.698	12.903	13.108	13.313	13.518	13.723	13.927	14.132
70	14.337	14.542	14.746	14.951	15.156	15.361	15.566	15.771	15.975	16.180
80	16.385	16.590	16.794	16.999	17.204	17.409	17.614	17.819	18.023	18.228
90	18.433	18.638	18.842	19.047	19.252	19.457	19.662	19.867	20.071	20.276
100	20.481	20.686	20.890	21.095	21.300	21.505	21.710	21.915	22.119	22.324

Pounds per Square Inch into Atmospheric Pressure.

Lbs. pr in².	0	1	2	3	4	5	6	7	8	9
	At.	At.	At.	At.	At.	At.	At.	At.	At.	At.
0	0.0000	0.0680	0.1361	0.2041	0.2722	0.3402	0.4082	0.4763	0.5443	0.6124
10	0.6804	0.7484	0.8165	0.8845	0.9526	1.0206	1.0886	1.1567	1.2247	1.2928
20	1.3608	1.4288	1.4969	1.5649	1.6330	1.7010	1.7690	1.8371	1.9051	1.9732
30	2.0413	2.1093	2.1774	2.2454	2.3135	2.3814	2.4495	2.5176	2.5856	2.6537
40	2.7217	2.7897	2.8578	2.9258	2.9939	3.0619	3.1299	3.1980	3.2660	3.3341
50	3.4021	3.4701	3.5382	3.6062	3.6743	3.7423	3.8103	3.8784	3.9464	4.0145
60	4.0825	4.1505	4.2186	4.2866	4.3547	4.4227	4.4907	4.5588	4.6268	4.6949
70	4.7630	4.8310	4.8991	4.9671	5.0352	5.1031	5.1712	5.2393	5.3073	5.3754
80	5.4434	5.5114	5.5795	5.6475	5.7156	5.7836	5.8516	5.9197	5.9877	6.0558
90	6.1238	6.1918	6.2599	6.3279	6.3960	6.4640	6.5320	6.6001	6.6681	6.7362
100	6.8042	6.8722	6.9403	7.0083	7.0764	7.1444	7.2124	7.2805	7.3485	7.4166

Atmospheric Pressure into Pounds per Square Inch.

Atm. pres.	0	1	2	3	4	5	6	7	8	9
	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²
0	0.0000	14.697	29.393	44.090	58.787	73.483	88.180	102.87	117.57	132.27
10	146.97	161.67	176.36	191.06	205.76	220.45	235.15	249.84	264.54	279.24
20	293.93	308.63	323.32	338.02	352.72	367.41	382.11	396.80	411.50	426.20
30	440.90	455.60	470.29	484.99	499.69	514.38	529.08	543.77	558.47	573.17
40	587.87	602.57	617.26	631.96	646.66	661.35	676.05	690.74	705.44	720.14
50	734.83	749.53	764.22	778.92	793.62	808.31	823.01	837.70	852.40	867.10
60	881.80	896.50	911.19	925.89	940.59	955.28	969.98	984.67	999.37	1014.1
70	1028.7	1043.4	1058.1	1072.8	1087.5	1102.2	1116.9	1131.6	1146.3	1161.0
80	1175.7	1190.4	1205.1	1219.8	1234.5	1249.2	1263.9	1278.6	1293.3	1308.0
90	1322.7	1337.4	1352.1	1366.8	1381.5	1396.2	1410.9	1425.6	1439.3	1455.0
100	1469.7	1484.4	1499.1	1513.8	1528.5	1543.2	1557.9	1572.6	1586.3	1602.0

Pounds per Square Inch into Kilogrammes per Square Centimetre.

Lbs. pr in².	0	1	2	3	4	5	6	7	8	9
	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²	K.cm²
0	0.0000	0.0703	0.1406	0.2109	0.2812	0.3515	0.4218	0.4921	0.5625	0.6328
10	0.7031	0.7734	0.8437	0.9140	0.9843	1.0546	1.1249	1.1952	1.2655	1.3358
20	1.4062	1.4765	1.5468	1.6171	1.6874	1.7577	1.8280	1.8983	1.9686	2.0389
30	2.1092	2.1795	2.2498	2.3202	2.3905	2.4608	2.5311	2.6014	2.6717	2.7420
40	2.8123	2.8826	2.9529	3.0232	3.0935	3.1639	3.2342	3.3045	3.3748	3.4451
50	3.5154	3.5857	3.6560	3.7263	3.7966	3.8669	3.9372	4.0075	4.0779	4.1482
60	4.2185	4.2888	4.3591	4.4294	4.4997	4.5700	4.6403	4.7106	4.7809	4.8512
70	4.9216	4.9919	5.0622	5.1325	5.2028	5.2731	5.3434	5.4137	5.4840	5.5543
80	5.6246	5.6949	5.7652	5.8356	5.9059	5.9762	6.0465	6.1168	6.1871	6.2574
90	6.3277	6.3980	6.4683	6.5386	6.6089	6.6793	6.7496	6.8199	6.8902	6.9605
100	7.0308	7.1011	7.1714	7.2417	7.3120	7.3823	7.4526	7.5229	7.5933	7.6636

Kilogrammes per Square Centimetre into Pounds per Square Inch.

K. per cm².	0	1	2	3	4	5	6	7	8	9
	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²	Lb.in²
0	0.0000	14.223	28.446	42.670	56.893	71.116	85.339	99.562	113.78	128.01
10	142.23	156.45	170.68	184.90	199.12	213.35	227.57	241.79	256.02	270.24
20	284.46	298.69	312.91	327.13	341.36	355.58	369.80	384.03	398.25	412.47
30	426.70	440.92	455.14	469.36	483.59	497.81	512.03	526.26	540.48	554.70
40	568.93	583.15	597.37	611.60	625.82	640.04	654.27	668.49	682.71	696.94
50	711.16	725.38	739.61	753.83	768.05	782.28	796.50	810.72	824.94	839.17
60	853.39	867.61	881.84	896.06	910.28	924.51	938.73	952.95	967.18	981.40
70	995.62	1009.8	1024.1	1038.3	1052.5	1066.7	1081.0	1095.2	1109.4	1123.6
80	1137.8	1152.1	1166.3	1180.5	1194.7	1209.0	1223.2	1237.4	1251.6	1265.9
90	1280.1	1294.3	1308.5	1322.7	1337.0	1351.2	1365.4	1379.6	1393.9	1408.1
100	1422.3	1436.5	1450.8	1465.0	1479.2	1493.4	1507.7	1521.9	1536.1	1550.3

Conversion of English Pounds into Kilogrammes.

Eng. lbs.	0	1	2	3	4	5	6	7	8	9
	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.
0	0.000	0.453	0.907	1.361	1.814	2.268	2.722	3.175	3.629	4.082
10	4.536	4.989	5.443	5.897	6.350	6.804	7.258	7.711	8.165	8.618
20	9.072	9.525	9.979	10.43	10.89	11.34	11.79	12.25	12.70	13.15
30	13.61	14.06	14.52	14.97	15 42	15.88	16.33	16.78	17.24	17.69
40	18.14	18.59	19.05	19.50	19.95	20.41	20.86	21.31	21.77	22.22
50	22.68	23.13	23.59	24.04	24.49	24.95	25.40	25.85	26.31	26.76
60	27.22	27.67	28.13	28.58	29.03	29.49	29.94	30.39	30.85	31.30
70	31.75	32.20	32.66	33.11	33.56	34.02	34.47	34.92	35.38	35.83
80	36.29	36.74	37.20	37.65	38.10	38.56	39.01	39.46	39.92	40.37
90	40.82	41.27	41.73	42.18	42.63	43.09	43.54	43.99	44.45	44.90
100	45.36	45.81	46.27	46.72	47.17	47.63	48.08	48.53	48.99	49.44

Conversion of Kilogrammes into English Pounds.

Fr. kilo.	0	1	2	3	4	5	6	7	8	9
	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.	Lbs.
0	0.000	2.205	4.410	6.615	8.820	11.02	13.23	15.43	17.64	19.84
10	22.05	24.25	26.46	28.67	30.87	33.07	35.28	37.48	39.69	41.89
20	44.10	46.30	48.51	50.72	52.92	55.12	57.33	59.53	61.74	63.94
30	66.15	68.35	70.56	72.77	74.97	77.17	79.38	81.58	83.79	85.99
40	88.20	90.40	92.61	94.82	97.02	99.22	101.4	103.6	105.8	108.0
50	110.2	112.5	114.6	116.8	119.0	121.2	123.4	125.6	127.8	130.0
60	132.3	134.5	136.7	138.9	141.1	143.3	145.5	147.7	149.9	152.1
70	154.3	156.5	158.7	160.9	163.1	165.3	167.5	169.7	171.9	174.1
80	176.4	178.6	180.8	183.0	185.2	187.4	189.6	191.8	194.0	196.2
90	198.4	200.6	202.8	205.0	207.2	209.4	211.6	213.8	216.0	218.2
100	220.5	222.7	224.9	227.1	229.3	231.5	233.7	235.9	238.1	240.3

Conversion of English Tons into Metric Tons.

Eng. tons.	0	1	2	3	4	5	6	7	8	9
	M. ton	M. ton	M. ton	M. ton	M. ton	M. ton	M. ton	M. ton	M. ton	M. ton
0	0.000	1.016	2.032	3.048	4.064	5.080	6.096	7.112	8.128	9.144
10	10.16	11.18	12.19	13.21	14.12	15.24	16.26	17.27	18.29	19.30
20	20.32	21.34	22.35	23.37	24.38	25.40	26.42	27.43	28.45	29.46
30	30.48	31.50	32.51	33.53	34.54	35.56	36.58	37.59	38.61	39.62
40	40.64	41.66	42.67	43.69	44.70	45.74	46.74	47.75	48.77	49.78
50	50.80	51.82	52.83	53.85	54.86	55.88	56.90	57.90	58.93	59.94
60	60.96	61.97	62.99	64.01	65.02	66.04	67.06	68.07	69.09	70.10
70	71.12	72.14	73.15	74.17	75.18	76.20	77.22	78.23	79.25	80.26
80	81.28	82.29	83.31	84.33	85.34	86.36	87.38	88.39	89.41	90.42
90	91.44	92.46	93.47	94.49	95.50	96.52	97.54	98.55	99.57	100.6
100	101.6	102.6	103.6	104.6	105.7	106.7	107.7	108.7	109.7	110.7

Conversion of Metric Tons into English Tons.

Fr. m. ton.	0	1	2	3	4	5	6	7	8	9
	E. ton	E. ton	E. ton	E. ton	E. ton	E. ton	E. ton	E. ton	E. ton	E. ton
0	0.000	0.984	1.969	2.953	3.937	4.921	5.906	6.890	7.874	8.858
10	9.843	10.83	11.81	12.79	13.78	14.76	15.75	16.73	17.72	18.70
20	19.69	20.67	21.66	22.64	23.63	24.61	25.60	26.58	27.56	28.55
30	29.53	30.51	31.50	32.48	33.47	34.45	35.44	36.42	37.40	38.39
40	39.37	40.35	41.34	42.32	43.31	44.29	45.28	46.26	47.24	48.23
50	49.21	50.19	51.18	52.16	53.15	54.13	55.12	56.10	57.08	58.07
60	59.06	60.04	61.03	62.01	63.00	63.98	64.97	65.95	66.93	67.92
70	68.90	69.88	70.87	71.85	72.84	73.82	74.81	75.79	76.77	77.76
80	78.74	79.72	80.71	81.69	82.68	83.66	84.65	85.63	86.61	87.60
90	88.58	89.56	90.55	91.53	92.52	93.50	94.49	95.47	96.45	97.44
100	98.43	99.41	100.4	101.4	102.4	103.3	104.3	105.3	106.3	107.3

Conversion of English Ounces Avoirdupois into French Grammes.

Eng. ozs.	0	1	2	3	4	5	6	7	8	9
	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams
0	0.0000	28.348	56.697	85.046	113.39	141.74	170.09	198.44	226.79	255.14
10	283.48	311.83	340.18	368.52	396.87	425.22	453.57	481.92	510.27	538.62
20	566.97	595.32	623.67	652.01	680.36	708.71	737.06	765.41	793.76	822.11
30	850.46	878.81	907.16	935.50	963.85	992.20	1020.5	1048.9	1077.2	1105.6
40	1133.9	1162.2	1190.6	1218.9	1247.3	1275.6	1304.0	1332.3	1360.7	1389.0
50	1417.4	1445.7	1474.1	1502.4	1530.8	1559.1	1587.5	1615.8	1644.2	1672.5
60	1700.9	1729.2	1756.6	1785.9	1814.3	1842.9	1871.0	1899.3	1927.7	1956.0
70	1984.4	2012.7	2041.1	2079.4	2097.8	2126.1	2154.5	2182.8	2211.2	2239.5
80	2267.9	2296.2	2324.6	2352.9	2381.3	2409.6	2438.0	2466.3	2494.7	2523.0
90	2551.4	2579.7	2608.1	2636.4	2664.8	2693.1	2721.5	2739.8	2778.2	2806.5
100	2834.8	2863.1	2891.5	2919.8	2948.2	2976.5	3004.9	3033.2	3061.6	3089.9

Conversion of French Grammes into English Ounces Avoirdupois.

Fr. grams.	0	1	2	3	4	5	6	7	8	9
	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.	Ozs.
0	0.0000	0.0353	0.0705	0.1058	0.1411	0.1768	0.2116	0.2469	0.2822	0.3175
10	0.3527	0.3880	0.4232	0.4585	0.4938	0.5295	0.5643	0.5996	0.6349	0.6702
20	0.7055	0.7408	0.7760	0.8113	0.8466	0.8823	0.9171	0.9524	0.9877	1.0230
30	1.0582	1.0935	1.1287	1.1640	1.1993	1.2350	1.2698	1.3051	1.3404	1.3757
40	1.4110	1.4463	1.4815	1.5168	1.5521	1.5878	1.6226	1.6579	1.6932	1.7285
50	1.7687	1.8040	1.8392	1.8745	1.9098	1.9455	1.9803	2.0156	2.0509	2.0862
60	2.1165	2.1518	2.1870	2.2223	2.2576	2.2933	2.3281	2.3634	2.3987	2.4340
70	2.4692	2.5045	2.5397	2.5750	2.6103	2.6460	2.6808	2.7161	2.7514	2.7867
80	2.8220	2.8573	2.8925	2.9278	2.9631	2.9988	3.0336	3.0689	3.1042	3.1395
90	3.1747	3.2100	3.2452	3.2805	3.3158	3.3515	3.3863	3.4216	3.4569	3.4922
100	3.5275	3.5628	3.5980	3.6333	3.6686	3.7043	3.7391	3.7744	3.8097	3.8450

Conversion of English Grains Troy into French Grammes.

Eng. grains	0	1	2	3	4	5	6	7	8	9
	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams	Grams
0	0.0000	0.0648	0.1296	0.1944	0.2592	0.3240	0.3888	0.4535	0.5183	0.5831
10	0.6479	0.7127	0.7775	0.8423	0.9071	0.9719	1.0367	1.1014	1.1662	1.2310
20	1.2959	1.3607	1.4255	1.4903	1.5551	1.6199	1.6847	1.7494	1.8142	1.8890
30	1.9438	2.0086	2.0734	2.1382	2.2030	2.2678	2.3326	2.3973	2.4621	2.5269
40	2.5918	2.6566	2.7214	2.7862	2.8510	2.9158	2.9806	3.0453	3.1101	3.1749
50	3.2398	3.3046	3.3694	3.4342	3.4990	3.5638	3.6286	3.6933	3.7581	3.8229
60	3.8877	3.9525	4.0173	4.0821	4.1469	4.2117	4.2765	4.3412	4.4060	4.4708
70	4.5357	4.6005	4.6653	4.7301	4.7949	4.8597	4.9245	4.9892	5.0540	5.1188
80	5.1830	5.2484	5.3132	5.3780	5.4428	5.5076	5.5724	5.6371	5.7019	5.7667
90	5.8316	5.8964	5.9612	6.0260	6.0908	6.1556	6.2204	6.2851	6.3499	6.4147
100	6.4795	6.5443	6.6091	6.6739	6.7387	6.8035	6.8683	6.9330	6.9978	7.0626

Conversion of French Grammes into English Grains Troy.

Fr. grams.	0	1	2	3	4	5	6	7	8	9
	Grs.	Grs.	Grs.	Grs.	Grs.	Grs.	Grs.	Grs.	Grs.	Grs.
0	0.0000	15.433	30.866	46.299	61.732	77.165	92.599	108.03	123.46	138.90
10	154.33	169.76	185.19	200.63	216.06	231.49	246.93	262.36	277.79	293.23
20	308.66	324.09	339.52	354.96	370.39	385.82	401.26	416.69	432.12	447.56
30	462.99	478.42	493.86	509.29	524.72	540.15	555.59	571.02	586.45	601.89
40	617.65	632.75	648.18	663.95	679.38	694.81	709.92	725.35	740.78	756.22
50	771.65	787.08	802.52	817.95	833.38	848.82	864.25	879.68	895.11	910.55
60	925.99	941.42	956.85	972.29	987.72	1003.1	1018.6	1034.0	1049.4	1064.9
70	1080.3	1095.7	1111.2	1126.6	1142.0	1157.5	1172.9	1188.3	1203.7	1219.2
80	1234.6	1250.0	1265.5	1280.1	1296.3	1311.8	1327.2	1342.6	1358.1	1373.5
90	1389.0	1404.4	1419.8	1435.3	1450.7	1466.1	1481.6	1497.0	1512.4	1527.9
100	1543.3	1558.7	1574.1	1589.6	1605.0	1620.4	1635.9	1651.3	1666.7	1682.2

Horse-power into Cheval-vapeur.

H.-power.	0	1	2	3	4	5	6	7	8	9
	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.	C.-v.
0	0.0000	1.0136	2.0272	3.0408	4.0544	5.0680	6.0816	7.0952	8.1088	9.1224
10	10.136	11.150	12.163	13.176	14.190	15.204	16.218	17.231	18.245	19.258
20	20.272	21.308	22.299	23.313	24.326	25.240	26.354	27.367	28.381	29.394
30	30.408	31.422	32.435	33.449	34.462	35.476	36.490	37.503	38.517	39.530
40	40.544	41.557	42.571	43.585	44.598	45.612	46.626	47.639	48.653	49.666
50	50.680	51.693	52.707	53.721	54.734	55.748	56.762	57.775	58.789	59.802
60	60.816	61.829	62.843	63.857	64.870	65.884	66.898	67.911	68.925	69.938
70	70.952	71.965	72.979	73.993	75.006	76.020	77.034	78.047	79.061	80.074
80	81.088	82.102	83.115	84.129	85.142	86.156	87.170	88.183	89.197	90.210
90	91.224	92.338	93.251	94.265	95.278	96.292	97.306	98.319	99.333	100.34
100	101.36	102.37	103.30	104.40	105.41	106.43	107.44	108.45	109.47	110.48

Cheval-vapeur into Horse-power.

Chev.-vap.	0	1	2	3	4	5	6	7	8	9
	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.	H.-p.
0	0.0000	0.9863	1.9726	2.9589	3.9452	4.9315	5.9178	6.9041	7.8904	8.8767
10	9.8630	10.849	11.835	12.822	13.808	14.794	15.781	16.767	17.753	18.739
20	19.726	20.712	21.698	22.685	23.671	24.657	25.644	26.630	27.616	28.602
30	29.589	30.575	31.561	32.548	33.534	34.520	35.507	36.493	37.479	38.465
40	39.452	40.438	41.424	42.411	43.397	44.383	45.370	46.356	47.342	48.328
50	49.315	50.301	51.287	52.274	53.260	54.246	55.233	56.219	57.205	58.191
60	59.178	60.164	61.150	62.137	63.123	64.109	65.096	66.082	67.068	68.054
70	69.041	70.027	71.013	71.990	72.986	73.972	74.959	75.945	76.941	77.917
80	78.904	79.890	80.876	81.863	82.849	83.835	84.822	85.808	86.794	87.780
90	88.767	89.753	90.739	91.726	92.712	93.698	94.785	95.671	96.657	97.643
100	98.630	99.616	100.60	101.59	102.57	103.56	104.55	105.53	106.52	107.50

Foot-pounds into Kilogrammetres.

Foot-lbs.	0	1	2	3	4	5	6	7	8	9
	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.	Kgm.
0	0.0000	0.1382	0.2764	0.4146	0.5528	0.6910	0.8292	0.9674	1.1056	1.2438
10	1.3820	1.5202	1.6584	1.7966	1.9348	2.0731	2.2112	2.3494	2.4876	2.6259
20	2.7640	2.9022	3.0404	3.1786	3.3168	3.4552	3.5933	3.7315	3.8696	4.0078
30	4.1460	4.2842	4.4224	4.5606	4.6988	4.8370	4.9751	5.1134	5.2517	5.3897
40	5.5280	5.6666	5.8044	5.9426	6.0808	6.2191	6.3572	6.4954	6.6336	6.7718
50	6.9100	7.0482	7.1864	7.3246	7.4628	7.6010	7.7393	7.8775	8.0155	8.1538
60	8.2920	8.4303	8.5684	8.7066	8.8448	8.9830	9.1212	9.2594	9.3976	9.5359
70	9.6740	9.8122	9.9504	10.088	10.227	10.365	10.503	10.641	10.779	10.918
80	11.056	11.194	11.322	11.570	11.609	11.747	11.885	12.023	12.161	12.300
90	12.438	12.576	12.714	12.855	12.991	13.129	13.267	13.405	13.544	13.682
100	13.820	13.958	14.096	14.235	14.373	14.511	14.649	14.787	14.925	14.064

Kilogrammetres into Foot-pounds.

Kgm.	0	1	2	3	4	5	6	7	8	9
	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.	Ft.-lb.
0	0.0000	7.2334	14.467	21.700	28.934	36.166	43.400	50.734	57.868	65.100
10	72.334	79.567	87.101	94.034	101.27	108.50	115.74	123.07	130.20	137.43
20	144.67	151.90	158.43	166.37	173.60	180.84	188.08	195.40	202.54	209.77
30	217.00	224.23	231.77	238.70	245.93	253.17	260.41	267.73	274.87	282.10
40	289.34	296.57	304.11	311.04	318.27	325.50	332.75	340.07	347.21	354.44
50	361.66	368.89	376.43	383.36	390.59	397.82	405.07	412.39	419.53	426.76
60	434.00	441.23	448.77	455.70	462.93	470.17	477.41	484.73	491.87	499.10
70	507.34	514.57	522.11	529.04	536.27	543.50	550.75	558.07	565.21	572.44
80	578.68	585.91	593.45	599.38	607.61	614.85	622.09	629.41	636.55	643.78
90	651.00	658.23	665.77	672.70	679.93	687.17	694.41	701.73	708.87	716.10
100	723.34	730.57	738.11	745.04	752.27	759.51	766.75	774.07	781.21	788.44

Conversion of Foot-tons into Tonnes-metres.

Foot-tons.	0	1	2	3	4	5	6	7	8	9
	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.	T.-m.
0	0.0000	0.3097	0.6194	0.9291	1.2382	1.5484	1.8581	2.1678	2.4775	2.7872
10	3.0969	3.3166	3.7163	4.0260	4.3356	4.6453	4.9550	5.2667	5.5744	5.8841
20	6.1938	6.4135	6.8132	7.1229	7.4325	7.7422	8.0519	8.3636	8.6713	8.9810
30	9.2906	9.6003	9.9100	10.219	10.529	10.839	11.149	11.460	11.768	12.078
40	12.387	12.697	13.006	13.316	13.626	13.935	14.245	14.557	14.864	15.174
50	15.484	15.794	16.103	16.413	16.723	17.032	17.342	17.654	17.961	18.271
60	18.581	18.891	19.200	19.510	19.820	20.129	20.439	20.751	21.058	21.368
70	21.678	21.988	22.297	22 607	22.917	23.226	23.536	23.848	24.155	24.465
80	24.775	25.085	25.394	25.704	26.014	26.323	26.633	26.945	27.252	27.562
90	27.872	28.182	28.491	28.801	29.111	29.420	29.730	30.042	30.349	30.659
100	30.969	31.279	31.588	31.898	32.208	32.517	32.827	33.139	33.446	33.756

Conversion of Tonnes-metres into Foot-tons.

T.-metres.	0	1	2	3	4	5	6	7	8	9
	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.	F.-tn.
0	0.0000	3.2290	6.4581	9.6871	12.916	16.145	19.374	22.603	25.832	29.061
10	32.290	35.519	38.758	41.977	45.206	48.435	51.664	54.893	58.122	61.351
20	64.581	67.810	71.049	74.268	77.497	80.726	83.955	87.184	90.413	93.642
30	96.871	100.10	103.34	106.56	109.79	113.01	116.24	119.47	122.70	125.93
40	129.16	133.39	135.63	138.85	142.07	145.30	148.53	151.76	154.99	158.22
50	161.45	164.68	167.92	171.14	174.36	177.59	180.82	184.05	187.28	190.51
60	193.74	196.97	200.21	203.43	206.65	209.88	213.11	216.34	219.57	222.80
70	226.03	229.26	232.50	235.72	238.94	242.17	245.40	248.63	251.86	255.09
80	258.32	261.55	264.79	268.01	271.23	274.46	277.69	280.92	284.15	287.38
90	290.61	293.84	297.08	300.30	303.52	306.75	309.98	313.21	316.44	319.67
100	322.90	326.13	329.37	332.59	335.81	339.04	342.27	345.50	348.73	351.96

British Thermal Units into French Calories.

B. T. U.	0	1	2	3	4	5	6	7	8	9
	Cal.	Cal.	Cal.	Cal.	Cal.	Cal.	Cal.	Cal.	Cal.	Cal.
0	0.0000	0.2520	0.5040	0.7560	1.0080	1.2600	1.5120	1.7640	2.0160	2.2680
10	2 5200	2.7720	3.0240	3.2760	3.5280	3.7800	4.0320	4.2840	4.5360	4.7880
20	5.0399	5.2919	5.5439	5.7959	6.0478	6.2699	6.5419	6.8039	7.0559	7.3079
30	7.5600	7.8120	8.0640	8.3160	8.5680	8.8200	9.0720	9.3340	9.5760	9.8280
40	10.080	10.332	10.584	10.836	11.088	11.340	11.512	11.844	12.096	12.348
50	12.600	12.852	13.104	13.356	13.608	13.860	14.112	14.364	14.616	14.868
60	15.120	15.372	15.624	15.876	16.128	16.380	16.632	16.884	17.136	17.388
70	17.640	17.892	18.144	18.396	18.648	18.900	19.152	19.404	19.656	19.908
80	20.160	20.412	20.664	20.916	21.168	21.420	21.672	21.924	22.176	22.428
90	22.680	22.932	23.184	23.436	23.688	23.940	24.192	24.444	24.696	24.948
100	25.200	25.452	25.704	25.956	26.208	26.460	26.712	26.964	27.216	27.468

French Calories into British Thermal Units.

Calories.	0	1	2	3	4	5	6	7	8	9
	T. U.	T. U.	T. U.	T. U.	T. U.	T. U.	T. U.	T. U.	T. U.	T. U.
0	0.0000	3.9683	7.9366	11.905	15.873	19.842	23.810	27.778	31.746	35.715
10	39.683	43.651	47.620	51.598	55.520	59.525	63.493	67.461	71.429	75.398
20	79.366	83.334	87.303	91.271	95.203	99.208	103.17	107.14	111.11	115.08
30	119.05	123.02	126.98	130.95	134.89	138.89	142.86	146.83	150.80	154.77
40	158.73	162.70	166.66	170.62	174.57	178.57	182.54	186.51	190.48	194.45
50	198.42	202.39	206.35	210.39	214.26	218.26	222.23	226.20	230.16	234.14
60	238.10	242.07	246.03	250.00	253.94	258.94	261.91	265.88	269.85	273.82
70	277.78	281.75	285.72	289.68	293.62	297.62	301.59	305.56	309.53	313.50
80	317.46	321.43	325.40	329.36	333.29	337.30	341.27	345.24	349.20	353.18
90	357.15	361.12	365.09	369.05	372.98	376.99	380.96	384.93	388.90	392.87
100	396.83	400.80	404.77	408.73	412.67	416.67	420.64	424.61	428.58	432.55

ALGEBRA.

For the detailed operations of Algebra the reader is referred to the numerous good text-books upon the subject, and only a few of the more important and generally practical matters will here be given in convenient form for reference.

Remembering that multiplication is represented in algebra by placing the two quantities next each other, without any intermediate sign, we have $aa = a^2$, $aaa = a^3$, etc.; also a multiplied by b is written ab, a divided by b is written $\frac{a}{b}$, etc.

From an examination of these facts we are able to place the rules regarding exponents in a form in which they can be conveniently remembered.

$$aaa = a^3; \text{ dividing this by } a \text{ we get}$$
$$aa = a^2; \text{ dividing again by } a \text{ we get}$$
$$a = a.$$

In each case we see that dividing any power of a by a is simply subtracting unity from the exponent. Proceeding, we see that

$$\frac{a}{a} = a^{1-1} = a^0 = 1;$$
$$\frac{a^0}{a} = a^{0-1} = a^{-1} = \frac{1}{a};$$
$$\frac{a^{-1}}{a} = a^{-1-1} = a^{-2} = \frac{1}{a^2}, \text{ etc.}$$

This shows why a negative exponent to any quantity means the reciprocal of the same power with a positive exponent.

Binomial Theorem.

The binomial theorem enables any power of the sum or difference of two quantities to be determined. For any value of n we have

$$(a \pm b)^n = a^n \pm na^{n-1}b + \frac{n(n-1)}{1 \cdot 2} a^{n-2}b^2 \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^{n-3}b^3 + \ldots$$

An examination of this will show that the right-hand side consists of the quantities a and b arranged according to the ascending and descending powers. Thus, when $n = 2$, we have $a^2 + ab + b^2$; for n^3 we have $a^3 + a^2b + ab^2 + b^3$, and so on.

The coefficients must be computed for each power, or they may be tabulated as below.

Table of Binomial Coefficients.

Exponents.	Terms.												
	0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	1											
2	1	2	1										
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	10	10	5	1							
6	1	6	15	20	15	6	1						
7	1	7	21	35	35	21	7	1					
8	1	8	28	56	70	56	28	8	1				
9	1	9	36	84	126	126	84	36	9	1			
10	1	10	45	120	210	252	210	120	45	10	1		
11	1	11	55	165	330	462	462	330	165	55	11	1	
12	1	12	66	220	495	792	924	792	495	220	66	12	1

Thus,

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4;$$

$$(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b7.$$

ARITHMETICAL PROGRESSION.

Arithmetical Progression is a series of numbers, as 2, 4, 6, 8, 10, 12, etc., or 18, 15, 12, 9, 6, 3, in which every successive term is increased or diminished by a constant number.

Letters denote

$a =$ the first term of the series.
$b =$ any other term whose number from a is n.
$n =$ number of terms within a and b.
$d =$ the difference between any two adjacent terms.
$S =$ the sum of all the terms.

In the series 2, 5, 8, 11, $a = 2$, $b = 11$, $n = 4$, $d = 3$, and $S = 26$.

☞ When the series is decreasing, take the first term $= b$ and the last term $= a$.

The accompanying table contains all the formulas or questions in Arithmetical Progressions, and the nature of the question will tell which formula is to be used.

Formulas for Arithmetical Progressions.

1. $a = b - d(n-1).$
2. $a = \frac{2S}{n} - b.$
3. $a = \frac{S}{n} - \frac{d}{2}(n-1).$
4. $b = a + d(n-1).$
5. $b = \frac{2S}{n} - a.$
6. $b = \frac{S}{n} + \frac{d}{2}(n-1).$
7. $n = \frac{b-a}{d} + 1.$
8. $n = \frac{2S}{a+b}.$
9. $d = \frac{b-a}{n-1}.$
10. $d = \frac{(b+a)(b-a)}{2S-a-b}.$
11. $d = \frac{2(S-an)}{n(n-1)}.$
12. $d = \frac{2(bn-S)}{n(n-1)}.$
13. $S = \frac{n(a+b)}{2}.$
14. $S = \frac{(a+b)(b+d-a)}{2d}.$
15. $S = n\left[a + \frac{d}{2}(n-1)\right].$
16. $S = n\left[b - \frac{d}{2}(n-1)\right].$
17. $a = \frac{d}{2} \pm \sqrt{\left(b+\frac{d}{2}\right)^2 - 2dS}.$
18. $b = -\frac{d}{2} \pm \sqrt{\left(a-\frac{d}{2}\right)^2 + 2dS}.$
19. $n = \frac{1}{2} - \frac{a}{d} \pm \sqrt{\left(\frac{1}{2}-\frac{a}{d}\right)^2 + \frac{2S}{d}}.$
20. $n = \frac{1}{2} + \frac{b}{d} \pm \sqrt{\left(\frac{1}{2}+\frac{b}{d}\right)^2 - \frac{2S}{d}}.$

GEOMETRICAL PROGRESSION.

Geometrical Progression is a series of numbers, as 2 : 4 : 8 : 16 : 32 :, etc., or 729 : 243 : 81 : 27 : 9 :, etc., in which every successive term is multiplied or divided by a constant factor.

Notation.

$a =$ the first term of the series;
$b =$ any other term whose number from a is n;
$n =$ number of terms within a and b, inclusive;
$r =$ ratio, or the factor by which the terms are multiplied or divided;
$S =$ Sum of the terms.

In the series $1 : 3 : 9 : 27$:, $a = 1$, $b = 27$, $n = 4$, $r = 3$, $S = 40$, inclusive.

The accompanying table contains all the formulas or questions in Geometrical Progressions. The nature of the question will tell which formula is to be used.

Formulas for Geometrical Progressions.

1. $a = \dfrac{b}{r^{n-1}}$.
2. $a = S - r(S - b)$.
3. $a = S\dfrac{r-1}{r^n - 1}$.
4. $b = ar^{n-1}$.
5. $b = S - \dfrac{S-a}{r}$.
6. $b = S\left(\dfrac{r-1}{r^n-1}\right)r^{n-1}$.
7. $r = \sqrt[n-1]{\dfrac{b}{a}}$.
8. $r = \dfrac{S-a}{S-b}$.
9. $ar^n + S - rS - a = 0$.
10. $S = \dfrac{br - a}{r-1}$.
11. $S = \dfrac{a(r^n - 1)}{r-1}$.
12. $S = \dfrac{b(r^n-1)}{(r-1)r^{n-1}}$.
13. $n = 1 + \dfrac{\log. b - \log. a}{\log. r}$.
14. $n = 1 + \dfrac{\log. b - \log. a}{\log. (S-a) - \log. (S-b)}$.
15. $n = \dfrac{\log. [a + S(r-1)] - \log. a}{\log. r}$.
16. $n = 1 + \dfrac{\log. b - \log. [br - S(r-1)]}{\log. r}$.
17. $S = \dfrac{b\sqrt[n-1]{b} - a\sqrt[n-1]{a}}{\sqrt[n-1]{b} - \sqrt[n-1]{a}}$.

SPECIAL SERIES.

Among the great variety of series occurring in practical mathematics the following will be found convenient for reference:

1. $1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots n = \dfrac{n(n+1)}{2}$.
2. $2 + 4 + 6 + 8 + \ldots\ldots\ldots\ldots 2n = n(n+1)$.
3. $1 + 3 + 5 + 7 + \ldots\ldots\ldots\ldots (2n-1) = n^2$.
4. $1^2 + 2^2 + 3^2 + 4^2 \ldots\ldots\ldots\ldots n^2 = \dfrac{n(n+1)(2n+1)}{1 \cdot 2 \cdot 3}$.
5. $1^3 + 2^3 + 3^3 + 4^3 \ldots\ldots\ldots\ldots n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$.

EQUATIONS.

Equations of the first degree need not be discussed here. Their solution may be found in any elementary algebra.

Equations of the second degree may be reduced to one of three forms, and solved respectively as follows:

1. $$x^2 + px + q = 0;\ x = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}.$$

$$ax^2 + bx + c = 0;\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

2. $$x^{2n} + px^n + q = 0;\ x = \sqrt[n]{-\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}}.$$

3. When $x \pm y = s$ and $xy = p$, we have

$$x = \frac{s + \sqrt{s^2 \mp 4p}}{2};\ y = \pm\frac{s - \sqrt{s^2 \mp 4p}}{2}.$$

LOGARITHMS.

There are four fundamental rules for operations with powers:

$$a^m \,.\, a^n = a^{m+n}.$$

That is, the product of any two powers of a number is equal to the number raised to a power whose exponent is the *sum* of the exponents of the two factors.

$$\frac{a^m}{a^n} = a^{m-n}.$$

Or, the quotient of two powers is equal to the number raised to a power whose exponent is the *difference* of the exponents of divisor and dividend.

$$(a^n)^m = a^{mn}.$$

Or, any power may be raised to a higher power by multiplying the two exponents.

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}.$$

Or, any root of any power may be extracted by *dividing* the exponent by the index of the root.

If we take any number, such as 2, and use it as the base of a geometrical series, we will see that the exponents form an arithmetical series. Thus, the exponent of $1 = 0$, of $2 = 1$, of $4 = 2$, of $8 = 3$, etc.; or, proceeding, we may arrange the following little table:

Powers.	Exponents.	Powers.	Exponents.	Powers.	Exponents.
1	0	1024	10	1048576	20
2	1	2048	11	2097152	21
4	2	4096	12	4194304	22
8	3	8192	13	8388608	23
16	4	16384	14	16777216	24
32	5	32768	15		
64	6	65536	16		
128	7	131072	17		
256	8	262144	18		
512	9	524288	19		

Suppose now we wish to multiply 128 by 512, we see that $128 = 2^7$ and $512 = 2^9$; hence, $128 \times 512 = 2^{7-9} = 2^{16}$, and in the table, opposite the

exponent 16, we find the power 65536, which is the product of the two factors, obtained by the simple addition of the exponents.

Again, $\frac{512}{128} = \frac{2^9}{2^7} = 2^{9-7} = 2^2 = 4.$

To raise a number to a power, such as 16 to the fifth power, we have $16 = 2^4$ and $(2^4)^5 = 2^{20} = 1048576$.

Again, the seventh root of 2097152 is formed as follows:

$$2097152 = 2^{21} \text{ and } \sqrt[7]{2^{21}} = 2^{\frac{21}{7}} = 2^3 = 8.$$

In the small table of the powers of 2 given above there are many gaps, because only those powers which have *whole* exponents are given. For all the numbers between 16 and 32, for example, the exponents will be decimals, and will be greater than 4 and less than 5, etc. In practice, the base used is not 2, but 10, and all the intermediate exponents have been computed to many decimals, these forming a table of logarithms.

Table of Logarithms of Numbers.

Pages 82 to 104 give the *mantissas*, or decimal portions of the logarithms, of all whole numbers from 1 to 10009. The *characteristics*, or whole numbers, which, with these decimals, form the complete logarithms, are found as follows:

The logarithm of 1 = 0, of 10 = 1, of 100 = 2, of 1000 = 3, etc.; hence, the logarithm of any number between 100 and 1000 must lie between 2 and 3, and be greater than 2 and less than 3, and so for any number. Therefore we have the rule that the whole portion of a logarithm of any number is one less than there are figures in the number. The decimal portion for any number below 10009 is taken directly from the table. Thus,

$$\log. 365 = 2.56229,$$

the decimal portion, 56229, being found directly opposite 365 in the table, and the whole portion being 2, or 1 less than the number of places in 365.

In like manner we have

$$\log. 36.5 = 1.56229,$$
$$\log. 3.65 = 0.56229.$$

The mantissa, or decimal portion, is always positive, but the characteristic is negative when the number is less than unity. Thus,

$$\log. 0.365 = \bar{1}.56229,$$
$$\log. 0.0365 = \bar{2}.56229,$$
$$\log. 0.00365 = \bar{3}.56229,$$

the minus being placed *over* the characteristic to show that it applies to that portion only, and not to the mantissa.

If the given number has more than three places, the mantissa is found in the body of the table. Thus, the logarithm of 1873 = 3.27253, the figures 0.27 being found opposite 183, and the 253 on the same horizontal line under 3.

If the last three figures of the mantissa are preceded by an asterisk, the first two figures are to be taken from the next line *below*, in the first column. Thus,

$$\log. 3897 = 3.59073,$$

in which, opposite 389, we find 59, and then, passing on under 7, we find *073, the asterisk indicating that we are to go one line below, taking out 59, not 58, for the first two figures of the mantissa, giving us 0.59073, as above.

The table, as will be seen, enables the logarithm of any number of *four* places to be taken out at once. If the number of which the logarithm is required has more than four places, the logarithm can be found from the table, as follows:

In the column at the extreme right of each page, under the heading P. P. (Proportional Parts), will be found in the black figures the *differences* between any logarithm and the next succeeding logarithm for the adjoin-

ing portions of the table. The smaller figures in the same column form little multiplication tables, in which these differences are multiplied by 0.1, 0.2, 0.3, etc.

The use of these proportional parts and their decimal parts is best shown by actual example. Suppose it is desired to find the logarithm of 18702. Opposite 187 and under 0 in the table we find the mantissa, 0.27184. The proportional part, or difference at this point between one logarithm and the next, is 23, or, in other words, there is a difference of 23 between the last two figures of the logarithm of 1870 and 1871. For 0.1 difference in the number, the difference in the logarithms would be 2.3; for 0.2, it would be 4.6, etc., as shown in the small table under 23 in the column P. P. For 2 points additional, therefore, we simply add 4.6 to the logarithm of 1870, and we have the logarithm of 18702. Thus,

$$
\begin{aligned}
\text{log. } 1870 &= 0.27184 \\
\text{p. p. for } 2 &= \underline{4.6} \\
\text{log. } 18702 &= 4.271886, \text{ or} \\
&\ 4.27189
\end{aligned}
$$

Again, let it be required to find the logarithm of 35.797.

$$
\begin{aligned}
\text{log. } 35.79 &= 1.55376 \qquad \text{p. p.} = 12 \\
\text{p. p. for } 7 &= \underline{8.4} \\
\text{log. } 35.797 &= 1.553844
\end{aligned}
$$

If the given number has six or more figures the method is the same, except that the proportional part is reduced one-tenth for each additional figure. Thus, the logarithm of 3725.96 is found as follows:

$$
\begin{aligned}
\text{log. } 3725 &= 3.57113 \qquad \text{p. p.} = 11 \\
\text{p. p. for } 9 &= 9.9 \\
\text{p. p. for } 6 &= \underline{0.66} \\
\text{log. } 3725.96 &= 3.5712356, \text{ or } 3.57124
\end{aligned}
$$

The operation of finding the number corresponding to a given logarithm is the reverse of the preceding. Thus, the number corresponding to the logarithm 2.73924 is found as follows:

In the table the next smaller logarithm is

	73918, and its number		= 584500
The given log. =	73924		..
and the difference =	6		..
The nearest difference in the table =	5.6 =	corresponding to	7
Subtracting	0.4	corresponding to	5
		Hence, the number is	584575

Since the characteristic = 2, there must be one more place before the decimal point; hence,

log. 2.73924 = num. 584.575

Num. 100 to 139. Log. 000 to 145.

N	L	0	1	2	3	4	5	6	7	8	9
100	00	000	043	087	130	173	217	260	303	346	389
101		432	475	518	561	604	647	689	732	775	817
102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	407	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
109		743	782	822	862	902	941	981	*021	*060	*100
110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	166	189	222	254	287	320	352
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	44	43	42	41	40	39	38	37	36	35
1	4.4	4.3	4.2	4.1	4.0	3.9	3.8	3.7	3.6	3.5
2	8.8	8.6	8.4	8.2	8.0	7.8	7.6	7.4	7.2	7.0
3	13.2	12.9	12.6	12.3	12.0	11.7	11.4	11.1	10.8	10.5
4	17.6	17.2	16.8	16.4	16.0	15.6	15.2	14.8	14.4	14.0
5	22.0	21.5	21.0	20.5	20.0	19.5	19.0	18.5	18.0	17.5
6	26.4	25.8	25.2	24.6	24.0	23.4	22.8	22.2	21.6	21.0
7	30.8	30.1	29.4	28.7	28.0	27.3	26.6	25.9	25.2	24.5
8	35.2	34.4	33.6	32.8	32.0	31.2	30.4	29.6	28.8	28.0
9	39.6	38.7	37.8	36.9	36.0	35.1	34.2	33.3	32.4	31.5

Num. 140 to 179. Log. 146 to 255.

N	L	0	1	2	3	4	5	6	7	8	9
140	14	613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		455	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
150		609	638	667	696	725	754	782	811	840	869
151		898	926	955	984	*013	*041	*070	*099	*127	*156
152	18	184	213	241	270	298	327	355	384	412	441
153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	167	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	779
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
177		797	822	846	871	895	920	944	969	993	*018
178	25	042	066	091	115	139	164	188	212	237	261
179		285	310	334	358	382	406	431	455	479	503
180		527	551	575	600	624	648	672	696	720	744
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	34	33
1	3.4	3.3
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	32	31
1	3.2	3.1
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	30	29
1	3.0	2.9
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	28	27
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

P. P.

Num. 180 to 219. Log. 255 to 342.

N	L	0	1	2	3	4	5	6	7	8	9
180	25	527	551	575	600	624	648	672	696	720	744
181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
183		245	269	293	316	340	364	387	411	435	458
184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	988	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	852
190		875	898	921	944	967	989	*012	*035	*058	*081
191	28	103	126	149	171	194	217	240	262	285	307
192		330	353	375	398	421	443	466	488	511	533
193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
211		428	449	469	490	510	531	552	572	593	613
212		634	654	675	695	715	736	756	777	797	818
213		838	858	879	899	919	940	960	980	*001	*021
214	33	041	062	082	102	122	143	163	183	203	224
215		244	264	284	304	325	345	365	385	405	425
216		445	465	486	506	526	546	566	586	606	626
217		646	666	686	706	726	746	766	786	806	826
218		846	866	885	905	925	945	965	985	*005	*025
219	34	044	064	084	104	124	143	163	183	203	223
220		242	262	282	301	321	341	361	380	400	420
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	24	23	22	21	20	19
1	2.4	2.3	2.2	2.1	2.0	1.9
2	4.8	4.6	4.4	4.2	4.0	3.8
3	7.2	6.9	6.6	6.3	6.0	5.7
4	9.6	9.2	8.8	8.4	8.0	7.6
5	12.0	11.5	11.0	10.5	10.0	9.5
6	14.4	13.8	13.2	12.6	12.0	11.4
7	16.8	16.1	15.4	14.7	14.0	13.3
8	19.2	18.4	17.6	16.8	16.0	15.2
9	21.6	20.7	19.8	18.9	18.0	17.1

P. P.

Num. 220 to 259. Log. 342 to 414.

N	L	0	1	2	3	4	5	6	7	8	9
220	34	242	262	282	301	321	341	361	380	400	420
221		439	459	479	498	518	537	557	577	596	616
222		635	655	674	694	713	733	753	772	792	811
223		830	850	869	889	908	928	947	967	986	*005
224	35	025	044	064	083	102	122	141	160	180	199
225		218	238	257	276	295	315	334	353	372	392
226		411	430	449	468	488	507	526	545	564	583
227		603	622	641	660	679	698	717	736	755	774
228		793	813	832	851	870	889	908	927	946	965
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36	173	192	211	229	248	267	286	305	324	342
231		361	380	399	418	436	455	474	493	511	530
232		549	568	586	605	624	642	661	680	698	717
233		736	754	773	791	810	829	847	866	884	903
234		922	940	959	977	996	*014	*033	*051	*070	*088
235	37	107	125	144	162	181	199	218	236	254	273
236		291	310	328	346	365	38	401	420	438	457
237		475	493	511	530	548	566	5	603	621	639
238		658	676	694	712	731	749	767	785	803	822
239		840	858	876	894	912	931	949	967	985	*003
240	38	021	039	057	075	093	112	130	148	166	184
241		202	220	238	256	274	292	310	328	346	364
242		382	399	417	435	453	471	489	507	525	543
243		561	578	596	614	632	650	668	686	703	721
244		739	757	775	792	810	828	846	863	881	899
245		917	934	952	970	987	*005	*023	*041	*058	*076
246	39	094	111	129	146	164	182	199	217	235	252
247		270	287	305	322	340	358	375	393	410	428
248		445	463	480	498	515	533	550	568	585	602
249		620	637	655	672	690	707	724	742	759	777
250		794	811	829	846	863	881	898	915	933	950
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40	140	157	175	192	209	226	243	261	278	295
253		312	329	346	364	381	398	415	432	449	466
254		483	500	518	535	552	569	586	603	620	637
255		654	671	688	705	722	739	756	773	790	807
256		824	841	858	875	892	909	926	943	960	976
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41	162	179	196	212	229	246	263	280	296	313
259		330	347	363	380	397	414	430	447	464	481
260		497	514	531	547	564	581	597	614	631	647
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	20	19	18	17
1	2.0	1.9	1.8	1.7
2	4.0	3.8	3.6	3.4
3	6.0	5.7	5.4	5.1
4	8.0	7.6	7.2	6.8
5	10.0	9.5	9.0	8.5
6	12.0	11.4	10.8	10.2
7	14.0	13.3	12.6	11.9
8	16.0	15.2	14.4	13.6
9	18.0	17.1	16.2	15.3

P. P.

Num. 260 to 299. Log. 414 to 476.

N	L	0	1	2	3	4	5	6	7	8	9
260	41	497	514	531	547	564	581	597	614	631	647
261		664	681	697	714	731	747	764	780	797	814
262		830	847	863	880	896	913	929	946	963	979
263		996	*012	*029	*045	*062	*078	*095	*111	*127	144
264	42	160	177	193	210	226	243	259	275	292	308
265		325	341	357	374	390	406	423	439	455	472
266		488	504	521	537	553	570	586	602	619	635
267		651	667	684	700	716	732	749	765	781	797
268		813	830	846	862	878	894	911	927	943	959
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43	136	152	169	185	201	217	233	249	265	281
271		297	313	329	345	361	377	393	409	425	441
272		457	473	489	505	521	537	553	569	584	600
273		616	632	648	664	680	696	712	727	743	759
274		775	791	807	823	838	854	870	886	902	917
275		933	949	965	981	996	*012	*028	*044	*059	*075
276	44	091	107	122	138	154	170	185	201	217	232
277		248	264	279	295	311	326	342	358	373	389
278		404	420	436	451	467	483	498	514	529	545
279		560	576	592	607	623	638	654	669	685	700
280		716	731	747	762	778	793	809	824	840	855
281		871	886	902	917	932	948	963	979	994	*010
282	45	025	040	056	071	086	102	117	133	148	163
283		179	194	209	225	240	255	271	286	301	317
284		332	347	362	378	393	408	423	439	454	469
285		484	500	515	530	545	561	576	591	606	621
286		637	652	667	682	697	712	728	743	758	773
287		788	803	818	834	849	864	879	894	909	924
288		939	954	969	984	*000	*015	*030	*045	*060	*075
289	46	090	105	120	135	150	165	180	195	210	225
290		240	255	270	285	300	315	330	345	359	374
291		389	404	419	434	449	464	479	494	509	523
292		538	553	568	583	598	613	627	642	657	672
293		687	702	716	731	746	761	776	790	805	820
294		835	850	864	879	894	909	923	938	953	967
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47	129	144	159	173	188	202	217	232	246	261
297		276	290	305	319	334	349	363	378	392	407
298		422	436	451	465	480	494	509	524	538	553
299		567	582	596	611	625	640	654	669	683	698
300		712	727	741	756	770	784	799	813	828	842
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

P. P.

Num. 300 to 339. Log. 477 to 531.

N	L	0	1	2	3	4	5	6	7	8	9
300	47	712	727	741	756	770	784	799	813	828	842
301		857	871	885	900	914	929	943	958	972	986
302	48	001	015	029	044	058	073	087	101	116	130
303		144	159	173	187	202	216	230	244	259	273
304		287	302	316	330	344	359	373	387	401	416
305		430	444	458	473	487	501	515	530	544	558
306		572	586	601	615	629	643	657	671	686	700
307		714	728	742	756	770	785	799	813	827	841
308		855	869	883	897	911	926	940	954	968	982
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122
310	49	136	150	164	178	192	206	220	234	248	262
311		276	290	304	318	332	346	360	374	388	402
312		415	429	443	457	471	485	499	513	527	541
313		554	568	582	596	610	624	638	651	665	679
314		693	707	721	734	748	762	776	790	803	817
315		831	845	859	872	886	900	914	927	941	955
316		969	982	996	*010	*024	*037	*051	*065	*079	*092
317	50	106	120	133	147	161	174	188	202	215	229
318		243	256	270	284	297	311	325	338	352	365
319		379	393	406	420	433	447	461	474	488	501
320		515	529	542	556	569	583	596	610	623	637
321		651	664	678	691	705	718	732	745	759	772
322		786	799	813	826	840	853	866	880	893	907
323		920	934	947	961	974	987	*001	*014	*028	*041
324	51	055	068	081	095	108	121	135	148	162	175
325		188	202	215	228	242	255	268	282	295	308
326		322	335	348	362	375	388	402	415	428	441
327		455	468	481	495	508	521	534	548	561	574
328		587	601	614	627	640	654	667	680	693	706
329		720	733	746	759	772	786	799	812	825	838
330		851	865	878	891	904	917	930	943	957	970
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101
332	52	114	127	140	153	166	179	192	205	218	231
333		244	257	270	284	297	310	323	336	349	362
334		375	388	401	414	427	440	453	466	479	492
335		504	517	530	543	556	569	582	595	608	621
336		634	647	660	673	686	699	711	724	737	750
337		763	776	789	802	815	827	840	853	866	879
338		892	905	917	930	943	956	969	982	994	*007
339	53	020	033	046	058	071	084	097	110	122	135
340		148	161	173	186	199	212	224	237	250	263
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

P. P.

Num. 340 to 379. Log. 531 to 579.

N	L	0	1	2	3	4	5	6	7	8	9
340	53	148	161	173	186	199	212	224	237	250	263
341		275	288	301	314	326	339	352	364	377	390
342		403	415	428	441	453	466	479	491	504	517
343		529	542	555	567	580	593	605	618	631	643
344		656	668	681	694	706	719	732	744	757	769
345		782	794	807	820	832	845	857	870	882	895
346		908	920	933	945	958	970	983	995	*008	*020
347	54	033	045	058	070	083	095	108	120	133	145
348		158	170	183	195	208	220	233	245	258	270
349		283	295	307	320	332	345	357	370	382	394
350		407	419	432	444	456	469	481	494	506	518
351		531	543	555	568	580	593	605	617	630	642
352		654	667	679	691	704	716	728	741	753	765
353		777	790	802	814	827	839	851	864	876	888
354		900	913	925	937	949	962	974	986	998	*011
355	55	023	035	047	060	072	084	096	108	121	133
356		145	157	169	182	194	206	218	230	242	255
357		267	279	291	303	315	328	340	352	364	376
358		388	400	413	425	437	449	461	473	485	497
359		509	522	534	546	558	570	582	594	606	618
360		630	642	654	666	678	691	703	715	727	739
361		751	763	775	787	799	811	823	835	847	859
362		871	883	895	907	919	931	943	955	967	979
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56	110	122	134	146	158	170	182	194	205	217
365		229	241	253	265	277	289	301	312	324	336
366		348	360	372	384	396	407	419	431	443	455
367		467	478	490	502	514	526	538	549	561	573
368		585	597	608	620	632	644	656	667	679	691
369		703	714	726	738	750	761	773	785	797	808
370		820	832	844	855	867	879	891	902	914	926
371		937	949	961	972	984	996	*008	*019	*031	*043
372	57	054	066	078	089	101	113	124	136	148	159
373		171	183	194	206	217	229	241	252	264	276
374		287	299	310	322	334	345	357	368	380	392
375		403	415	426	438	449	461	473	484	496	507
376		519	530	542	553	565	576	588	600	611	623
377		634	646	657	669	680	692	703	715	726	738
378		749	761	772	784	795	807	818	830	841	852
379		864	875	887	898	910	921	933	944	955	967
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

P. P.

Num. 380 to 419. Log. 579 to 623.

N	L	0	1	2	3	4	5	6	7	8	9
380	57	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58	092	104	115	127	138	149	161	172	184	195
382		206	218	229	240	252	263	274	286	297	309
383		320	331	343	354	365	377	388	399	410	422
384		433	444	456	467	478	490	501	512	524	535
385		546	557	569	580	591	602	614	625	636	647
386		659	670	681	692	704	715	726	737	749	760
387		771	782	794	805	816	827	838	850	861	872
388		883	894	906	917	928	939	950	961	973	984
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59	106	118	129	140	151	162	173	184	195	207
391		218	229	240	251	262	273	284	295	306	318
392		329	340	351	362	373	384	395	406	417	428
393		439	450	461	472	483	494	506	517	528	539
394		550	561	572	583	594	605	616	627	638	649
395		660	671	682	693	704	715	726	737	748	759
396		770	780	791	802	813	824	835	846	857	868
397		879	890	901	912	923	934	945	956	966	977
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60	097	108	119	130	141	152	163	173	184	195
400		206	217	228	239	249	260	271	282	293	304
401		314	325	336	347	358	369	379	390	401	412
402		423	433	444	455	466	477	487	498	509	520
403		531	541	552	563	574	584	595	606	617	627
404		638	649	660	670	681	692	703	713	724	735
405		746	756	767	778	788	799	810	821	831	842
406		853	863	874	885	895	906	917	927	938	949
407		959	970	981	991	*002	*013	*023	*034	*045	*055
408	61	066	077	087	098	109	119	130	140	151	162
409		172	183	194	204	215	225	236	247	257	268
410		278	289	300	310	321	331	342	352	363	374
411		384	395	405	416	426	437	448	458	469	479
412		490	500	511	521	532	542	553	563	574	584
413		595	606	616	627	637	648	658	669	679	690
414		700	711	721	731	742	752	763	773	784	794
415		805	815	826	836	847	857	868	878	888	899
416		909	920	930	941	951	962	972	982	993	*003
417	62	014	024	034	045	055	066	076	086	097	107
418		118	128	138	149	159	170	180	190	201	211
419		221	232	242	252	263	273	284	294	304	315
420		325	335	346	356	366	377	387	397	408	418
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

P. P.

Num. 420 to 459. Log. 623 to 662.

N	L	0	1	2	3	4	5	6	7	8	9
420	62	325	335	346	356	366	377	387	397	408	418
421		428	439	449	459	469	480	490	500	511	521
422		531	542	552	562	572	583	593	603	613	624
423		634	644	655	665	675	685	696	706	716	726
424		737	747	757	767	778	788	798	808	818	829
425		839	849	859	870	880	890	900	910	921	931
426		941	951	961	972	982	992	*002	*012	*022	*033
427	63	043	053	063	073	083	094	104	114	124	134
428		144	155	165	175	185	195	205	215	225	236
429		246	256	266	276	286	296	306	317	327	337
430		347	357	367	377	387	397	407	417	428	438
431		448	458	468	478	488	498	508	518	528	538
432		548	558	568	579	589	599	609	619	629	639
433		649	659	669	679	689	699	709	719	729	739
434		749	759	769	779	789	799	809	819	829	839
435		849	859	869	879	889	899	909	919	929	939
436		949	959	969	979	988	998	*008	*018	*028	*038
437	64	048	058	068	078	088	098	108	118	128	137
438		147	157	167	177	187	197	207	217	227	237
439		246	256	266	276	286	296	306	316	326	335
440		345	355	365	375	385	395	404	414	424	434
441		444	454	464	473	483	493	503	513	523	532
442		542	552	562	572	582	591	601	611	621	631
443		640	650	660	670	680	689	699	709	719	729
444		738	748	758	768	777	787	797	807	816	826
445		836	846	856	865	875	885	895	904	914	924
446		933	943	953	963	972	982	992	*002	*011	*021
447	65	031	040	050	060	070	079	089	099	108	118
448		128	137	147	157	167	176	186	196	205	215
449		225	234	244	254	263	273	283	292	302	312
450		321	331	341	350	360	369	379	389	398	408
451		418	427	437	447	456	466	475	485	495	504
452		514	523	533	543	552	562	571	581	591	600
453		610	619	629	639	648	658	667	677	686	696
454		706	715	725	734	744	753	763	772	782	792
455		801	811	820	830	839	849	858	868	877	887
456		896	906	916	925	935	944	954	963	973	982
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077
458	66	087	096	106	115	124	134	143	153	162	172
459		181	191	200	210	219	229	238	247	257	266
460		276	285	295	304	314	323	332	342	351	361
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

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1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Num. 460 to 499. Log. 662 to 698.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
460	66	276	285	295	304	314	323	332	342	351	361	
461		370	380	389	398	408	417	427	436	445	455	
462		464	474	483	492	502	511	521	530	539	549	
463		558	567	577	586	596	605	614	624	633	642	
464		652	661	671	680	689	699	708	717	727	736	
465		745	755	764	773	783	792	801	811	820	829	
466		839	848	857	867	876	885	894	904	913	922	
467		932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	089	099	108	
469		117	127	136	145	154	164	173	182	191	201	**10**
470		210	219	228	237	247	256	265	274	284	293	1 | 1.0
471		302	311	321	330	339	348	357	367	376	385	2 | 2.0
472		394	403	413	422	431	440	449	459	468	477	3 | 3.0
473		486	495	504	514	523	532	541	550	560	569	4 | 4.0
474		578	587	596	605	614	624	633	642	651	660	5 | 5.0
475		669	679	688	697	706	715	724	733	742	752	6 | 6.0
476		761	770	779	788	797	806	815	825	834	843	7 | 7.0
477		852	861	870	879	888	897	906	916	925	934	8 | 8.0
478		943	952	961	970	979	988	997	*006	*015	*024	9 | 9.0
479	68	034	043	052	061	070	079	088	097	106	115	
480		124	133	142	151	160	169	178	187	196	205	
481		215	224	233	242	251	260	269	278	287	296	
482		305	314	323	332	341	350	359	368	377	386	
483		395	404	413	422	431	440	449	458	467	476	
484		485	494	502	511	520	529	538	547	556	565	
485		574	583	592	601	610	619	628	637	646	655	**9**
486		664	673	681	690	699	708	717	726	735	744	1 | 0.9
487		753	762	771	780	789	797	806	815	824	833	2 | 1.8
488		842	851	860	869	878	886	895	904	913	922	3 | 2.7
489		931	940	949	958	966	975	984	993	*002	*011	4 | 3.6
490	69	020	028	037	046	055	064	073	082	090	099	5 | 4.5
491		108	117	126	135	144	152	162	170	179	188	6 | 5.4
492		197	205	214	223	232	241	249	258	267	276	7 | 6.3
493		285	294	302	311	320	329	338	346	355	364	8 | 7.2
494		373	381	390	399	408	417	425	434	443	452	9 | 8.1
495		461	469	478	487	496	504	513	522	531	539	
496		548	557	566	574	583	592	601	609	618	627	
497		636	644	653	662	671	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	801	
499		810	819	827	836	845	854	862	871	880	888	
500		897	906	914	923	932	940	949	958	966	975	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

Num. 500 to 539. Log. 698 to 732.

N	L	0	1	2	3	4	5	6	7	8	9
500	69	897	906	914	922	932	940	949	958	966	975
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062
502	70	070	079	088	096	105	114	122	131	140	148
503		157	165	174	183	191	200	209	217	226	234
504		243	252	260	269	278	286	295	303	312	321
505		329	338	346	355	364	372	381	389	398	406
506		415	424	432	441	449	458	467	475	484	492
507		501	509	518	526	535	544	552	561	569	578
508		586	595	603	612	621	629	638	646	655	663
509		672	680	689	697	706	714	723	731	740	749
510		757	766	774	783	791	800	808	817	825	834
511		842	851	859	868	876	885	893	902	910	919
512		927	935	944	952	961	969	978	986	995	*003
513	71	012	020	029	037	046	054	063	071	079	088
514		096	105	113	122	130	139	147	155	164	172
515		181	189	198	206	214	223	231	240	248	257
516		265	273	282	290	299	307	315	324	332	341
517		349	357	366	374	383	391	399	408	416	425
518		433	441	450	458	466	475	483	492	500	508
519		517	525	533	542	550	559	567	575	584	592
520		600	609	617	625	634	642	650	659	667	675
521		684	692	700	709	717	725	734	742	750	759
522		767	775	784	792	800	809	817	825	834	842
523		850	858	867	875	883	892	900	908	917	925
524		933	941	950	958	966	975	983	991	999	*008
525	72	016	024	032	041	049	057	066	074	082	090
526		099	107	115	123	132	140	148	156	165	173
527		181	189	198	206	214	222	230	239	247	255
528		263	272	280	288	296	304	313	321	329	337
529		346	354	362	370	378	387	395	403	411	419
530		428	436	444	452	460	469	477	485	493	501
531		509	518	526	534	542	550	558	567	575	583
532		591	599	607	616	624	632	640	648	656	665
533		673	681	689	697	705	713	722	730	738	746
534		754	762	770	779	787	795	803	811	819	827
535		835	843	852	860	868	876	884	892	900	908
536		916	925	933	941	949	957	965	973	981	989
537		997	*006	*014	*022	*030	*038	*046	*054	062	*070
538	73	078	086	094	102	111	119	127	135	143	151
539		159	167	175	183	191	199	207	215	223	231
540		239	247	255	263	272	280	288	296	304	312
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

P. P.

Num. 540 to 579. Log. 732 to 763.

N	L	0	1	2	3	4	5	6	7	8	9
540	73	239	247	255	263	272	280	288	296	304	312
541		320	328	336	344	352	360	368	376	384	392
542		400	408	416	424	432	440	448	456	464	472
543		480	488	496	504	512	520	528	536	544	552
544		560	568	576	584	592	600	608	616	624	632
545		640	648	656	664	672	679	687	695	703	711
546		719	727	735	743	751	759	767	775	783	791
547		799	807	815	823	830	838	846	854	862	870
548		878	886	894	902	910	918	926	933	941	949
549		957	965	973	981	989	997	*005	*013	*020	*028
550	74	036	044	052	060	068	076	084	092	099	107
551		115	123	131	139	147	155	162	170	178	186
552		194	202	210	218	225	233	241	249	257	265
553		273	280	288	296	304	312	320	327	335	343
554		351	359	367	374	382	390	398	406	414	421
555		429	437	445	453	461	468	476	484	492	500
556		507	515	523	531	539	547	554	562	570	578
557		586	593	601	609	617	624	632	640	648	656
558		663	671	679	687	695	702	710	718	726	733
559		741	749	757	764	772	780	788	796	803	811
560		819	827	834	842	850	858	865	873	881	889
561		896	904	912	920	927	935	943	950	958	966
562		974	981	989	997	*005	*012	*020	*028	*035	*043
563	75	051	059	066	074	082	089	097	105	113	120
564		128	136	143	151	159	166	174	182	189	197
565		205	213	220	228	236	243	251	259	266	274
566		282	289	297	305	312	320	328	335	343	351
567		358	366	374	381	389	397	404	412	420	427
568		435	442	450	458	465	473	481	488	496	504
569		511	519	526	534	542	549	557	565	572	580
570		587	595	603	610	618	626	633	641	648	656
571		664	671	679	686	694	702	709	717	724	732
572		740	747	755	762	770	778	785	793	800	808
573		815	823	831	838	846	853	861	868	876	884
574		891	899	906	914	921	929	937	944	952	959
575		967	974	982	989	997	*005	*012	*020	*027	*035
576	76	042	050	057	065	072	080	087	095	103	110
577		118	125	133	140	148	155	163	170	178	185
578		193	200	208	215	223	230	238	245	253	260
579		268	275	283	290	298	305	313	320	328	335
580		343	350	358	365	373	380	388	395	403	410
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

P. P.

Num. 580 to 619. Log. 763 to 792.

N	L	0	1	2	3	4	5	6	7	8	9
580	76	343	350	358	365	373	380	388	395	403	410
581		418	425	433	440	448	455	462	470	477	485
582		492	500	507	515	522	530	537	545	552	559
583		567	574	582	589	597	604	612	619	626	634
584		641	649	656	664	671	678	686	693	701	708
585		716	723	730	738	745	753	760	768	775	782
586		790	797	805	812	819	827	834	842	849	856
587		864	871	879	886	893	901	908	916	923	930
588		938	945	953	960	967	975	982	989	997	*004
589	77	012	019	026	034	041	048	056	063	070	078
590		085	093	100	107	115	122	129	137	144	151
591		159	166	173	181	188	195	203	210	217	225
592		232	240	247	254	262	269	276	283	291	298
593		305	313	320	327	335	342	349	357	364	371
594		379	386	393	401	408	415	422	430	437	444
595		452	459	466	474	481	488	495	503	510	517
596		525	532	539	546	554	561	568	576	583	590
597		597	605	612	619	627	634	641	648	656	663
598		670	677	685	692	699	706	714	721	728	735
599		743	750	757	764	772	779	786	793	801	808
600		815	822	830	837	844	851	859	866	873	880
601		887	895	902	909	916	924	931	938	945	952
602		960	967	974	981	988	996	*003	*010	*017	*025
603	78	032	039	046	053	061	068	075	082	089	097
604		104	111	118	125	132	140	147	154	161	168
605		176	183	190	197	204	211	219	226	233	240
606		247	254	262	269	276	283	290	297	305	312
607		319	326	333	340	347	355	362	369	376	383
608		390	398	405	412	419	426	433	440	447	455
609		462	469	476	483	490	497	504	512	519	526
610		533	540	547	554	561	569	576	583	590	597
611		604	611	618	625	633	640	647	654	661	668
612		675	682	689	696	704	711	718	725	732	739
613		746	753	760	767	774	781	789	796	802	810
614		817	824	831	838	845	852	859	866	873	880
615		888	895	902	909	916	923	930	937	944	951
616		958	965	972	979	986	993	*000	*007	*014	*021
617	79	029	036	043	050	057	064	071	078	085	092
618		099	106	113	120	127	134	141	148	155	162
619		169	176	183	190	197	204	211	218	225	232
620		239	246	253	260	267	274	281	288	295	302
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	8
1	0.8
2	1.5
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

P. P.

Num. 620 to 659. Log. 792 to 819.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
620	79	239	246	253	260	267	274	281	288	295	302	
621		309	316	323	330	337	344	351	358	365	372	
622		379	386	393	400	407	414	421	428	435	442	
623		449	456	463	470	477	484	491	498	505	511	
624		518	525	532	539	546	553	560	567	574	581	
625		588	595	602	609	616	623	630	637	644	650	
626		657	664	671	678	685	692	699	706	713	720	
627		727	734	741	748	754	761	768	775	782	789	
628		796	803	810	817	824	831	837	844	851	858	
629		865	872	879	886	893	900	906	913	920	927	
630		934	941	948	955	962	969	975	982	989	996	
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	085	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	
637		414	421	428	434	441	448	455	462	468	475	
638		482	489	496	502	509	516	523	530	536	543	
639		550	557	564	570	577	584	591	598	604	611	
640		618	625	632	638	645	652	659	665	672	679	
641		686	693	699	706	713	720	726	733	740	747	
642		754	760	767	774	781	787	794	801	808	814	
643		821	828	835	841	848	855	862	868	875	882	
644		889	895	902	909	916	922	929	936	943	949	
645		956	963	969	976	983	990	996	*003	*010	*017	
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	238	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

Num. 660 to 699. Log. 819 to 845.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
660	81	954	961	968	974	981	987	994	*000	*007	*014	7
661	82	020	027	033	040	046	053	060	066	073	079	1 0.7
662		086	092	099	105	112	119	125	132	138	145	2 1.4
663		151	158	164	171	178	184	191	197	204	210	3 2.1
664		217	223	230	236	243	249	256	263	269	276	4 2.8
665		282	289	295	302	308	315	321	328	334	341	5 3.5
666		347	354	360	367	373	380	387	393	400	406	6 4.2
667		413	419	426	432	439	445	452	458	465	471	7 4.9
668		478	484	491	497	504	510	517	523	530	536	8 5.6
669		543	549	556	562	569	575	582	588	595	601	9 6.3
670		607	614	620	627	633	640	646	653	659	666	
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	6
678		123	129	136	142	149	155	161	168	174	181	1 0.6
679		187	193	200	206	213	219	225	232	238	245	2 1.2
680		251	257	264	270	276	283	289	296	302	308	3 1.8
681		315	321	327	334	340	347	353	359	366	372	4 2.4
682		378	385	391	398	404	410	417	423	429	436	5 3.0
683		442	448	455	461	467	474	480	487	493	499	6 3.6
684		506	512	518	525	531	537	544	550	556	563	7 4.2
685		569	575	582	588	594	601	607	613	620	626	8 4.8
686		632	639	645	651	658	664	670	677	683	689	9 5.4
687		696	702	708	715	721	727	734	740	746	753	
688		759	765	771	778	784	790	797	803	809	816	
689		822	828	835	841	847	853	860	866	872	879	
690		885	891	897	904	910	916	923	929	935	942	
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

Num. 700 to 739. Log. 845 to 869.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
700	84	510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	
709		065	071	077	083	089	095	101	107	114	120	
710		126	132	138	144	150	156	163	169	175	181	
711		187	193	199	205	211	217	224	230	236	242	
712		248	254	260	266	272	278	285	291	297	303	
713		309	315	321	327	333	339	345	352	358	364	
714		370	376	382	388	394	400	406	412	418	425	
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724		974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Num. 740 to 779. Log. 869 to 892.

N	L	0	1	2	3	4	5	6	7	8	9
740	86	923	929	935	941	947	953	958	964	970	976
741		982	988	994	999	*005	*011	*017	*023	*029	*035
742	87	040	046	052	058	064	070	075	081	087	093
743		099	105	111	116	122	128	134	140	146	151
744		157	163	169	175	181	186	192	198	204	210
745		216	221	227	233	239	245	251	256	262	268
746		274	280	286	291	297	303	309	315	320	326
747		332	338	344	349	355	361	367	373	379	384
748		390	396	402	408	413	419	425	431	437	442
749		448	454	460	466	471	477	483	489	495	500
750		506	512	518	523	529	535	541	547	552	558
751		564	570	576	581	587	593	599	604	610	616
752		622	628	633	639	645	651	656	662	668	674
753		679	685	691	697	703	708	714	720	726	731
754		737	743	749	754	760	766	772	777	783	789
755		795	800	806	812	818	823	829	835	841	846
756		852	858	864	869	875	881	887	892	898	904
757		910	915	921	927	933	938	944	950	955	961
758		967	973	978	984	990	996	*001	*007	*013	*018
759	88	024	030	036	041	047	053	058	064	070	076
760		081	087	093	098	104	110	116	121	127	133
761		138	144	150	156	161	167	173	178	184	190
762		195	201	207	213	218	224	230	235	241	247
763		252	258	264	270	275	281	287	292	298	304
764		309	315	321	326	332	338	343	349	355	360
765		366	372	377	383	389	395	400	406	412	417
766		423	429	434	440	446	451	457	463	468	474
767		480	485	491	497	502	508	513	519	525	530
768		536	542	547	553	559	564	570	576	581	587
769		593	598	604	610	615	621	627	632	638	643
770		649	655	660	666	672	677	683	689	694	700
771		705	711	717	722	728	734	739	745	750	756
772		762	767	773	779	784	790	795	801	807	812
773		818	824	829	835	840	846	852	857	863	868
774		874	880	885	891	897	902	908	913	919	925
775		930	936	941	947	953	958	964	969	975	981
776		986	992	997	*003	*009	*014	*020	*025	*031	*037
777	89	042	048	053	059	064	070	076	081	087	092
778		098	104	109	115	120	126	131	137	143	148
779		154	159	165	170	176	182	187	193	198	204
780		209	215	221	226	232	237	243	248	254	260
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Num. 780 to 819. Log. 892 to 913.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
780	89	209	215	221	226	232	237	243	248	254	260	
781		265	271	276	282	287	293	298	304	310	315	
782		321	326	332	337	343	348	354	360	365	371	
783		376	382	387	393	398	404	409	415	421	426	
784		432	437	443	448	454	459	465	470	476	481	
785		487	492	498	504	509	515	520	526	531	537	
786		542	548	553	559	564	570	575	581	586	592	
787		597	603	609	614	620	625	631	636	642	647	
788		653	658	664	669	675	680	686	691	697	702	
789		708	713	719	724	730	735	741	746	752	757	
790		763	768	774	779	785	790	796	801	807	812	
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	
811		902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	
813	91	009	014	020	025	030	036	041	046	052	057	
814		062	068	073	078	084	089	094	100	105	110	
815		116	121	126	132	137	142	148	153	158	164	
816		169	174	180	185	190	196	201	206	212	217	
817		222	228	233	238	243	249	254	259	265	270	
818		275	281	286	291	297	302	307	312	318	323	
819		328	334	339	344	350	355	360	365	371	376	
820		381	387	392	397	403	408	413	418	424	429	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Num. 820 to 859. Log. 913 to 934.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
820	91	381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	
831		960	965	971	976	981	986	991	997	*002	*007	
832	92	012	018	023	028	033	038	044	049	054	059	
833		065	070	075	080	085	091	096	101	106	111	
834		117	122	127	132	137	143	148	153	158	163	
835		169	174	179	184	189	195	200	205	210	215	
836		221	226	231	236	241	247	252	257	262	267	
837		273	278	283	288	293	298	304	309	314	319	
838		324	330	335	340	345	350	355	361	366	371	
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
851		993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	425	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Num. 860 to 899. Log. 934 to 954.

N	L	0	1	2	3	4	5	6	7	8	9
860	93	450	455	460	465	470	475	480	485	490	495
861		500	505	510	515	520	526	531	536	541	546
862		551	556	561	566	571	576	581	586	591	596
863		601	606	611	616	621	626	631	636	641	646
864		651	656	661	666	671	676	682	687	692	697
865		702	707	712	717	722	727	732	737	742	747
866		752	757	762	767	772	777	782	787	792	797
867		802	807	812	817	822	827	832	837	842	847
868		852	857	862	867	872	877	882	887	892	897
869		902	907	912	917	922	927	932	937	942	947
870		952	957	962	967	972	977	982	987	992	997
871	94	002	007	012	017	022	027	032	037	042	047
872		052	057	062	067	072	077	082	086	091	096
873		101	106	111	116	121	126	131	136	141	146
874		151	156	161	166	171	176	181	186	191	196
875		201	206	211	216	221	226	231	236	240	245
876		250	255	260	265	270	275	280	285	290	295
877		300	305	310	315	320	325	330	335	340	345
878		349	354	359	364	369	374	379	384	389	394
879		399	404	409	414	419	424	429	433	438	443
880		448	453	458	463	468	473	478	483	488	493
881		498	503	507	512	517	522	527	532	537	542
882		547	552	557	562	567	571	576	581	586	591
883		596	601	606	611	616	621	626	630	635	640
884		645	650	655	660	665	670	675	680	685	689
885		694	699	704	709	714	719	724	729	734	738
886		743	748	753	758	763	768	773	778	783	787
887		792	797	802	807	812	817	822	827	832	836
888		841	846	851	856	861	866	871	876	880	885
889		890	895	900	905	910	915	919	924	929	934
890		939	944	949	954	959	963	968	973	978	983
891		988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95	036	041	046	051	056	061	066	071	075	080
893		085	090	095	100	105	109	114	119	124	129
894		134	139	143	148	153	158	163	168	173	177
895		182	187	192	197	202	207	211	216	221	226
896		231	236	240	245	250	255	260	265	270	274
897		279	284	289	294	299	303	308	313	318	323
898		328	332	337	342	347	352	357	361	366	371
899		376	381	386	390	395	400	405	410	415	419
900		424	429	434	439	444	448	453	458	463	468
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Num. 900 to 939. Log. 954 to 973.

N	L	0	1	2	3	4	5	6	7	8	9	P. P.
900	95	424	429	434	439	444	448	453	458	463	468	
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96	047	052	057	061	066	071	076	080	085	090	
914		095	099	104	109	114	118	123	128	133	137	
915		142	147	152	156	161	166	171	175	180	185	
916		190	194	199	204	209	213	218	223	227	232	
917		237	242	246	251	256	261	265	270	275	280	
918		284	289	294	298	303	308	313	317	322	327	
919		332	336	341	346	350	355	360	365	369	374	
920		379	384	388	393	398	402	407	412	417	421	
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97	035	039	044	049	053	058	063	067	072	077	
935		081	086	090	095	100	104	109	114	118	123	
936		128	132	137	142	146	151	155	160	165	169	
937		174	179	183	188	192	197	202	206	211	216	
938		220	225	230	234	239	243	248	253	257	262	
939		267	271	276	280	285	290	294	299	304	308	
940		313	317	322	327	331	336	340	345	350	354	
N	L	0	1	2	3	4	5	6	7	8	9	P. P.

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Num. 940 to 979. Log. 973 to 991.

N	L	0	1	2	3	4	5	6	7	8	9
940	97	313	317	322	327	331	336	340	345	350	354
941		359	364	368	373	377	382	387	391	396	400
942		405	410	414	419	424	428	433	437	442	447
943		451	456	460	465	470	474	479	483	488	493
944		497	502	506	511	516	520	525	529	534	539
945		543	548	552	557	562	566	571	575	580	585
946		589	594	598	603	607	612	617	621	626	630
947		635	640	644	649	653	658	663	667	672	676
948		681	685	690	695	699	704	708	713	717	722
949		727	731	736	740	745	749	754	759	763	768
950		772	777	782	786	791	795	800	804	809	813
951		818	823	827	832	836	841	845	850	855	859
952		864	868	873	877	882	886	891	896	900	905
953		909	914	918	923	928	932	937	941	946	950
954		955	959	964	968	973	978	982	987	991	996
955	98	000	005	009	014	019	023	028	032	037	041
956		046	050	055	059	064	068	073	078	082	087
957		091	096	100	105	109	114	118	123	127	132
958		137	141	146	150	155	159	164	168	173	177
959		182	186	191	195	200	204	209	214	218	223
960		227	232	236	241	245	250	254	259	263	268
961		272	277	281	286	290	295	299	304	308	313
962		318	322	327	331	336	340	345	349	354	358
963		363	367	372	376	381	385	390	394	399	403
964		408	412	417	421	426	430	435	439	444	448
965		453	457	462	466	471	475	480	484	489	493
966		498	502	507	511	516	520	525	529	534	538
967		543	547	552	556	561	565	570	574	579	583
968		588	592	597	601	605	610	614	619	623	628
969		632	637	641	646	650	655	659	664	668	673
970		677	682	686	691	695	700	704	709	713	717
971		722	726	731	735	740	744	749	753	758	762
972		767	771	776	780	784	789	793	798	802	807
973		811	816	820	825	829	834	838	843	847	851
974		856	860	865	869	874	878	883	887	892	896
975		900	905	909	914	918	923	927	932	936	941
976		945	949	954	958	963	967	972	976	981	985
977		989	994	998	*003	*007	*012	*016	*021	*025	*029
978	99	034	038	043	047	052	056	061	065	069	074
979		078	083	087	092	096	100	105	109	114	118
980		123	127	131	136	140	145	149	154	158	162
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Num. 980 to 1000. Log. 991 to 999.

N	L	0	1	2	3	4	5	6	7	8	9
980	99	123	127	131	136	140	145	149	154	158	162
981		167	171	176	180	185	189	193	198	202	207
982		211	216	220	224	229	233	238	242	247	251
983		255	260	264	269	273	277	282	286	291	295
984		300	304	308	313	317	322	326	330	335	339
985		344	348	352	357	361	366	370	374	379	383
986		388	392	396	401	405	410	414	419	423	427
987		432	436	441	445	449	454	458	463	467	471
988		476	480	484	489	493	498	502	506	511	515
989		520	524	528	533	537	542	546	550	555	559
990		564	568	572	577	581	585	590	594	599	603
991		607	612	616	621	625	629	634	638	642	647
992		651	656	660	664	669	673	677	682	686	691
993		695	699	704	708	712	717	721	726	730	734
994		739	743	747	752	756	760	765	769	774	778
995		782	787	791	795	800	804	808	813	817	822
996		826	830	835	839	843	848	852	856	861	865
997		870	874	878	883	887	891	896	900	904	909
998		913	917	922	926	930	935	939	944	948	952
999		957	961	965	970	974	978	983	987	991	996
1000	000	000	043	087	130	174	217	260	304	347	391
N	L	0	1	2	3	4	5	6	7	8	9

P. P.

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Logarithms of Important Numbers.

Number.	Logarithm.
$\pi = 3.141\ 593$	$0.497\ 150$
$\frac{4}{3}\pi = 4.188\ 790$	$0.622\ 089$
$\frac{1}{6}\pi = 0.523\ 599$	$\bar{1}.718\ 999$
$\frac{1}{\pi} = 0.318\ 310$	$\bar{1}.502\ 850$
$\pi^2 = 9.869\ 604$	$0.994\ 300$
$\frac{1}{\pi^2} = 0.101\ 321$	$\bar{1}.005\ 700$
$\sqrt{\pi} = 1.772\ 454$	$0.248\ 575$
$\frac{1}{\sqrt{\pi}} = 0.564\ 190$	$\bar{1}.751\ 425$
$\sqrt[3]{\pi} = 1.464\ 592$	$0.165\ 717$
$\frac{1}{\sqrt[3]{\pi}} = 0.682\ 784$	$\bar{1}.834\ 283$
$\sqrt[3]{\frac{6}{\pi}} = 1.240\ 701$	$0.093\ 667$

GEOMETRY.

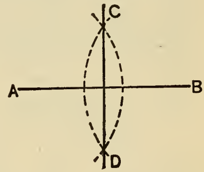
No attempt will be made to give the successive propositions of geometry, as these can be found in the standard text-books. Instead will be given such constructions as will be found useful to the engineer, followed by the mensuration of bodies in one, two, and three dimensions.

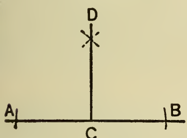
A straight line is usually best obtained by the use of a straight edge, such as a T square, or one of the sides of a draughtsman's triangle. In some cases, however, when very long centre lines are required, it is not advisable to place too much reliance upon any long straight edge. A fine thread tightly stretched between points may be used to advantage, a comparatively short straight edge being used to connect points marked off upon the line of the thread.

Right angles are best obtained by use of a draughtsman's triangle or set square, but where this is not available, or where the angle is to be laid out upon a large scale, as upon the ground or on a floor, the following constructions may be found useful:

To divide a given line, *AB*, into two equal parts, and to erect a perpendicular through the middle:

With the ends, *A* and *B*, as centres, draw the dotted circle arcs with a radius greater than half the line. Through the crossings of the arcs draw the perpendicular, *CD*, which divides the line into two equal parts.



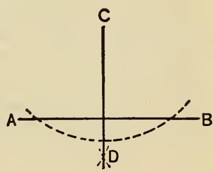


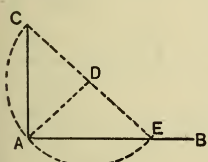
From a given point, *C*, on the line, *AB*, to erect a perpendicular, *CD:*

With *C* as a centre, find two points, *A* and *B*, on the line at equal distances from *C*. With *A* and *B* as centres, draw the dotted circle arcs at *D*. From the crossing, *D*, draw the required perpendicular, *DC*.

From a given point, *C*, at a distance from the line, *AB*, to draw a perpendicular to the line:

With *C* as a centre, draw the dotted circle arc so that it cuts the line at *A* and *B*. With *A* and *B* as centres, draw the dotted cross arcs at *D* with equal radii. Draw the required perpendicular through *C* and *D*.

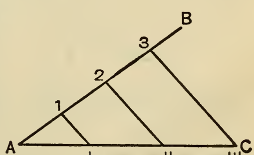




At the end, *A*, of a given line, *AB*, to erect a perpendicular, *AC:*

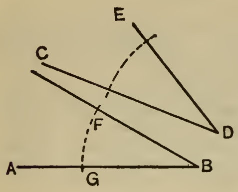
With any point, *D*, as a centre at a distance from the line, and with *AD* as radius, draw the dotted circle arc so that it cuts the line at *E;* through *E* and *D* draw the diameter, *EC;* then join *C* and *A*, which will give the required perpendicular.

The division of a line into any required number of parts may best be done by continual bisection as far as possible. When the limit has been reached in this manner the portions thus obtained may be divided by trial and error, using fine dividers with screw adjustment, or the following construction may be used:



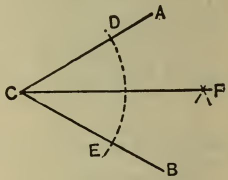
Let it be required to divide *AC* into three equal parts. From *A* draw any convenient line, *AB*, and on it step off with the dividers any equal spaces, 1, 2, 3. Then join 3 with *C*, and draw from 1 and 2 lines parallel to 3*C;* these will divide *AC* into three equal parts at I, II, and III.

Constructions with Angles.



On a given line, AB, and at the point, B, to construct an angle equal to the angle, CDE:

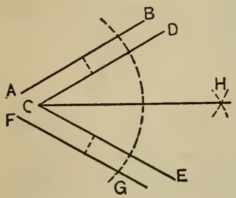
With D as a centre, draw the dotted arc, CE; and with the same radius and B as a centre, draw the arc, GF; then make GF equal to CE; then join BF, which will form the required angle, $FBG = CDE$.



To divide the angle, ACB, into two equal parts:

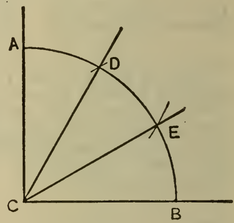
With C as a centre, draw the dotted arc, DE; with D and E as centres, draw the cross arcs at F with equal radii. Join CF, which divides the angle into the required parts.

Angles: $ACF = FCB = \frac{1}{2}(ACB)$.



To divide an angle into two equal parts when the lines do not extend to a meeting point:

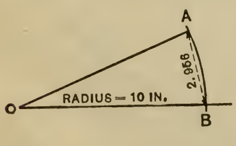
Draw the lines CD and CE parallel and at equal distances from the lines AB and FG. With C as a centre, draw the dotted arc, BG; and with B and G as centres, draw the cross arcs, H. Join CH, which divides the angle into the required equal parts.



To trisect a *right* angle, ACB:

With any convenient radius, CB, strike a circular arc with C as a centre. With the dividers open to the same radius sweep short arcs from A and B as centres. These will intersect the first arc at D and E. Through E and D draw lines passing through C. These lines will trisect the right angle, ACB.

Angles may be laid off by means of a protractor graduated in degrees and subdivisions, but unless the instrument is accurately made and carefully used the results are not very reliable. A more accurate method is to use a table of chords. With the dividers open to any convenient distance sweep an arc. Multiply the radius used by the tabular value in the table on page 107 for the required angle, and sweep the distance as a chord upon the arc. The remaining side of the angle may then be drawn.



A convenient radius is 10 inches. Considering this as units, one inch will be 0.1, one-tenth of an inch will be 0.01, and one-hundredth of an inch will be 0.001, and the chord may be taken directly from the table: Thus, for an angle of 17° we have from the table a value for the chord of 0.2956, or for a radius of 10 the chord is 2.956, and the angle, $AOB = 17°$, laid out far more accurately than would be possible with any but the most elaborate protractor of large size. When the chord can be taken on a vernier scale the length can readily be taken to as high a degree of precision as can be laid out on a drawing-board. Intermediate values may be taken by direct proportion.

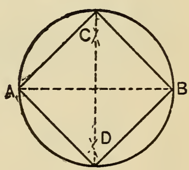
The chord for any angle is equal to twice the sine of half of the angle.

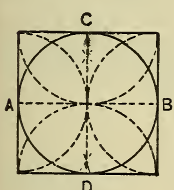
Table of Chords.

Deg.	Chord.	Deg.	Chord.	Deg.	Chord.	Deg.	Chord.	Deg.	Chord.
1	0.0175	19	0.3301	37	0.6346	55	0.9235	73	1.1896
2	0.0349	20	0.3473	38	0.6511	56	0.9389	74	1.2036
3	0.0524	21	0.3645	39	0.6676	57	0.9543	75	1.2175
4	0.0698	22	0.3816	40	0.6840	58	0.9696	76	1.2313
5	0.0872	23	0.3987	41	0.7004	59	0.9848	77	1.2450
6	0.1047	24	0.4158	42	0.7167	60	1.0000	78	1.2586
7	0.1221	25	0.4329	43	0.7330	61	1.0151	79	1.2722
8	0.1395	26	0.4499	44	0.7492	62	1.0301	80	1.2856
9	0.1569	27	0.4669	45	0.7654	63	1.0450	81	1.2989
10	0.1743	28	0.4838	46	0.7815	64	1.0598	82	1.3121
11	0.1917	29	0.5008	47	0.7975	65	1.0746	83	1.3252
12	0.2091	30	0.5176	48	0.8135	66	1.0893	84	1.3383
13	0.2264	31	0.5345	49	0.8294	67	1.1039	85	1.3512
14	0.2437	32	0.5513	50	0.8452	68	1.1184	86	1.3640
15	0.2611	33	0.5680	51	0.8610	69	1.1328	87	1.3767
16	0.2783	34	0.5847	52	0.8767	70	1.1472	88	1.3893
17	0.2956	35	0.6014	53	0.8924	71	1.1614	89	1.4018
18	0.3129	36	0.6180	54	0.9080	72	1.1756	90	1.4142

Construction of Polygons.

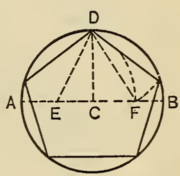
To inscribe a square in a given circle:
Draw the diameter, *AB*, and through the centre erect the perpendicular, *CD*, and complete the square as shown in the illustration.

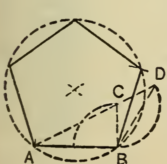




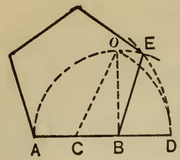
To describe a square about a given circle:
Draw the diameters, *AB* and *CD*, at right angles to one another; with the radius of the circle, and *A*, *B*, *C*, and *D* as centres, draw the four dotted half circles which cross one another in the corners of the square, and thus solve the problem.

To inscribe a pentagon in a given circle:
Draw the diameter, *AB*, and from the centre, *C*, erect the perpendicular, *CD*. Bisect the radius, *AC*, at *E;* with *E* as centre, and *DE* as radius, draw the arc, *DF*, and the straight line, *DF*, is the length of the side of the pentagon.



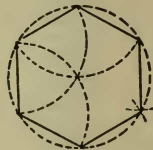


To construct a pentagon on a given side, *AB:*
From *B* erect *BC* perpendicular to and half the length of *AB;* draw a line from *A* through *C* and beyond; with *C* as a centre and *CB* as radius, draw the arc, *BD*, cutting this last line at *D;* then the chord, *BD*, is the radius of the circle circumscribing the pentagon. With *A* and *B* as centres, and *BD* as radius, draw the cross in the centre.



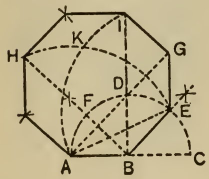
To construct a pentagon on a given side, *AB*, without resort to its centre:

From *B* erect *Bo* perpendicular and equal to *AB;* with *C* as centre and *Co* as radius, draw the arc, *Do;* then *AD* is the diagonal of the pentagon. With *AD* as radius and *A* as centre, draw the arc, *DE;* and with *B* as centre and *AB* as radius, finish the cross, *E*, and thus complete the pentagon.



To construct a hexagon in a given circle:

The radius of the circle is equal to the side of the hexagon.

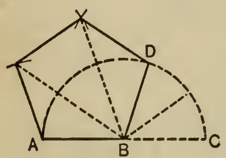


To construct an octagon on the given line, *AB:*

Prolong *AB* through *B.* With *B* as centre and *AB* as radius, draw semicircle, *AFDEC;* from *B*, draw *BI* at right angles to *AB;* divide the angles, *ABD* and *DBC*, each into two equal parts; then *BE* is one side of the octagon. With *A* and *E* as centres, and radius *AE*, draw the arcs, *HKE* and *AKI*, which determine the points *H* and *I*, and thus complete the octagon as shown in the illustration.

To cut off the corners of a square so as to make a regular octagon:

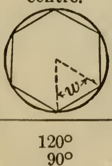
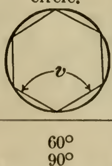
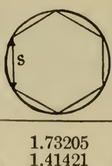
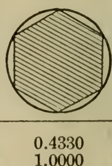
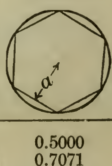
With the corners as centres, draw circle arcs through the centre of the square to the sides, which determines the sides of the octagon.



To construct any regular polygon on a given line, *AB*, without resort to its centre:

Extend *AB* through *B*, and, with *B* as a centre, draw the half circle, *ADC*. Divide the half circle into as many parts as the number of sides in the polygon, and complete the construction as shown in the illustration.

Table of Polygons.

No. of sides.	Angle of centre. 	Angle of circle. 	Side. 	Area. 	Apothem. 
3	120°	60°	1.73205	0.4330	0.5000
4	90°	90°	1.41421	1.0000	0.7071
5	72°	108°	1.17557	1.7205	0.8090
6	60°	120°	1.00000	2.5980	0.8660
7	51° 25′ 42″	128° 17′	0.86776	3.6339	0.9009
8	45°	135°	0.76536	4.8284	0.9238
9	40°	140°	0.68404	6.1820	0.9396
10	36°	144°	0.61801	7.6942	0.9510
11	32° 43′ 38″	147° 47′	0.56346	9.3656	0.9595
12	30°	150°	0.51763	11.1961	0.9659

To find the length of side of any polygon, multiply the radius of the circumscribing circle by the tabular number. To find the arc, multiply the square of the side by the tabular number. To find the apothem, multiply radius of circumscribing circle by tabular number.

The Circle.

Notation.

d = diameter of the circle.
r = radius of the circle.
p = periphery or circumference.
a = area of a circle or part thereof.
b = length of a circle-arc.
c = chord of a segment, length of.
h = height of a segment.
s = side of a regular polygon.
v = centre angle.
w = polygon angle.

All measures must be expressed in terms of the same unit.

Formulas for the Circle.

Periphery or Circumference.	*Diameter and Radius.*	*Area of the Circle.*
$p = \pi d = 3.14d.$	$d = \frac{p}{\pi} = \frac{p}{3.14}.$	$a = \frac{\pi d^2}{4} = 0.7854d^2.$
$p = 2\pi r = 6.28r.$	$r = \frac{p}{2\pi} = \frac{p}{6.28}.$	$a = \pi r^2 = 3.14r^2.$
$p = 2\sqrt{\pi a} = 3.54\sqrt{a}.$	$d = 2\sqrt{\frac{a}{\pi}} = 1.128\sqrt{a}.$	$a = \frac{p^2}{4\pi} = \frac{p^2}{12.56}.$
$p = \frac{2a}{r} = \frac{4a}{d}.$	$r = \sqrt{\frac{a}{\pi}} = 0.564\sqrt{a}.$	$a = \frac{pr}{2} = \frac{pd}{4}.$

$$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279\ 502\ 884\ 197\ 169\ 399$$

$2\pi = 6.283\ 185$	$\frac{1}{4}\pi = 0.785\ 398$	$\frac{1}{\pi} = 0.318\ 310$	$\frac{360}{\pi} = 114.5915$
$3\pi = 9.424\ 778$	$\frac{1}{3}\pi = 1.047\ 197$	$\frac{2}{\pi} = 0.636\ 619$	$\pi^2 = 9.869\ 650$
$4\pi = 12.566\ 370$	$\frac{1}{2}\pi = 1.570\ 796$	$\frac{3}{\pi} = 0.954\ 929$	$\sqrt{\pi} = 1.772\ 453$
$5\pi = 15.707\ 963$	$\frac{1}{8}\pi = 0.392\ 699$	$\frac{4}{\pi} = 1.273\ 239$	$\sqrt{\frac{1}{\pi}} = 0.564\ 189$
$6\pi = 18.849\ 556$	$\frac{1}{6}\pi = 0.523\ 599$	$\frac{6}{\pi} = 1.909\ 859$	$\sqrt{\frac{\pi}{2}} = 1.253\ 314$
$7\pi = 21.991\ 148$	$\frac{1}{12}\pi = 0.261\ 799$	$\frac{8}{\pi} = 2.546\ 478$	$\sqrt{\frac{2}{\pi}} = 0.797\ 884$
$8\pi = 25.132\ 741$	$\frac{2}{3}\pi = 2.094\ 394$	$\frac{12}{\pi} = 3.819\ 718$	
$9\pi = 28.274\ 334$	$\frac{1}{360}\pi = 0.008\ 726$		

Log. $\pi = 0.497\ 149\ 872\ 69413$

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
1	3.1416	0.7854	51	160.22	2042.8	101	317.30	8011.9
2	6.2832	3.1416	52	163.36	2123.7	102	320.44	8171.3
3	9.4248	7.0686	53	166.50	2206.2	103	323.58	8332.3
4	12.566	12.5664	54	169.65	2290.2	104	326.73	8494.9
5	15.708	19.6350	55	172.79	2375.8	105	329.87	8659.0
6	18.850	28.2743	56	175.93	2463.0	106	333.01	8824.7
7	21.991	38.4845	57	179.07	2551.8	107	336.15	8992.0
8	25.133	50.2655	58	182.21	2642.1	108	339.29	9160.9
9	28.274	63.6173	59	185.35	2734.0	109	342.43	9331.3
10	31.416	78.54	60	188.50	2827.4	110	345.58	9503.3
11	34.558	95.03	61	191.64	2922.5	111	348.72	9676.9
12	37.699	113.10	62	194.78	3019.1	112	351.86	9852.0
13	40.841	132.73	63	197.92	3117.2	113	355.00	10028.8
14	43.982	153.94	64	201.06	3217.0	114	358.14	10207.0
15	47.124	176.71	65	204.20	3318.3	115	361.28	10386.9
16	50.265	201.06	66	207.35	3421.2	116	364.42	10568.3
17	53.407	226.98	67	210.49	3525.7	117	367.57	10751.3
18	56.549	254.47	68	213.63	3631.7	118	370.71	10935.9
19	59.690	283.53	69	216.77	3739.3	119	373.85	11122.0
20	62.832	314.16	70	219.91	3848.5	120	376.99	11310
21	65.973	346.36	71	223.05	3959.2	121	380.13	11499
22	69.115	380.13	72	226.19	4071.5	122	383.27	11690
23	72.257	415.48	73	229.34	4185.4	123	386.42	11882
24	75.398	452.39	74	232.48	4300.8	124	389.56	12076
25	78.540	490.87	75	235.62	4417.9	125	392.70	12272
26	81.681	530.93	76	238.76	4536.5	126	395.84	12469
27	84.823	572.56	77	241.90	4656.6	127	398.98	12668
28	87.965	615.75	78	245.04	4778.4	128	402.12	12868
29	91.106	660.52	79	248.19	4901.7	129	405.27	13070
30	94.248	706.86	80	251.33	5026.6	130	408.41	13273
31	97.389	754.77	81	254.47	5153.0	131	411.55	13478
32	100.53	804.25	82	257.61	5281.0	132	414.69	13685
33	103.67	855.30	83	260.75	5410.6	133	417.83	13893
34	106.81	907.92	84	263.89	5541.8	134	420.97	14103
35	109.96	962.11	85	267.04	5674.5	135	424.12	14314
36	113.10	1017.88	86	270.18	5808.8	136	427.26	14527
37	116.24	1075.21	87	273.32	5944.7	137	430.40	14741
38	119.38	1134.11	88	276.46	6082.1	138	433.54	14957
39	122.52	1194.59	89	279.60	6221.1	139	436.68	15175
40	125.66	1256.63	90	282.74	6361.7	140	439.82	15394
41	128.81	1320.25	91	285.88	6503.9	141	442.96	15615
42	131.95	1385.44	92	289.03	6647.6	142	446.11	15837
43	135.09	1452.20	93	292.17	6792.9	143	449.25	16061
44	138.23	1520.52	94	295.31	6939.8	144	452.39	16286
45	141.37	1590.43	95	298.45	7088.2	145	455.53	16513
46	144.51	1661.90	96	301.59	7238.2	146	458.67	16742
47	147.65	1734.94	97	304.73	7389.8	147	461.81	16972
48	150.80	1809.55	98	307.88	7543.0	148	464.96	17203
49	153.94	1885.74	99	311.02	7697.7	149	468.10	17437
50	157.08	1963.50	100	314.16	7854.0	150	471.24	17671

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
151	474.38	17908	201	631.46	31731	251	788.54	49481
152	477.52	18146	202	634.60	32047	252	791.68	49876
153	480.66	18385	203	637.74	32365	253	794.82	50273
154	483.81	18627	204	640.89	32685	254	797.96	50671
155	486.95	18869	205	644.03	33006	255	801.11	51071
156	490.09	19113	206	647.17	33329	256	804.25	51472
157	493.23	19359	207	650.31	33654	257	807.39	51875
158	496.37	19607	208	653.45	33979	258	810.53	52279
159	499.51	19856	209	656.59	34307	259	813.67	52685
160	502.65	20106	210	659.73	34636	260	816.81	53093
161	505.80	20358	211	662.88	34967	261	819.96	53502
162	508.94	20612	212	666.02	35299	262	823.10	53913
163	512.08	20867	213	669.16	35633	263	826.24	54325
164	515.22	21124	214	672.30	35968	264	829.38	54739
165	518.36	21382	215	675.44	36305	265	832.52	55155
166	521.50	21642	216	678.58	36644	266	835.66	55572
167	524.65	21904	217	681.73	36984	267	838.81	55990
168	527.79	22167	218	684.87	37325	268	841.95	56410
169	530.93	22432	219	688.01	37668	269	845.09	56832
170	534.07	22698	220	691.15	38013	270	848.23	57256
171	537.21	22966	221	694.29	38360	271	851.37	57680
172	540.35	23235	222	697.43	38708	272	854.51	58107
173	543.50	23506	223	700.58	39057	273	857.66	58535
174	546.64	23779	224	703.72	39408	274	860.80	58965
175	549.78	24053	225	706.86	39761	275	863.94	59396
176	552.92	24328	226	710.00	40115	276	867.08	59828
177	556.06	24606	227	713.14	40471	277	870.22	60263
178	559.20	24885	228	716.28	40828	278	873.36	60699
179	562.35	25165	229	719.42	41187	279	876.50	61136
180	565.49	25447	230	722.57	41548	280	879.65	61575
181	568.63	25730	231	725.71	41910	281	882.79	62016
182	571.77	26016	232	728.85	42273	282	885.93	62458
183	574.91	26302	233	731.99	42638	283	889.07	62902
184	578.05	26590	234	735.13	43005	284	892.21	63347
185	581.19	26880	235	738.27	43374	285	895.35	63794
186	584.34	27172	236	741.42	43744	286	898.50	64242
187	587.48	27465	237	744.56	44115	287	901.64	64692
188	590.62	27759	238	747.70	44488	288	904.78	65144
189	593.76	28055	239	750.84	44863	289	907.92	65597
190	596.90	28353	240	753.98	45239	290	911.06	66052
191	600.04	28652	241	757.12	45617	291	914.20	66508
192	603.19	28953	242	760.27	45996	292	917.35	66966
193	606.33	29255	243	763.41	46377	293	920.49	67426
194	609.47	29559	244	766.55	46759	294	923.63	67887
195	612.61	29865	245	769.69	47144	295	926.77	68349
196	615.75	30172	246	772.83	47529	296	929.91	68813
197	618.89	30481	247	775.97	47916	297	933.05	69279
198	622.04	30791	248	779.12	48305	298	936.19	69747
199	625.18	31103	249	782.26	48695	299	939.34	70215
200	628.32	31416	250	785.40	49087	300	942.48	70686

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
301	945.62	71158	351	1102.70	96 762	401	1259.78	126 293
302	948.76	71631	352	1105.84	97 314	402	1262.92	126 923
303	951.90	72107	353	1108.98	97 868	403	1266.06	127 556
304	955.04	72583	354	1112.12	98 423	404	1269.20	128 190
305	958.19	73062	355	1115.27	98 980	405	1272.35	128 825
306	961.33	73542	356	1118.41	99 538	406	1275.49	129 462
307	964.47	74023	357	1121.55	100 098	407	1278.63	130 100
308	967.61	74506	358	1124.69	100 660	408	1281.77	130 741
309	970.75	74991	359	1127.83	101 223	409	1284.91	131 382
310	973.89	75477	360	1130.97	101 788	410	1288.05	132 025
311	977.04	75964	361	1134.11	102 354	411	1291.19	132 670
312	980.18	76454	362	1137.26	102 922	412	1294.34	133 317
313	983.32	76945	363	1140.40	103 491	413	1297.48	133 965
314	986.46	77437	364	1143.54	104 062	414	1300.62	134 614
315	989.60	77931	365	1146.68	104 635	415	1303.76	135 265
316	992.74	78427	366	1149.82	105 209	416	1306.90	135 918
317	995.88	78924	367	1152.96	105 785	417	1310.04	136 572
318	999.03	79423	368	1156.11	106 362	418	1313.19	137 228
319	1002.17	79923	369	1159.25	106 941	419	1316.33	137 885
320	1005.31	80425	370	1162.39	107 521	420	1319.47	138 544
321	1008.45	80928	371	1165.53	108 103	421	1322.61	139 205
322	1011.59	81433	372	1168.67	108 687	422	1325.75	139 867
323	1014.73	81940	373	1171.81	109 272	423	1328.89	140 531
324	1017.88	82448	374	1174.96	109 858	424	1332.04	141 196
325	1021.02	82958	375	1178.10	110 447	425	1335.18	141 863
326	1024.16	83469	376	1181.24	111 036	426	1338.32	142 531
327	1027.30	83982	377	1184.38	111 628	427	1341.46	143 201
328	1030.44	84496	378	1187.52	112 221	428	1344.60	143 872
329	1033.58	85012	379	1190.66	112 815	429	1347.74	144 545
330	1036.73	85530	380	1193.81	113 411	430	1350.88	145 220
331	1039.87	86049	381	1196.95	114 009	431	1354.03	145 896
332	1043.01	86570	382	1200.09	114 608	432	1357.17	146 574
333	1046.15	87092	383	1203.23	115 209	433	1360.31	147 254
334	1049.29	87616	384	1206.37	115 812	434	1363.45	147 934
335	1052.43	88141	385	1209.51	116 416	435	1366.59	148 617
336	1055.58	88668	386	1212.65	117 021	436	1369.73	149 301
337	1058.72	89197	387	1215.80	117 628	437	1372.88	149 987
338	1061.86	89727	388	1218.94	118 237	438	1376.02	150 674
339	1065.00	90259	389	1222.08	118 847	439	1379.16	151 363
340	1068.14	90792	390	1225.22	119 459	440	1382.30	152 053
341	1071.28	91327	391	1228.36	120 072	441	1385.44	152 745
342	1074.42	91863	392	1231.50	120 687	442	1388.58	153 439
343	1077.57	92401	393	1234.65	121 304	443	1391.73	154 134
344	1080.71	92941	394	1237.79	121 922	444	1394.87	154 830
345	1083.85	93482	395	1240.93	122 542	445	1398.01	155 528
346	1086.99	94025	396	1244.07	123 163	446	1401.15	156 228
347	1090.13	94569	397	1247.21	123 786	447	1404.29	156 930
348	1093.27	95115	398	1250.35	124 410	448	1407.43	157 633
349	1096.42	95662	399	1253.50	125 036	449	1410.58	158 337
350	1099.56	96211	400	1256.64	125 664	450	1413.72	159 043

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
451	1416.86	159 751	501	1573.94	197 136	551	1731.02	238 448
452	1420.00	160 460	502	1577.08	197 923	552	1734.16	239 314
453	1423.14	161 171	503	1580.22	198 713	553	1737.40	240 182
454	1426.28	161 883	504	1583.36	199 504	554	1740.44	241 051
455	1429.42	162 597	505	1586.50	200 296	555	1743.58	241 922
456	1432.57	163 313	506	1589.65	201 090	556	1746.73	242 795
457	1435.71	164 030	507	1592.79	201 886	557	1749.87	243 669
458	1438.85	164 748	508	1595.93	202 683	558	1753.01	244 545
459	1441.99	165 468	509	1599.07	203 482	559	1756.15	245 422
460	1445.13	166 190	510	1602.21	204 282	560	1759.29	246 301
461	1448.27	166 914	511	1605.35	205 084	561	1762.43	247 181
462	1451.42	167 639	512	1608.50	205 887	562	1765.58	248 063
463	1454.56	168 365	513	1611.64	206 692	563	1768.72	248 947
464	1457.70	169 093	514	1614.78	207 499	564	1771.86	249 832
465	1460.84	169 823	515	1617.92	208 307	565	1775.00	250 719
466	1463.98	170 554	516	1621.06	209 117	566	1778.14	251 607
467	1467.12	171 287	517	1624.20	209 928	567	1781.28	252 497
468	1470.27	172 021	518	1627.35	210 741	568	1784.42	253 388
469	1473.41	172 757	519	1630.49	211 556	569	1787.57	254 281
470	1476.55	173 494	520	1633.63	212 372	570	1790.71	255 176
471	1479.69	174 234	521	1636.77	213 189	571	1793.85	256 072
472	1482.83	174 974	522	1639.91	214 008	572	1796.99	256 970
473	1485.97	175 716	523	1643.05	214 829	573	1800.13	257 869
474	1489.11	176 460	524	1646.20	215 651	574	1803.27	258 770
475	1492.26	177 205	525	1649.34	216 475	575	1806.42	259 672
476	1495.40	177 952	526	1652.48	217 301	576	1809.56	260 576
477	1498.54	178 701	527	1655.62	218 128	577	1812.70	261 482
478	1501.68	179 451	528	1658.76	218 956	578	1815.84	262 389
479	1504.82	180 203	529	1661.90	219 787	579	1818.98	263 298
480	1507.96	180 956	530	1665.04	220 618	580	1822.12	264 208
481	1511.11	181 711	531	1668.19	221 452	581	1825.27	265 120
482	1514.25	182 467	532	1671.33	222 287	582	1828.41	266 033
483	1517.39	183 225	533	1674.47	223 123	583	1831.55	266 948
484	1520.53	183 984	534	1677.61	223 961	584	1834.69	267 865
485	1523.67	184 745	535	1680.75	224 801	585	1837.83	268 783
486	1526.81	185 508	536	1683.89	225 642	586	1840.97	269 702
487	1529.96	186 272	537	1687.04	226 484	587	1844.11	270 624
488	1533.10	187 038	538	1690.18	227 329	588	1847.26	271 547
489	1536.24	187 805	539	1693.32	228 175	589	1850.40	272 471
490	1539.38	188 574	540	1696.46	229 022	590	1853.54	273 397
491	1542.52	189 345	541	1699.60	229 871	591	1856.68	274 325
492	1545.66	190 117	542	1702.74	230 722	592	1859.82	275 254
493	1548.81	190 890	543	1705.88	231 574	593	1862.96	276 184
494	1551.95	191 665	544	1709.03	232 428	594	1866.11	277 117
495	1555.09	192 442	545	1712.17	233 283	595	1869.25	278 051
496	1558.23	193 221	546	1715.31	234 140	596	1872.39	278 986
497	1561.37	194 000	547	1718.45	234 998	597	1875.53	279 923
498	1564.51	194 782	548	1721.59	235 858	598	1878.67	280 862
499	1567.65	195 565	549	1724.73	236 720	599	1881.81	281 802
500	1570.80	196 350	550	1727.88	237 583	600	1884.96	282 743

8

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
601	1888.10	283 687	651	2045.18	332 853	701	2202.26	385 945
602	1891.24	284 631	652	2048.32	333 876	702	2205.40	387 047
603	1894.38	285 578	653	2051.46	334 901	703	2208.54	388 151
604	1897.52	286 526	654	2054.60	335 927	704	2211.68	389 256
605	1900.66	287 475	655	2057.74	336 955	705	2214.82	390 363
606	1903.81	288 426	656	2060.88	337 985	706	2217.96	391 471
607	1906.95	289 379	657	2064.03	339 016	707	2221.11	392 580
608	1910.09	290 333	658	2067.17	340 049	708	2224.25	393 692
609	1913.23	291 289	659	2070.31	341 083	709	2227.39	394 805
610	1916.37	292 247	660	2073.45	342 119	710	2230.53	395 919
611	1919.51	293 206	661	2076.59	343 157	711	2233.67	397 035
612	1922.65	294 166	662	2079.73	344 196	712	2236.81	398 153
613	1925.80	295 128	663	2082.88	345 237	713	2239.96	399 272
614	1928.94	296 092	664	2086.02	346 279	714	2243.10	400 393
615	1932.08	297 057	665	2089.16	347 323	715	2246.24	401 515
616	1935.22	298 024	666	2092.30	348 368	716	2249.38	402 639
617	1938.36	298 992	667	2095.44	349 415	717	2252.52	403 765
618	1941.50	299 962	668	2098.58	350 464	718	2255.66	404 892
619	1944.65	300 934	669	2101.73	351 514	719	2258.81	406 020
620	1947.79	301 907	670	2104.87	352 565	720	2261.95	407 150
621	1950.93	302 882	671	2108.01	353 618	721	2265.09	408 282
622	1954.07	303 858	672	2111.15	354 673	722	2268.23	409 416
623	1957.21	304 836	673	2114.29	355 730	723	2271.37	410 550
624	1960.35	305 815	674	2117.43	356 788	724	2274.51	411 687
625	1963.50	306 796	675	2120.58	357 847	725	2277.65	412 825
626	1966.64	307 779	676	2123.72	358 908	726	2280.80	413 965
627	1969.78	308 763	677	2126.86	359 971	727	2283.94	415 106
628	1972.92	309 748	678	2130.00	361 035	728	2287.08	416 248
629	1976.06	310 736	679	2133.14	362 101	729	2290.22	417 393
630	1979.20	311 725	680	2136.28	363 168	730	2293.36	418 539
631	1982.35	312 715	681	2139.42	364 237	731	2296.50	419 686
632	1985.49	313 707	682	2142.57	365 308	732	2299.65	420 835
633	1988.63	314 700	683	2145.71	366 380	733	2302.79	421 986
634	1991.77	315 696	684	2148.85	367 453	734	2305.93	423 139
635	1994.91	316 692	685	2151.99	368 528	735	2309.07	424 292
636	1998.05	317 690	686	2155.13	369 605	736	2312.21	425 447
637	2001.19	318 690	687	2158.27	370 684	737	2315.35	426 604
638	2004.34	319 692	688	2161.42	371 764	738	2318.50	427 762
639	2007.48	320 695	689	2164.56	372 845	739	2321.64	428 922
640	2010.62	321 699	690	2167.70	373 928	740	2324.78	430 084
641	2013.67	322 705	691	2170.84	375 013	741	2327.92	431 247
642	2016.90	323 713	692	2173.98	376 099	742	2331.06	432 412
643	2020.04	324 722	693	2177.12	377 187	743	2334.30	433 578
644	2023.19	325 733	694	2180.27	378 276	744	2337.34	434 746
645	2026.33	326 745	695	2183.41	379 367	745	2340.49	435 916
646	2029.47	327 759	696	2186.55	380 459	746	2343.63	437 087
647	2032.61	328 775	697	2189.69	381 554	747	2346.77	438 259
648	2035.75	329 792	698	2192.83	382 649	748	2349.91	439 433
649	2038.89	330 810	699	2195.97	383 746	749	2353.05	440 609
650	2042.04	331 831	700	2199.11	384 845	750	2356.19	441 786

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
751	2359.34	442 965	801	2516.42	503 912	851	2673.50	568 786
752	2362.48	444 146	802	2519.56	505 171	852	2676.64	570 124
753	2365.62	445 328	803	2522.70	506 432	853	2679.78	571 463
754	2368.76	446 511	804	2525.84	507 694	854	2682.92	572 803
755	2371.90	447 697	805	2528.98	508 958	855	2686.06	574 146
756	2375.04	448 883	806	2532.12	510 223	856	2689.20	575 490
757	2378.19	450 072	807	2535.27	511 490	857	2692.34	576 835
758	2381.33	451 262	808	2538.41	512 758	858	2695.49	578 182
759	2384.47	452 453	809	2541.55	514 028	859	2698.63	579 530
760	2387.61	453 646	810	2544.69	515 300	860	2701.77	580 880
761	2390.75	454 841	811	2547.83	516 573	861	2704.91	582 232
762	2393.89	456 037	812	2550.97	517 848	862	2708.05	583 585
763	2397.04	457 234	813	2554.11	519 124	863	2711.19	584 940
764	2400.18	458 434	814	2557.26	520 402	864	2714.34	586 297
765	2403.32	459 635	815	2560.40	521 681	865	2717.48	587 655
766	2406.46	460 837	816	2563.54	522 962	866	2720.62	589 014
767	2409.60	462 041	817	2566.68	524 245	867	2723.76	590 375
768	2412.74	463 247	818	2569.82	525 529	868	2726.90	591 738
769	2415.88	464 454	819	2572.96	526 814	869	2730.04	593 102
770	2419.03	465 663	820	2576.11	528 102	870	2733.19	594 468
771	2422.17	466 873	821	2579.25	529 391	871	2736.33	595 835
772	2425.31	468 085	822	2582.39	530 681	872	2739.47	597 204
773	2428.45	469 298	823	2585.53	531 973	873	2742.61	598 575
774	2431.59	470 513	824	2588.67	533 267	874	2745.75	599 947
775	2434.73	471 730	825	2591.81	534 562	875	2748.89	601 320
776	2437.88	472 948	826	2594.96	535 858	876	2752.04	602 696
777	2441.02	474 168	827	2598.10	537 157	877	2755.18	604 073
778	2444.16	475 389	828	2601.24	538 456	878	2758.32	605 451
779	2447.30	476 612	829	2604.38	539 758	879	2761.46	606 831
780	2450.44	477 836	830	2607.52	541 061	880	2764.60	608 212
781	2453.58	479 062	831	2610.66	542 365	881	2767.74	609 595
782	2456.73	480 290	832	2613.81	543 671	882	2770.88	610 980
783	2459.87	481 519	833	2616.95	544 979	883	2774.03	612 366
784	2463.01	482 750	834	2620.09	546 288	884	2777.17	613 754
785	2466.15	483 982	835	2623.23	547 599	885	2780.31	615 143
786	2469.29	485 216	836	2626.37	548 912	886	2783.45	616 534
787	2472.43	486 451	837	2629.51	550 226	887	2786.59	617 927
788	2475.58	487 688	838	2632.65	551 541	888	2789.73	619 321
789	2478.72	488 927	839	2635.80	552 858	889	2792.88	620 717
790	2481.86	490 167	840	2638.94	554 177	890	2796.02	622 114
791	2485.00	491 409	841	2642.08	555 497	891	2799.16	623 513
792	2488.14	492 652	842	2645.22	556 819	892	2802.30	624 913
793	2491.28	493 897	843	2648.36	558 142	893	2805.44	626 315
794	2494.42	495 143	844	2651.50	559 467	894	2808.58	627 718
795	2497.57	496 391	845	2654.65	560 794	895	2811.73	629 124
796	2500.71	497 641	846	2657.79	562 122	896	2814.87	630 530
797	2503.85	498 892	847	2660.93	563 452	897	2818.01	631 938
798	2506.99	500 145	848	2664.07	564 783	898	2821.15	633 348
799	2510.13	501 399	849	2667.21	566 116	899	2824.29	634 760
800	2513.27	502 655	850	2670.35	567 450	900	2827.43	636 173

Diameter.	Circum.	Area.	Diameter.	Circum.	Area.	Diameter.	Circum.	Area.
901	2830.58	637 587	934	2934.25	685 147	967	3037.92	734 417
902	2833.72	639 003	935	2937.39	686 615	968	3041.06	735 937
903	2836.86	640 421	936	2940.53	688 084	969	3044.20	737 458
904	2840.00	641 840	937	2943.67	689 555	970	3047.34	738 981
905	2843.14	643 261	938	2946.81	691 028	971	3050.49	740 506
906	2846.28	644 683	939	2949.96	692 502	972	3053.63	742 032
907	2849.42	646 107	940	2953.10	693 978	973	3056.77	743 559
908	2852.57	647 533	941	2956.24	695 455	974	3059.91	745 088
909	2855.71	648 960	942	2959.38	696 934	975	3063.05	746 619
910	2858.85	650 388	943	2962.52	698 415	976	3066.19	748 151
911	2861.99	651 818	944	2965.66	699 897	977	3069.34	749 685
912	2865.13	653 250	945	2968.81	701 380	978	3072.48	751 221
913	2868.27	654 684	946	2971.95	702 865	979	3075.62	752 758
914	2871.42	656 118	947	2975.09	704 352	980	3078.76	754 296
915	2874.56	657 555	948	2978.23	705 840	981	3081.90	755 837
916	2877.70	658 993	949	2981.37	707 330	982	3085.04	757 378
917	2880.84	660 433	950	2984.51	708 822	983	3088.19	758 922
918	2883.98	661 874	951	2987.65	710 315	984	3091.33	760 466
919	2887.12	663 317	952	2990.80	711 809	985	3094.47	762 013
920	2890.27	664 761	953	2993.94	713 307	986	3097.61	763 561
921	2893.41	666 207	954	2997.08	714 803	987	3100.75	765 111
922	2896.55	667 654	955	3000.22	716 303	988	3103.89	766 662
923	2899.69	669 103	956	3003.36	717 804	989	3107.04	768 215
924	2902.83	670 554	957	3006.50	719 306	990	3110.18	769 769
925	2905.97	672 006	958	3009.65	720 810	991	3113.32	771 325
926	2909.11	673 460	959	3012.79	722 316	992	3116.46	772 882
927	2912.26	674 915	960	3015.93	723 823	993	3119.60	774 441
928	2915.40	676 372	961	3019.07	725 332	994	3122.74	776 002
929	2918.54	677 831	962	3022.21	726 842	995	3125.88	777 564
930	2921.68	679 291	963	3025.35	728 354	996	3129.03	779 128
931	2924.82	680 752	964	3028.50	729 867	997	3132.17	780 693
932	2927.96	682 216	965	3031.64	731 382	998	3135.31	782 260
933	2931.11	683 680	966	3034.78	732 899	999	3138.45	783 828

NOTE.—When it is desired to find the circumference corresponding to any diameter not in the table, point off as many places in the circumference as have been pointed off in the diameter, and point off twice as many places in this area as have been pointed off in the diameter. Thus:

Diameters.	*Circumferences.*	*Areas.*
9.16	28.777	65.8993
91.6	287.77	6 589.93
916.	2877.7	658 993.
9160.	28777.	65 899 321.

When it is desired to find the circumference or area for any diameter consisting of a whole number and a decimal, it may be done by taking the difference between the tabular figures for the diameters between which the given diameter lies and multiplying this difference by the decimal and adding the result to the tabular value corresponding to the next lower diameter. Thus:

Required the circumference for the diameter 916.27?

We have

$$\begin{aligned} \text{Circumference } 917 &= 2880.84 \\ \text{Circumference } 916 &= \underline{2877.70} \\ \text{Difference,} \quad & 3.14 \end{aligned}$$

$$3.14 \times 27 = 0.8478$$

$$\text{Circumference } 916.27 = 2877.70 + 0.85 = 2878.55$$

For the area corresponding to the same diameter we have

$$\begin{aligned} \text{Area } 917 &= 660433 \\ \text{Area } 916 &= \underline{658993} \\ \text{Difference,} \quad & 1440 \end{aligned}$$

$$1440 \times 0.27 = 388.8$$

$$\text{Area } 916.27 = 658993 + 388.8 = 659381.8$$

Arcs and Segments of Circles.

The table starting below enables the following values to be determined: angle at centre $= v$, radius $= r$, length of arc $= b$, area of segment $= a$, surface of spherical segment $= \mathbf{a}$, volume of spherical segment $= \mathbf{c}$, and length of chord $= c$.

The quantities given are the height or versedsine of the arc $= h$, and the length of the chord $= c$.

To use the table, divide the length of the chord by the height. Look for the nearest value to this quotient in the first, or extreme left-hand column, and opposite this value will be found the corresponding values for the various coefficients, k, for a chord of unit length. These values, multiplied by the length of the given chord, will give the required lengths; by the square of the chord, will give the required surfaces; and by the cube of the chord, will give the required volume.

Thus, for a chord, $c = 25$, and height, $h = 5$, we have

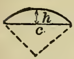
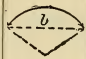
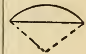
$$\frac{25}{5} = 5.$$

The nearest value to this in the table is 5.0134.
We then have

Angle at centre,	$v = 87°$;
Radius,	$r = 25 \times 0.72637 = 18.159$;
Length of arc,	$b = 25 \times 1.1027 = 27.567$;
Area of segment,	$a = 25^2 \times 0.13704 = 85.65$;
Surface of spherical segment,	$\mathbf{a} = 25^2 \times 0.91036 = 568.97$;
Volume of spherical segment,	$\mathbf{c} = 25^3 \times 0.08340 = 1303.12$.

Chord div. by height.	Centre angle v.	Radius $r = kc$.	Cir. arc $b = kc$.	Area seg. $a = kc^2$.	Surface $\mathbf{a} = kc^2$.	Solidity $\mathbf{c} = kc^3$.	Chord $c = kr$.
458.08	1	57.296	1.0000	.00109	.78539	.00085	.01744
229.18	2	28.649	1.0000	.00218	.78549	.00172	.03490
152.77	3	19.101	1.0000	.00327	.78562	.00255	.05234
114.57	4	14.327	1.0000	.00436	.78574	.00310	.06978
84.747	5	11.462	1.0001	.00647	.78586	.00401	.08722
76.375	6	9.5530	1.0003	.00741	.78599	.00514	.10466
65.943	7	8.1902	1.0004	.00910	.78621	.00592	.12208
57.273	8	7.1678	1.0006	.01089	.78630	.00686	.13950
50.902	9	6.3728	1.0008	.01254	.78665	.00772	.15690
45.807	10	5.7368	1.0011	.01407	.78695	.00857	.17430

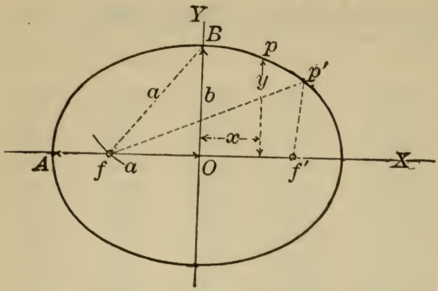
Chord div. by height.	Centre angle v.	Radius $r = kc$.	Cir. arc $b = kc$.	Area seg. $a = kc^2$.	Surface $\mathbf{a} = kc^2$.	Solidity $\mathbf{c} = kc^3$.	Chord $c = kr$.
41.203	11	5.2167	1.0013	.01552	.78730	.00964	.19168
38.133	12	4.7834	1.0016	.01695	.78762	.01031	.20904
35.221	13	4.4168	1.0019	.01841	.78794	.01114	.22640
32.742	14	4.1027	1.0023	.02000	.78832	.01199	.24372
30.514	15	3.8307	1.0027	.02157	.78889	.01288	.26104
28.601	16	3.5927	1.0029	.02269	.78909	.01375	.27834
26.915	17	3.3827	1.0034	.02434	.78969	.01462	.29560
25.412	18	3.1962	1.0039	.02592	.79028	.01542	.31286
24.068	19	3.0293	1.0044	.02744	.79084	.01635	.33008
22.860	20	2.8793	1.0048	.02878	.79140	.01722	.34728
21.760	21	2.7440	1.0054	.03040	.79234	.01802	.36446
20.777	22	2.6222	1.0059	.03178	.79300	.01897	.38160
19.862	23	2.5080	1.0066	.03343	.79340	.01984	.39872
19.028	24	2.4050	1.0072	.03493	.79416	.02072	.41582
18.261	25	2.3101	1.0078	.03639	.79486	.02159	.43286
17.553	26	2.2233	1.0084	.03784	.79530	.02248	.44990
16.970	27	2.1418	1.0091	.03970	.79639	.02315	.46688
16.288	28	2.0673	1.0101	.04115	.79748	.02424	.48384
15.721	29	1.9969	1.0105	.04230	.79811	.02511	.50076
15.191	30	1.9319	1.0113	.04385	.79907	.02600	.51762
14.970	31	1.8710	1.0121	.04476	.80002	.02692	.53446
14.230	32	1.8140	1.0129	.04710	.80098	.02778	.55126
13.796	33	1.7605	1.0138	.04842	.80181	.02866	.56802
13.382	34	1.7102	1.0146	.04989	.80300	.02956	.58479
12.994	35	1.6628	1.0155	.05137	.80405	.03046	.60140
12.733	36	1.6184	1.0167	.05311	.80531	.03137	.61802
12.473	37	1.5758	1.0174	.05401	.80622	.03226	.63460
11.931	38	1.5358	1.0184	.05628	.80713	.03328	.65112
11.621	39	1.4979	1.0194	.05755	.80850	.03418	.66760
11.342	40	1.4619	1.0204	.05899	.80987	.03506	.68404
11.060	41	1.4266	1.0207	.06001	.81046	.03589	.70040
10.791	42	1.3952	1.0226	.06196	.81240	.03680	.71672
10.534	43	1.3643	1.0237	.06359	.81377	.03773	.73300
10.289	44	1.3347	1.0248	.06574	.81505	.03864	.74920
10.043	45	1.3066	1.0260	.06628	.81756	.03890	.76536
9.8303	46	1.2797	1.0272	.06826	.81795	.04050	.78146
9.6153	47	1.2539	1.0290	.06998	.81939	.04143	.79748
9.4092	48	1.2289	1.0297	.07138	.82064	.04247	.81346
9.2113	49	1.2057	1.0309	.07290	.82244	.04330	.82938
9.0214	50	1.1831	1.0323	.07453	.82384	.04424	.84522
8.8387	51	1.1614	1.0336	.07611	.82562	.04519	.86102
8.6629	52	1.1406	1.0349	.07758	.82729	.04614	.87674
8.4462	53	1.1206	1.0364	.07959	.82896	.04685	.89238
8.3306	54	1.1014	1.0378	.08083	.83072	.04805	.90798
8.1733	55	1.0828	1.0393	.08246	.83249	.04901	.92348

Chord div. by height.	Centre angle v.	Radius $r = kc$.	Cir. arc $b = kc$.	Area seg. $a = kc^2$.	Surface $\mathbf{a} = kc^2$.	Solidity $\mathbf{c} = kc^3$.	Chord $c = kr$.
							
8.0215	56	1.0650	1.0407	.08400	.83422	.05002	.93894
7.8750	57	1.0478	1.0422	.08579	.83602	.05098	.95430
7.7334	58	1.0313	1.0431	.08680	.83796	.05191	.96960
7.5895	59	1.0154	1.0454	.08891	.84064	.05299	.98484
7.4565	60	1.0000	1.0470	.09106	.84266	.05400	1.0000
7.3358	61	.98515	1.0486	.09209	.84380	.05466	1.0150
7.2118	62	.97080	1.0503	.09375	.84581	.05583	1.0300
7.0914	63	.95694	1.0520	.09540	.84791	.05684	1.0450
6.9748	64	.94352	1.0537	.09697	.84996	.05784	1.0598
6.8616	65	.93058	1.0555	.09865	.85215	.05885	1.0746
6.7512	66	.91804	1.0573	.10036	.85441	.05987	1.0892
6.6453	67	.90590	1.0591	.10201	.85640	.06088	1.1038
6.5469	68	.89415	1.0610	.10367	.85815	.06181	1.1184
6.4902	69	.88276	1.0629	.10520	.86082	.06201	1.1328
6.3431	70	.87172	1.0648	.10710	.86350	.06396	1.1471
6.2400	71	.86102	1.0668	.10887	.86699	.06515	1.1614
6.1553	72	.85065	1.0687	.11046	.86834	.06604	1.1755
6.0652	73	.84058	1.0708	.11225	.87081	.06709	1.1896
5.9773	74	.83082	1.0728	.11385	.87344	.06815	1.2036
5.8918	75	.82134	1.0749	.11563	.87590	.06921	1.2175
5.8084	76	.81213	1.0770	.11736	.87853	.07037	1.2313
5.7271	77	.80319	1.0792	.11910	.88120	.07136	1.2450
5.6478	78	.79449	1.0814	.12072	.88389	.07244	1.2586
5.5704	79	.78606	1.0836	.12281	.88677	.07352	1.2721
5.4949	80	.77786	1.0859	.12441	.88949	.07462	1.2855
5.4254	81	.76988	1.0882	.12660	.89161	.07512	1.2989
5.3492	82	.76212	1.0905	.12793	.89520	.07683	1.3121
5.2705	83	.75458	1.0920	.12958	.89958	.07819	1.3252
5.2101	84	.74724	1.0953	.13157	.90095	.07907	1.3383
5.1429	85	.74009	1.0977	.13330	.90420	.07960	1.3512
5.0772	86	.73314	1.1012	.13546	.90734	.08102	1.3639
5.0134	87	.72637	1.1027	.13704	.91036	.08340	1.3767
4.9501	88	.71978	1.1054	.13893	.91363	.08436	1.3893
4.8886	89	.71336	1.1079	.14078	.91696	.08530	1.4018
4.8216	90	.70710	1.1105	.14279	.92210	.08621	1.4142
4.7694	91	.70101	1.1132	.14449	.92352	.08716	1.4265
4.7117	92	.69508	1.1159	.14643	.92476	.08798	1.4387
4.6615	93	.68930	1.1186	.14817	.92914	.08932	1.4507
4.5999	94	.68366	1.1211	.15009	.93385	.09076	1.4627
4.5453	95	.67817	1.1242	.15211	.93746	.09197	1.4745
4.4845	96	.67282	1.1271	.15375	.94272	.09348	1.4863
4.4398	97	.66760	1.1300	.15600	.94470	.09442	1.4979
4.3859	98	.66250	1.1329	.15801	.94852	.09567	1.5094
4.3383	99	.65754	1.1359	.15995	.95236	.09693	1.5208
4.2862	100	.65270	1.1382	.16180	.95682	.09831	1.5321

Chord div. by height.	Centre angle v.	Radius $r = kc$.	Cir. arc $b = kc$.	Area seg. $a = kc^2$.	Surface $\mathbf{a} = kc^2$.	Solidity $\mathbf{c} = kc^3$.	Chord $c = kr$.
4.2406	101	.64798	1.1420	.16393	.96011	.09956	1.5432
4.1930	102	.64338	1.1451	.16610	.96412	.10076	1.5543
4.1570	103	.63889	1.1483	.16925	.96568	.10215	1.5652
4.1006	104	.63450	1.1515	.17001	.97246	.10343	1.5760
4.0555	105	.63023	1.1547	.17204	.97643	.10471	1.5867
4.0113	106	.62607	1.1580	.17414	.98067	.10601	1.5973
3.9679	107	.62200	1.1614	.17619	.98495	.10735	1.6077
3.9252	108	.61803	1.1648	.17832	.98931	.10870	1.6180
3.8832	109	.61416	1.1682	.18041	.99376	.11007	1.6282
3.8419	110	.61039	1.1716	.18257	.99827	.11149	1.6383
3.8013	111	.60670	1.1752	.18472	1.0028	.11284	1.6482
3.7612	112	.60325	1.1790	.18696	1.0077	.11426	1.6581
3.7221	113	.59960	1.1823	.18900	1.0122	.11566	1.6677
3.6837	114	.59618	1.1859	.19117	1.0169	.11709	1.6773
3.6454	115	.59284	1.1897	.19339	1.0218	.11853	1.6867
3.6086	116	.58959	1.1934	.19559	1.0266	.11995	1.6961
3.5712	117	.58641	1.1972	.19787	1.0317	.12145	1.7053
3.5349	118	.58331	1.2011	.20009	1.0368	.12294	1.7143
3.4992	119	.58030	1.2050	.20227	1.0417	.12444	1.7232
3.4641	120	.57735	1.2089	.20453	1.0472	.12596	1.7320
3.4296	121	.57450	1.2130	.20678	1.0525	.12748	1.7407
3.3953	122	.57168	1.2177	.20945	1.0578	.12903	1.7492
3.3616	123	.56895	1.2213	.21175	1.0634	.13060	1.7576
3.3285	124	.56628	1.2253	.21399	1.0690	.13218	1.7659
3.2940	125	.56370	1.2295	.21538	1.0753	.13391	1.7740
3.2637	126	.56116	1.2338	.21859	1.0803	.13558	1.7820
3.2319	127	.55870	1.2381	.22121	1.0862	.13701	1.7898
3.2006	128	.55630	1.2425	.22370	1.0921	.13866	1.7976
3.1716	129	.55396	1.2470	.22617	1.0974	.14028	1.8051
3.1393	130	.55169	1.2515	.22865	1.1040	.14202	1.8126
3.1093	131	.54947	1.2561	.23113	1.1104	.14371	1.8199
3.0805	132	.54732	1.2607	.23372	1.1164	.14537	1.8271
3.0555	133	.54522	1.2654	.23603	1.1212	.14676	1.8341
3.0216	134	.54318	1.2701	.23892	1.1295	.14894	1.8410
2.9777	135	.54120	1.2749	.24198	1.1363	.15209	1.8477
2.9651	136	.53927	1.2798	.24364	1.1428	.15252	1.8543
2.9374	137	.53740	1.2847	.24676	1.1495	.15422	1.8608
2.9115	138	.53557	1.2897	.24938	1.1558	.15605	1.8671
2.8829	139	.53380	1.2948	.25222	1.1634	.15807	1.8733
2.8562	140	.53209	1.2999	.25485	1.1705	.15996	1.8794
2.8299	141	.53042	1.3051	.25759	1.1777	.16201	1.8853
2.8038	142	.52881	1.3065	.25936	1.1851	.16381	1.8910
2.7781	143	.52724	1.3157	.26320	1.1925	.16577	1.8966
2.7527	144	.52573	1.3211	.26604	1.2000	.16776	1.9021
2.7276	145	.52426	1.3265	.26889	1.2077	.16965	1.9074

Chord div. by height.	Centre angle v.	Radius $r = kc$.	Cir. arc $b = kc$.	Area seg. $a = kc^2$.	Surface $\mathbf{a} = kc^2$.	Solidity $\mathbf{c} = kc^3$.	Chord $c = kr$.
2.7002	146	.52284	1.3320	.27196	1.2166	.17209	1.9126
2.6816	147	.52147	1.3377	.27449	1.2219	.17405	1.9176
2.6533	148	.52015	1.3433	.27772	1.2318	.17605	1.9225
2.6301	149	.51887	1.3491	.28168	1.2396	.17809	1.9272
2.6064	150	.51764	1.3549	.28369	1.2476	.18023	1.9318
2.5830	151	.51645	1.3608	.28674	1.2563	.18666	1.9363
2.5598	152	.51530	1.3668	.28983	1.2648	.18751	1.9406
2.5239	153	.51420	1.3729	.29397	1.2801	.18845	1.9447
2.5143	154	.51315	1.3790	.29607	1.2824	.18913	1.9487
2.4919	155	.51214	1.3852	.29928	1.2914	.19147	1.9526
2.4699	156	.51117	1.3919	.30259	1.3004	.19374	1.9563
2.4478	157	.51014	1.3973	.30560	1.3094	.19607	1.9598
2.4262	158	.50936	1.4043	.30905	1.3191	.19851	1.9632
2.4047	159	.50851	1.4109	.31239	1.3287	.20095	1.9663
2.3835	160	.50771	1.4175	.31575	1.3368	.20342	1.9696
2.3613	161	.50695	1.4243	.31931	1.3490	.20609	1.9725
2.3417	162	.50623	1.4311	.32263	1.3583	.20847	1.9753
2.3211	163	.50555	1.4380	.32618	1.3682	.21105	1.9780
2.3004	164	.50491	1.4450	.32969	1.3791	.21371	1.9805
2.2805	165	.50431	1.4520	.33327	1.3895	.21634	1.9829
2.2605	166	.50374	1.4592	.33684	1.4021	.21904	1.9851
2.2408	167	.50323	1.4665	.34048	1.4111	.22177	1.9871
2.2212	168	.50275	1.4739	.34422	1.4222	.22450	1.9890
2.2013	169	.50231	1.4813	.34802	1.4344	.22766	1.9908
2.1826	170	.50191	1.4889	.35230	1.4476	.23028	1.9924
2.1636	171	.50154	1.4966	.35563	1.4565	.23266	1.9938
2.1447	172	.50122	1.5044	.35953	1.4684	.23650	1.9951
2.1271	173	.50093	1.5123	.36337	1.4797	.23900	1.9962
2.1075	174	.50068	1.5202	.36747	1.4927	.24225	1.9972
2.0892	175	.50047	1.5283	.37152	1.5052	.24537	1.9981
2.0710	176	.50030	1.5365	.37562	1.5179	.24856	1.9988
2.0530	177	.50017	1.5448	.37974	1.5308	.25179	1.9993
2.0352	178	.50007	1.5533	.38401	1.5439	.25531	1.9996
2.0175	179	.50002	1.5618	.38828	1.5573	.25840	1.9999
2.0000	180	.50000	1.5708	.39269	1.5708	.26179	2.0000

The Ellipse.



Notation.

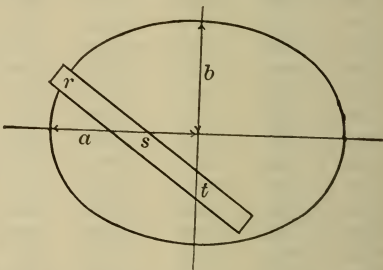
a = semi-major axis.
b = semi-minor axis.
f, f' = foci.
x = abscissa = horizontal distance from centre to base of vertical under any point, p, on perimeter.
y = ordinate = vertical distance from horizontal axis to point, p, on perimeter.

Equation of ellipses, referred to axes through centre:

$$a^2y^2 + b^2x^2 = a^2b^2.$$

Construction: given the semi-axes, a and b.

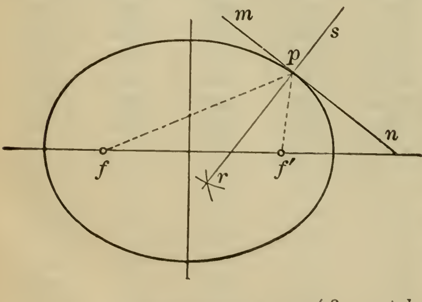
Find the foci, f, f', by taking the semi-major axis, a, in the dividers and sweeping arcs from B, intersecting the major axis at f and f'. By attaching a string to pins at f and f', and making the length of the string equal to $2a$, the curve can be drawn by moving a pencil around in the bight of the string.



Points on the perimeter of an ellipse may be found as follows:

Mark off on a straight-edged piece of paper the distances $rt = a$, $rs = b$; then, when t is on the minor axis and s on the major axis, r will be on a point in the curve, and so any number of points may be found.

To draw a normal or a tangent at any point, p, on the perimeter of an ellipse, draw lines, $fp, f'p$, from the point, p, to the two foci. A line, rs, bisecting the angle, fpf', will be the normal, and a line, mn, at right angles to the normal will be the tangent. The construction of the normal indicates the proper angles for joints in elliptical arches.



The perimeter of an ellipse can be accurately computed only by the summation of a series.

A good, approximate formula is that of Boussinesq, which is very close, when a is not more than three times greater than b.

$$\text{Perimeter} = S = 2\pi\left(\frac{3}{2}\cdot\frac{a+b}{2} - \frac{1}{2}\sqrt{ab}\right).$$

The quantity in the parenthesis is the radius of a circle of equivalent perimeter to an ellipse whose major and minor semi-axes are a and b.

Example. Let $a = 5$, $b = 2$.

$$S = 2\pi\left(\frac{3}{2} \cdot \frac{7}{2} - \frac{1}{2}\sqrt{10}\right) = 2\pi \times 3.6689 = 23.052.$$

The true perimeter of any ellipse may be computed from the following series:

$$\text{Perimeter} = S = \pi(a+b)\left[1 + \frac{1}{4}\left(\frac{a-b}{a+b}\right)^2 + \frac{1}{64}\left(\frac{a-b}{a+b}\right)^4 + \frac{1}{256}\left(\frac{a-b}{a+b}\right)^6 \cdots\right].$$

Calling the quantity within the brackets k, we have

$$S = \pi(a+b)k.$$

In the following table are given values of k for successive values of $\frac{a-b}{a+b}$, this rendering the application of the formula simple.

Example. Let $a = 7$, $b = 1$.

We have $$\frac{a-b}{a+b} = \frac{6}{8} = 0.75.$$

In the table, for 0.75 we have $k = 1.1466$; hence,

$$S = \pi \times 8 \times 1.1466 = 28.817.$$

By the Boussinesq formula we get

$$S = 29.388.$$

Perimeter of Ellipse.

VALUES OF k FOR SUCCESSIVE VALUES OF $\frac{a-b}{a+b}$.

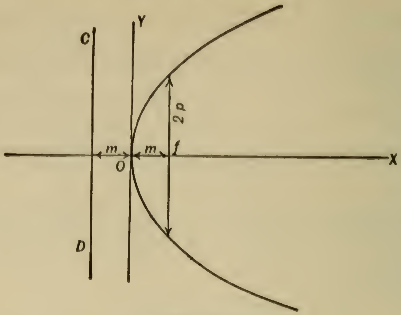
$\frac{a-b}{a+b}$	k	$\frac{a-b}{a+b}$	k	$\frac{a-b}{a+b}$	k	$\frac{a-b}{a+b}$	k	$\frac{a-b}{a+b}$	k
0.01	1.0000	0.21	1.0110	0.41	1.0431	0.61	1.0954	0.81	1.1721
0.02	1.0001	0.22	1.0122	0.42	1.0450	0.62	1.0986	0.82	1.1768
0.03	1.0002	0.23	1.0133	0.43	1.0472	0.63	1.1016	0.83	1.1813
0.04	1.0004	0.24	1.0145	0.44	1.0494	0.64	1.1048	0.84	1.1859
0.05	1.0006	0.25	1.0158	0.45	1.0516	0.65	1.1083	0.85	1.1903
0.06	1.0009	0.26	1.0173	0.46	1.0538	0.66	1.1115	0.86	1.1950
0.07	1.0012	0.27	1.0186	0.47	1.0561	0.67	1.1157	0.87	1.2000
0.08	1.0016	0.28	1.0200	0.48	1.0585	0.68	1.1193	0.88	1.2049
0.09	1.0020	0.29	1.0215	0.49	1.0608	0.69	1.1229	0.89	1.2100
0.10	1.0025	0.30	1.0226	0.50	1.0635	0.70	1.1267	0.90	1.2154
0.11	1.0029	0.31	1.0245	0.51	1.0661	0.71	1.1306	0.91	1.2207
0.12	1.0034	0.32	1.0261	0.52	1.0686	0.72	1.1345	0.92	1.2263
0.13	1.0040	0.33	1.0276	0.53	1.0713	0.73	1.1383	0.93	1.2315
0.14	1.0047	0.34	1.0291	0.54	1.0740	0.74	1.1423	0.94	1.2374
0.15	1.0054	0.35	1.0311	0.55	1.0768	0.75	1.1466	0.95	1.2430
0.16	1.0062	0.36	1.0331	0.56	1.0798	0.76	1.1509	0.96	1.2486
0.17	1.0070	0.37	1.0349	0.57	1.0827	0.77	1.1550	0.97	1.2546
0.18	1.0080	0.38	1.0369	0.58	1.0857	0.78	1.1593	0.98	1.2605
0.19	1.0090	0.39	1.0389	0.59	1.0889	0.79	1.1637	0.99	1.2665
0.20	1.0100	0.40	1.0404	0.60	1.0922	0.80	1.1677	1.00	1.2732

The area of an ellipse is readily found by the formula

$$\text{Area} = A = \pi ab.$$

This is simply obtained by taking the product, ab, as the diameter of a circle and looking up the corresponding area in the table of circles, pages 110–116.

The Parabola.



Notation.

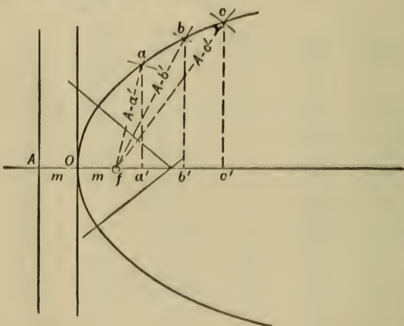
x = abscissa for any point on the curve.
y = ordinate.
f = focus.
O = vertex.
p = semi-parameter = double ordinate through focus.
CD = directrix.
$m = \frac{1}{2}p$ = distance of focus from vertex = distance of directrix from vertex.

Equation:

$$y^2 = 2px;$$
$$y = \sqrt{2px}.$$

Construction of Parabola.

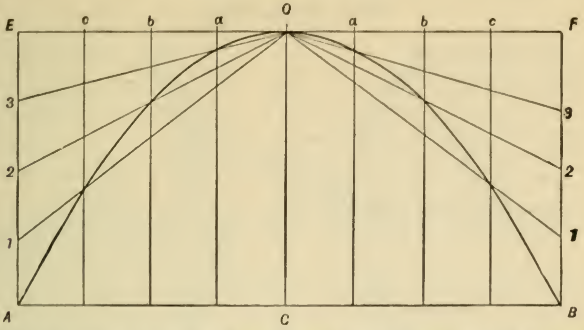
Given position of vertex, O, and focus, f:



Take the distance, $m = fO$, and lay it off from O to A; A will then be the point where the directrix cuts the horizontal axis. At any point, a', erect a vertical, and with the distance, Aa', in the dividers, sweep an arc with f as a centre; the intersection of this arc with the vertical will be a point in the curve. In like manner the points b, c, or any others may be found.

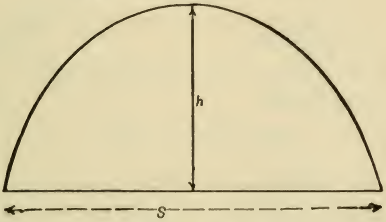
Given the rise and span of the curve:

Lay off the span, A–B, and height, O–C; divide A–E and B–F into any number of equal parts, 1–2–3, and EO and OF into the same number of equal parts. Join 1–2–3 with O, and the intersection of these lines with



verticals through a, b, c, etc., will be points in the curve. The accuracy of the curve will depend upon the number of divisions.

Length of Parabolic Curve.



Let h = height, s = span, l = length of curve.

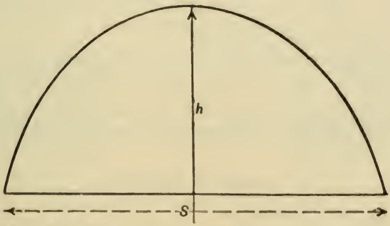
$$l = s\left[1 + \frac{8}{3}\left(\frac{h}{s}\right)^2 - \frac{32}{5}\left(\frac{h}{s}\right)^4\right].$$

This is a close approximation when the rise is small in proportion to the span.

The exact formula for the length of an arc of a parabola from the vertex to a point whose co-ordinates are x and y is

$$l = \frac{p}{2}\left[\sqrt{\frac{2x}{p}\left(1 + \frac{2x}{p}\right)} + \text{hyp. log.}\left(\sqrt{\frac{2x}{p}} + \sqrt{1 + \frac{2x}{p}}\right)\right].$$

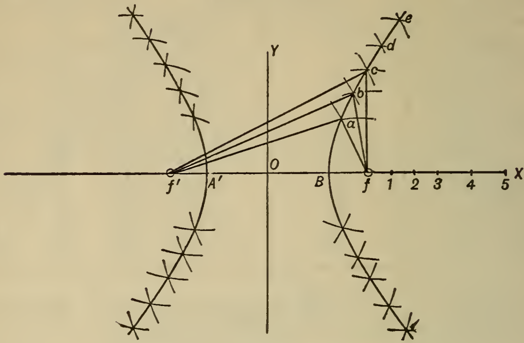
Area of Parabola.



Let s = span, h = height.

Area = $\frac{2}{3}sh$.

The Hyperbola.



Notation.

x = abscissa for any point on the curve.
y = ordinate.
f, f' = foci.
A, B = vertices, A–B = transverse axis.

Equation of the hyperbola:

$$a^2y^2 - b^2x^2 = -a^2b^2.$$

Construction of the Curve.

Given the transverse axis, A–B, and the foci, f, f':

Take any points, 1, 2, 3, 4, etc., on the axis, OX, and make $fa = B1$, $f'a = A1$, $fb = B2$, $f'b = A2$, etc., thus obtaining as many points on the curves as may be required.

Cycloidal Curves.

Cycloidal curves are those generated by the path of a point on a circle which rolls upon a given line. They are principally used for tooth profiles in wheel gearing.

We may consider the usual forms of cycloidal curves as generated by one circle rolling upon another, the rolling circle being called the *generating* circle, and the stationary one the *base* circle.

When the base circle is of infinitely great diameter it may be considered as a straight line, and the curve is the *orthocycloid*, usually called the common cycloid. When the generating circle rolls on the outside of the base circle, the curve is called the *epicycloid;* when it rolls inside of the base circle it is called the *hypocycloid.*

When the rolling circle is of infinitely great diameter it may be considered as a straight line, and the curve is called an *involute*, or more correctly an *evolute.*

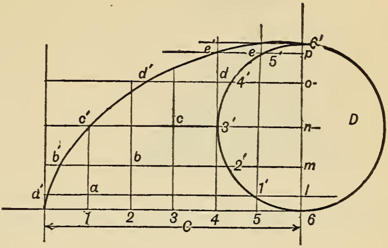
We shall here give only the geometrical methods of construction of the four curves, taking up their applications in connection with the practical constructions.

Common Cycloid.

Let D be the generating circle:

Lay off C, equal to one-half the circumference of the circle, D. Divide C and the half circumference of D into the same number of equal parts,

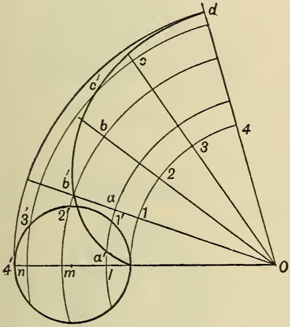
1, 2, 3, 4, 5, 6, and 1′, 2′, 3′, 4′, 5′, 6′. Erect ordinates from 1, 2, 3, etc., and draw horizontals from 1′, 2′, 3′, etc. Then make $aa' = 1'l$, $bb' = 2'm$, $cc' = 3'n$, $d'd = 4'o$, $ee' = 5'p$. Then a', b', c', etc., will be points on the *curve.*

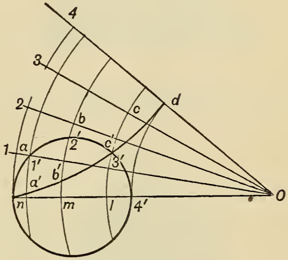


Epicycloid and Hypocycloid.

The construction of both epicycloid and hypocycloid is similar to that of the common cycloid, modified only by the fact that the base is circular instead of straight. The following construction applies to both curves, the only change being that due to the rolling being external and internal.

In each case the arc, 1–4, on the base circle is made equal in length to the semi-circumference of the generating circle. Radial lines are drawn



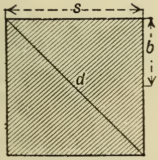


from the centre of the base circle through 1, 2, 3, 4, and arcs struck from O through 1′, 2′, 3′, 4′. Then $aa' = 1'n$, $bb' = 2'm$, $cc' = 3'l$, and the curve is drawn through a', b', c', d.

Areas of Plane Figures.

a = area; other dimensions as in the figures.

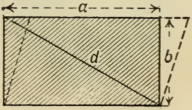
Square.



$\mathbf{a} = \mathbf{s}^2 = 4b^2$,

$\mathbf{a} = 0.5d^2$.

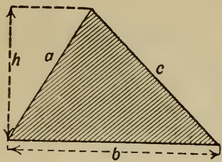
Rectangle.



$\mathbf{a} = ab$,

$\mathbf{a} = b\sqrt{d^2 - b^2}$.

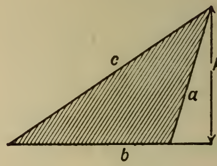
Triangle.



$$\mathbf{a} = \frac{bh}{2} = \tfrac{1}{2}bh,$$

$$\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}.$$

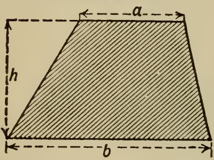
Triangle.



$$\mathbf{a} = \tfrac{1}{2}bh,$$

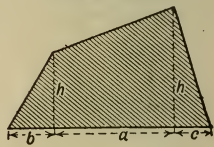
$$\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2b}\right)^2}.$$

Trapezoid.



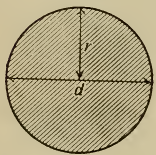
$$\mathbf{a} = \tfrac{1}{2}h(a + b).$$

Trapezium.



$$\mathbf{a} = \tfrac{1}{2}(a[h + h'] + bh' + ch).$$

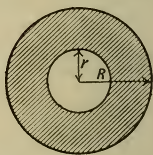
Circle.



$$\mathbf{a} = \pi r^2 = 0.7854d^2,$$

$$\mathbf{a} = \frac{pr}{2} = 0.0796P^2.$$

Circle Ring.



$$\mathbf{a} = \pi(R^2 - r^2) = \pi(R + r)(R - r),$$

$$\mathbf{a} = 0.7854(D^2 - d^2).$$

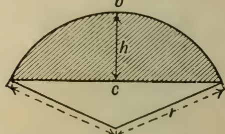
Or take the difference between the areas of the inner and outer circles, as found in the tables of areas of circles.

Sector.

$$\mathbf{a} = \tfrac{1}{2}br,$$

$$\mathbf{a} = \frac{\pi r^2 v}{360} = \frac{r^2 v}{114.5}.$$

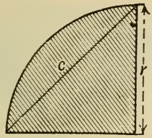
Segment.



$$\mathbf{a} = \tfrac{1}{2}[br - c\,(r - h)],$$

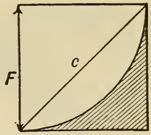
$$\mathbf{a} = \frac{\pi r^2 v}{360} \mp \frac{c}{2}(r - h).$$

Quadrant.



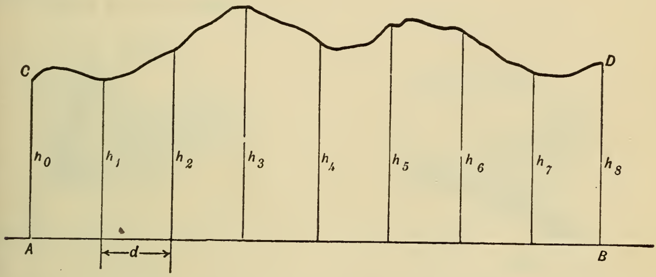
a $= 0.785r^2 = 0.3927c^2$.

Spandrel.



a $= 0.215r^2 = 0.1075c^2$.

The area of any irregular figure is best found by Simpson's Rule, as follows:



Divide the base, AB, into any even number of parts, d (in the illustration, 8 parts), and erect the ordinates, h_0, h_1, h_2, etc. Then the area, **a**, of the figure, $ABCD$, will be

$$\mathbf{a} = \frac{d}{3}(h_0 + 4h_1 + 2h_2 + 4h_3 + 2h_4 + 4h_5 + 2h_6 + 4h_7 + h_8).$$

It will be observed that the coefficients of the ordinates are alternately 4 and 2, with the exception of the first and last.

When the figure is drawn to scale, the area is best measured by a planimeter, but if this is not available, Simpson's Rule is practically as correct as any. The degree of accuracy will naturally depend upon the number of divisions taken.

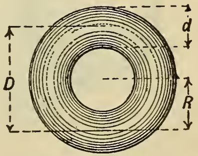
Surfaces of Solids.

Sphere.

a $= 4\pi r^2 = 12.5664r^2 = \pi d^2$.

The surface of any sphere may readily be found by multiplying the area of a circle of the same diameter by 4, using the Table of Areas of Circles.

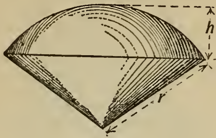
Torus, or Ring of Circular Cross Section.



a $= 4\pi^2 Rr = 39.4784Rr$,

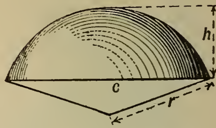
a $= 9.8696Dd$.

Sector of a Sphere.



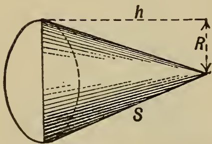
$$\mathbf{a} = \frac{\pi r}{2}(4h + c).$$

Circle Zone.



$$\mathbf{a} = 2\pi rh = \frac{\pi}{4}(c^2 + 4h^2).$$

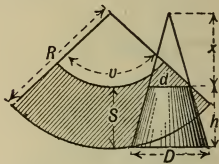
Cone.



$$\mathbf{a} = \pi Rs,$$

$$\mathbf{a} = \pi R\sqrt{R^2 + h^2}.$$

Frustum of a Cone.



$$x = \frac{dh}{D - d}, \qquad R = s + \frac{ds}{D - d},$$

$$\mathbf{a} = \frac{\pi s}{2}(D + d),$$

$$v = \frac{180D}{R} = \frac{180(D - d)}{s}.$$

Volumes of Solids.

$\mathbf{c}$ = content of the various bodies in terms of the dimensions given in the figures.

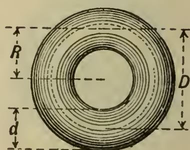
Sphere.

$$\mathbf{c} = \frac{4\pi r^3}{3} = 4.18879r^3,$$

$$\mathbf{c} = \frac{\pi d^3}{6} = 0.5236d^3.$$

For Table of Volumes of Spheres, see page 132.

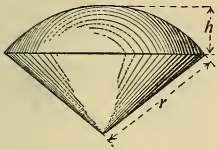
Torus.



$$\mathbf{c} = 2\pi^2 Rr^2 = 19.74Rr^2$$

$$\mathbf{c} = 2.463Dd^2.$$

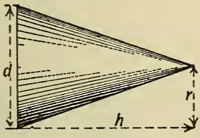
Sphere Sector.



$$c = \tfrac{2}{3}\pi r^2 h = 2.0944 r^2 h,$$

$$c = \tfrac{2}{3}\pi r^2 (r \mp \sqrt{r^2 - \tfrac{1}{4}c^2}).$$

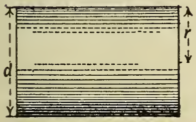
Cone.



$$c = \frac{\pi r^2 h}{3} = 1.047 r^2 h,$$

$$c = 0.2618 d^2 h.$$

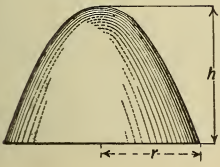
Cylinder.



$$c = \pi r^2 h = 0.785 d^2 h,$$

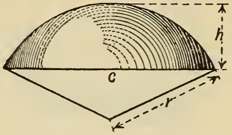
$$c = \frac{p^2 h}{4\pi} = 0.0796 p^2 h.$$

Paraboloid.



$$c = \tfrac{1}{2}\pi r^2 h = 1.5707 r^2 h.$$

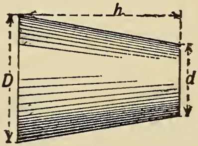
Segment of a Sphere.



$$c = \pi h^2 (r - \tfrac{1}{3}h),$$

$$c = \pi h^2 \left(\frac{c^2 + 4h^2}{8h} - \tfrac{1}{3}h \right).$$

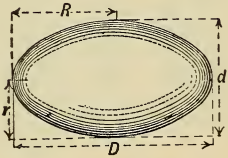
Conic Frustum.



$$c = \tfrac{1}{3}\pi h (R^2 + Rr + r^2),$$

$$c = \tfrac{1}{12}\pi h (D^2 + Dd + d^2).$$

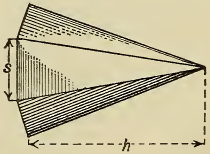
Ellipsoid.



$$c = 0.424\pi^2 R r^2 = 4.1847 R r^2,$$

$$c = 0.053\pi^2 D d^2 = 0.5231 D d^2.$$

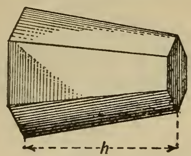
Pyramid.



$$c = \tfrac{1}{3} a h,$$

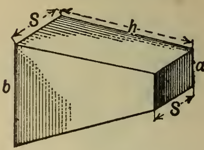
$$c = \frac{nsh}{6}\sqrt{r^2 - \frac{s^2}{4}}.$$

Pyramidic Frustum.



$$c = \frac{h}{3}(A + \mathbf{a} + \sqrt{A\mathbf{a}}).$$

Wedge Frustum.



$$c = \frac{hs}{2}(a + b).$$

Volumes of Spheres.

D = diameter.

D	Volume.	D	Volume.	D	Volume.	D	Volume.	D	Volume.
1	0.523599	21	4849.048	41	36086.95	61	118846.9	81	278261.8
2	4.188790	22	5575.280	42	38792.38	62	124788.2	82	288695.6
3	14.13717	23	6370.626	43	41629.77	63	130924.3	83	299387.0
4	33.51032	24	7238.228	44	44602.24	64	137258.3	84	310339.1
5	65.44984	25	8181.230	45	47712.93	65	143793.3	85	321555.1
6	113.0974	26	9202.770	46	50965.00	66	150532.5	86	333038.2
7	179.5943	27	10306.00	47	54361.60	67	157479.1	87	344791.4
8	268.0826	28	11494.04	48	57905.83	68	164636.2	88	356818.0
9	381.7035	29	12770.05	49	61600.86	69	172006.9	89	369120.9
10	523.5988	30	14137.17	50	65449.84	70	179594.3	90	381703.5
11	696.9100	31	15598.53	51	69455.90	71	187401.7	91	394568.8
12	904.7785	32	17157.25	52	73622.17	72	195432.2	92	407720.0
13	1150.347	33	18816.56	53	77951.80	73	203688.8	93	421160.4
14	1436.755	34	20579.52	54	82447.94	74	212174.8	94	434892.8
15	1767.146	35	22449.29	55	87113.74	75	220893.3	95	448920.4
16	2144.660	36	24429.02	56	91952.32	76	229847.3	96	463246.7
17	2572.441	37	26521.84	57	96966.82	77	239040.1	97	477874.4
18	3053.628	38	28730.91	58	102160.4	78	248474.8	98	492807.0
19	3591.364	39	31059.35	59	107536.2	79	258154.6	99	508047.3
20	4188.790	40	33510.32	60	113097.4	80	268082.6	100	523598.8

TRIGONOMETRY.

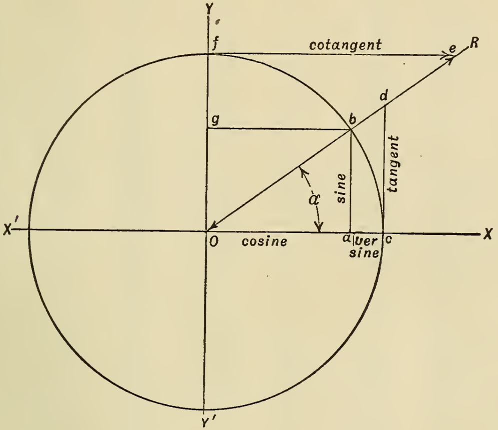
Angular Functions.

In order to obtain a clear idea of the various functions by which angular values may be expressed in terms of straight lines, let it be supposed that we have a straight line, $X'X$, and that from a point, O, on this line we have an arm, OR, which may be moved like a crank about O as a centre. The arm, OR, will then make various angles with the line, $X'X$, according to the position which is given to it.

If we take a radius, Oc = unity on any convenient scale, and describe a circle about O, we find that there are a number of ways in which we can measure the angle, α, which the arm, OR, makes with the line, $X'X$.

Thus, we may erect a perpendicular from c until it reaches OR at d, and the distance, cd, will be the *tangent* of the angle, α (written *tan* α). Or, we may drop a perpendicular from b to OX at α, and we have ab, the *sine* of the angle, α. Again, we may measure the distance, Oa, the *cosine* of α; or ac, the *versed-sine* of α; or fc, the *cotangent* of α. If we had given any one of the distances, measured on the same scale as the radius, Oc, we can construct the angle, α.

By supposing the arm, OR, to be gradually moved about O, so that the angle, α, steadily increases, we may observe the manner in which these functions vary. At first, when the angle is equal to zero, the sine, tangent, and versed-sine are also equal to zero, while the cosine and secant are both equal to the radius, or equal to unity. As the angle increases the sine, tangent, and versed-sine increase, while the cosine diminishes. At 45° the



sine and cosine are equal to each other and equal to $\frac{1}{2}\sqrt{2} = 0.7071$, while the tangent and cotangent are also equal to each other and also equal to the radius or unity. At 90° the cosine and cotangent become equal to zero and the sine equals the radius. For angles between 90° and 180° the cosine and cotangent become negative; between 180° and 270° the sine and cosine, tangent and cotangent, are all negative; and between 270° and 360° the sine and tangent are negative, the cosine and cotangent positive. Distances measured above $X'X$ and to the right of YY' are positive; those measured to the left of YY' and below $X'X$ are negative.

Referring again to the diagram, the functions are:

ab = sine,	ac = versed sine,	cd = tangent,	Od = secant,
aO = cosine,	fg = coversed sine,	fe = cotangent,	Oe = cosecant.

Trigonometric Tables.

In the following tables the values of the various angular functions are given for every degree and minute of the quadrant for a radius of unity. If any other radius is used, the tabular values are to be multiplied by the actual length of the radius. These tables of so-called Natural Functions are followed by tables of the Logarithmic Angular Functions, these being the logarithms of the natural functions. If the computations are made by the ordinary processes of multiplication and division, the natural functions are used, and if logarithms of numbers are used, the logarithms of the angular functions are to be used with them.

In the logarithmic functions the characteristics have been increased by 10, in order to avoid negative characteristics; hence, the corresponding number of tens are to be subtracted from the final result.

0° Natural Trigonometrical Functions. 179°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.00000	1.0000	Infinite.	.00000	Infinite.	1.0000	.00000	1.0000	60
1	. 0029	.99971	3437.7	. 0029	3437.7	.0000	. 0000	.0000	59
2	. 0058	. 9942	1718.9	. 0058	1718.9	.0000	. 0000	.0000	58
3	. 0087	. 9913	1145.9	. 0087	1145.9	.0000	. 0000	.0000	57
4	. 0116	. 9884	859.44	. 0116	859.44	.0000	. 0000	.0000	56
5	.00145	.99854	687.55	.00145	687.55	1.0000	.00000	1.0000	55
6	. 0174	. 9825	572.96	. 0174	572.96	.0000	. 0000	.0000	54
7	. 0204	. 9796	491.11	. 0204	491.11	.0000	. 0000	.0000	53
8	. 0233	. 9767	429.72	. 0233	429.72	.0000	. 0000	.0000	52
9	. 0262	. 9738	381.97	. 0262	381.97	.0000	. 0000	.0000	51
10	.00291	.99709	343.77	.00291	343.77	1.0000	.00000	.99999	50
11	. 0320	. 9680	312.52	. 0320	312.52	.0000	. 0000	. 9999	49
12	. 0349	. 9651	286.48	. 0349	286.48	.0000	. 0001	. 9999	48
13	. 0378	. 9622	64.44	. 0378	64.44	.0000	. 0001	. 9999	47
14	. 0407	. 9593	45.55	. 0407	45.55	.0000	. 0001	. 9999	46
15	.00436	.99564	229.18	.00436	229.18	1.0000	.00001	.99999	45
16	. 0465	. 9534	14.86	. 0465	14.86	.0000	. 0001	. 9999	44
17	. 0494	. 9505	02.22	. 0494	02.22	.0000	. 0001	. 9999	43
18	. 0524	. 9476	190.99	. 0524	190.98	.0000	. 0001	. 9999	42
19	. 0553	. 9447	80.93	. 0553	80.93	.0000	. 0001	. 9998	41
20	.00582	.99418	171.89	.00582	171.88	1.0000	.00002	.99998	40
21	. 0611	. 9389	63.70	. 0611	63.70	.0000	. 0002	. 9998	39
22	. 0640	. 9360	56.26	. 0640	56.26	.0000	. 0002	. 9998	38
23	. 0669	. 9331	49.47	. 0669	49.46	.0000	. 0002	. 9998	37
24	. 0698	. 9302	43.24	. 0698	43.24	.0000	. 0002	. 9997	36
25	.00727	.99273	137.51	.00727	137.51	1.0000	.00003	.99997	35
26	. 0756	. 9244	32.22	. 0756	32.22	.0000	. 0003	. 9997	34
27	. 0785	. 9215	27.32	. 0785	27.32	.0000	. 0003	. 9997	33
28	. 0814	. 9185	22.78	. 0814	22.77	.0000	. 0003	. 9997	32
29	. 0843	. 9156	18.54	. 0844	18.54	.0000	. 0003	. 9996	31
30	.00873	.99127	114.59	.00873	114.59	1.0000	.00004	.99996	30
31	. 0902	. 9098	10.90	. 0902	10.89	.0000	. 0004	. 9996	29
32	. 0931	. 9069	07.43	. 0931	07.43	.0000	. 0004	. 9996	28
33	. 0960	. 9040	04.17	. 0960	04.17	.0000	. 0005	. 9995	27
34	. 0989	. 9011	01.11	. 0989	01.11	.0000	. 0005	. 9995	26
35	.01018	.98982	98.223	.01018	98.218	1.0000	.00005	.99995	25
36	. 1047	. 8953	5.495	. 1047	5.489	.0000	. 0005	. 9994	24
37	. 1076	. 8924	2.914	. 1076	2.908	.0000	. 0006	. 9994	23
38	. 1105	. 8895	0.469	. 1105	0.463	.0001	. 0006	. 9994	22
39	. 1134	. 8865	88.149	. 1134	88.143	.0001	. 0006	. 9993	21
40	.01163	.98836	85.946	.01164	85.940	1.0001	.00007	.99993	20
41	. 1193	. 8807	3.849	. 1193	3.843	.0001	. 0007	. 9993	19
42	. 1222	. 8778	1.853	. 1222	1.847	.0001	. 0007	. 9992	18
43	. 1251	. 8749	79.950	. 1251	79.943	.0001	. 0008	. 9992	17
44	. 1280	. 8720	8.133	. 1280	8.126	.0001	. 0008	. 9992	16
45	.01309	.98691	76.396	.01309	76.390	1.0001	.00008	.99991	15
46	. 1338	. 8662	4.736	. 1338	4.729	.0001	. 0009	. 9991	14
47	. 1367	. 8633	3.146	. 1367	3.139	.0001	. 0009	. 9991	13
48	. 1396	. 8604	1.622	. 1396	1.615	.0001	. 0010	. 9990	12
49	. 1425	. 8575	0.160	. 1425	0.153	.0001	. 0010	. 9990	11
50	.01454	.98546	68.757	.01454	68.750	1.0001	.00010	.99989	10
51	. 1483	. 8516	7.409	. 1484	7.402	.0001	. 0011	. 9989	9
52	. 1512	. 8487	6.113	. 1513	6.105	.0001	. 0011	. 9988	8
53	. 1542	. 8458	4.866	. 1542	4.858	.0001	. 0012	. 9988	7
54	. 1571	. 8429	3.664	. 1571	3.657	.0001	. 0012	. 9988	6
55	.01600	.98400	62.507	.01600	62.499	1.0001	.00013	.99987	5
56	. 1629	. 8371	1.391	. 1629	1.383	.0001	. 0013	. 9987	4
57	. 1658	. 8342	0.314	. 1658	0.306	.0001	. 0014	. 9987	3
58	. 1687	. 8313	59.274	. 1687	59.266	.0001	. 0014	. 9986	2
59	. 1716	. 8284	8.270	. 1716	8.261	.0001	. 0015	. 9985	1
60	. 1745	. 8255	7.299	. 1745	7.290	.0001	. 0015	. 9985	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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1° **Natural Trigonometrical Functions.** 178°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.01745	.98255	57.299	.01745	57.290	1.0001	.00015	.99985	60
1	. 1774	. 8226	6.359	. 1775	6.350	.0001	. 0016	. 9984	59
2	. 1803	. 8196	5.450	. 1804	5.441	.0001	. 0016	. 9984	58
3	. 1832	. 8167	4.570	. 1833	4.561	.0002	. 0017	. 9983	57
4	. 1861	. 8138	3.718	. 1862	3.708	.0002	. 0017	. 9983	56
5	.01891	.98109	52.891	.01891	52.882	1.0002	.00018	.99982	55
6	. 1920	. 8080	2.090	. 1920	2.081	.0002	. 0018	. 9981	54
7	. 1949	. 8051	1.313	. 1949	1.303	.0002	. 0019	. 9981	53
8	. 1978	. 8022	0.558	. 1978	0.548	.0002	. 0019	. 9980	52
9	. 2007	. 7993	49.826	. 2007	49.816	.0002	. 0020	. 9980	51
10	.02036	.97964	49.114	.02036	49.104	1.0002	.00021	.99979	50
11	. 2065	. 7935	8.422	. 2066	8.412	.0002	. 0021	. 9979	49
12	. 2094	. 7906	7.750	. 2095	7.739	.0002	. 0022	. 9978	48
13	. 2123	. 7877	7.096	. 2124	7.085	.0002	. 0022	. 9977	47
14	. 2152	. 7847	6.460	. 2153	6.449	.0002	. 0023	. 9977	46
15	.02181	.97818	45.840	.02182	45.829	1.0002	.00024	.99976	45
16	. 2210	. 7789	5.237	. 2211	5.226	.0002	. 0024	. 9975	44
17	. 2240	. 7760	4.650	. 2240	4.638	.0002	. 0025	. 9975	43
18	. 2269	. 7731	4.077	. 2269	4.066	.0002	. 0026	. 9974	42
19	. 2298	. 7702	3.520	. 2298	3.508	.0003	. 0026	. 9974	41
20	.02327	.97673	42.976	.02327	42.964	1.0003	.00027	.99973	40
21	. 2356	. 7644	2.445	. 2357	2.433	.0003	. 0028	. 9972	39
22	. 2385	. 7615	1.928	. 2386	1.916	.0003	. 0028	. 9971	38
23	. 2414	. 7586	1.423	. 2415	1.410	.0003	. 0029	. 9971	37
24	. 2443	. 7557	0.930	. 2444	0.917	.0003	. 0030	. 9970	36
25	.02472	.97528	40 448	.02473	40.436	1.0003	.00030	.99969	35
26	. 2501	. 7499	39.978	. 2502	39.965	.0003	. 0031	. 9969	34
27	. 2530	. 7469	9.518	. 2531	9.506	.0003	. 0032	. 9968	33
28	. 2559	. 7440	9.069	. 2560	9.057	.0003	. 0033	. 9967	32
29	. 2589	. 7411	8.631	. 2589	8.618	.0003	. 0033	. 9966	31
30	.02618	.97382	38.201	.02618	38.188	1.0003	.00034	.99966	30
31	. 2647	. 7353	7.782	. 2648	7.769	.0003	. 0035	. 9965	29
32	. 2676	. 7324	7.371	. 2677	7.358	.0003	. 0036	. 9964	28
33	. 2705	. 7295	6.969	. 2706	6.956	.0004	. 0036	. 9963	27
34	. 2734	. 7266	6.576	. 2735	6.563	.0004	. 0037	. 9963	26
35	.02763	.97237	36.191	.02764	36.177	1.0004	.00038	.99962	25
36	. 2792	. 7208	5.814	. 2793	5.800	.0004	. 0039	. 9961	24
37	. 2821	. 7179	5.445	. 2822	5.431	.0004	. 0040	. 9960	23
38	. 2850	. 7150	5.084	. 2851	5.069	.0004	. 0041	. 9959	22
39	. 2879	. 7121	4.729	. 2880	4.715	.0004	. 0041	. 9958	21
40	.02908	.97091	34.382	.02910	34.368	1.0004	.00042	.99958	20
41	. 2937	. 7062	4.042	. 2939	4.027	.0004	. 0043	. 9957	19
42	. 2967	. 7033	3.708	. 2968	3.693	.0004	. 0044	. 9956	18
43	. 2996	. 7004	3.381	. 2997	3.366	.0004	. 0045	. 9955	17
44	. 3025	. 6975	3.060	. 3026	3.045	.0004	. 0046	. 9954	16
45	.03054	.96946	32.745	.03055	32.730	1.0005	.00046	.99953	15
46	. 3083	. 6917	2.437	. 3084	2.421	.0005	. 0047	. 9952	14
47	. 3112	. 6888	2.134	. 3113	2.118	.0005	. 0048	. 9951	13
48	. 3141	. 6859	1.836	. 3143	1.820	.0005	. 0049	. 9951	12
49	. 3170	. 6830	1.544	. 3172	1.528	.0005	. 0050	. 9950	11
50	.03199	.96801	31.257	.03201	31.241	1.0005	.00051	.99949	10
51	. 3228	. 6772	0.976	. 3230	0.960	.0005	. 0052	. 9948	9
52	. 3257	. 6743	0.699	. 3259	0.683	.0005	. 0053	. 9947	8
53	. 3286	. 6713	0.428	. 3288	0.411	.0005	. 0054	. 9946	7
54	. 3315	. 6684	0.161	. 3317	0.145	.0005	. 0055	. 9945	6
55	.03344	.96655	29.899	.03346	29.882	1.0005	.00056	.99944	5
56	. 3374	. 6626	9.641	. 3375	9.624	.0006	. 0057	. 9943	4
57	. 3403	. 6597	9.388	. 3405	9.371	.0006	. 0058	. 9942	3
58	. 3432	. 6568	9.139	. 3434	9.122	.0006	. 0059	. 9941	2
59	. 3461	. 6539	8.894	. 3463	8.877	.0006	. 0060	. 9940	1
60	. 3490	. 6510	8.654	. 3492	8.636	.0006	. 0061	. 9939	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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2°			Natural Trigonometrical Functions.						177°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.03490	.96510	28.654	.03492	28.636	1.0006	.00061	.99939	60
1	. 3519	. 6481	8.417	. 3521	8.399	.0006	. 0062	. 9938	59
2	. 3548	. 6452	8.184	. 3550	8.166	.0006	. 0063	. 9937	58
3	. 3577	. 6423	7.955	. 3579	7.937	.0006	. 0064	. 9936	57
4	. 3606	. 6394	7.730	. 3608	7.712	.0006	. 0065	. 9935	56
5	.03635	.96365	27.508	.03638	27.490	1.0007	.00066	.99934	55
6	. 3664	. 6336	7.290	. 3667	7.271	.0007	. 0067	. 9933	54
7	. 3693	. 6306	7.075	. 3696	7.056	.0007	. 0068	. 9932	53
8	. 3722	. 6277	6.864	. 3725	6.845	.0007	. 0069	. 9931	52
9	. 3751	. 6248	6.655	. 3754	6.637	.0007	. 0070	. 9930	51
10	.03781	.96219	26.450	.03783	26.432	1.0007	.00071	.99928	50
11	. 3810	. 6190	6.249	. 3812	6.230	.0007	. 0073	. 9927	49
12	. 3839	. 6161	6.050	. 3842	6.031	.0007	. 0074	. 9926	48
13	. 3868	. 6132	5.854	. 3871	5.835	.0007	. 0075	. 9925	47
14	. 3897	. 6103	5.661	. 3900	5.642	.0008	. 0076	. 9924	46
15	.03926	.96074	25.471	.03929	25.452	1.0008	.00077	.99923	45
16	. 3955	. 6045	5.284	. 3958	5.264	.0008	. 0078	. 9922	44
17	. 3984	. 6016	5.100	. 3987	5.080	.0008	. 0079	. 9921	43
18	. 4013	. 5987	4.918	. 4016	4.898	.0008	. 0080	. 9919	42
19	. 4042	. 5958	4.739	. 4045	4.718	.0008	. 0082	. 9918	41
20	.04071	.95929	24.562	.04075	24.542	1.0008	.00083	.99917	40
21	. 4100	. 5900	4.388	. 4104	4.367	.0008	. 0084	. 9916	39
22	. 4129	. 5870	4.216	. 4133	4.196	.0008	. 0085	. 9915	38
23	. 4158	. 5841	4.047	. 4162	4.026	.0009	. 0086	. 9913	37
24	. 4187	. 5812	3.880	. 4191	3.859	.0009	. 0088	. 9912	36
25	.04217	.95783	23.716	.04220	23.694	1.0009	.00089	.99911	35
26	. 4246	. 5754	3.553	. 4249	3.532	.0009	. 0090	. 9910	34
27	. 4275	. 5725	3.393	. 4279	3.372	.0009	. 0091	. 9908	33
28	. 4304	. 5696	3.235	. 4308	3.214	.0009	. 0093	. 9907	32
29	. 4333	. 5667	3.079	. 4337	3.058	.0009	. 0094	. 9906	31
30	.04362	.95638	22.925	.04366	22.904	1.0009	.00095	.99905	30
31	. 4391	. 5609	2.774	. 4395	2.752	.0010	. 0096	. 9903	29
32	. 4420	. 5580	2.624	. 4424	2.602	.0010	. 0098	. 9902	28
33	. 4449	. 5551	2.476	. 4453	2.454	.0010	. 0099	. 9901	27
34	. 4478	. 5522	2.330	. 4483	2.308	.0010	. 0100	. 9900	26
35	.04507	.95493	22.186	.04512	22.164	1.0010	.00102	.99898	25
36	. 4536	. 5464	2.044	. 4541	2.022	.0010	. 0103	. 9897	24
37	. 4565	. 5435	1.904	. 4570	1.881	.0010	. 0104	. 9896	23
38	. 4594	. 5405	1.765	. 4599	1.742	.0010	. 0106	. 9894	22
39	. 4623	. 5376	1.629	. 4628	1.606	.0011	. 0107	. 9893	21
40	.04652	.95347	21.494	.04657	21.470	1.0011	.00108	.99892	20
41	. 4681	. 5318	1.360	. 4687	1.337	.0011	. 0110	. 9890	19
42	. 4711	. 5289	1.228	. 4716	1.205	.0011	. 0111	. 9889	18
43	. 4740	. 5260	1.098	. 4745	1.075	.0011	. 0112	. 9888	17
44	. 4769	. 5231	0.970	. 4774	0.946	.0011	. 0114	. 9886	16
45	.04798	.95202	20.843	.04803	20.819	1.0011	.00115	.99885	15
46	. 4827	. 5173	0.717	. 4832	0.693	.0012	. 0116	. 9883	14
47	. 4856	. 5144	0.593	. 4862	0.569	.0012	. 0118	. 9882	13
48	. 4885	. 5115	0.471	. 4891	0.446	.0012	. 0119	. 9881	12
49	. 4914	. 5086	0.350	. 4920	0.325	.0012	. 0121	. 9879	11
50	.04943	.95057	20.230	.04949	20.205	1.0012	.00122	.99878	10
51	. 4972	. 5028	0.112	. 4978	0.087	.0012	. 0124	. 9876	9
52	. 5001	. 4999	19.995	. 5007	19.970	.0012	. 0125	. 9875	8
53	. 5030	. 4970	9.880	. 5037	9.854	.0013	. 0127	. 9873	7
54	. 5059	. 4941	9.766	. 5066	9.740	.0013	. 0128	. 9872	6
55	.05088	.94912	19.653	.05095	19.627	1.0013	.00129	.99870	5
56	. 5117	. 4883	9.541	. 5124	9.515	.0013	. 0131	. 9869	4
57	. 5146	. 4853	9.431	. 5153	9.405	.0013	. 0132	. 9867	3
58	. 5175	. 4824	9.322	. 5182	9.296	.0013	. 0134	. 9866	2
59	. 5204	. 4795	9.214	. 5212	9.188	.0013	. 0135	. 9864	1
60	. 5234	. 4766	9.107	. 5241	9.081	.0014	. 0137	. 9863	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
92°									87°

3° Natural Trigonometrical Functions. 176°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.05234	.94766	19.107	.05241	19.081	1.0014	.00137	.99863	60
1	. 5263	. 4737	9.002	. 5270	8.975	.0014	. 0138	. 9861	59
2	. 5292	. 4708	8.897	. 5299	8.871	.0014	. 0140	. 9860	58
3	. 5321	. 4679	8.794	. 5328	8.768	.0014	. 0142	. 9858	57
4	. 5350	. 4650	8.692	. 5357	8.665	.0014	. 0143	. 9857	56
5	.05379	.94621	18.591	.05387	18.564	1.0014	.00145	.99855	55
6	. 5408	. 4592	8.491	. 5416	8.464	.0015	. 0146	. 9854	54
7	. 5437	. 4563	8.393	. 5445	8.365	.0015	. 0148	. 9852	53
8	. 5466	. 4534	8.295	. 5474	8.268	.0015	. 0149	. 9850	52
9	. 5495	. 4505	8.198	. 5503	8.171	.0015	. 0151	. 9849	51
10	.05524	.94476	18.103	.05532	18.075	1.0015	.00153	.99847	50
11	. 5553	. 4447	8.008	. 5562	7.980	.0015	. 0154	. 9846	49
12	. 5582	. 4418	7.914	. 5591	7.886	.0016	. 0156	. 9844	48
13	. 5611	. 4389	7.821	. 5620	7.793	.0016	. 0157	. 9842	47
14	. 5640	. 4360	7.730	. 5649	7.701	.0016	. 0159	. 9841	46
15	.05669	.94331	17.639	.05678	17.610	1.0016	.00161	.99839	45
16	. 5698	. 4302	7.549	. 5707	7.520	.0016	. 0162	. 9837	44
17	. 5727	. 4273	7.460	. 5737	7.431	.0016	. 0164	. 9836	43
18	. 5756	. 4244	7.372	. 5766	7.343	.0017	. 0166	. 9834	42
19	. 5785	. 4214	7.285	. 5795	7.256	.0017	. 0167	. 9832	41
20	.05814	.94185	17.198	.05824	17.169	1.0017	.00169	.99831	40
21	. 5843	. 4156	7.113	. 5853	7.084	.0017	. 0171	. 9829	39
22	. 5872	. 4127	7.028	. 5883	6.999	.0017	. 0172	. 9827	38
23	. 5902	. 4098	6.944	. 5912	6.915	.0017	. 0174	. 9826	37
24	. 5931	. 4069	6.861	. 5941	6.832	.0018	. 0176	. 9824	36
25	.05960	.94040	16.779	.05970	16.750	1.0018	.00178	.99822	35
26	. 5989	. 4011	6.698	. 5999	6.668	.0018	. 0179	. 9820	34
27	. 6018	. 3982	6.617	. 6029	6.587	.0018	. 0181	. 9819	33
28	. 6047	. 3953	6.538	. 6058	6.507	.0018	. 0183	. 9817	32
29	. 6076	. 3924	6.459	. 6087	6.428	.0018	. 0185	. 9815	31
30	.06105	.93895	16.380	.06116	16.350	1.0019	.00186	.99813	30
31	. 6134	. 3866	6.303	. 6145	6.272	.0019	. 0188	. 9812	29
32	. 6163	. 3837	6.226	. 6175	6.195	.0019	. 0190	. 9810	28
33	. 6192	. 3808	6.150	. 6204	6.119	.0019	. 0192	. 9808	27
34	. 6221	. 3777	6.075	. 6233	6.043	.0019	. 0194	. 9806	26
35	.06250	.93750	16.000	.06262	15.969	1.0019	.00195	.99804	25
36	. 6279	. 3721	5.926	. 6291	5.894	.0020	. 0197	. 9803	24
37	. 6308	. 3692	5.853	. 6321	5.821	.0020	. 0199	. 9801	23
38	. 6337	. 3663	5.780	. 6350	5.748	.0020	. 0201	. 9799	22
39	. 6366	. 3634	5.708	. 6379	5.676	.0020	. 0203	. 9797	21
40	.06395	.93605	15.637	.06408	15.605	1.0020	.00205	.99795	20
41	. 6424	. 3576	5.566	. 6437	5.534	.0021	. 0206	. 9793	19
42	. 6453	. 3547	5.496	. 6467	5.464	.0021	. 0208	. 9791	18
43	. 6482	. 3518	5.427	. 6496	5.394	.0021	. 0210	. 9790	17
44	. 6511	. 3489	5.358	. 6525	5.325	.0021	. 0212	. 9788	16
45	.06540	.93460	15.290	.06554	15.257	1.0021	.00214	.99786	15
46	. 6569	. 3431	5.222	. 6583	5.189	.0022	. 0216	. 9784	14
47	. 6598	. 3402	5.155	. 6613	5.122	.0022	. 0218	. 9782	13
48	. 6627	. 3373	5.089	. 6642	5.056	.0022	. 0220	. 9780	12
49	. 6656	. 3343	5.023	. 6671	4.990	.0022	. 0222	. 9778	11
50	.06685	.93314	14.958	.06700	14.924	1.0022	.00224	.99776	10
51	. 6714	. 3285	4.893	. 6730	4.860	.0023	. 0226	. 9774	9
52	. 6743	. 3256	4.829	. 6759	4.795	.0023	. 0228	. 9772	8
53	. 6772	. 3227	4.765	. 6788	4.732	.0023	. 0230	. 9770	7
54	. 6801	. 3198	4.702	. 6817	4.668	.0023	. 0231	. 9768	6
55	.06830	.93169	14.640	.06846	14.606	1.0023	.00233	.99766	5
56	. 6859	. 3140	4.578	. 6876	4.544	.0024	. 0235	. 9764	4
57	. 6888	. 3111	4.517	. 6905	4.482	.0024	. 0237	. 9762	3
58	. 6918	. 3082	4.456	. 6934	4.421	.0024	. 0239	. 9760	2
59	. 6947	. 3053	4.395	. 6963	4.361	.0024	. 0241	. 9758	1
60	. 6976	. 3024	4.335	. 6993	4.301	.0024	. 0243	. 9756	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	.

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4° **Natural Trigonometrical Functions.** 175°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.06976	.93024	14.335	.06993	14.301	1.0024	.00243	.99756	60
1	. 7005	. 2995	4.276	. 7022	4.241	.0025	. 0246	. 9754	59
2	. 7034	. 2966	4.217	. 7051	4.182	.0025	. 0248	. 9752	58
3	. 7063	. 2937	4.159	. 7080	4.123	.0025	. 0250	. 9750	57
4	. 7092	. 2908	4.101	. 7110	4.065	.0025	. 0252	. 9748	56
5	.07121	.92879	14.043	.07139	14.008	1.0025	.00254	.99746	55
6	. 7150	. 2850	3.986	. 7168	3.951	.0026	. 0256	. 9744	54
7	. 7179	. 2821	3.930	. 7197	3.894	.0026	. 0258	. 9742	53
8	. 7208	. 2792	3.874	. 7226	3.838	.0026	. 0260	. 9740	52
9	. 7237	. 2763	3.818	. 7256	3.782	.0026	. 0262	. 9738	51
10	.07266	.92734	13.763	.07285	13.727	1.0026	.00264	.99736	50
11	. 7295	. 2705	3.708	. 7314	3.672	.0027	. 0266	. 9733	49
12	. 7324	. 2676	3.654	. 7343	3.617	.0027	. 0268	. 9731	48
13	. 7353	. 2647	3.600	. 7373	3.563	.0027	. 0271	. 9729	47
14	. 7382	. 2618	3.547	. 7402	3.510	.0027	. 0273	. 9727	46
15	.07411	.92589	13.494	.07431	13.457	1.0027	.00275	.99725	45
16	. 7440	. 2560	3.441	. 7460	3.404	.0028	. 0277	. 9723	44
17	. 7469	. 2531	3.389	. 7490	3.351	.0028	. 0279	. 9721	43
18	. 7498	. 2502	3.337	. 7519	3.299	.0028	. 0281	. 9718	42
19	. 7527	. 2473	3.286	. 7548	3.248	.0028	. 0284	. 9716	41
20	.07556	.92444	13.235	.07577	13.197	1.0029	.00286	.99714	40
21	. 7585	. 2415	3.184	. 7607	3.146	.0029	. 0288	. 9712	39
22	. 7614	. 2386	3.134	. 7636	3.096	.0029	. 0290	. 9710	38
23	. 7643	. 2357	3.084	. 7665	3.046	.0029	. 0292	. 9707	37
24	. 7672	. 2328	3.034	. 7694	2.996	.0029	. 0295	. 9705	36
25	.07701	.92299	12.985	.07724	12.947	1.0030	.00297	.99703	35
26	. 7730	. 2270	2.937	. 7753	2.898	.0030	. 0299	. 9701	34
27	. 7759	. 2241	2.888	. 7782	2.849	.0030	. 0301	. 9698	33
28	. 7788	. 2212	2.840	. 7812	2.801	.0030	. 0304	. 9696	32
29	. 7817	. 2183	2.793	. 7841	2.754	.0031	. 0306	. 9694	31
30	.07846	.92154	12.745	.07870	12.706	1.0031	.00308	.99692	30
31	. 7875	. 2125	2.698	. 7899	2.659	.0031	. 0310	. 9689	29
32	. 7904	. 2096	2.652	. 7929	2.612	.0031	. 0313	. 9687	28
33	. 7933	. 2067	2.606	. 7958	2.566	.0032	. 0315	. 9685	27
34	. 7962	. 2038	2.560	. 7987	2.520	.0032	. 0317	. 9682	26
35	.07991	.92009	12.514	.08016	12.474	1.0032	.00320	.99680	25
36	. 8020	. 1980	2.469	. 8046	2.429	.0032	. 0322	. 9678	24
37	. 8049	. 1951	2.424	. 8075	2.384	.0032	. 0324	. 9675	23
38	. 8078	. 1922	2.379	. 8104	2.339	.0033	. 0327	. 9673	22
39	. 8107	. 1893	2.335	. 8134	2.295	.0033	. 0329	. 9671	21
40	.08136	.91864	12.291	.08163	12.250	1.0033	.00331	.99668	20
41	. 8165	. 1835	2.248	. 8192	2.207	.0033	. 0334	. 9666	19
42	. 8194	. 1806	2.204	. 8221	2.163	.0034	. 0336	. 9664	18
43	. 8223	. 1777	2.161	. 8251	2.120	.0034	. 0339	. 9661	17
44	. 8252	. 1748	2.118	. 8280	2.077	.0034	. 0341	. 9659	16
45	.08281	.91719	12.076	.08309	12.035	1.0034	.00343	.99656	15
46	. 8310	. 1690	2.034	. 8339	1.992	.0035	. 0346	. 9654	14
47	. 8339	. 1661	1.992	. 8368	1.950	.0035	. 0348	. 9652	13
48	. 8368	. 1632	1.950	. 8397	1.909	.0035	. 0351	. 9649	12
49	. 8397	. 1603	1.909	. 8426	1.867	.0035	. 0353	. 9647	11
50	.08426	.91574	11.868	.08456	11.826	1.0036	.00356	.99644	10
51	. 8455	. 1545	1.828	. 8485	1.785	.0036	. 0358	. 9642	9
52	. 8484	. 1516	1.787	. 8514	1.745	.0036	. 0360	. 9639	8
53	. 8513	. 1487	1.747	. 8544	1.704	.0036	. 0363	. 9637	7
54	. 8542	. 1458	1.707	. 8573	1.664	.0037	. 0365	. 9634	6
55	.08571	.91429	11.668	.08602	11.625	1.0037	.00368	.99632	5
56	. 8600	. 1400	1.628	. 8632	1.585	.0037	. 0370	. 9629	4
57	. 8629	. 1371	1.589	. 8661	1.546	.0037	. 0373	. 9627	3
58	. 8658	. 1342	1.550	. 8690	1.507	.0038	. 0375	. 9624	2
59	. 8687	. 1313	1.512	. 8719	1.468	.0038	. 0378	. 9622	1
60	. 8715	. 1284	1.474	. 8749	1.430	.0038	. 0380	. 9619	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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5° **Natural Trigonometrical Functions.** 174°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.08715	.91284	11.474	.08749	11.430	1.0038	.00380	.99619	60
1	. 8744	. 1255	1.436	. 8778	1.392	.0038	. 0383	. 9617	59
2	. 8773	. 1226	1.398	. 8807	1.354	.0039	. 0386	. 9614	58
3	. 8802	. 1197	1.360	. 8837	1.316	.0039	. 0388	. 9612	57
4	. 8831	. 1168	1.323	. 8866	1.279	.0039	. 0391	. 9609	56
5	.08860	.91139	11.286	.08895	11.242	1.0039	.00393	.99607	55
6	. 8889	. 1110	1.249	. 8925	1.205	.0040	. 0396	. 9604	54
7	. 8918	. 1082	1.213	. 8954	1.168	.0040	. 0398	. 9601	53
8	. 8947	. 1053	1.176	. 8983	1.132	.0040	. 0401	. 9599	52
9	. 8976	. 1024	1.140	. 9013	1.095	.0040	. 0404	. 9596	51
10	.09005	.90995	11.104	.09042	11.059	1.0041	.00406	.99594	50
11	. 9034	. 0966	1.069	. 9071	1.024	.0041	. 0409	. 9591	49
12	. 9063	. 0937	1.033	. 9101	0.988	.0041	. 0411	. 9588	48
13	. 9092	. 0908	0.998	. 9130	0.953	.0041	. 0414	. 9586	47
14	. 9121	. 0879	0.963	. 9159	0.918	.0042	. 0417	. 9583	46
15	.09150	.90850	10.929	.09189	10.883	1.0042	.00419	.99580	45
16	. 9179	. 0821	0.894	. 9218	0.848	.0042	. 0422	. 9578	44
17	. 9208	. 0792	0.860	. 9247	0.814	.0043	. 0425	. 9575	43
18	. 9237	. 0763	0.826	. 9277	0.780	.0043	. 0427	. 9572	42
19	. 9266	. 0734	0.792	. 9306	0.746	.0043	. 0430	. 9570	41
20	.09295	.90705	10.758	.09335	10.712	1.0043	.00433	.99567	40
21	. 9324	. 0676	0.725	. 9365	0.678	.0044	. 0436	. 9564	39
22	. 9353	. 0647	0.692	. 9394	0.645	.0044	. 0438	. 9562	38
23	. 9382	. 0618	0.659	. 9423	0.612	.0044	. 0441	. 9559	37
24	. 9411	. 0589	0.626	. 9453	0.579	.0044	. 0444	. 9556	36
25	.09440	.90560	10.593	.09482	10.546	1.0045	.00446	.99553	35
26	. 9469	. 0531	0.561	. 9511	0.514	.0045	. 0449	. 9551	34
27	. 9498	. 0502	0.529	. 9541	0.481	.0045	. 0452	. 9548	33
28	. 9527	. 0473	0.497	. 9570	0.449	.0046	. 0455	. 9545	32
29	. 9556	. 0444	0.465	. 9599	0.417	.0046	. 0458	. 9542	31
30	.09584	.90415	10.433	.09629	10.385	1.0046	.00460	.99540	30
31	. 9613	. 0386	0.402	. 9658	0.354	.0046	. 0463	. 9537	29
32	. 9642	. 0357	0.371	. 9688	0.322	.0047	. 0466	. 9534	28
33	. 9671	. 0328	0.340	. 9717	0.291	.0047	. 0469	. 9531	27
34	. 9700	. 0300	0.309	. 9746	0.260	.0047	. 0472	. 9528	26
35	.09729	.90271	10.278	.09776	10.229	1.0048	.00474	.99525	25
36	. 9758	. 0242	0.248	. 9805	0.199	.0048	. 0477	. 9523	24
37	. 9787	. 0213	0.217	. 9834	0.168	.0048	. 0480	. 9520	23
38	. 9816	. 0184	0.187	. 9864	0.138	.0048	. 0483	. 9517	22
39	. 9845	. 0155	0.157	. 9893	0.108	.0049	. 0486	. 9514	21
40	.09874	.90126	10.127	.09922	10.078	1.0049	.00489	.99511	20
41	. 9903	. 0097	0.098	. 9952	0.048	.0049	. 0491	. 9508	19
42	. 9932	. 0068	0.068	. 9981	0.019	.0050	. 0494	. 9505	18
43	. 9961	. 0039	0.039	.10011	9.9893	.0050	. 0497	. 9503	17
44	. 9990	. 0010	0.010	. 0040	.9601	.0050	. 0500	. 9500	16
45	.10019	.89981	9.9812	.10069	9.9310	1.0050	.00503	.99497	15
46	. 0048	. 9952	.9525	. 0099	.9021	.0051	. 0506	. 9494	14
47	. 0077	. 9923	.9239	. 0128	.8734	.0051	. 0509	. 9491	13
48	. 0106	. 9894	.8955	. 0158	.8448	.0051	. 0512	. 9488	12
49	. 0134	. 9865	.8672	. 0187	.8164	.0052	. 0515	. 9485	11
50	.10163	.89836	9.8391	.10216	9.7882	1.0052	.00518	.99482	10
51	. 0192	. 9807	.8112	. 0246	.7601	.0052	. 0521	. 9479	9
52	. 0221	. 9779	.7834	. 0275	.7322	.0053	. 0524	. 9476	8
53	. 0250	. 9750	.7558	. 0305	.7044	.0053	. 0527	. 9473	7
54	. 0279	. 9721	.7283	. 0334	.6768	.0053	. 0530	. 9470	6
55	.10308	.89692	9.7010	.10363	9.6493	1.0053	.00533	.99467	5
56	. 0337	. 9663	.6739	. 0393	.6220	.0054	. 0536	. 9464	4
57	. 0366	. 9634	.6469	. 0422	.5949	.0054	. 0539	. 9461	3
58	. 0395	. 9605	.6200	. 0452	.5679	.0054	. 0542	. 9458	2
59	. 0424	. 9576	.5933	. 0481	.5411	.0055	. 0545	. 9455	1
60	. 0453	. 9547	.5668	. 0510	.5144	.0055	. 0548	. 9452	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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6° Natural Trigonometrical Functions. 173°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.10453	.89547	9.5668	.10510	9.5144	1.0055	.00548	.99452	60
1	. 0482	. 9518	.5404	. 0540	.4878	.0055	. 0551	. 9449	59
2	. 0511	. 9489	.5141	. 0569	.4614	.0056	. 0554	. 9446	58
3	. 0540	. 9460	.4880	. 0599	.4351	.0056	. 0557	. 9443	57
4	. 0568	. 9431	.4620	. 0628	.4090	.0056	. 0560	. 9440	56
5	.10597	.89402	9.4362	.10657	9.3831	1.0057	.00563	.99437	55
6	. 0626	. 9373	.4105	. 0687	.3572	.0057	. 0566	. 9434	54
7	. 0655	. 9345	.3850	. 0716	.3315	.0057	. 0569	. 9431	53
8	. 0684	. 9316	.3596	. 0746	.3060	.0057	. 0572	. 9428	52
9	. 0713	. 9287	.3343	. 0775	.2806	.0058	. 0575	. 9424	51
10	.10742	.89258	9.3092	.10805	9.2553	1.0058	.00579	.99421	50
11	. 0771	. 9229	.2842	. 0834	.2302	.0058	. 0582	. 9418	49
12	. 0800	. 9200	.2593	. 0863	.2051	.0059	. 0585	. 9415	48
13	. 0829	. 9171	.2346	. 0893	.1803	.0059	. 0588	. 9412	47
14	. 0858	. 9142	.2100	. 0922	.1555	.0059	. 0591	. 9409	46
15	.10887	.89113	9.1855	.10952	9.1309	1.0060	.00594	.99406	45
16	. 0916	. 9084	.1612	. 0981	.1064	.0060	. 0597	. 9402	44
17	. 0944	. 9055	.1370	. 1011	.0821	.0060	. 0601	. 9399	43
18	. 0973	. 9026	.1129	. 1040	.0579	.0061	. 0604	. 9396	42
19	. 1002	. 8998	.0890	. 1069	.0338	.0061	. 0607	. 9393	41
20	.11031	.88969	9.0651	.11099	9.0098	1.0061	.00610	.99390	40
21	. 1060	. 8940	.0414	. 1128	8.9860	.0062	. 0613	. 9386	39
22	. 1089	. 8911	.0179	. 1158	.9623	.0062	. 0617	. 9383	38
23	. 1118	. 8882	8.9944	. 1187	.9387	.0062	. 0620	. 9380	37
24	. 1147	. 8853	.9711	. 1217	.9152	.0063	. 0623	. 9377	36
25	.11176	.88824	8.9479	.11246	8.8918	1.0063	.00626	.99373	35
26	. 1205	. 8795	.9248	. 1276	.8686	.0063	. 0630	. 9370	34
27	. 1234	. 8766	.9018	. 1305	.8455	.0064	. 0633	. 9367	33
28	. 1262	. 8737	.8790	. 1335	.8225	.0064	. 0636	. 9364	32
29	. 1291	. 8708	.8563	. 1364	.7996	.0064	. 0639	. 9360	31
30	.11320	.88680	8.8337	.11393	8.7769	1.0065	.00643	.99357	30
31	. 1349	. 8651	.8112	. 1423	.7542	.0065	. 0646	. 9354	29
32	. 1378	. 8622	.7888	. 1452	.7317	.0065	. 0649	. 9350	28
33	. 1407	. 8593	.7665	. 1482	.7093	.0066	. 0653	. 9347	27
34	. 1436	. 8564	.7444	. 1511	.6870	.0066	. 0656	. 9344	26
35	.11465	.88535	8.7223	.11541	8.6648	1.0066	.00659	.99341	25
36	. 1494	. 8506	.7004	. 1570	.6427	.0067	. 0663	. 9337	24
37	. 1523	. 8477	.6786	. 1600	.6208	.0067	. 0666	. 9334	23
38	. 1551	. 8448	.6569	. 1629	.5989	.0067	. 0669	. 9330	22
39	. 1580	. 8420	.6353	. 1659	.5772	.0068	. 0673	. 9327	21
40	.11609	.88391	8.6138	.11688	8.5555	1.0068	.00676	.99324	20
41	. 1638	. 8362	.5924	. 1718	.5340	.0068	. 0679	. 9320	19
42	. 1667	. 8333	.5711	. 1747	.5126	.0069	. 0683	. 9317	18
43	. 1696	. 8304	.5499	. 1777	.4913	.0069	. 0686	. 9314	17
44	. 1725	. 8272	.5289	. 1806	.4701	.0069	. 0690	. 9310	16
45	.11754	.88246	8.5079	.11836	8.4489	1.0070	.00693	.99307	15
46	. 1783	. 8217	.4871	. 1865	.4279	.0070	. 0696	. 9303	14
47	. 1811	. 8188	.4663	. 1895	.4070	.0070	. 0700	. 9300	13
48	. 1840	. 8160	.4457	. 1924	.3862	.0071	. 0703	. 9296	12
49	. 1869	. 8131	.4251	. 1954	.3655	.0071	. 0707	. 9293	11
50	.11898	.88102	8.4046	.11983	8.3449	1.0071	.00710	.99290	10
51	. 1927	. 8073	.3843	. 2013	.3244	.0072	. 0714	. 9286	9
52	. 1956	. 8044	.3640	. 2042	.3040	.0072	. 0717	. 9283	8
53	. 1985	. 8015	.3439	. 2072	.2837	.0073	. 0721	. 9279	7
54	. 2014	. 7986	.3238	. 2101	.2635	.0073	. 0724	. 9276	6
55	.12042	.87957	8.3039	.12131	8.2434	1.0073	.00728	.99272	5
56	. 2071	. 7928	.2840	. 2160	.2234	.0074	. 0731	. 9269	4
57	. 2100	. 7900	.2642	. 2190	.2035	.0074	. 0735	. 9265	3
58	. 2129	. 7871	.2446	. 2219	.1837	.0074	. 0738	. 9262	2
59	. 2158	. 7842	.2250	. 2249	.1640	.0075	. 0742	. 9258	1
60	. 2187	. 7813	.2055	. 2278	.1443	.0075	. 0745	. 9255	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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7° **Natural Trigonometrical Functions.** 172°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.12187	.87813	8.2055	.12278	8.1443	1.0075	.00745	.99255	60
1	. 2216	. 7787	.1861	. 2308	.1248	.0075	. 0749	. 9251	59
2	. 2245	. 7755	.1668	. 2337	.1053	.0076	. 0752	. 9247	58
3	. 2273	. 7726	.1476	. 2367	.0860	.0076	. 0756	. 9244	57
4	. 2302	. 7697	.1285	. 2396	.0667	.0076	. 0760	. 9240	56
5	.12331	.87669	8.1094	.12426	8.0476	1.0077	.00763	.99237	55
6	. 2360	. 7640	.0905	. 2456	.0285	.0077	. 0767	. 9233	54
7	. 2389	. 7611	.0717	. 2485	.0095	.0078	. 0770	. 9229	53
8	. 2418	. 7582	.0529	. 2515	7.9906	.0078	. 0774	. 9226	52
9	. 2447	. 7553	.0342	. 2544	.9717	.0078	. 0778	. 9222	51
10	.12476	.87524	8.0156	.12574	7.9530	1.0079	.00781	.99219	50
11	. 2504	. 7495	7.9971	. 2603	.9344	.0079	. 0785	. 9215	49
12	. 2533	. 7467	.9787	. 2633	.9158	.0079	. 0788	. 9211	48
13	. 2562	. 7438	.9604	. 2662	.8973	.0080	. 0792	. 9208	47
14	. 2591	. 7409	.9421	. 2692	.8789	.0080	. 0796	. 9204	46
15	.12620	.87380	7.9240	.12722	7.8606	1.0080	.00799	.99200	45
16	. 2649	. 7351	.9059	. 2751	.8424	.0081	. 0803	. 9197	44
17	. 2678	. 7322	.8879	. 2781	.8243	.0081	. 0807	. 9193	43
18	. 2706	. 7293	.8700	. 2810	.8062	.0082	. 0810	. 9189	42
19	. 2735	. 7265	.8522	. 2840	.7882	.0082	. 0814	. 9186	41
20	.12764	.87236	7.8344	.12869	7.7703	1.0082	.00818	.99182	40
21	. 2793	. 7207	.8168	. 2899	.7525	.0083	. 0822	. 9178	39
22	. 2822	. 7178	.7992	. 2928	.7348	.0083	. 0825	. 9174	38
23	. 2851	. 7149	.7817	. 2958	.7171	.0084	. 0829	. 9171	37
24	. 2879	. 7120	.7642	. 2988	.6996	.0084	. 0833	. 9167	36
25	.12908	.87091	7.7469	.13017	7.6821	1.0084	.00837	.99163	35
26	. 2937	. 7063	.7296	. 3047	.6646	.0085	. 0840	. 9160	34
27	. 2966	. 7034	.7124	. 3076	.6473	.0085	. 0844	. 9156	33
28	. 2995	. 7005	.6953	. 3106	.6300	.0085	. 0848	. 9152	32
29	. 3024	. 6976	.6783	. 3136	.6129	.0086	. 0852	. 9148	31
30	.13053	.86947	7.6613	.13165	7.5957	1.0086	.00855	.99144	30
31	. 3081	. 6918	.6444	. 3195	.5787	.0087	. 0859	. 9141	29
32	. 3110	. 6890	.6276	. 3224	.5617	.0087	. 0863	. 9137	28
33	. 3139	. 6861	.6108	. 3254	.5449	.0087	. 0867	. 9133	27
34	. 3168	. 6832	.5942	. 3284	.5280	.0088	. 0871	. 9129	26
35	.13197	.86803	7.5776	.13313	7.5113	1.0088	.00875	.99125	25
36	. 3226	. 6774	.5611	. 3343	.4946	.0089	. 0878	. 9121	24
37	. 3254	. 6745	.5446	. 3372	.4780	.0089	. 0882	. 9118	23
38	. 3283	. 6717	.5282	. 3402	.4615	.0089	. 0886	. 9114	22
39	. 3312	. 6688	.5119	. 3432	.4451	.0090	. 0890	. 9110	21
40	.13341	.86659	7.4957	.13461	7.4287	1.0090	.00894	.99106	20
41	. 3370	. 6630	.4795	. 3491	.4124	.0090	. 0898	. 9102	19
42	. 3399	. 6601	.4634	. 3520	.3961	.0091	. 0902	. 9098	18
43	. 3427	. 6572	.4474	. 3550	.3800	.0091	. 0905	. 9094	17
44	. 3456	. 6544	.4315	. 3580	.3639	.0092	. 0909	. 9090	16
45	.13485	.86515	7.4156	.13609	7.3479	1.0092	.00913	.99086	15
46	. 3514	. 6486	.3998	. 3639	.3319	.0092	. 0917	. 9083	14
47	. 3543	. 6457	.3840	. 3669	.3160	.0093	. 0921	. 9079	13
48	. 3571	. 6428	.3683	. 3698	.3002	.0093	. 0925	. 9075	12
49	. 3600	. 6400	.3527	. 3728	.2844	.0094	. 0929	. 9070	11
50	.13629	.86371	7.3372	.13757	7.2687	1.0094	.00933	.99067	10
51	. 3658	. 6342	.3217	. 3787	.2531	.0094	. 0937	. 9063	9
52	. 3687	. 6313	.3063	. 3817	.2375	.0095	. 0941	. 9059	8
53	. 3716	. 6284	.2909	. 3846	.2220	.0095	. 0945	. 9055	7
54	. 3744	. 6255	.2757	. 3876	.2066	.0096	. 0949	. 9051	6
55	.13773	.86227	7.2604	.13906	7.1912	1.0096	.00953	.99047	5
56	. 3802	. 6198	.2453	. 3935	.1759	.0097	. 0957	. 9043	4
57	. 3831	. 6169	.2302	. 3965	.1607	.0097	. 0961	. 9039	3
58	. 3860	. 6140	.2152	. 3995	.1455	.0097	. 0965	. 9035	2
59	. 3888	. 6111	.2002	. 4024	.1304	.0098	. 0969	. 9031	1
60	. 3917	. 6083	.1853	. 4054	.1154	.0098	. 0973	. 9027	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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8° **Natural Trigonometrical Functions.** 171°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.13917	.86083	7.1853	.14054	7.1154	1.0098	.00973	.99027	60
1	. 3946	. 6054	.1704	. 4084	.1004	.0099	. 0977	. 9023	59
2	. 3975	. 6025	.1557	. 4113	.0854	.0099	. 0981	. 9019	58
3	. 4004	. 5996	.1409	. 4143	.0706	.0099	. 0985	. 9015	57
4	. 4032	. 5967	.1263	. 4173	.0558	.0100	. 0989	. 9010	56
5	.14061	.85939	7.1117	.14202	7.0410	1.0100	.00993	.99006	55
6	. 4090	. 5910	.0972	. 4232	.0264	.0101	. 0998	. 9002	54
7	. 4119	. 5881	.0827	. 4262	.0117	.0101	. 1002	. 8998	53
8	. 4148	. 5852	.0683	. 4291	6.9972	.0102	. 1006	. 8994	52
9	. 4176	. 5823	.0539	. 4321	.9827	.0102	. 1010	. 8990	51
10	.14205	.85795	7.0396	.14351	6.9682	1.0102	.01014	.98986	50
11	. 4234	. 5766	.0254	. 4380	.9538	.0103	. 1018	. 8982	49
12	. 4263	. 5737	.0112	. 4410	.9395	.0103	. 1022	. 8978	48
13	. 4292	. 5708	6.9971	. 4440	.9252	.0104	. 1026	. 8973	47
14	. 4320	. 5679	.9830	. 4470	.9110	.0104	. 1031	. 8969	46
15	.14349	.85651	6.9690	.14499	6.8969	1.0104	.01035	.98965	45
16	. 4378	. 5622	.9550	. 4529	.8828	.0105	. 1039	. 8961	44
17	. 4407	. 5593	.9411	. 4559	.8687	.0105	. 1043	. 8957	43
18	. 4436	. 5564	.9273	. 4588	.8547	.0106	. 1047	. 8952	42
19	. 4464	. 5536	.9135	. 4618	.8408	.0106	. 1052	. 8948	41
20	.14493	.85507	6.8998	.14648	6.8269	1.0107	.01056	.98944	40
21	. 4522	. 5478	.8861	. 4677	.8131	.0107	. 1060	. 8940	39
22	. 4551	. 5449	.8725	. 4707	.7993	.0107	. 1064	. 8936	38
23	. 4579	. 5420	.8589	. 4737	.7856	.0108	. 1068	. 8931	37
24	. 4608	. 5392	.8454	. 4767	.7720	.0108	. 1073	. 8927	36
25	.14637	.85363	6.8320	.14796	6.7584	1.0109	.01077	.98923	35
26	. 4666	. 5334	.8185	. 4826	.7448	.0109	. 1081	. 8919	34
27	. 4695	. 5305	.8052	. 4856	.7313	.0110	. 1085	. 8914	33
28	. 4723	. 5277	.7919	. 4886	.7179	.0110	. 1090	. 8910	32
29	. 4752	. 5248	.7787	. 4915	.7045	.0111	. 1094	. 8906	31
30	.14781	.85219	6.7655	.14945	6.6911	1.0111	.01098	.98901	30
31	. 4810	. 5190	.7523	. 4975	.6779	.0111	. 1103	. 8897	29
32	. 4838	. 5161	.7392	. 5004	.6646	.0112	. 1107	. 8893	28
33	. 4867	. 5133	.7262	. 5034	.6514	.0112	. 1111	. 8889	27
34	. 4896	. 5104	.7132	. 5064	.6383	.0113	. 1116	. 8884	26
35	.14925	.85075	6.7003	.15094	6.6252	1.0113	.01120	.98880	25
36	. 4953	. 5046	.6874	. 5123	.6122	.0114	. 1124	. 8876	24
37	. 4982	. 5018	.6745	. 5153	.5992	.0114	. 1129	. 8871	23
38	. 5011	. 4989	.6617	. 5183	.5863	.0115	. 1133	. 8867	22
39	. 5040	. 4960	.6490	. 5213	.5734	.0115	. 1137	. 8862	21
40	.15068	.84931	6.6363	.15243	6.5605	1.0115	.01142	.98858	20
41	. 5097	. 4903	.6237	. 5272	.5478	.0116	. 1146	. 8854	19
42	. 5126	. 4874	.6111	. 5302	.5350	.0116	. 1151	. 8849	18
43	. 5155	. 4845	.5985	. 5332	.5223	.0117	. 1155	. 8845	17
44	. 5183	. 4816	.5860	. 5362	.5097	.0117	. 1159	. 8840	16
45	.15212	.84788	6.5736	.15391	6.4971	1.0118	.01164	.98836	15
46	. 5241	. 4759	.5612	. 5421	.4845	.0118	. 1168	. 8832	14
47	. 5270	. 4730	.5488	. 5451	.4720	.0119	. 1173	. 8827	13
48	. 5298	. 4701	.5365	. 5481	.4596	.0119	. 1177	. 8823	12
49	. 5328	. 4672	.5243	. 5511	.4472	.0119	. 1182	. 8818	11
50	.15356	.84644	6.5121	.15540	6.4348	1.0120	.01186	.98814	10
51	. 5385	. 4615	.4999	. 5570	.4225	.0120	. 1190	. 8809	9
52	. 5413	. 4586	.4878	. 5600	.4103	.0121	. 1195	. 8805	8
53	. 5442	. 4558	.4757	. 5630	.3980	.0121	. 1199	. 8800	7
54	. 5471	. 4529	.4637	. 5659	.3859	.0122	. 1204	. 8796	6
55	.15500	.84500	6.4517	.15689	6.3737	1.0122	.01208	.98791	5
56	. 5528	. 4471	.4398	. 5719	.3616	.0123	. 1213	. 8787	4
57	. 5557	. 4443	.4279	. 5749	.3496	.0123	. 1217	. 8782	3
58	. 5586	. 4414	.4160	. 5779	.3376	.0124	. 1222	. 8778	2
59	. 5615	. 4385	.4042	. 5809	.3257	.0124	. 1227	. 8773	1
60	. 5643	. 4356	.3924	. 5838	.3137	.0125	. 1231	. 8769	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Sine.	Vrs. cos.	M.

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9° Natural Trigonometrical Functions. 170°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.15643	.84356	6.3924	.15838	6.3137	1.0125	.01231	.98769	60
1	. 5672	. 4328	.3807	. 5868	.3019	.0125	. 1236	. 8764	59
2	. 5701	. 4299	.3690	. 5898	.2901	.0125	. 1240	. 8760	58
3	. 5730	. 4270	.3574	. 5928	.2783	.0126	. 1245	. 8755	57
4	. 5758	. 4242	.3458	. 5958	.2665	.0126	. 1249	. 8750	56
5	.15787	.84213	6.3343	.15987	6.2548	1.0127	.01254	.98746	55
6	. 5816	. 4184	.3228	. 6017	.2432	.0127	. 1259	. 8741	54
7	. 5844	. 4155	.3113	. 6047	.2316	.0128	. 1263	. 8737	53
8	. 5873	. 4127	.2999	. 6077	.2200	.0128	. 1268	. 8732	52
9	. 5902	. 4098	.2885	. 6107	.2085	.0129	. 1272	. 8727	51
10	.15931	.84069	6.2772	.16137	6.1970	1.0129	.01277	.98723	50
11	. 5959	. 4041	.2659	. 6167	.1856	.0130	. 1282	. 8718	49
12	. 5988	. 4012	.2546	. 6196	.1742	.0130	. 1286	. 8714	48
13	. 6017	. 3983	.2434	. 6226	.1628	.0131	. 1291	. 8709	47
14	. 6045	. 3954	.2322	. 6256	.1515	.0131	. 1296	. 8704	46
15	.16074	.83926	6.2211	.16286	6.1402	1.0132	.01300	.98700	45
16	. 6103	. 3897	.2100	. 6316	.1290	.0132	. 1305	. 8695	44
17	. 6132	. 3868	.1990	. 6346	.1178	.0133	. 1310	. 8690	43
18	. 6160	. 3840	.1880	. 6376	.1066	.0133	. 1314	. 8685	42
19	. 6189	. 3811	.1770	. 6405	.0955	.0134	. 1319	. 8681	41
20	.16218	.83782	6.1661	.16435	6.0844	1.0134	.01324	.98676	40
21	. 6246	. 3753	.1552	. 6465	.0734	.0135	. 1328	. 8671	39
22	. 6275	. 3725	.1443	. 6495	.0624	.0135	. 1333	. 8667	38
23	. 6304	. 3696	.1335	. 6525	.0514	.0136	. 1338	. 8662	37
24	. 6333	. 3667	.1227	. 6555	.0405	.0136	. 1343	. 8657	36
25	.16361	.83639	6.1120	.16585	6.0296	1.0136	.01347	.98652	35
26	. 6390	. 3610	.1013	. 6615	.0188	.0137	. 1352	. 8648	34
27	. 6419	. 3581	.0906	. 6644	.0080	.0137	. 1357	. 8643	33
28	. 6447	. 3553	.0800	. 6674	5.9972	.0138	. 1362	. 8638	32
29	. 6476	. 3524	.0694	. 6704	.9865	.0138	. 1367	. 8633	31
30	.16505	.83495	6.0588	.16734	5.9758	1.0139	.01371	.98628	30
31	. 6533	. 3466	.0483	. 6764	.9651	.0139	. 1376	. 8624	29
32	. 6562	. 3438	.0379	. 6794	.9545	.0140	. 1381	. 8619	28
33	. 6591	. 3409	.0274	. 6824	.9439	.0140	. 1386	. 8614	27
34	. 6619	. 3380	.0170	. 6854	.9333	.0141	. 1391	. 8609	26
35	.16648	.83352	6.0066	.16884	5.9228	1.0141	.01395	.98604	25
36	. 6677	. 3323	5.9963	. 6914	.9123	.0142	. 1400	. 8600	24
37	. 6705	. 3294	.9860	. 6944	.9019	.0142	. 1405	. 8595	23
38	. 6734	. 3266	.9758	. 6973	.8915	.0143	. 1410	. 8590	22
39	. 6763	. 3237	.9655	. 7003	.8811	.0143	. 1415	. 8585	21
40	.16791	.83208	5.9554	.17033	5.8708	1.0144	.01420	.98580	20
41	. 6820	. 3180	.9452	. 7063	.8605	.0144	. 1425	. 8575	19
42	. 6849	. 3151	.9351	. 7093	.8502	.0145	. 1430	. 8570	18
43	. 6878	. 3122	.9250	. 7123	.8400	.0145	. 1434	. 8565	17
44	. 6906	. 3094	.9150	. 7153	.8298	.0146	. 1439	. 8560	16
45	.16935	.83065	5.9049	.17183	5.8196	1.0146	.01444	.98556	15
46	. 6964	. 3036	.8950	. 7213	.8095	.0147	. 1449	. 8551	14
47	. 6992	. 3008	.8850	. 7243	.7994	.0147	. 1454	. 8546	13
48	. 7021	. 2979	.8751	. 7273	.7894	.0148	. 1459	. 8541	12
49	. 7050	. 2950	.8652	. 7303	.7793	.0148	. 1464	. 8536	11
50	.17078	.82922	5.8554	.17333	5.7694	1.0149	.01469	.98531	10
51	. 7107	. 2893	.8456	. 7363	.7594	.0150	. 1474	. 8526	9
52	. 7136	. 2864	.8358	. 7393	.7495	.0150	. 1479	. 8521	8
53	. 7164	. 2836	.8261	. 7423	.7396	.0151	. 1484	. 8516	7
54	. 7193	. 2807	.8163	. 7453	.7297	.0151	. 1489	. 8511	6
55	.17221	.82778	5.8067	.17483	5.7199	1.0152	.01494	.98506	5
56	. 7250	. 2750	.7970	. 7513	.7101	.0152	. 1499	. 8501	4
57	. 7279	. 2721	.7874	. 7543	.7004	.0153	. 1504	. 8496	3
58	. 7307	. 2692	.7778	. 7573	.6906	.0153	. 1509	. 8491	2
59	. 7336	. 2664	.7683	. 7603	.6809	.0154	. 1514	. 8486	1
60	. 7365	. 2635	.7588	. 7633	.6713	.0154	. 1519	. 8481	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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10° Natural Trigonometrical Functions. 169°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.17365	.82635	5.7588	.17633	5.6713	1.0154	.01519	.98481	60
1	.7393	.2606	.7493	.7663	.6616	.0155	.1524	.8476	59
2	.7422	.2578	.7398	.7693	.6520	.0155	.1529	.8471	58
3	.7451	.2549	.7304	.7723	.6425	.0156	.1534	.8465	57
4	.7479	.2521	.7210	.7753	.6329	.0156	.1539	.8460	56
5	.17508	.82492	5.7117	.17783	5.6234	1.0157	.01544	.98455	55
6	.7537	.2463	.7023	.7813	.6140	.0157	.1550	.8450	54
7	.7565	.2435	.6930	.7843	.6045	.0158	.1555	.8445	53
8	.7594	.2406	.6838	.7873	.5951	.0158	.1560	.8440	52
9	.7622	.2377	.6745	.7903	.5857	.0159	.1565	.8435	51
10	.17651	.82349	5.6653	.17933	5.5764	1.0159	.01570	.98430	50
11	.7680	.2320	.6561	.7963	.5670	.0160	.1575	.8425	49
12	.7708	.2291	.6470	.7993	.5578	.0160	.1580	.8419	48
13	.7737	.2263	.6379	.8023	.5485	.0161	.1585	.8414	47
14	.7766	.2234	.6288	.8053	.5393	.0162	.1591	.8409	46
15	.17794	.82206	5.6197	.18083	5.5301	1.0162	.01596	.98404	45
16	.7823	.2177	.6107	.8113	.5209	.0163	.1601	.8399	44
17	.7852	.2148	.6017	.8143	.5117	.0163	.1606	.8394	43
18	.7880	.2120	.5928	.8173	.5026	.0164	.1611	.8388	42
19	.7909	.2091	.5838	.8203	.4936	.0164	.1617	.8383	41
20	.17937	.82062	5.5749	.18233	5.4845	1.0165	.01622	.98378	40
21	.7966	.2034	.5660	.8263	.4755	.0165	.1627	.8373	39
22	.7995	.2005	.5572	.8293	.4665	.0166	.1632	.8368	38
23	.8023	.1977	.5484	.8323	.4575	.0166	.1638	.8362	37
24	.8052	.1948	.5396	.8353	.4486	.0167	.1643	.8357	36
25	.18080	.81919	5.5308	.18383	5.4396	1.0167	.01648	.98352	35
26	.8109	.1891	.5221	.8413	.4308	.0168	.1653	.8347	34
27	.8138	.1862	.5134	.8444	.4219	.0169	.1659	.8341	33
28	.8166	.1834	.5047	.8474	.4131	.0169	.1664	.8336	32
29	.8195	.1805	.4960	.8504	.4043	.0170	.1669	.8331	31
30	.18223	.81776	5.4874	.18534	5.3955	1.0170	.01674	.98325	30
31	.8252	.1748	.4788	.8564	.3868	.0171	.1680	.8320	29
32	.8281	.1719	.4702	.8594	.3780	.0171	.1685	.8315	28
33	.8309	.1691	.4617	.8624	.3694	.0172	.1690	.8309	27
34	.8338	.1662	.4532	.8654	.3607	.0172	.1696	.8304	26
35	.18366	.81633	5.4447	.18684	5.3521	1.0173	.01701	.98299	25
36	.8395	.1605	.4362	.8714	.3434	.0174	.1706	.8293	24
37	.8424	.1576	.4278	.8745	.3349	.0174	.1712	.8288	23
38	.8452	.1548	.4194	.8775	.3263	.0175	.1717	.8283	22
39	.8481	.1519	.4110	.8805	.3178	.0175	.1722	.8277	21
40	.18509	.81490	5.4026	.18835	5.3093	1.0176	.01728	.98272	20
41	.8538	.1462	.3943	.8865	.3008	.0176	.1733	.8267	19
42	.8567	.1433	.3860	.8895	.2923	.0177	.1739	.8261	18
43	.8595	.1405	.3777	.8925	.2839	.0177	.1744	.8256	17
44	.8624	.1376	.3695	.8955	.2755	.0178	.1749	.8250	16
45	.18652	.81348	5.3612	.18985	5.2671	1.0179	.01755	.98245	15
46	.8681	.1319	.3530	.9016	.2588	.0179	.1760	.8240	14
47	.8709	.1290	.3449	.9046	.2505	.0180	.1766	.8234	13
48	.8738	.1262	.3367	.9076	.2422	.0180	.1771	.8229	12
49	.8767	.1233	.3286	.9106	.2339	.0181	.1777	.8223	11
50	.18795	.81205	5.3205	.19136	5.2257	1.0181	.01782	.98218	10
51	.8824	.1176	.3124	.9166	.2174	.0182	.1788	.8212	9
52	.8852	.1147	.3044	.9197	.2092	.0182	.1793	.8207	8
53	.8881	.1119	.2963	.9227	.2011	.0183	.1799	.8201	7
54	.8909	.1090	.2883	.9257	.1929	.0184	.1804	.8196	6
55	.18938	.81062	5.2803	.19287	5.1848	1.0184	.01810	.98190	5
56	.8967	.1033	.2724	.9317	.1767	.0185	.1815	.8185	4
57	.8995	.1005	.2645	.9347	.1686	.0185	.1821	.8179	3
58	.9024	.0976	.2566	.9378	.1606	.0186	.1826	.8174	2
59	.9052	.0948	.2487	.9408	.1525	.0186	.1832	.8168	1
60	.9081	.0919	.2408	.9438	.1445	.0187	.1837	.8163	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

100° 79°

11° **Natural Trigonometrical Functions.** **168°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.19081	.80919	5.2408	.19438	5.1445	1.0187	.01837	.98163	60
1	. 9109	. 0890	.2330	. 9468	.1366	.0188	. 1843	. 8157	59
2	. 9138	. 0862	.2252	. 9498	.1286	.0188	. 1848	. 8152	58
3	. 9166	. 0833	.2174	. 9529	.1207	.0189	. 1854	. 8146	57
4	. 9195	. 0805	.2097	. 9559	.1128	.0189	. 1859	. 8140	56
5	.19224	.80776	5.2019	.19589	5.1049	1.0190	.01865	.98135	55
6	. 9252	. 0748	.1942	. 9619	.0970	.0191	. 1871	. 8129	54
7	. 9281	. 0719	.1865	. 9649	.0892	.0191	. 1876	. 8124	53
8	. 9309	. 0691	.1788	. 9680	.0814	.0192	. 1882	. 8118	52
9	. 9338	. 0662	.1712	. 9710	.0736	.0192	. 1887	. 8112	51
10	.19366	.80634	5.1636	.19740	5.0658	1.0193	.01893	.98107	50
11	. 9395	. 0605	.1560	. 9770	.0581	.0193	. 1899	. 8101	49
12	. 9423	. 0576	.1484	. 9800	.0504	.0194	. 1904	. 8095	48
13	. 9452	. 0548	.1409	. 9831	.0427	.0195	. 1910	. 8090	47
14	. 9480	. 0519	.1333	. 9861	.0350	.0195	. 1916	. 8084	46
15	.19509	.80491	5.1258	.19891	5.0273	1.0196	.01921	.98078	45
16	. 9537	. 0462	.1183	. 9921	.0197	.0196	. 1927	. 8073	44
17	. 9566	. 0434	.1109	. 9952	.0121	.0197	. 1933	. 8067	43
18	. 9595	. 0405	.1034	. 9982	.0045	.0198	. 1938	. 8061	42
19	. 9623	. 0377	.0960	.20012	4.9969	.0198	. 1944	. 8056	41
20	.19652	.80348	5.0886	.20042	4.9894	1.0199	.01950	.98050	40
21	. 9680	. 0320	.0812	. 0073	.9819	.0199	. 1956	. 8044	39
22	. 9709	. 0291	.0739	. 0103	.9744	.0200	. 1961	. 8039	38
23	. 9737	. 0263	.0666	. 0133	.9669	.0201	. 1967	. 8033	37
24	. 9766	. 0234	.0593	. 0163	.9594	.0201	. 1973	. 8027	36
25	.19794	.80206	5.0520	.20194	4.9520	1.0202	.01979	.98021	35
26	. 9823	. 0177	.0447	. 0224	.9446	.0202	. 1984	. 8016	34
27	. 9851	. 0149	.0375	. 0254	.9372	.0203	. 1990	. 8010	33
28	. 9880	. 0120	.0302	. 0285	.9298	.0204	. 1996	. 8004	32
29	. 9908	. 0092	.0230	. 0315	.9225	.0204	. 2002	. 7998	31
30	.19937	.80063	5.0158	.20345	4.9151	1.0205	.02007	.97992	30
31	. 9965	. 0035	.0087	. 0375	.9078	.0205	. 2013	. 7987	29
32	. 9994	. 0006	.0015	. 0406	.9006	.0206	. 2019	. 7981	28
33	.20022	.79978	4.9944	. 0436	.8933	.0207	. 2025	. 7975	27
34	. 0051	. 9949	.9873	. 0466	.8860	.0207	. 2031	. 7969	26
35	.20079	.79921	4.9802	.20497	4.8788	1.0208	.02037	.97963	25
36	. 0108	. 9892	.9732	. 0527	.8716	.0208	. 2042	. 7957	24
37	. 0136	. 9863	.9661	. 0557	.8644	.0209	. 2048	. 7952	23
38	. 0165	. 9835	.9591	. 0588	.8573	.0210	. 2054	. 7946	22
39	. 0193	. 9807	.9521	. 0618	.8501	.0210	. 2060	. 7940	21
40	.20222	.79778	4.9452	.20648	4.8430	1.0211	.02066	.97934	20
41	. 0250	. 9750	.9382	. 0679	.8359	.0211	. 2072	. 7928	19
42	. 0279	. 9721	.9313	. 0709	.8288	.0212	. 2078	. 7922	18
43	. 0307	. 9693	.9243	. 0739	.8217	.0213	. 2084	. 7916	17
44	. 0336	. 9664	.9175	. 0770	.8147	.0213	. 2089	. 7910	16
45	.20364	.79636	4.9106	.20800	4.8077	1.0214	.02095	.97904	15
46	. 0393	. 9607	.9037	. 0830	.8007	.0215	. 2101	. 7899	14
47	. 0421	. 9579	.8969	. 0861	.7937	.0215	. 2107	. 7893	13
48	. 0450	. 9550	.8901	. 0891	.7867	.0216	. 2113	. 7887	12
49	. 0478	. 9522	.8833	. 0921	.7798	.0216	. 2119	. 7881	11
50	.20506	.79493	4.8765	.20952	4.7728	1.0217	.02125	.97875	10
51	. 0535	. 9465	.8697	. 0982	.7659	.0218	. 2131	. 7869	9
52	. 0563	. 9436	.8630	. 1012	.7591	.0218	. 2137	. 7863	8
53	. 0592	. 9408	.8563	. 1043	.7522	.0219	. 2143	. 7857	7
54	. 0620	. 9379	.8496	. 1073	.7453	.0220	. 2149	. 7851	6
55	.20649	.79351	4.8429	.21104	4.7385	1.0220	.02155	.97845	5
56	. 0677	. 9323	.8362	. 1134	.7317	.0221	. 2161	. 7839	4
57	. 0706	. 9294	.8296	. 1164	.7249	.0221	. 2167	. 7833	3
58	. 0734	. 9266	.8229	. 1195	.7181	.0222	. 2173	. 7827	2
59	. 0763	. 9237	.8163	. 1225	.7114	.0223	. 2179	. 7821	1
60	. 0791	. 9209	.8097	. 1256	.7046	.0223	. 2185	. 7815	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

101° **78°**

12° Natural Trigonometrical Functions. 167°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.20791	.79209	4.8097	.21256	4.7046	1.0223	.02185	.97815	60
1	. 0820	. 9180	.8032	. 1286	.6979	.0224	. 2191	. 7809	59
2	. 0848	. 9152	.7966	. 1316	.6912	.0225	. 2197	. 7803	58
3	. 0876	. 9123	.7901	. 1347	.6845	.0225	. 2203	. 7806	57
4	. 0905	. 9105	.7835	. 1377	.6778	.0226	. 2209	. 7790	56
5	.20933	.79066	4.7770	.21408	4.6712	1.0226	.02215	.97784	55
6	. 0962	. 9038	.7706	. 1438	.6646	.0227	. 2222	. 7778	54
7	. 0990	. 9010	.7641	. 1468	.6580	.0228	. 2228	. 7772	53
8	. 1019	. 8981	.7576	. 1499	.6514	.0228	. 2234	. 7766	52
9	. 1047	. 8953	.7512	. 1529	.6448	.0229	. 2240	. 7760	51
10	.21076	.78924	4.7448	.21560	4.6382	1.0230	.02246	.97754	50
11	. 1104	. 8896	.7384	. 1590	.6317	.0230	. 2252	. 7748	49
12	. 1132	. 8867	.7320	. 1621	.6252	.0231	. 2258	. 7741	48
13	. 1161	. 8839	.7257	. 1651	.6187	.0232	. 2264	. 7735	47
14	. 1189	. 8811	.7193	. 1682	.6122	.0232	. 2271	. 7729	46
15	.21218	.78782	4.7130	.21712	4.6057	1.0233	.02277	.97723	45
16	. 1246	. 8754	.7067	. 1742	.5993	.0234	. 2283	. 7717	44
17	. 1275	. 8725	.7004	. 1773	.5928	.0234	. 2289	. 7711	43
18	. 1303	. 8697	.6942	. 1803	.5864	.0235	. 2295	. 7704	42
19	. 1331	. 8668	.6879	. 1834	.5800	.0235	. 2302	. 7698	41
20	.21360	.78640	4.6817	.21864	4.5736	1.0236	.02308	.97692	40
21	. 1388	. 8612	.6754	. 1895	.5673	.0237	. 2314	. 7686	39
22	. 1417	. 8583	.6692	. 1925	.5609	.0237	. 2320	. 7680	38
23	. 1445	. 8555	.6631	. 1956	.5546	.0238	. 2326	. 7673	37
24	. 1473	. 8526	.6569	. 1986	.5483	.0239	. 2333	. 7667	36
25	.21502	.78508	4.6507	.22017	4.5420	1.0239	.02339	.97661	35
26	. 1530	. 8470	.6446	. 2047	.5357	.0240	. 2345	. 7655	34
27	. 1559	. 8441	.6385	. 2078	.5294	.0241	. 2351	. 7648	33
28	. 1587	. 8413	.6324	. 2108	.5232	.0241	. 2358	. 7642	32
29	. 1615	. 8384	.6263	. 2139	.5169	.0242	. 2364	. 7636	31
30	.21644	.78356	4.6202	.22169	4.5107	1.0243	.02370	.97630	30
31	. 1672	. 8328	.6142	. 2200	.5045	.0243	. 2377	. 7623	29
32	. 1701	. 8299	.6081	. 2230	.4983	.0244	. 2383	. 7617	28
33	. 1729	. 8271	.6021	. 2261	.4921	.0245	. 2389	. 7611	27
34	. 1757	. 8242	.5961	. 2291	.4860	.0245	. 2396	. 7604	26
35	.21786	.78214	4.5901	.22322	4.4799	1.0246	.02402	.97598	25
36	. 1814	. 8186	.5841	. 2353	.4737	.0247	. 2408	. 7592	24
37	. 1843	. 8154	.5782	. 2383	.4676	.0247	. 2415	. 7585	23
38	. 1871	. 8129	.5722	. 2414	.4615	.0248	. 2421	. 7579	22
39	. 1899	. 8100	.5663	. 2444	.4555	.0249	. 2427	. 7573	21
40	.21928	.78072	4.5604	.22475	4.4494	1.0249	.02434	.97566	20
41	. 1956	. 8043	.5545	. 2505	.4434	.0250	. 2440	. 7560	19
42	. 1985	. 8015	.5486	. 2536	.4373	.0251	. 2446	. 7553	18
43	. 2013	. 7987	.5428	. 2566	.4313	.0251	. 2453	. 7547	17
44	. 2041	. 7959	.5369	. 2597	.4253	.0252	. 2459	. 7541	16
45	.22070	.77930	4.5311	.22628	4.4194	1.0253	.02466	.97534	15
46	. 2098	. 7902	.5253	. 2658	.4134	.0253	. 2472	. 7528	14
47	. 2126	. 7873	.5195	. 2689	.4074	.0254	. 2479	. 7521	13
48	. 2155	. 7845	.5137	. 2719	.4015	.0255	. 2485	. 7515	12
49	. 2183	. 7817	.5079	. 2750	.3956	.0255	. 2491	. 7508	11
50	.22211	.77788	4.5021	.22781	4.3897	1.0256	.02498	.97502	10
51	. 2240	. 7760	.4964	. 2811	.3838	.0257	. 2504	. 7495	9
52	. 2268	. 7732	.4907	. 2842	.3779	.0257	. 2511	. 7489	8
53	. 2297	. 7703	.4850	. 2872	.3721	.0258	. 2517	. 7483	7
54	. 2325	. 7675	.4793	. 2903	.3662	.0259	. 2524	. 7476	6
55	.22353	.77647	4.4736	.22934	4.3604	1.0260	.02530	.97470	5
56	. 2382	. 7618	.4679	. 2964	.3546	.0260	. 2537	. 7463	4
57	. 2410	. 7590	.4623	. 2995	.3488	.0261	. 2543	. 7457	3
58	. 2438	. 7561	.4566	. 3025	.3430	.0262	. 2550	. 7450	2
59	. 2467	. 7533	.4510	. 3056	.3372	.0262	. 2556	. 7443	1
60	. 2495	. 7505	.4454	. 3087	.3315	.0263	. 2563	. 7437	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

102° 77°

13° Natural Trigonometrical Functions. 166°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.22495	.77505	4.4454	.23087	4.3315	1.0263	.02563	.97437	60
1	. 2523	. 7476	.4398	. 3117	.3257	.0264	. 2569	. 7430	59
2	. 2552	. 7448	.4342	. 3148	.3200	.0264	. 2576	. 7424	58
3	. 2580	. 7420	.4287	. 3179	.3143	.0265	. 2583	. 7417	57
4	. 2608	. 7391	.4231	. 3209	.3086	.0266	. 2589	. 7411	56
5	.22637	.77363	4.4176	.23240	4.3029	1.0266	.02596	.97404	55
6	. 2665	. 7335	.4121	. 3270	.2972	.0267	. 2602	. 7398	54
7	. 2693	. 7306	.4065	. 3301	.2916	.0268	. 2609	. 7391	53
8	. 2722	. 7278	.4011	. 3332	.2859	.0268	. 2616	. 7384	52
9	. 2750	. 7250	.3956	. 3363	.2803	.0269	. 2622	. 7378	51
10	.22778	.77221	4.3901	.23393	4.2747	1.0270	.02629	.97371	50
11	. 2807	. 7193	.3847	. 3424	.2691	.0271	. 2635	. 7364	49
12	. 2835	. 7165	.3792	. 3455	.2635	.0271	. 2642	. 7358	48
13	. 2863	. 7136	.3738	. 3485	.2579	.0272	. 2649	. 7351	47
14	. 2892	. 7108	.3684	. 3516	.2524	.0273	. 2655	. 7344	46
15	.22920	.77080	4.3630	.23547	4.2468	1.0273	.02662	.97338	45
16	. 2948	. 7052	.3576	. 3577	.2413	.0274	. 2669	. 7331	44
17	. 2977	. 7023	.3522	. 3608	.2358	.0275	. 2675	. 7324	43
18	. 3005	. 6995	.3469	. 3639	.2303	.0276	. 2682	. 7318	42
19	. 3033	. 6967	.3415	. 3670	.2248	.0276	. 2689	. 7311	41
20	.23061	.76938	4.3362	.23700	4.2193	1.0277	.02695	.97304	40
21	. 3090	. 6910	.3309	. 3731	.2139	.0278	. 2702	. 7298	39
22	. 3118	. 6882	.3256	. 3762	.2084	.0278	. 2709	. 7291	38
23	. 3146	. 6853	.3203	. 3793	.2030	.0279	. 2716	. 7284	37
24	. 3175	. 6825	.3150	. 3823	.1976	.0280	. 2722	. 7277	36
25	.23203	.76797	4.3098	.23854	4.1921	1.0280	.02729	.97271	35
26	. 3231	. 6769	.3045	. 3885	.1867	.0281	. 2736	. 7264	34
27	. 3260	. 6740	.2993	. 3916	.1814	.0282	. 2743	. 7257	33
28	. 3288	. 6712	.2941	. 3946	.1760	.0283	. 2749	. 7250	32
29	. 3316	. 6684	.2888	. 3977	.1706	.0283	. 2756	. 7244	31
30	.23344	.76655	4.2836	.24008	4.1653	1.0284	.02763	.97237	30
31	. 3373	. 6627	.2785	. 4039	.1600	.0285	. 2770	. 7230	29
32	. 3401	. 6599	.2733	. 4069	.1546	.0285	. 2777	. 7223	28
33	. 3429	. 6571	.2681	. 4100	.1493	.0286	. 2783	. 7216	27
34	. 3458	. 6542	.2630	. 4131	.1440	.0287	. 2790	. 7210	26
35	.23486	.76514	4.2579	.24162	4.1388	1.0288	.02797	.97203	25
36	. 3514	. 6486	.2527	. 4192	.1335	.0288	. 2804	. 7196	24
37	. 3542	. 6457	.2476	. 4223	.1282	.0289	. 2811	. 7189	23
38	. 3571	. 6429	.2425	. 4254	.1230	.0290	. 2818	. 7182	22
39	. 3599	. 6401	.2375	. 4285	.1178	.0291	. 2824	. 7175	21
40	.23627	.76373	4.2324	.24316	4.1126	1.0291	.02831	.97169	20
41	. 3655	. 6344	.2273	. 4346	.1073	.0292	. 2838	. 7162	19
42	. 3684	. 6316	.2223	. 4377	.1022	.0293	. 2845	. 7155	18
43	. 3712	. 6288	.2173	. 4408	.0970	.0293	. 2852	. 7148	17
44	. 3740	. 6260	.2122	. 4439	.0918	.0294	. 2859	. 7141	16
45	.23768	.76231	4.2072	.24470	4.0867	1.0295	.02866	.97134	15
46	. 3797	. 6203	.2022	. 4501	.0815	.0296	. 2873	. 7127	14
47	. 3825	. 6175	.1972	. 4531	.0764	.0296	. 2880	. 7120	13
48	. 3853	. 6147	.1923	. 4562	.0713	.0297	. 2886	. 7113	12
49	. 3881	. 6118	.1873	. 4593	.0662	.0298	. 2893	. 7106	11
50	.23910	.76090	4.1824	.24624	4.0611	1.0299	.02900	.97099	10
51	. 3938	. 6062	.1774	. 4655	.0560	.0299	. 2907	. 7092	9
52	. 3966	. 6034	.1725	. 4686	.0509	.0300	. 2914	. 7086	8
53	. 3994	. 6005	.1676	. 4717	.0458	.0301	. 2921	. 7079	7
54	. 4023	. 5977	.1627	. 4747	.0408	.0302	. 2928	. 7072	6
55	.24051	.75949	4.1578	.24778	4.0358	1.0302	.02935	.97065	5
56	. 4079	. 5921	.1529	. 4809	.0307	.0303	. 2942	. 7058	4
57	. 4107	. 5892	.1481	. 4840	.0257	.0304	. 2949	. 7051	3
58	. 4136	. 5864	.1432	. 4871	.0207	.0305	. 2956	. 7044	2
59	. 4164	. 5836	.1384	. 4902	.0157	.0305	. 2963	. 7037	1
60	. 4192	. 5808	.1336	. 4933	.0108	.0306	. 2970	. 7029	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

103° 76°

14° Natural Trigonometrical Functions. 165°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.24192	.75808	4.1336	.24933	4.0108	1.0306	.02970	.97029	60
1	. 4220	. 5779	.1287	. 4964	.0058	.0307	. 2977	. 7022	59
2	. 4249	. 5751	.1239	. 4995	.0009	.0308	. 2984	. 7015	58
3	. 4277	. 5723	.1191	. 5025	3.9959	.0308	. 2991	. 7008	57
4	. 4305	. 5695	.1144	. 5056	.9910	.0309	. 2999	. 7001	56
5	.24333	.75667	4.1096	.25087	3.9861	1.0310	.03006	.96994	55
6	. 4361	. 5638	.1048	. 5118	.9812	.0311	. 3013	. 6987	54
7	. 4390	. 5610	.1001	. 5149	.9763	.0311	. 3020	. 6980	53
8	. 4418	. 5582	.0953	. 5180	.9714	.0312	. 3027	. 6973	52
9	. 4446	. 5554	.0906	. 5211	.9665	.0313	. 3034	. 6966	51
10	.24474	.75526	4.0859	.25242	3.9616	1.0314	.03041	.96959	50
11	. 4502	. 5497	.0812	. 5273	.9568	.0314	. 3048	. 6952	49
12	. 4531	. 5469	.0765	. 5304	.9520	.0315	. 3055	. 6944	48
13	. 4559	. 5441	.0718	. 5335	.9471	.0316	. 3063	. 6937	47
14	. 4587	. 5413	.0672	. 5366	.9423	.0317	. 3070	. 6930	46
15	.24615	.75385	4.0625	.25397	3.9375	1.0317	.03077	.96923	45
16	. 4643	. 5356	.0579	. 5428	.9327	.0318	. 3084	. 6916	44
17	. 4672	. 5328	.0532	. 5459	.9279	.0319	. 3091	. 6909	43
18	. 4700	. 5300	.0486	. 5490	.9231	.0320	. 3098	. 6901	42
19	. 4728	. 5272	.0440	. 5521	.9184	.0320	. 3106	. 6894	41
20	.24756	.75244	4.0394	.25552	3.9136	1.0321	.03113	.96887	40
21	. 4784	. 5215	.0348	. 5583	.9089	.0322	. 3120	. 6880	39
22	. 4813	. 5187	.0302	. 5614	.9042	.0323	. 3127	. 6873	38
23	. 4841	. 5159	.0256	. 5645	.8994	.0323	. 3134	. 6865	37
24	. 4869	. 5131	.0211	. 5676	.8947	.0324	. 3142	. 6858	36
25	.24897	.75103	4.0165	.25707	3.8900	1.0325	.03149	.96851	35
26	. 4925	. 5075	.0120	. 5738	.8853	.0326	. 3156	. 6844	34
27	. 4953	. 5046	.0074	. 5769	.8807	.0327	. 3163	. 6836	33
28	. 4982	. 5018	.0029	. 5800	.8760	.0327	. 3171	. 6829	32
29	. 5010	. 4990	3.9984	. 5831	.8713	.0328	. 3178	. 6822	31
30	.25038	.74962	3.9939	.25862	3.8667	1.0329	.03185	.96815	30
31	. 5066	. 4934	.9894	. 5893	.8621	.0330	. 3192	. 6807	29
32	. 5094	. 4906	.9850	. 5924	.8574	.0330	. 3200	. 6800	28
33	. 5122	. 4877	.9805	. 5955	.8528	.0331	. 3207	. 6793	27
34	. 5151	. 4849	.9760	. 5986	.8482	.0332	. 3214	. 6785	26
35	.25179	.74821	3.9716	.26017	3.8436	1.0333	.03222	.96778	25
36	. 5207	. 4793	.9672	. 6048	.8390	.0334	. 3229	. 6771	24
37	. 5235	. 4765	.9627	. 6079	.8345	.0334	. 3236	. 6763	23
38	. 5263	. 4737	.9583	. 6110	.8299	.0335	. 3244	. 6756	22
39	. 5291	. 4709	.9539	. 6141	.8254	.0336	. 3251	. 6749	21
40	.25319	.74680	3.9495	.26172	3.8208	1.0337	.03258	.96741	20
41	. 5348	. 4652	.9451	. 6203	.8163	.0338	. 3266	. 6734	19
42	. 5376	. 4624	.9408	. 6234	.8118	.0338	. 3273	. 6727	18
43	. 5404	. 4596	.9364	. 6266	.8073	.0339	. 3281	. 6719	17
44	. 5432	. 4568	.9320	. 6297	.8027	.0340	. 3288	. 6712	16
45	.25460	.74540	3.9277	.26328	3.7983	1.0341	.03295	.96704	15
46	. 5488	. 4512	.9234	. 6359	.7938	.0341	. 3303	. 6697	14
47	. 5516	. 4483	.9190	. 6390	.7893	.0342	. 3310	. 6690	13
48	. 5544	. 4455	.9147	. 6421	.7848	.0343	. 3318	. 6682	12
49	. 5573	. 4427	.9104	. 6452	.7804	.0344	. 3325	. 6675	11
50	.25601	.74399	3.9061	.26483	3.7759	1.0345	.03332	.96667	10
51	. 5629	. 4371	.9018	. 6514	.7715	.0345	. 3340	. 6660	9
52	. 5657	. 4344	.8976	. 6546	.7671	.0346	. 3347	. 6652	8
53	. 5685	. 4315	.8933	. 6577	.7627	.0347	. 3355	. 6645	7
54	. 5713	. 4287	.8890	. 6608	.7583	.0348	. 3362	. 6638	6
55	.25741	.74259	3.8848	.26639	3.7539	1.0349	.03370	.96630	5
56	. 5769	. 4230	.8805	. 6670	.7495	.0349	. 3377	. 6623	4
57	. 5798	. 4202	.8763	. 6701	.7451	.0350	. 3385	. 6615	3
58	. 5826	. 4174	.8721	. 6732	.7407	.0351	. 3392	. 6608	2
59	. 5854	. 4146	.8679	. 6764	.7364	.0352	. 3400	. 6600	1
60	. 5882	. 4118	.8637	. 6795	.7320	.0353	. 3407	. 6592	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

104° 75°

15° Natural Trigonometrical Functions. 164°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.25882	.74118	3.8637	.26795	3.7320	1.0353	.03407	.96592	60
1	. 5910	. 4090	.8595	. 6826	.7277	.0353	. 3415	. 6585	59
2	. 5938	. 4062	.8553	. 6857	.7234	.0354	. 3422	. 6577	58
3	. 5966	. 4034	.8512	. 6888	.7191	.0355	. 3430	. 6570	57
4	. 5994	. 4006	.8470	. 6920	.7147	.0356	. 3438	. 6562	56
5	.26022	.73978	3.8428	.26951	3.7104	1.0357	.03445	.96555	55
6	. 6050	. 3949	.8387	. 6982	.7062	.0358	. 3453	. 6547	54
7	. 6078	. 3921	.8346	. 7013	.7019	.0358	. 3460	. 6540	53
8	. 6107	. 3893	.8304	. 7044	.6976	.0359	. 3468	. 6532	52
9	. 6135	. 3865	.8263	. 7076	.6933	.0360	. 3475	. 6524	51
10	.26163	.73837	3.8222	.27107	3.6891	1.0361	.03483	.96517	50
11	. 6191	. 3809	.8181	. 7138	.6848	.0362	. 3491	. 6509	49
12	. 6219	. 3781	.8140	. 7169	.6806	.0362	. 3498	. 6502	48
13	. 6247	. 3753	.8100	. 7201	.6764	.0363	. 3506	. 6494	47
14	. 6275	. 3725	.8059	. 7232	.6722	.0364	. 3514	. 6486	46
15	.26303	.73697	3.8018	.27263	3.6679	1.0365	.03521	.96479	45
16	. 6331	. 3669	.7978	. 7294	.6637	.0366	. 3529	. 6471	44
17	. 6359	. 3641	.7937	. 7326	.6596	.0367	. 3536	. 6463	43
18	. 6387	. 3613	.7897	. 7357	.6554	.0367	. 3544	. 6456	42
19	. 6415	. 3585	.7857	. 7388	.6512	.0368	. 3552	. 6448	41
20	.26443	.73556	3.7816	.27419	3.6470	1.0369	.03560	.96440	40
21	. 6471	. 3528	.7776	. 7451	.6429	.0370	. 3567	. 6433	39
22	. 6499	. 3500	.7736	. 7482	.6387	.0371	. 3575	. 6425	38
23	. 6527	. 3472	.7697	. 7513	.6346	.0371	. 3583	. 6417	37
24	. 6556	. 3444	.7657	. 7544	.6305	.0372	. 3590	. 6409	36
25	.26584	.73416	3.7617	.27576	3.6263	1.0373	.03598	.96402	35
26	. 6612	. 3388	.7577	. 7607	.6222	.0374	. 3606	. 6394	34
27	. 6640	. 3360	.7538	. 7638	.6181	.0375	. 3614	. 6386	33
28	. 6668	. 3332	.7498	. 7670	.6140	.0376	. 3621	. 6378	32
29	. 6696	. 3304	.7459	. 7701	.6100	.0376	. 3629	. 6371	31
30	.26724	.73276	3.7420	.27732	3.6059	1.0377	.03637	.96363	30
31	. 6752	. 3248	.7380	. 7764	.6018	.0378	. 3645	. 6355	29
32	. 6780	. 3220	.7341	. 7795	.5977	.0379	. 3652	. 6347	28
33	. 6808	. 3192	.7302	. 7826	.5937	.0380	. 3660	. 6340	27
34	. 6836	. 3164	.7263	. 7858	.5896	.0381	. 3668	. 6332	26
35	.26864	.73136	3.7224	.27889	3.5856	1.0382	.03676	.96324	25
36	. 6892	. 3108	.7186	. 7920	.5816	.0382	. 3684	. 6316	24
37	. 6920	. 3080	.7147	. 7952	.5776	.0383	. 3691	. 6308	23
38	. 6948	. 3052	.7108	. 7983	.5736	.0384	. 3699	. 6301	22
39	. 6976	. 3024	.7070	. 8014	.5696	.0385	. 3707	. 6293	21
40	.27004	.72996	3.7031	.28046	3.5656	1.0386	.03715	.96285	20
41	. 7032	. 2968	.6993	. 8077	.5616	.0387	. 3723	. 6277	19
42	. 7060	. 2940	.6955	. 8109	.5576	.0387	. 3731	. 6269	18
43	. 7088	. 2912	.6917	. 8140	.5536	.0388	. 3739	. 6261	17
44	. 7116	. 2884	.6878	. 8171	.5497	.0389	. 3746	. 6253	16
45	.27144	.72856	3.6840	.28203	3.5457	1.0390	.03754	.96245	15
46	. 7172	. 2828	.6802	. 8234	.5418	.0391	. 3762	. 6238	14
47	. 7200	. 2800	.6765	. 8266	.5378	.0392	. 3770	. 6230	13
48	. 7228	. 2772	.6727	. 8297	.5339	.0393	. 3778	. 6222	12
49	. 7256	. 2744	.6689	. 8328	.5300	.0393	. 3786	. 6214	11
50	.27284	.72716	3.6651	.28360	3.5261	1.0394	.03794	.96206	10
51	. 7312	. 2688	.6614	. 8391	.5222	.0395	. 3802	. 6198	9
52	. 7340	. 2660	.6576	. 8423	.5183	.0396	. 3810	. 6190	8
53	. 7368	. 2632	.6539	. 8454	.5144	.0397	. 3818	. 6182	7
54	. 7396	. 2604	.6502	. 8486	.5105	.0398	. 3826	. 6174	6
55	.27424	.72576	3.6464	.28517	3.5066	1.0399	.03834	.96166	5
56	. 7452	. 2548	.6427	. 8549	.5028	.0399	. 3842	. 6158	4
57	. 7480	. 2520	.6390	. 8580	.4989	.0400	. 3850	. 6150	3
58	. 7508	. 2492	.6353	. 8611	.4951	.0401	. 3858	. 6142	2
59	. 7536	. 2464	.6316	. 8643	.4912	.0402	. 3866	. 6134	1
60	. 7564	. 2436	.6279	. 8674	.4874	.0403	. 3874	. 6126	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

105° 74°

16° Natural Trigonometrical Functions. 163°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.27564	.72436	3.6279	.28674	3.4874	1.0403	.03874	.96126	60
1	. 7592	. 2408	.6243	. 8706	.4836	.0404	. 3882	. 6118	59
2	. 7620	. 2380	.6206	. 8737	.4798	.0405	. 3890	. 6110	58
3	. 7648	. 2352	.6169	. 8769	.4760	.0406	. 3898	. 6102	57
4	. 7675	. 2324	.6133	. 8800	.4722	.0406	. 3906	. 6094	56
5	.27703	.72296	3.6096	.28832	3.4684	1.0407	.03914	.96086	55
6	. 7731	. 2268	.6060	. 8863	.4646	.0408	. 3922	. 6078	54
7	. 7759	. 2240	.6024	. 8895	.4608	.0409	. 3930	. 6070	53
8	. 7787	. 2213	.5987	. 8926	.4570	.0410	. 3938	. 6062	52
9	. 7815	. 2185	.5951	. 8958	.4533	.0411	. 3946	. 6054	51
10	.27843	.72157	3.5915	.28990	3.4495	1.0412	.03954	.96045	50
11	. 7871	. 2129	.5879	. 9021	.4458	.0413	. 3962	. 6037	49
12	. 7899	. 2101	.5843	. 9053	.4420	.0413	. 3971	. 6029	48
13	. 7927	. 2073	.5807	. 9084	.4383	.0414	. 3979	. 6021	47
14	. 7955	. 2045	.5772	. 9116	.4346	.0415	. 3987	. 6013	46
15	.27983	.72017	3.5736	.29147	3.4308	1.0416	.03995	.96005	45
16	. 8011	. 1989	.5700	. 9179	.4271	.0417	. 4003	. 5997	44
17	. 8039	. 1961	.5665	. 9210	.4234	.0418	. 4011	. 5989	43
18	. 8067	. 1933	.5629	. 9242	.4197	.0419	. 4019	. 5980	42
19	. 8094	. 1905	.5594	. 9274	.4160	.0420	. 4028	. 5972	41
20	.28122	.71877	3.5559	.29305	3.4124	1.0420	.04036	.95964	40
21	. 8150	. 1849	.5523	. 9337	.4087	.0421	. 4044	. 5956	39
22	. 8178	. 1822	.5488	. 9368	.4050	.0422	. 4052	. 5948	38
23	. 8206	. 1794	.5453	. 9400	.4014	.0423	. 4060	. 5940	37
24	. 8234	. 1766	.5418	. 9432	.3977	.0424	. 4069	. 5931	36
25	.28262	.71738	3.5383	.29463	3.3941	1.0425	.04077	.95923	35
26	. 8290	. 1710	.5348	. 9495	.3904	.0426	. 4085	. 5915	34
27	. 8318	. 1682	.5313	. 9526	.3868	.0427	. 4093	. 5907	33
28	. 8346	. 1654	.5279	. 9558	.3832	.0428	. 4101	. 5898	32
29	. 8374	. 1626	.5244	. 9590	.3795	.0428	. 4110	. 5890	31
30	.28401	.71608	3.5209	.29621	3.3759	1.0429	.04118	.95882	30
31	. 8429	. 1570	.5175	. 9653	.3723	.0430	. 4126	. 5874	29
32	. 8457	. 1543	.5140	. 9685	.3687	.0431	. 4134	. 5865	28
33	. 8485	. 1515	.5106	. 9716	.3651	.0432	. 4143	. 5857	27
34	. 8513	. 1487	.5072	. 9748	.3616	.0433	. 4151	. 5849	26
35	.28541	.71459	3.5037	.29780	3.3580	1.0434	.04159	.95840	25
36	. 8569	. 1431	.5003	. 9811	.3544	.0435	. 4168	. 5832	24
37	. 8597	. 1403	.4969	. 9843	.3509	.0436	. 4176	. 5824	23
38	. 8624	. 1375	.4935	. 9875	.3473	.0437	. 4184	. 5816	22
39	. 8652	. 1347	.4901	. 9906	.3438	.0438	. 4193	. 5807	21
40	.28680	.71320	3.4867	.29938	3.3402	1.0438	.04201	.95799	20
41	. 8708	. 1292	.4833	. 9970	.3367	.0439	. 4209	. 5791	19
42	. 8736	. 1264	.4799	.30001	.3332	.0440	. 4218	. 5782	18
43	. 8764	. 1236	.4766	. 0033	.3296	.0441	. 4226	. 5774	17
44	. 8792	. 1208	.4732	. 0065	.3261	.0442	. 4234	. 5765	16
45	.28820	.71180	3.4698	.30096	3.3226	1.0443	.04243	.95757	15
46	. 8847	. 1152	.4665	. 0128	.3191	.0444	. 4251	. 5749	14
47	. 8875	. 1125	.4632	. 0160	.3156	.0445	. 4260	. 5740	13
48	. 8903	. 1097	.4598	. 0192	.3121	.0446	. 4268	. 5732	12
49	. 8931	. 1069	.4565	. 0223	.3087	.0447	. 4276	. 5723	11
50	.28959	.71041	3.4532	.30255	3.3052	1.0448	.04285	.95715	10
51	. 8987	. 1013	.4498	. 0287	.3017	.0448	. 4293	. 5707	9
52	. 9014	. 0985	.4465	. 0319	.2983	.0449	. 4302	. 5698	8
53	. 9042	. 0958	.4432	. 0350	.2948	.0450	. 4310	. 5690	7
54	. 9070	. 0930	.4399	. 0382	.2914	.0451	. 4319	. 5681	6
55	.29098	.70902	3.4366	.30414	3.2879	1.0452	.04327	.95673	5
56	. 9126	. 0874	.4334	. 0446	.2845	.0453	. 4335	. 5664	4
57	. 9154	. 0846	.4301	. 0478	.2811	.0454	. 4344	. 5656	3
58	. 9181	. 0818	.4268	. 0509	.2777	.0455	. 4352	. 5647	2
59	. 9209	. 0791	.4236	. 0541	.2742	.0456	. 4361	. 5639	1
60	. 9237	. 0763	.4203	. 0573	.2708	.0457	. 4369	. 5630	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

106° 73°

17° Natural Trigonometrical Functions. 162°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.29237	.70763	3.4203	.30573	3.2708	1.0457	.04369	.95630	60
1	. 9265	. 0735	.4170	. 0605	.2674	.0458	. 4378	. 5622	59
2	. 9293	. 0707	.4138	. 0637	.2640	.0459	. 4386	. 5613	58
3	. 9321	. 0679	.4106	. 0668	.2607	.0460	. 4395	. 5605	57
4	. 9348	. 0651	.4073	. 0700	.2573	.0461	. 4404	. 5596	56
5	.29376	.70624	3.4041	.30732	3.2539	1.0461	.04412	.95588	55
6	. 9404	. 0596	.4009	. 0764	.2505	.0462	. 4421	. 5579	54
7	. 9432	. 0568	.3977	. 0796	.2472	.0463	. 4426	. 5571	53
8	. 9460	. 0540	.3945	. 0828	.2438	.0464	. 4438	. 5562	52
9	. 9487	. 0512	.3913	. 0859	.2405	.0465	. 4446	. 5554	51
10	.29515	.70485	3.3881	.30891	3.2371	1.0466	.04455	.95545	50
11	. 9543	. 0457	.3849	. 0923	.2338	.0467	. 4463	. 5536	49
12	. 9571	. 0429	.3817	. 0955	.2305	.0468	. 4472	. 5528	48
13	. 9598	. 0401	.3785	. 0987	.2271	.0469	. 4481	. 5519	47
14	. 9626	. 0374	.3754	. 1019	.2238	.0470	. 4489	. 5511	46
15	.29654	.70346	3.3722	.31051	3.2205	1.0471	.04498	.95502	45
16	. 9682	. 0318	.3690	. 1083	.2172	.0472	. 4507	. 5493	44
17	. 9710	. 0290	.3659	. 1115	.2139	.0473	. 4515	. 5485	43
18	. 9737	. 0262	.3627	. 1146	.2106	.0474	. 4524	. 5476	42
19	. 9765	. 0235	.3596	. 1178	.2073	.0475	. 4532	. 5467	41
20	.29793	.70207	3.3565	.31210	3.2041	1.0476	.04541	.95459	40
21	. 9821	. 0179	.3534	. 1242	.2008	.0477	. 4550	. 5450	39
22	. 9848	. 0151	.3502	. 1274	.1975	.0478	. 4558	. 5441	38
23	. 9876	. 0124	.3471	. 1306	.1942	.0478	. 4567	. 5433	37
24	. 9904	. 0096	.3440	. 1338	.1910	.0479	. 4576	. 5424	36
25	.29932	.70068	3.3409	.31370	3.1877	1.0480	.04585	.95415	35
26	. 9959	. 0040	.3378	. 1402	.1845	.0481	. 4593	. 5407	34
27	. 9987	. 0013	.3347	. 1434	.1813	.0482	. 4602	. 5398	33
28	.30015	.69982	.3316	. 1466	.1780	.0483	. 4611	. 5389	32
29	. 0043	. 9957	.3286	. 1498	.1748	.0484	. 4619	. 5380	31
30	.30070	.69929	3.3255	.31530	3.1716	1.0485	.04628	.95372	30
31	. 0098	. 9902	.3224	. 1562	.1684	.0486	. 4637	. 5363	29
32	. 0126	. 9874	.3194	. 1594	.1652	.0487	. 4646	. 5354	28
33	. 0154	. 9846	.3163	. 1626	.1620	.0488	. 4654	. 5345	27
34	. 0181	. 9818	.3133	. 1658	.1588	.0489	. 4663	. 5337	26
35	.30209	.69791	3.3102	.31690	3.1556	1.0490	.04672	.95328	25
36	. 0237	. 9763	.3072	. 1722	.1524	.0491	. 4681	. 5319	24
37	. 0265	. 9735	.3042	. 1754	.1492	.0492	. 4690	. 5310	23
38	. 0292	. 9707	.3011	. 1786	.1460	.0493	. 4698	. 5301	22
39	. 0320	. 9680	.2981	. 1818	.1429	.0494	. 4707	. 5293	21
40	.30348	.69652	3.2951	.31850	3.1397	1.0495	.04716	.95284	20
41	. 0375	. 9624	.2921	. 1882	.1366	.0496	. 4725	. 5275	19
42	. 0403	. 9597	.2891	. 1914	.1334	.0497	. 4734	. 5266	18
43	. 0431	. 9569	.2861	. 1946	.1303	.0498	. 4743	. 5257	17
44	. 0459	. 9541	.2831	. 1978	.1271	.0499	. 4751	. 5248	16
45	.30486	.69513	3.2801	.32010	3.1240	1.0500	.04760	.95239	15
46	. 0514	. 9486	.2772	. 2042	.1209	.0501	. 4769	. 5231	14
47	. 0542	. 9458	.2742	. 2074	.1177	.0502	. 4778	. 5222	13
48	. 0569	. 9430	.2712	. 2106	.1146	.0503	. 4787	. 5213	12
49	. 0597	. 9403	.2683	. 2138	.1115	.0504	. 4796	. 5204	11
50	.30625	.69375	3.2653	.32171	3.1084	1.0505	.04805	.95195	10
51	. 0653	. 9347	.2624	. 2203	.1053	.0506	. 4814	. 5186	9
52	. 0680	. 9320	.2594	. 2235	.1022	.0507	. 4823	. 5177	8
53	. 0708	. 9292	.2565	. 2267	.0991	.0508	. 4832	. 5168	7
54	. 0736	. 9264	.2535	. 2299	.0960	.0509	. 4840	. 5159	6
55	.30763	.69237	3.2506	.32331	3.0930	1.0510	.04849	.95150	5
56	. 0791	. 9209	.2477	. 2363	.0899	.0511	. 4858	. 5141	4
57	. 0819	. 9181	.2448	. 2395	.0868	.0512	. 4867	. 5132	3
58	. 0846	. 9154	.2419	. 2428	.0838	.0513	. 4876	. 5124	2
59	. 0874	. 9126	.2390	. 2460	.0807	.0514	. 4885	. 5115	1
60	. 0902	. 9098	.2361	. 2492	.0777	.0515	. 4894	. 5106	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

107° 72°

18° **Natural Trigonometrical Functions.** **161°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.30902	.69098	3.2361	.32492	3.0777	1.0515	.04894	.95106	60
1	. 0929	. 9071	.2332	. 2524	.0746	.0516	. 4903	. 5097	59
2	. 0957	. 9043	.2303	. 2556	.0716	.0517	. 4912	. 5088	58
3	. 0985	. 9015	.2274	. 2588	.0686	.0518	. 4921	. 5079	57
4	. 1012	. 8988	.2245	. 2621	.0655	.0519	. 4930	. 5070	56
5	.31040	.68960	3.2216	.32653	3.0625	1.0520	.04939	.95061	55
6	. 1068	. 8932	.2188	. 2685	.0595	.0521	. 4948	. 5051	54
7	. 1095	. 8905	.2159	. 2717	.0565	.0522	. 4957	. 5042	53
8	. 1123	. 8877	.2131	. 2749	.0535	.0523	. 4966	. 5033	52
9	. 1150	. 8849	.2102	. 2782	.0505	.0524	. 4975	. 5024	51
10	.31178	.68822	3.2074	.32814	3.0475	1.0525	.04985	.95015	50
11	. 1206	. 8794	.2045	. 2846	.0445	.0526	. 4994	. 5006	49
12	. 1233	. 8766	.2017	. 2878	.0415	.0527	. 5003	. 4997	48
13	. 1261	. 8739	.1989	. 2910	.0385	.0528	. 5012	. 4988	47
14	. 1289	. 8711	.1960	. 2943	.0356	.0529	. 5021	. 4979	46
15	.31316	.68684	3.1932	.32975	3.0326	1.0530	.05030	.94970	45
16	. 1344	. 8656	.1904	. 3007	.0296	.0531	. 5039	. 4961	44
17	. 1372	. 8628	.1876	. 3039	.0267	.0532	. 5048	. 4952	43
18	. 1399	. 8601	.1848	. 3072	.0237	.0533	. 5057	. 4942	42
19	. 1427	. 8573	.1820	. 3104	.0208	.0534	. 5066	. 4933	41
20	.31454	.68545	3.1792	.33136	3.0178	1.0535	.05076	.94924	40
21	. 1482	. 8518	.1764	. 3169	.0149	.0536	. 5085	. 4915	39
22	. 1510	. 8490	.1736	. 3201	.0120	.0537	. 5094	. 4906	38
23	. 1537	. 8463	.1708	. 3233	.0090	.0538	. 5103	. 4897	37
24	. 1565	. 8435	.1681	. 3265	.0061	.0539	. 5112	. 4888	36
25	.31592	.68407	3.1653	.33298	3.0032	1.0540	.05121	.94878	35
26	. 1620	. 8380	.1625	. 3330	.0003	.0541	. 5131	. 4869	34
27	. 1648	. 8352	.1598	. 3362	2.9974	.0542	. 5140	. 4860	33
28	. 1675	. 8325	.1570	. 3395	.9945	.0543	. 5149	. 4851	32
29	. 1703	. 8297	.1543	. 3427	.9916	.0544	. 5158	. 4841	31
30	.31730	.68269	3.1515	.33459	2.9887	1.0545	.05168	.94832	30
31	. 1758	. 8242	.1488	. 3492	.9858	.0546	. 5177	. 4823	29
32	. 1786	. 8214	.1461	. 3524	.9829	.0547	. 5186	. 4814	28
33	. 1813	. 8187	.1433	. 3557	.9800	.0548	. 5195	. 4805	27
34	. 1841	. 8159	.1406	. 3589	.9772	.0549	. 5205	. 4795	26
35	.31868	.68132	3.1379	.33621	2.9743	1.0550	.05214	.94786	25
36	. 1896	. 8104	.1352	. 3654	.9714	.0551	. 5223	. 4777	24
37	. 1923	. 8076	.1325	. 3686	.9686	.0552	. 5232	. 4767	23
38	. 1951	. 8049	.1298	. 3718	.9657	.0553	. 5242	. 4758	22
39	. 1978	. 8021	.1271	. 3751	.9629	.0554	. 5251	. 4749	21
40	.32006	.67994	3.1244	.33783	2.9600	1.0555	.05260	.94740	20
41	. 2034	. 7966	.1217	. 3816	.9572	.0556	. 5270	. 4730	19
42	. 2061	. 7939	.1190	. 3848	.9544	.0557	. 5279	. 4721	18
43	. 2089	. 7911	.1163	. 3880	.9515	.0558	. 5288	. 4712	17
44	. 2116	. 7884	.1137	. 3913	.9487	.0559	. 5297	. 4702	16
45	.32144	.67856	3.1110	.33945	2.9459	1.0560	.05307	.94693	15
46	. 2171	. 7828	.1083	. 3978	.9431	.0561	. 5316	. 4684	14
47	. 2199	. 7801	.1057	. 4010	.9403	.0562	. 5326	. 4674	13
48	. 2226	. 7773	.1030	. 4043	.9375	.0563	. 5335	. 4665	12
49	. 2254	. 7746	.1004	. 4075	.9347	.0565	. 5344	. 4655	11
50	.32282	.67718	3.0977	.34108	2.9319	1.0566	.05354	.94646	10
51	. 2309	. 7691	.0951	. 4140	.9291	.0567	. 5363	. 4637	9
52	. 2337	. 7663	.0925	. 4173	.9263	.0568	. 5373	. 4627	8
53	. 2364	. 7636	.0898	. 4205	.9235	.0569	. 5382	. 4618	7
54	. 2392	. 7608	.0872	. 4238	.9208	.0570	. 5391	. 4608	6
55	.32419	.67581	3.0846	.34270	2.9180	1.0571	.05401	.94599	5
56	. 2447	. 7553	.0820	. 4303	.9152	.0572	. 5410	. 4590	4
57	. 2474	. 7526	.0793	. 4335	.9125	.0573	. 5420	. 4580	3
58	. 2502	. 7498	.0767	. 4368	.9097	.0574	. 5429	. 4571	2
59	. 2529	. 7471	.0741	. 4400	.9069	.0575	. 5439	. 4561	1
60	. 2557	. 7443	.0715	. 4433	.9042	.0576	. 5448	. 4552	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

108° **71°**

19° **Natural Trigonometrical Functions.** 160°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.32557	.67443	3.0715	.34433	2.9042	1.0576	.05448	.94552	60
1	. 2584	. 7416	.0690	. 4465	.9015	.0577	. 5458	. 4542	59
2	. 2612	. 7388	.0664	. 4498	.8987	.0578	. 5467	. 4533	58
3	. 2639	. 7361	.0638	. 4530	.8960	.0579	. 5476	. 4523	57
4	. 2667	. 7333	.0612	. 4563	.8933	.0580	. 5486	. 4514	56
5	.32694	.67306	3.0586	.34595	2.8905	1.0581	.05495	.94504	55
6	. 2722	. 7278	.0561	. 4628	.8878	.0582	. 5505	. 4495	54
7	. 2749	. 7251	.0535	. 4661	.8851	.0584	. 5515	. 4485	53
8	. 2777	. 7223	.0509	. 4693	.8824	.0585	. 5524	. 4476	52
9	. 2804	. 7196	.0484	. 4726	.8797	.0586	. 5534	. 4466	51
10	.32832	.67168	3.0458	.34758	2.8770	1.0587	.05543	.94457	50
11	. 2859	. 7141	.0433	. 4791	.8743	.0588	. 5553	. 4447	49
12	. 2887	. 7113	.0407	. 4824	.8716	.0589	. 5562	. 4438	48
13	. 2914	. 7086	.0382	. 4856	.8689	.0590	. 5572	. 4428	47
14	. 2942	. 7058	.0357	. 4889	.8662	.0591	. 5581	. 4418	46
15	.32969	.67031	3.0331	.34921	2.8636	1.0592	.05591	.94409	45
16	. 2996	. 7003	.0306	. 4954	.8609	.0593	. 5601	. 4399	44
17	. 3024	. 6976	.0281	. 4987	.8582	.0594	. 5610	. 4390	43
18	. 3051	. 6948	.0256	. 5019	.8555	.0595	. 5620	. 4380	42
19	. 3079	. 6921	.0231	. 5052	.8529	.0596	. 5629	. 4370	41
20	.33106	.66894	3.0206	.35085	2.8502	1.0598	.05639	.94361	40
21	. 3134	. 6866	.0181	. 5117	.8476	.0599	. 5649	. 4351	39
22	. 3161	. 6839	.0156	. 5150	.8449	.0600	. 5658	. 4341	38
23	. 3189	. 6811	.0131	. 5183	.8423	.0601	. 5668	. 4332	37
24	. 3216	. 6784	.0106	. 5215	.8396	.0602	. 5678	. 4322	36
25	.33243	.66756	3.0081	.35248	2.8370	1.0603	.05687	.94313	35
26	. 3271	. 6729	.0056	. 5281	.8344	.0604	. 5697	. 4303	34
27	. 3298	. 6701	.0031	. 5314	.8318	.0605	. 5707	. 4293	33
28	. 3326	. 6674	.0007	. 5346	.8291	.0606	. 5716	. 4283	32
29	. 3353	. 6647	2.9982	. 5379	.8265	.0607	. 5726	. 4274	31
30	.33381	.66619	2.9957	.35412	2.8239	1.0608	.05736	.94264	30
31	. 3408	. 6592	.9933	. 5445	.8213	.0609	. 5745	. 4254	29
32	. 3435	. 6564	.9908	. 5477	.8187	.0611	. 5755	. 4245	28
33	. 3463	. 6537	.9884	. 5510	.8161	.0612	. 5765	. 4235	27
34	. 3490	. 6510	.9859	. 5543	.8135	.0613	. 5775	. 4225	26
35	.33518	.66482	2.9835	.35576	2.8109	1.0614	.05784	.94215	25
36	. 3545	. 6455	.9810	. 5608	.8083	.0615	. 5794	. 4206	24
37	. 3572	. 6427	.9786	. 5641	.8057	.0616	. 5804	. 4196	23
38	. 3600	. 6400	.9762	. 5674	.8032	.0617	. 5814	. 4186	22
39	. 3627	. 6373	.9738	. 5707	.8006	.0618	. 5823	. 4176	21
40	.33655	.66345	2.9713	.35739	2.7980	1.0619	.05833	.94167	20
41	. 3682	. 6318	.9689	. 5772	.7954	.0620	. 5843	. 4157	19
42	. 3709	. 6290	.9665	. 5805	.7929	.0622	. 5853	. 4147	18
43	. 3737	. 6263	.9641	. 5838	.7903	.0623	. 5863	. 4137	17
44	. 3764	. 6236	.9617	. 5871	.7878	.0624	. 5872	. 4127	16
45	.33792	.66208	2.9593	.35904	2.7852	1.0625	.05882	.94118	15
46	. 3819	. 6181	.9569	. 5936	.7827	.0626	. 5892	. 4108	14
47	. 3846	. 6153	.9545	. 5969	.7801	.0627	. 5902	. 4098	13
48	. 3874	. 6126	.9521	. 6002	.7776	.0628	. 5912	. 4088	12
49	. 3901	. 6099	.9497	. 6035	.7751	.0629	. 5922	. 4078	11
50	.33928	.66071	2.9474	.36068	2.7725	1.0630	.05932	.94068	10
51	. 3956	. 6044	.9450	. 6101	.7700	.0632	. 5941	. 4058	9
52	. 3983	. 6017	.9426	. 6134	.7675	.0633	. 5951	. 4049	8
53	. 4011	. 5989	.9402	. 6167	.7650	.0634	. 5961	. 4039	7
54	. 4038	. 5962	.9379	. 6199	.7625	.0635	. 5971	. 4029	6
55	.34065	.65935	2.9355	.36232	2.7600	1.0636	.05981	.94019	5
56	. 4093	. 5907	.9332	. 6265	.7574	.0637	. 5991	. 4009	4
57	. 4120	. 5880	.9308	. 6298	.7549	.0638	. 6001	. 3999	3
58	. 4147	. 5853	.9285	. 6331	.7524	.0639	. 6011	. 3989	2
59	. 4175	. 5825	.9261	. 6364	.7500	.0641	. 6021	. 3979	1
60	. 4202	. 5798	.9238	. 6397	.7475	.0642	. 6031	. 3969	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

109° 70°

20°			Natural Trigonometrical Functions.					159°	
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.34202	.65798	2.9238	.36397	2.7475	1.0642	.06031	.93969	60
1	. 4229	. 5771	.9215	. 6430	.7450	.0643	. 6041	. 3959	59
2	. 4257	. 5743	.9191	. 6463	.7425	.0644	. 6051	. 3949	58
3	. 4284	. 5716	.9168	. 6496	.7400	.0645	. 6061	. 3939	57
4	. 4311	. 5689	.9145	. 6529	.7376	.0646	. 6071	. 3929	56
5	.34339	.65661	2.9122	.36562	2.7351	1.0647	.06080	.93919	55
6	. 4366	. 5634	.9098	. 6595	.7326	.0648	. 6090	. 3909	54
7	. 4393	. 5607	.9075	. 6628	.7302	.0650	. 6100	. 3899	53
8	. 4421	. 5579	.9052	. 6661	.7277	.0651	. 6110	. 3889	52
9	. 4448	. 5552	.9029	. 6694	.7252	.0652	. 6121	. 3879	51
10	.34475	.65525	2.9006	.36727	2.7228	1.0653	.06131	.93869	50
11	. 4502	. 5497	.8983	. 6760	.7204	.0654	. 6141	. 3859	49
12	. 4530	. 5470	.8960	. 6793	.7179	.0655	. 6151	. 3849	48
13	. 4557	. 5443	.8937	. 6826	.7155	.0656	. 6161	. 3839	47
14	. 4584	. 5415	.8915	. 6859	.7130	.0658	. 6171	. 3829	46
15	.34612	.65388	2.8892	.36892	2.7106	1.0659	.06181	.93819	45
16	. 4639	. 5361	.8869	. 6925	.7082	.0660	. 6191	. 3809	44
17	. 4666	. 5334	.8846	. 6958	.7058	.0661	. 6201	. 3799	43
18	. 4693	. 5306	.8824	. 6991	.7033	.0662	. 6211	. 3789	42
19	. 4721	. 5279	.8801	. 7024	.7009	.0663	. 6221	. 3779	41
20	.34748	.65252	2.8778	.37057	2.6985	1.0664	.06231	.93769	40
21	. 4775	. 5225	.8756	. 7090	.6961	.0666	. 6241	. 3758	39
22	. 4803	. 5197	.8733	. 7123	.6937	.0667	. 6251	. 3748	38
23	. 4830	. 5170	.8711	. 7156	.6913	.0668	. 6262	. 3738	37
24	. 4857	. 5143	.8688	. 7190	.6889	.0669	. 6272	. 3728	36
25	.34884	.65115	2.8666	.37223	2.6865	1.0670	.06282	.93718	35
26	. 4912	. 5088	.8644	. 7256	.6841	.0671	. 6292	. 3708	34
27	. 4939	. 5061	.8621	. 7289	.6817	.0673	. 6302	. 3698	33
28	. 4966	. 5034	.8599	. 7322	.6794	.0674	. 6312	. 3687	32
29	. 4993	. 5006	.8577	. 7355	.6770	.0675	. 6323	. 3677	31
30	.35021	.64979	2.8554	.37388	2.6746	1.0676	.06333	.93667	30
31	. 5048	. 4952	.8532	. 7422	.6722	.0677	. 6343	. 3657	29
32	. 5075	. 4925	.8510	. 7455	.6699	.0678	. 6353	. 3647	28
33	. 5102	. 4897	.8488	. 7488	.6675	.0679	. 6363	. 3637	27
34	. 5130	. 4870	.8466	. 7521	.6652	.0681	. 6373	. 3626	26
35	.35157	.64843	2.8444	.37554	2.6628	1.0682	.06384	.93616	25
36	. 5184	. 4816	.8422	. 7587	.6604	.0683	. 6394	. 3606	24
37	. 5211	. 4789	.8400	. 7621	.6581	.0684	. 6404	. 3596	23
38	. 5239	. 4761	.8378	. 7654	.6558	.0685	. 6414	. 3585	22
39	. 5266	. 4734	.8356	. 7687	.6534	.0686	. 6425	. 3575	21
40	.35293	.64707	2.8334	.37720	2.6511	1.0688	.06435	.93565	20
41	. 5320	. 4680	.8312	. 7754	.6487	.0689	. 6445	. 3555	19
42	. 5347	. 4652	.8290	. 7787	.6464	.0690	. 6456	. 3544	18
43	. 5375	. 4625	.8269	. 7820	.6441	.0691	. 6466	. 3534	17
44	. 5402	. 4598	.8247	. 7853	.6418	.0692	. 6476	. 3524	16
45	.35429	.64571	2.8225	.37887	2.6394	1.0694	.06486	.93513	15
46	. 5456	. 4544	.8204	. 7920	.6371	.0695	. 6497	. 3503	14
47	. 5483	. 4516	.8182	. 7953	.6348	.0696	. 6507	. 3493	13
48	. 5511	. 4189	.8160	. 7986	.6325	.0697	. 6517	. 3482	12
49	. 5538	. 4462	.8139	. 8020	.6302	.0698	. 6528	. 3472	11
50	.35565	.64435	2.8117	.38053	2.6279	1.0699	.06538	.93462	10
51	. 5592	. 4408	.8096	. 8086	.6256	.0701	. 6548	. 3451	9
52	. 5619	. 4380	.8074	. 8120	.6233	.0702	. 6559	. 3441	8
53	. 5647	. 4353	.8053	. 8153	.6210	.0703	. 6569	. 3431	7
54	. 5674	. 4326	.8032	. 8186	.6187	.0704	. 6579	. 3420	6
55	.35701	.64299	2.8010	.38220	2.6164	1.0705	.06590	.93410	5
56	. 5728	. 4272	.7989	. 8253	.6142	.0707	. 6600	. 3400	4
57	. 5755	. 4245	.7968	. 8286	.6119	.0708	. 6611	. 3389	3
58	. 5782	. 4217	.7947	. 8320	.6096	.0709	. 6621	. 3379	2
59	. 5810	. 4190	.7925	. 8353	.6073	.0710	. 6631	. 3368	1
60	. 5837	. 4163	.7904	. 8386	.6051	.0711	. 6642	. 3358	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
110°								69°	

21° **Natural Trigonometrical Functions.** **158°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.35837	.64163	2.7904	.38386	2.6051	1.0711	.06642	.93358	60
1	. 5864	. 4136	.7883	. 8420	.6028	.0713	. 6652	. 3348	59
2	. 5891	. 4109	.7862	. 8453	.6006	.0714	. 6663	. 3337	58
3	. 5918	. 4082	.7841	. 8486	.5983	.0715	. 6673	. 3327	57
4	. 5945	. 4055	.7820	. 8520	.5960	.0716	. 6684	. 3316	56
5	.35972	.64027	2.7799	.38553	2.5938	1.0717	.06694	.93306	55
6	. 6000	. 4000	.7778	. 8587	.5916	.0719	. 6705	. 3295	54
7	. 6027	. 3973	.7757	. 8620	.5893	.0720	. 6715	. 3285	53
8	. 6054	. 3946	.7736	. 8654	.5871	.0721	. 6726	. 3274	52
9	. 6081	. 3919	.7715	. 8687	.5848	.0722	. 6736	. 3264	51
10	.36108	.63892	2.7694	.38720	2.5826	1.0723	.06747	.93253	50
11	. 6135	. 3865	.7674	. 8754	.5804	.0725	. 6757	. 3243	49
12	. 6162	. 3837	.7653	. 8787	.5781	.0726	. 6768	. 3232	48
13	. 6189	. 3810	.7632	. 8821	.5759	.0727	. 6778	. 3222	47
14	. 6217	. 3783	.7611	. 8854	.5737	.0728	. 6789	. 3211	46
15	.36244	.63756	2.7591	.38888	2.5715	1.0729	.06799	.93201	45
16	. 6271	. 3729	.7570	. 8921	.5693	.0731	. 6810	. 3190	44
17	. 6298	. 3702	.7550	. 8955	.5671	.0732	. 6820	. 3180	43
18	. 6325	. 3675	.7529	. 8988	.5640	.0733	. 6831	. 3169	42
19	. 6352	. 3648	.7509	. 9022	.5627	.0734	. 6841	. 3158	41
20	.36379	.63621	2.7488	.39055	2.5605	1.0736	.06852	.93148	40
21	. 6406	. 3593	.7468	. 9089	.5583	.0737	. 6863	. 3137	39
22	. 6433	. 3566	.7447	. 9122	.5561	.0738	. 6873	. 3127	38
23	. 6460	. 3539	.7427	. 9156	.5539	.0739	. 6884	. 3116	37
24	. 6488	. 3512	.7406	. 9189	.5517	.0740	. 6894	. 3105	36
25	.36515	.63485	2.7386	.39223	2.5495	1.0742	.06905	.93095	35
26	. 6542	. 3458	.7366	. 9257	.5473	.0743	. 6916	. 3084	34
27	. 6569	. 3431	.7346	. 9290	.5451	.0744	. 6926	. 3074	33
28	. 6596	. 3404	.7325	. 9324	.5430	.0745	. 6937	. 3063	32
29	. 6623	. 3377	.7305	. 9357	.5408	.0747	. 6947	. 3052	31
30	.36650	.63350	2.7285	.39391	2.5386	1.0748	.06958	.93042	30
31	. 6677	. 3323	.7265	. 9425	.5365	.0749	. 6969	. 3031	29
32	. 6704	. 3296	.7245	. 9458	.5343	.0750	. 6979	. 3020	28
33	. 6731	. 3269	.7225	. 9492	.5322	.0751	. 6990	. 3010	27
34	. 6758	. 3242	.7205	. 9525	.5300	.0753	. 7001	. 2999	26
35	.36785	.63214	2.7185	.39559	2.5278	1.0754	.07012	.92988	25
36	. 6812	. 3187	.7165	. 9593	.5257	.0755	. 7022	. 2978	24
37	. 6839	. 3160	.7145	. 9626	.5236	.0756	. 7033	. 2967	23
38	. 6866	. 3133	.7125	. 9660	.5214	.0758	. 7044	. 2956	22
39	. 6893	. 3106	.7105	. 9694	.5193	.0759	. 7054	. 2945	21
40	.36921	.63079	2.7085	.39727	2.5171	1.0760	.07065	.92935	20
41	. 6948	. 3052	.7065	. 9761	.5150	.0761	. 7076	. 2924	19
42	. 6975	. 3025	.7045	. 9795	.5129	.0763	. 7087	. 2913	18
43	. 7002	. 2998	.7026	. 9828	.5108	.0764	. 7097	. 2902	17
44	. 7029	. 2971	.7006	. 9862	.5086	.0765	. 7108	. 2892	16
45	.37056	.62944	2.6986	.39896	2.5065	1.0766	.07119	.92881	15
46	. 7083	. 2917	.6967	. 9930	.5044	.0768	. 7130	. 2870	14
47	. 7110	. 2890	.6947	. 9963	.5023	.0769	. 7141	. 2859	13
48	. 7137	. 2863	.6927	. 9997	.5002	.0770	. 7151	. 2848	12
49	. 7164	. 2836	.6908	.40031	.4981	.0771	. 7162	. 2838	11
50	.37191	.62809	2.6888	.40065	2.4960	1.0773	.07173	.92827	10
51	. 7218	. 2782	.6869	. 0098	.4939	.0774	. 7184	. 2816	9
52	. 7245	. 2755	.6849	. 0132	.4918	.0775	. 7195	. 2805	8
53	. 7272	. 2728	.6830	. 0166	.4897	.0776	. 7205	. 2794	7
54	. 7299	. 2701	.6810	. 0200	.4876	.0778	. 7216	. 2784	6
55	.37326	.62674	2.6791	.40233	2.4855	1.0779	.07227	.92773	5
56	. 7353	. 2647	.6772	. 0267	.4834	.0780	. 7238	. 2762	4
57	. 7380	. 2620	.6752	. 0301	.4813	.0781	. 7249	. 2751	3
58	. 7407	. 2593	.6733	. 0335	.4792	.0783	. 7260	. 2740	2
59	. 7434	. 2566	.6714	. 0369	.4772	.0784	. 7271	. 2729	1
60	. 7461	. 2539	.6695	. 0403	.4751	.0785	. 7282	. 2718	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

111° **68°**

22° **Natural Trigonometrical Functions.** **157°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.37461	.62539	2.6695	.40403	2.4751	1.0785	.07282	.92718	60
1	. 7488	. 2512	.6675	. 0436	.4730	.0787	. 7292	. 2707	59
2	. 7514	. 2485	.6656	. 0470	.4709	.0788	. 7303	. 2696	58
3	. 7541	. 2458	.6637	. 0504	.4689	.0789	. 7314	. 2686	57
4	. 7568	. 2431	.6618	. 0538	.4668	.0790	. 7325	. 2675	56
5	.37595	.62404	2.6599	.40572	2.4647	1.0792	.07336	.92664	55
6	. 7622	. 2377	.6580	. 0606	.4627	.0793	. 7347	. 2653	54
7	. 7649	. 2351	.6561	. 0640	.4606	.0794	. 7358	. 2642	53
8	. 7676	. 2324	.6542	. 0673	.4586	.0795	. 7369	. 2631	52
9	. 7703	. 2297	.6523	. 0707	.4565	.0797	. 7380	. 2620	51
10	.37730	.62270	2.6504	.40741	2.4545	1.0798	.07391	.92609	50
11	. 7757	. 2243	.6485	. 0775	.4525	.0799	. 7402	. 2598	49
12	. 7784	. 2216	.6466	. 0809	.4504	.0801	. 7413	. 2587	48
13	. 7811	. 2189	.6447	. 0843	.4484	.0802	. 7424	. 2576	47
14	. 7838	. 2162	.6428	. 0877	.4463	.0803	. 7435	. 2565	46
15	.37865	.62135	2.6410	.40911	2.4443	1.0804	.07446	.92554	45
16	. 7892	. 2108	.6391	. 0945	.4423	.0806	. 7457	. 2543	44
17	. 7919	. 2081	.6372	. 0979	.4403	.0807	. 7468	. 2532	43
18	. 7946	. 2054	.6353	. 1013	.4382	.0808	. 7479	. 2521	42
19	. 7972	. 2027	.6335	. 1047	.4362	.0810	. 7490	. 2510	41
20	.37999	.62000	2.6316	.41081	2.4342	1.0811	.07501	.92499	40
21	. 8026	. 1974	.6297	. 1115	.4322	.0812	. 7512	. 2488	39
22	. 8053	. 1947	.6279	. 1149	.4302	.0813	. 7523	. 2477	38
23	. 8080	. 1920	.6260	. 1183	.4282	.0815	. 7534	. 2466	37
24	. 8107	. 1893	.6242	. 1217	.4262	.0816	. 7545	. 2455	36
25	.38134	.61866	2.6223	.41251	2.4242	1.0817	.07556	.92443	35
26	. 8161	. 1839	.6205	. 1285	.4222	.0819	. 7567	. 2432	34
27	. 8188	. 1812	.6186	. 1319	.4202	.0820	. 7579	. 2421	33
28	. 8214	. 1785	.6168	. 1353	.4182	.0821	. 7590	. 2410	32
29	. 8241	. 1758	.6150	. 1387	.4162	.0823	. 7601	. 2399	31
30	.38268	.61732	2.6131	.41421	2.4142	1.0824	.07612	.92388	30
31	. 8295	. 1705	.6113	. 1455	.4122	.0825	. 7623	. 2377	29
32	. 8322	. 1678	.6095	. 1489	.4102	.0826	. 7634	. 2366	28
33	. 8349	. 1651	.6076	. 1524	.4083	.0828	. 7645	. 2354	27
34	. 8376	. 1624	.6058	. 1558	.4063	.0829	. 7657	. 2343	26
35	.38403	.61597	2.6040	.41592	2.4043	1.0830	.07668	.92332	25
36	. 8429	. 1570	.6022	. 1626	.4023	.0832	. 7679	. 2321	24
37	. 8456	. 1544	.6003	. 1660	.4004	.0833	. 7690	. 2310	23
38	. 8483	. 1517	.5985	. 1694	.3984	.0834	. 7701	. 2299	22
39	. 8510	. 1490	.5967	. 1728	.3964	.0836	. 7712	. 2287	21
40	.38537	.61463	2.5949	.41762	2.3945	1.0837	.07724	.92276	20
41	. 8564	. 1436	.5931	. 1797	.3925	.0838	. 7735	. 2265	19
42	. 8591	. 1409	.5913	. 1831	.3906	.0840	. 7746	. 2254	18
43	. 8617	. 1382	.5895	. 1865	.3886	.0841	. 7757	. 2242	17
44	. 8644	. 1356	.5877	. 1899	.3867	.0842	. 7769	. 2231	16
45	.38671	.61329	2.5859	.41933	2.3847	1.0844	.07780	.92220	15
46	. 8698	. 1302	.5841	. 1968	.3828	.0845	. 7791	. 2209	14
47	. 8725	. 1275	.5823	. 2002	.3808	.0846	. 7802	. 2197	13
48	. 8751	. 1248	.5805	. 2036	.3789	.0847	. 7814	. 2186	12
49	. 8778	. 1222	.5787	. 2070	.3770	.0849	. 7825	. 2175	11
50	.38805	.61195	2.5770	.42105	2.3750	1.0850	.07836	.92164	10
51	. 8832	. 1168	.5752	. 2139	.3731	.0851	. 7847	. 2152	9
52	. 8859	. 1141	.5734	. 2173	.3712	.0853	. 7859	. 2141	8
53	. 8886	. 1114	.5716	. 2207	.3692	.0854	. 7870	. 2130	7
54	. 8912	. 1088	.5699	. 2242	.3673	.0855	. 7881	. 2118	6
55	.38939	.61061	2.5681	.42276	2.3654	1.0857	.07893	.92107	5
56	. 8966	. 1034	.5663	. 2310	.3635	.0858	. 7904	. 2096	4
57	. 8993	. 1007	.5646	. 2344	.3616	.0859	. 7915	. 2084	3
58	. 9019	. 0980	.5628	. 2379	.3597	.0861	. 7927	. 2073	2
59	. 9046	. 0954	.5610	. 2413	.3577	.0862	. 7938	. 2062	1
60	. 9073	. 0927	.5593	. 2447	.3558	.0864	. 7949	. 2050	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

112° **67°**

23° **Natural Trigonometrical Functions.** **156°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.39073	.60927	2.5593	.42447	2.3558	1.0864	.07949	.92050	60
1	. 9100	. 0900	.5575	. 2482	.3539	.0865	. 7961	. 2039	59
2	. 9126	. 0873	.5558	. 2516	.3520	.0866	. 7972	. 2028	58
3	. 9153	. 0846	.5540	. 2550	.3501	.0868	. 7984	. 2016	57
4	. 9180	. 0820	.5523	. 2585	.3482	.0869	. 7995	. 2005	56
5	.39207	.60793	2.5506	.42619	2.3463	1.0870	.08006	.91993	55
6	. 9234	. 0766	.5488	. 2654	.3445	.0872	. 8018	. 1982	54
7	. 9260	. 0739	.5471	. 2688	.3426	.0873	. 8029	. 1971	53
8	. 9287	. 0713	.5453	. 2722	.3407	.0874	. 8041	. 1959	52
9	. 9314	. 0686	.5436	. 2757	.3388	.0876	. 8052	. 1948	51
10	.39341	.60659	2.5419	.42791	2.3369	1.0877	.08063	.91936	50
11	. 9367	. 0632	.5402	. 2826	.3350	.0878	. 8075	. 1925	49
12	. 9394	. 0606	.5384	. 2860	.3332	.0880	. 8086	. 1913	48
13	. 9421	. 0579	.5367	. 2894	.3313	.0881	. 8098	. 1902	47
14	. 9448	. 0552	.5350	. 2929	.3294	.0882	. 8109	. 1891	46
15	.39474	.60526	2.5333	.42963	2.3276	1.0884	.08121	.91879	45
16	. 9501	. 0499	.5316	. 2998	.3257	.0885	. 8132	. 1868	44
17	. 9528	. 0472	.5299	. 3032	.3238	.0886	. 8144	. 1856	43
18	. 9554	. 0445	.5281	. 3067	.3220	.0888	. 8155	. 1845	42
19	. 9581	. 0419	.5264	. 3101	.3201	.0889	. 8167	. 1833	41
20	.39608	.60392	2.5247	.43136	2.3183	1.0891	.08178	.91822	40
21	. 9635	. 0365	.5230	. 3170	.3164	.0892	. 8190	. 1810	39
22	. 9661	. 0339	.5213	. 3205	.3145	.0893	. 8201	. 1798	38
23	. 9688	. 0312	.5196	. 3239	.3127	.0895	. 8213	. 1787	37
24	. 9715	. 0285	.5179	. 3274	.3109	.0896	. 8224	. 1775	36
25	.39741	.60258	2.5163	.43308	2.3090	1.0897	.08236	.91764	35
26	. 9768	. 0232	.5146	. 3343	.3072	.0899	. 8248	. 1752	34
27	. 9795	. 0205	.5129	. 3377	.3053	.0900	. 8259	. 1741	33
28	. 9821	. 0178	.5112	. 3412	.3035	.0902	. 8271	. 1729	32
29	. 9848	. 0152	.5095	. 3447	.3017	.0903	. 8282	. 1718	31
30	.39875	.60125	2.5078	.43481	2.2998	1.0904	.08294	.91706	30
31	. 9901	. 0098	.5062	. 3516	.2980	.0906	. 8306	. 1694	29
32	. 9928	. 0072	.5045	. 3550	.2962	.0907	. 8317	. 1683	28
33	. 9955	. 0045	.5028	. 3585	.2944	.0908	. 8329	. 1671	27
34	. 9981	. 0018	.5011	. 3620	.2925	.0910	. 8340	. 1659	26
35	.40008	.59992	2.4995	.43654	2.2907	1.0911	.08352	.91648	25
36	. 0035	. 9965	.4978	. 3689	.2889	.0913	. 8364	. 1636	24
37	. 0061	. 9938	.4961	. 3723	.2871	.0914	. 8375	. 1625	23
38	. 0088	. 9912	.4945	. 3758	.2853	.0915	. 8387	. 1613	22
39	. 0115	. 9885	.4928	. 3793	.2835	.0917	. 8399	. 1601	21
40	.40141	.59858	2.4912	.43827	2.2817	1.0918	.08410	.91590	20
41	. 0168	. 9832	.4895	. 3862	.2799	.0920	. 8422	. 1578	19
42	. 0195	. 9805	.4879	. 3897	.2781	.0921	. 8434	. 1566	18
43	. 0221	. 9778	.4862	. 3932	.2763	.0922	. 8445	. 1554	17
44	. 0248	. 9752	.4846	. 3966	.2745	.0924	. 8457	. 1543	16
45	.40275	.59725	2.4829	.44001	2.2727	1.0925	.08469	.91531	15
46	. 0301	. 9699	.4813	. 4036	.2709	.0927	. 8480	. 1519	14
47	. 0328	. 9672	.4797	. 4070	.2691	.0928	. 8492	. 1508	13
48	. 0354	. 9645	.4780	. 4105	.2673	.0929	. 8504	. 1496	12
49	. 0381	. 9619	.4764	. 4140	.2655	.0931	. 8516	. 1484	11
50	.40408	.59592	2.4748	.44175	2.2637	1.0932	.08527	.91472	10
51	. 0434	. 9566	.4731	. 4209	.2619	.0934	. 8539	. 1461	9
52	. 0461	. 9539	.4715	. 4244	.2602	.0935	. 8551	. 1449	8
53	. 0487	. 9512	.4699	. 4279	.2584	.0936	. 8563	. 1437	7
54	. 0514	. 9486	.4683	. 4314	.2566	.0938	. 8575	. 1425	6
55	.40541	.59459	2.4666	.44349	2.2548	1.0939	.08586	.91414	5
56	. 0567	. 9433	.4650	. 4383	.2531	.0941	. 8598	. 1402	4
57	. 0594	. 9406	.4634	. 4418	.2513	.0942	. 8610	. 1390	3
58	. 0620	. 9379	.4618	. 4453	.2495	.0943	. 8622	. 1378	2
59	. 0647	. 9353	.4602	. 4488	.2478	.0945	. 8634	. 1366	1
60	. 0674	. 9326	.4586	. 4523	.2460	.0946	. 8645	. 1354	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

113° **66°**

24° **Natural Trigonometrical Functions.** **155°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.40674	.59326	2.4586	.44523	2.2460	1.0946	.08645	.91354	60
1	. 0700	. 9300	.4570	. 4558	.2443	.0948	. 8657	. 1343	59
2	. 0727	. 9273	.4554	. 4593	.2425	.0949	. 8669	. 1331	58
3	. 0753	. 9247	.4538	. 4627	.2408	.0951	. 8681	. 1319	57
4	. 0780	. 9220	.4522	. 4662	.2390	.0952	. 8693	. 1307	56
5	.40806	.59193	2.4506	.44697	2.2373	1.0953	.08705	.91295	55
6	. 0833	. 9167	.4490	. 4732	.2355	.0955	. 8716	. 1283	54
7	. 0860	. 9140	.4474	. 4767	.2338	.0956	. 8728	. 1271	53
8	. 0886	. 9114	.4458	. 4802	.2320	.0958	. 8740	. 1260	52
9	. 0913	. 9087	.4442	. 4837	.2303	.0959	. 8752	. 1248	51
10	.40939	.59061	2.4426	.44872	2.2286	1.0961	.08764	.91236	50
11	. 0966	. 9034	.4411	. 4907	.2268	.0962	. 8776	. 1224	49
12	. 0992	. 9008	.4395	. 4942	.2251	.0963	. 8788	. 1212	48
13	. 1019	. 8981	.4379	. 4977	.2234	.0965	. 8800	. 1200	47
14	. 1045	. 8955	.4363	. 5012	.2216	.0966	. 8812	. 1188	46
15	.41072	.58928	2.4347	.45047	2.2199	1.0968	.08824	.91176	45
16	. 1098	. 8901	.4332	. 5082	.2182	.0969	. 8836	. 1164	44
17	. 1125	. 8875	.4316	. 5117	.2165	.0971	. 8848	. 1152	43
18	. 1151	. 8848	.4300	. 5152	.2147	.0972	. 8860	. 1140	42
19	. 1178	. 8822	.4285	. 5187	.2130	.0973	. 8872	. 1128	41
20	.41204	.58795	2.4269	.45222	2.2113	1.0975	.08884	.91116	40
21	. 1231	. 8769	.4254	. 5257	.2096	.0976	. 8896	. 1104	39
22	. 1257	. 8742	.4238	. 5292	.2079	.0978	. 8908	. 1092	38
23	. 1284	. 8716	.4222	. 5327	.2062	.0979	. 8920	. 1080	37
24	. 1310	. 8689	.4207	. 5362	.2045	.0981	. 8932	. 1068	36
25	.41337	.58663	2.4191	.45397	2.2028	1.0982	.08944	.91056	35
26	. 1363	. 8636	.4176	. 5432	.2011	.0984	. 8956	. 1044	34
27	. 1390	. 8610	.4160	. 5467	.1994	.0985	. 8968	. 1032	33
28	. 1416	. 8584	.4145	. 5502	.1977	.0986	. 8980	. 1020	32
29	. 1443	. 8557	.4130	. 5537	.1960	.0988	. 8992	. 1008	31
30	.41469	.58531	2.4114	.45573	2.1943	1.0989	.09004	.90996	30
31	. 1496	. 8504	.4099	. 5608	.1926	.0991	. 9016	. 0984	29
32	. 1522	. 8478	.4083	. 5643	.1909	.0992	. 9028	. 0972	28
33	. 1549	. 8451	.4068	. 5678	.1892	.0994	. 9040	. 0960	27
34	. 1575	. 8425	.4053	. 5713	.1875	.0995	. 9052	. 0948	26
35	41602	.58398	2.4037	.45748	2.1859	1.0997	.09064	.90936	25
36	. 1628	. 8372	.4022	. 5783	.1842	.0998	. 9076	. 0924	24
37	. 1654	. 8345	.4007	. 5819	.1825	.1000	. 9088	. 0911	23
38	. 1681	. 8319	.3992	. 5854	.1808	.1001	. 9101	. 0899	22
39	. 1707	. 8292	.3976	. 5889	.1792	.1003	. 9113	. 0887	21
40	.41734	.58266	2.3961	.45924	2.1775	1.1004	.09125	.90875	20
41	. 1760	. 8240	.3946	. 5960	.1758	.1005	. 9137	. 0863	19
42	. 1787	. 8213	.3931	. 5995	.1741	.1007	. 9149	. 0851	18
43	. 1813	. 8187	.3916	. 6030	.1725	.1008	. 9161	. 0839	17
44	. 1839	. 8160	.3901	. 6065	.1708	.1010	. 9173	. 0826	16
45	.41866	.58134	2.3886	.46101	2.1692	1.1011	.09186	.90814	15
46	. 1892	. 8108	.3871	. 6136	.1675	.1013	. 9198	. 0802	14
47	. 1919	. 8081	.3856	. 6171	.1658	.1014	. 9210	. 0790	13
48	. 1945	. 8055	.3841	. 6206	.1642	.1016	. 9222	. 0778	12
49	. 1972	. 8028	.3826	. 6242	.1625	.1017	. 9234	. 0765	11
50	.41998	.58002	2.3811	.46277	2.1609	1.1019	.09247	.90753	10
51	. 2024	. 7975	.3796	. 6312	.1592	.1020	. 9259	. 0741	9
52	. 2051	. 7949	.3781	. 6348	.1576	.1022	. 9271	. 0729	8
53	. 2077	. 7923	.3766	. 6383	.1559	.1023	. 9283	. 0717	7
54	. 2103	. 7896	.3751	. 6418	.1543	.1025	. 9296	. 0704	6
55	.42130	.57870	2.3736	.46454	2.1527	1.1026	.09308	.90692	5
56	. 2156	. 7844	.3721	. 6489	.1510	.1028	. 9320	. 0680	4
57	. 2183	. 7817	.3706	. 6524	.1494	.1029	. 9332	. 0668	3
58	. 2209	. 7791	.3691	. 6560	.1478	.1031	. 9345	. 0655	2
59	. 2235	. 7764	.3677	. 6595	.1461	.1032	. 9357	. 0643	1
60	. 2262	. 7738	.3662	. 6631	.1445	.1034	. 9369	. 0631	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

114° **65°**

25° **Natural Trigonometrical Functions.** 154°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.42262	.57738	2.3662	.46631	2.1445	1.1034	.09369	.90631	60
1	. 2288	. 7712	.3647	. 6666	.1429	.1035	. 9381	. 0618	59
2	. 2314	. 7685	.3632	. 6702	.1412	.1037	. 9394	. 0606	58
3	. 2341	. 7659	.3618	. 6737	.1396	.1038	. 9406	. 0594	57
4	. 2367	. 7633	.3603	. 6772	.1380	.1040	. 9418	. 0581	56
5	.42394	.57606	2.3588	.46808	2.1364	1.1041	.09431	.90569	55
6	. 2420	. 7580	.3574	. 6843	.1348	.1043	. 9443	. 0557	54
7	. 2446	. 7554	.3559	. 6879	.1331	.1044	. 9455	. 0544	53
8	. 2473	. 7527	.3544	. 6914	.1315	.1046	. 9468	. 0532	52
9	. 2499	. 7501	.3530	. 6950	.1299	.1047	. 9480	. 0520	51
10	.42525	.57475	2.3515	.46985	2.1283	1.1049	.09492	.90507	50
11	. 2552	. 7448	.3501	. 7021	.1267	.1050	. 9505	. 0495	49
12	. 2578	. 7422	.3486	. 7056	.1251	.1052	. 9517	. 0483	48
13	. 2604	. 7396	.3472	. 7092	.1235	.1053	. 9530	. 0470	47
14	. 2630	. 7369	.3457	. 7127	.1219	.1055	. 9542	. 0458	46
15	.42657	.57343	2.3443	.47163	2.1203	1.1056	.09554	.90445	45
16	. 2683	. 7317	.3428	. 7199	.1187	.1058	. 9567	. 0433	44
17	. 2709	. 7290	.3414	. 7234	.1171	.1059	. 9579	. 0421	43
18	. 2736	. 7264	.3399	. 7270	.1155	.1061	. 9592	. 0408	42
19	. 2762	. 7238	.3385	. 7305	.1139	.1062	. 9604	. 0396	41
20	.42788	.57212	2.3371	.47341	2.1123	1.1064	.09617	.90383	40
21	. 2815	. 7185	.3356	. 7376	.1107	.1065	. 9629	. 0371	39
22	. 2841	. 7159	.3342	. 7412	.1092	.1067	. 9641	. 0358	38
23	. 2867	. 7133	.3328	. 7448	.1076	.1068	. 9654	. 0346	37
24	. 2893	. 7106	.3313	. 7483	.1060	.1070	. 9666	. 0333	36
25	.42920	.57080	2.3299	.47519	2.1044	1.1072	.09679	.90321	35
26	. 2946	. 7054	.3285	. 7555	.1028	.1073	. 9691	. 0308	34
27	. 2972	. 7028	.3271	. 7590	.1013	.1075	. 9704	. 0296	33
28	. 2998	. 7001	.3256	. 7626	.0997	.1076	. 9716	. 0283	32
29	. 3025	. 6975	.3242	. 7662	.0981	.1078	. 9729	. 0271	31
30	.43051	.56949	2.3228	.47697	2.0965	1.1079	.09741	.90258	30
31	. 3077	. 6923	.3214	. 7733	.0950	.1081	. 9754	. 0246	29
32	. 3104	. 6896	.3200	. 7769	.0934	.1082	. 9766	. 0233	28
33	. 3130	. 6870	.3186	. 7805	.0918	.1084	. 9779	. 0221	27
34	. 3156	. 6844	.3172	. 7840	.0903	.1085	. 9792	. 0208	26
35	.43182	.56818	2.3158	.47876	2.0887	1.1087	.09804	.90196	25
36	. 3208	. 6791	.3143	. 7912	.0872	.1088	. 9817	. 0183	24
37	. 3235	. 6765	.3129	. 7948	.0856	.1090	. 9829	. 0171	23
38	. 3261	. 6739	.3115	. 7983	.0840	.1092	. 9842	. 0158	22
39	. 3287	. 6713	.3101	. 8019	.0825	.1093	. 9854	. 0145	21
40	.43313	.56686	2.3087	.48055	2.0809	1.1095	.09867	.90133	20
41	. 3340	. 6660	.3073	. 8091	.0794	.1096	. 9880	. 0120	19
42	. 3366	. 6634	.3059	. 8127	.0778	.1098	. 9892	. 0108	18
43	. 3392	. 6608	.3046	. 8162	.0763	.1099	. 9905	. 0095	17
44	. 3418	. 6582	.3032	. 8198	.0747	.1101	. 9917	. 0082	16
45	.43444	.56555	2.3018	.48234	2.0732	1.1102	.09930	.90070	15
46	. 3471	. 6529	.3004	. 8270	.0717	.1104	. 9943	. 0057	14
47	. 3497	. 6503	.2990	. 8306	.0701	.1106	. 9955	. 0044	13
48	. 3523	. 6477	.2976	. 8342	.0686	.1107	. 9968	. 0032	12
49	. 3549	. 6451	.2962	. 8378	.0671	.1109	. 9981	. 0019	11
50	.43575	.56424	2.2949	.48414	2.0655	1.1110	.09993	.90006	10
51	. 3602	. 6398	.2935	. 8449	.0640	.1112	.10006	.89994	9
52	. 3628	. 6372	.2921	. 8485	.0625	.1113	. 0019	. 9981	8
53	. 3654	. 6346	.2907	. 8521	.0609	.1115	. 0031	. 9968	7
54	. 3680	. 6320	.2894	. 8557	.0594	.1116	. 0044	. 9956	6
55	.43706	.56294	2.2880	.48593	2.0579	1.1118	.10057	.89943	5
56	. 3732	. 6267	.2866	. 8629	.0564	.1120	. 0070	. 9930	4
57	. 3759	. 6241	.2853	. 8665	.0548	.1121	. 0082	. 9918	3
58	. 3785	. 6215	.2839	. 8701	.0533	.1123	. 0095	. 9905	2
59	. 3811	. 6189	.2825	. 8737	.0518	.1124	. 0108	. 9892	1
60	. 3837	. 6163	.2812	. 8773	.0503	.1126	. 0121	. 9879	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

115° 64°

26° **Natural Trigonometrical Functions.** **153°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.43837	.56163	2.2812	.48773	2.0503	1.1126	.10121	.89879	60
1	. 3863	. 6137	.2798	. 8809	.0488	.1127	. 0133	. 9867	59
2	. 3889	. 6111	.2784	. 8845	.0473	.1129	. 0146	. 9854	58
3	. 3915	. 6084	.2771	. 8881	.0458	.1131	. 0159	. 9841	57
4	. 3942	. 6058	.2757	. 8917	.0443	.1132	. 0172	. 9828	56
5	.43968	.56032	2.2744	.48953	2.0427	1.1134	.10184	.89815	55
6	. 3994	. 6006	.2730	. 8989	.0412	.1135	. 0197	. 9803	54
7	. 4020	. 5980	.2717	. 9025	.0397	.1137	. 0210	. 9790	53
8	. 4046	. 5954	.2703	. 9062	.0382	.1139	. 0223	. 9777	52
9	. 4072	. 5928	.2690	. 9098	.0367	.1140	. 0236	. 9764	51
10	.44098	.55902	2.2676	.49134	2.0352	1.1142	.10248	.89751	50
11	. 4124	. 5875	.2663	. 9170	.0338	.1143	. 0261	. 9739	49
12	. 4150	. 5849	.2650	. 9206	.0323	.1145	. 0274	. 9726	48
13	. 4177	. 5823	.2636	. 9242	.0308	.1147	. 0287	. 9713	47
14	. 4203	. 5797	.2623	. 9278	.0293	.1148	. 0300	. 9700	46
15	.44229	.55771	2.2610	.49314	2.0278	1.1150	.10313	.89687	45
16	. 4255	. 5745	.2596	. 9351	.0263	.1151	. 0326	. 9674	44
17	. 4281	. 5719	.2583	. 9387	.0248	.1153	. 0338	. 9661	43
18	. 4307	. 5693	.2570	. 9423	.0233	.1155	. 0351	. 9649	42
19	. 4333	. 5667	.2556	. 9459	.0219	.1156	. 0364	. 9636	41
20	.44359	.55641	2.2543	.49495	2.0204	1.1158	.10377	.89623	40
21	. 4385	. 5615	.2530	. 9532	.0189	.1159	. 0390	. 9610	39
22	. 4411	. 5589	.2517	. 9568	.0174	.1161	. 0403	. 9597	38
23	. 4437	. 5562	.2503	. 9604	.0159	.1163	. 0416	. 9584	37
24	. 4463	. 5536	.2490	. 9640	.0145	.1164	. 0429	. 9571	36
25	.44489	.55510	2.2477	.49677	2.0130	1.1166	.10442	.89558	35
26	. 4516	. 5484	.2464	. 9713	.0115	.1167	. 0455	. 9545	34
27	. 4542	. 5458	.2451	. 9749	.0101	.1169	. 0468	. 9532	33
28	. 4568	. 5432	.2438	. 9785	.0086	.1171	. 0481	. 9519	32
29	. 4594	. 5406	.2425	. 9822	.0071	.1172	. 0493	. 9506	31
30	.44620	.55380	2.2411	.49858	2.0058	1.1174	.10506	.89493	30
31	. 4646	. 5354	.2398	. 9894	.0042	.1176	. 0519	. 9480	29
32	. 4672	. 5328	.2385	. 9931	.0028	.1177	. 0532	. 9467	28
33	. 4698	. 5302	.2372	. 9967	.0013	.1179	. 0545	. 9454	27
34	. 4724	. 5276	.2359	.50003	1.9998	.1180	. 0558	. 9441	26
35	.44750	.55250	2.2346	.50040	1.9984	1.1182	.10571	.89428	25
36	. 4776	. 5224	.2333	. 0076	.9969	.1184	. 0584	. 9415	24
37	. 4802	. 5198	.2320	. 0113	.9955	.1185	. 0598	. 9402	23
38	. 4828	. 5172	.2307	. 0149	.9940	.1187	. 0611	. 9389	22
39	. 4854	. 5146	.2294	. 0185	.9926	.1189	. 0624	. 9376	21
40	.44880	.55120	2.2282	.50222	1.9912	1.1190	.10637	.89363	20
41	. 4906	. 5094	.2269	. 0258	.9897	.1192	. 0650	. 9350	19
42	. 4932	. 5068	.2256	. 0295	.9883	.1193	. 0663	. 9337	18
43	. 4958	. 5042	.2243	. 0331	.9868	.1195	. 0676	. 9324	17
44	. 4984	. 5016	.2230	. 0368	.9854	.1197	. 0689	. 9311	16
45	.45010	.54990	2.2217	.50404	1.9840	1.1198	.10702	.89298	15
46	. 5036	. 4964	.2204	. 0441	.9825	.1200	. 0715	. 9285	14
47	. 5062	. 4938	.2192	. 0477	.9811	.1202	. 0728	. 9272	13
48	. 5088	. 4912	.2179	. 0514	.9797	.1203	. 0741	. 9258	12
49	. 5114	. 4886	.2166	. 0550	.9782	.1205	. 0754	. 9245	11
50	.45140	.54860	2.2153	.50587	1.9768	1.1207	.10768	.89232	10
51	. 5166	. 4834	.2141	. 0623	.9754	.1208	. 0781	. 9219	9
52	. 5191	. 4808	.2128	. 0660	.9739	.1210	. 0794	. 9206	8
53	. 5217	. 4782	.2115	. 0696	.9725	.1212	. 0807	. 9193	7
54	. 5243	. 4756	.2103	. 0733	.9711	.1213	. 0820	. 9180	6
55	.45269	.54730	2.2090	.50769	1.9697	1.1215	.10833	.89166	5
56	. 5295	. 4705	.2077	. 0806	.9683	.1217	. 0846	. 9153	4
57	. 5321	. 4679	.2065	. 0843	.9668	.1218	. 0860	. 9140	3
58	. 5347	. 4653	.2052	. 0879	.9654	.1220	. 0873	. 9127	2
59	. 5373	. 4627	.2039	. 0916	.9640	.1222	. 0886	. 9114	1
60	. 5399	. 4601	.2027	. 0952	.9626	.1223	. 0899	. 9101	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

116° **63°**

Natural Trigonometrical Functions.

27°									152°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.45399	.54601	2.2027	.50952	1.9626	1.1223	.10899	.89101	60
1	. 5425	. 4575	.2014	. 0989	.9612	.1225	. 0912	. 9087	59
2	. 5451	. 4549	.2002	. 1026	.9598	.1226	. 0926	. 9074	58
3	. 5477	. 4523	.1989	. 1062	.9584	.1228	. 0939	. 9061	57
4	. 5503	. 4497	.1977	. 1099	.9570	.1230	. 0952	. 9048	56
5	.45528	.54471	2.1964	.51136	1.9556	1.1231	.10965	.89034	55
6	. 5554	. 4445	.1952	. 1172	.9542	.1233	. 0979	. 9021	54
7	. 5580	. 4420	.1939	. 1209	.9528	.1235	. 0992	. 9008	53
8	. 5606	. 4394	.1927	. 1246	.9514	.1237	. 1005	. 8995	52
9	. 5632	. 4368	.1914	. 1283	.9500	.1238	. 1018	. 8981	51
10	.45658	.54342	2.1902	.51319	1.9486	1.1240	.11032	.88968	50
11	. 5684	. 4316	.1889	. 1356	.9472	.1242	. 1045	. 8955	49
12	. 5710	. 4290	.1877	. 1393	.9458	.1243	. 1058	. 8942	48
13	. 5736	. 4264	.1865	. 1430	.9444	.1245	. 1072	. 8928	47
14	. 5761	. 4238	.1852	. 1466	.9430	.1247	. 1085	. 8915	46
15	.45787	.54213	2.1840	.51503	1.9416	1.1248	.11098	.88902	45
16	. 5813	. 4187	.1828	. 1540	.9402	.1250	. 1112	. 8888	44
17	. 5839	. 4161	.1815	. 1577	.9388	.1252	. 1125	. 8875	43
18	. 5865	. 4135	.1803	. 1614	.9375	.1253	. 1138	. 8862	42
19	. 5891	. 4109	.1791	. 1651	.9361	.1255	. 1152	. 8848	41
20	.45917	.54083	2.1778	.51687	1.9347	1.1257	.11165	.88835	40
21	. 5942	. 4057	.1766	. 1724	.9333	.1258	. 1178	. 8822	39
22	. 5968	. 4032	.1754	. 1761	.9319	.1260	. 1192	. 8808	38
23	. 5994	. 4006	.1742	. 1798	.9306	.1262	. 1205	. 8795	37
24	. 6020	. 3980	.1730	. 1835	.9292	.1264	. 1218	. 8781	36
25	.46046	.53954	2.1717	.51872	1.9278	1.1265	.11232	.88768	35
26	. 6072	. 3928	.1705	. 1909	.9264	.1267	. 1245	. 8755	34
27	. 6097	. 3902	.1693	. 1946	.9251	.1269	. 1259	. 8741	33
28	. 6123	. 3877	.1681	. 1983	.9237	.1270	. 1272	. 8728	32
29	. 6149	. 3851	.1669	. 2020	.9223	.1272	. 1285	. 8714	31
30	.46175	.53825	2.1657	.52057	1.9210	1.1274	.11299	.88701	30
31	. 6201	. 3799	.1645	. 2094	.9196	.1275	. 1312	. 8688	29
32	. 6226	. 3773	.1633	. 2131	.9182	.1277	. 1326	. 8674	28
33	. 6252	. 3748	.1620	. 2168	.9169	.1279	. 1339	. 8661	27
34	. 6278	. 3722	.1608	. 2205	.9155	.1281	. 1353	. 8647	26
35	.46304	.53696	2.1596	.52242	1.9142	1.1282	.11366	.88634	25
36	. 6330	. 3670	.1584	. 2279	.9128	.1284	. 1380	. 8620	24
37	. 6355	. 3645	.1572	. 2316	.9115	.1286	. 1393	. 8607	23
38	. 6381	. 3619	.1560	. 2353	.9101	.1287	. 1407	. 8593	22
39	. 6407	. 3593	.1548	. 2390	.9088	.1289	. 1420	. 8580	21
40	.46433	.53567	2.1536	.52427	1.9074	1.1291	.11434	.88566	20
41	. 6458	. 3541	.1525	. 2464	.9061	.1293	. 1447	. 8553	19
42	. 6484	. 3516	.1513	. 2501	.9047	.1294	. 1461	. 8539	18
43	. 6510	. 3490	.1501	. 2538	.9034	.1296	. 1474	. 8526	17
44	. 6536	. 3464	.1489	. 2575	.9020	.1298	. 1488	. 8512	16
45	.46561	.53438	2.1477	.52612	1.9007	1.1299	.11501	.88499	15
46	. 6587	. 3413	.1465	. 2650	.8993	.1301	. 1515	. 8485	14
47	. 6613	. 3387	.1453	. 2687	.8980	.1303	. 1528	. 8472	13
48	. 6639	. 3361	.1441	. 2724	.8967	.1305	. 1542	. 8458	12
49	. 6664	. 3336	.1430	. 2761	.8953	.1306	. 1555	. 8444	11
50	.46690	.53310	2.1418	.52798	1.8940	1.1308	.11569	.88431	10
51	. 6716	. 3284	.1406	. 2836	.8927	.1310	. 1583	. 8417	9
52	. 6741	. 3258	.1394	. 2873	.8913	.1312	. 1596	. 8404	8
53	. 6767	. 3233	.1382	. 2910	.8900	.1313	. 1610	. 8390	7
54	. 6793	. 3207	.1371	. 2947	.8887	.1315	. 1623	. 8376	6
55	.46819	.53181	2.1359	.52984	1.8873	1.1317	.11637	.88363	5
56	. 6844	. 3156	.1347	. 3022	.8860	.1319	. 1651	. 8349	4
57	. 6870	. 3130	.1335	. 3059	.8847	.1320	. 1664	. 8336	3
58	. 6896	. 3104	.1324	. 3096	.8834	.1322	. 1678	. 8322	2
59	. 6921	. 3078	.1312	. 3134	.8820	.1324	. 1691	. 8308	1
60	. 6947	. 3053	.1300	. 3171	.8807	.1326	. 1705	. 8295	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
117°									62°

28° **Natural Trigonometrical Functions.** **151°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.46947	.53053	2.1300	.53171	1.8807	1.1326	.11705	.88295	60
1	. 6973	. 3027	.1289	. 3208	.8794	.1327	. 1719	. 8281	59
2	. 6998	. 3001	.1277	. 3245	.8781	.1329	. 1732	. 8267	58
3	. 7024	. 2976	.1266	. 3283	.8768	.1331	. 1746	. 8254	57
4	. 7050	. 2950	.1254	. 3320	.8754	.1333	. 1760	. 8240	56
5	.47075	.52924	2.1242	.53358	1.8741	1.1334	.11774	.88226	55
6	. 7101	. 2899	.1231	. 3395	.8728	.1336	. 1787	. 8213	54
7	. 7127	. 2873	.1219	. 3432	.8715	.1338	. 1801	. 8199	53
8	. 7152	. 2847	.1208	. 3470	.8702	.1340	. 1815	. 8185	52
9	. 7178	. 2822	.1196	. 3507	.8689	.1341	. 1828	. 8171	51
10	.47204	.52796	2.1185	.53545	1.8676	1.1343	.11842	.88158	50
11	. 7229	. 2770	.1173	. 3582	.8663	.1345	. 1856	. 8144	49
12	. 7255	. 2745	.1162	. 3619	.8650	.1347	. 1870	. 8130	48
13	. 7281	. 2719	.1150	. 3657	.8637	.1349	. 1883	. 8117	47
14	. 7306	. 2694	.1139	. 3694	.8624	.1350	. 1897	. 8103	46
15	.47332	.52668	2.1127	.53732	1.8611	1.1352	.11911	·88089	45
16	. 7357	. 2642	.1116	. 3769	.8598	.1354	. 1925	. 8075	44
17	. 7383	. 2617	.1104	. 3807	.8585	.1356	. 1938	. 8061	43
18	. 7409	. 2591	.1093	. 3844	.8572	.1357	. 1952	. 8048	42
19	. 7434	. 2565	.1082	. 3882	.8559	.1359	. 1966	. 8034	41
20	.47460	.52540	2.1070	.53919	1.8546	1.1361	.11980	.88020	40
21	. 7486	. 2514	.1059	. 3957	.8533	.1363	. 1994	. 8006	39
22	. 7511	. 2489	.1048	. 3995	.8520	.1365	. 2007	. 7992	38
23	. 7537	. 2463	.1036	. 4032	.8507	.1366	. 2021	. 7979	37
24	. 7562	. 2437	.1025	. 4070	.8495	.1368	. 2035	. 7965	36
25	.47588	.52412	2.1014	.54107	1.8482	1.1370	.12049	.87951	35
26	. 7613	. 2386	.1002	. 4145	.8469	.1372	. 2063	. 7937	34
27	. 7639	. 2361	.0991	. 4183	.8456	.1373	. 2077	. 7923	33
28	. 7665	. 2335	.0980	. 4220	.8443	.1375	. 2090	. 7909	32
29	. 7690	. 2310	.0969	. 4258	.8430	.1377	. 2104	. 7895	31
30	.47716	.52284	2.0957	.54295	1.8418	1.1379	.12118	.87882	30
31	. 7741	. 2258	.0946	. 4333	.8405	.1381	. 2132	. 7868	29
32	. 7767	. 2233	.0935	. 4371	.8392	.1382	. 2146	. 7854	28
33	. 7792	. 2207	.0924	. 4409	.8379	.1384	. 2160	. 7840	27
34	. 7818	. 2182	.0912	. 4446	.8367	.1386	. 2174	. 7826	26
35	.47844	.52156	2.0901	.54484	1.8354	1.1388	.12188	.87812	25
36	. 7869	. 2131	.0890	. 4522	.8341	.1390	. 2202	. 7798	24
37	. 7895	. 2105	.0879	. 4559	.8329	.1391	. 2216	. 7784	23
38	. 7920	. 2080	.0868	. 4597	.8316	.1393	. 2229	. 7770	22
39	. 7946	. 2054	.0857	. 4635	.8303	.1395	. 2243	. 7756	21
40	.47971	.52029	2.0846	.54673	1.8291	1.1397	.12257	.87742	20
41	. 7997	. 2003	.0835	. 4711	.8278	.1399	. 2271	. 7728	19
42	. 8022	. 1978	.0824	. 4748	.8265	.1401	. 2285	. 7715	18
43	. 8048	. 1952	.0812	. 4786	.8253	.1402	. 2299	. 7701	17
44	. 8073	. 1927	.0801	. 4824	.8240	.1404	. 2313	. 7687	16
45	.48099	.51901	2.0790	.54862	1.8227	1.1406	.12327	.87673	15
46	. 8124	. 1876	.0779	. 4900	.8215	.1408	. 2341	. 7659	14
47	. 8150	. 1850	.0768	. 4937	.8202	.1410	. 2355	. 7645	13
48	. 8175	. 1825	.0757	. 4975	.8190	.1411	. 2369	. 7631	12
49	. 8201	. 1799	.0746	. 5013	.8177	.1413	. 2383	. 7617	11
50	.48226	.51774	2.0735	.55051	1.8165	1.1415	.12397	.87603	10
51	. 8252	. 1748	.0725	. 5089	.8152	.1417	. 2411	. 7588	9
52	. 8277	. 1723	.0714	. 5127	.8140	.1419	. 2425	. 7574	8
53	. 8303	. 1697	.0703	. 5165	.8127	.1421	. 2439	. 7560	7
54	. 8328	. 1672	.0692	. 5203	.8115	.1422	. 2453	. 7546	6
55	.48354	.51646	2.0681	.55241	1.8102	1.1424	.12468	.87532	5
56	. 8379	. 1621	.0670	. 5279	.8090	.1426	. 2482	. 7518	4
57	. 8405	. 1595	.0659	. 5317	.8078	.1428	. 2496	. 7504	3
58	. 8430	. 1570	.0648	. 5355	.8065	.1430	. 2510	. 7490	2
59	. 8455	. 1544	.0637	. 5393	.8053	.1432	. 2524	. 7476	1
60	. 8481	. 1519	.0627	. 5431	.8040	.1433	. 2538	. 7462	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

118° **61°**

29° **Natural Trigonometrical Functions.** 150°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.48481	.51519	2.0627	.55431	1.8040	1.1433	.12538	.87462	60
1	. 8506	. 1493	.0616	. 5469	.8028	.1435	. 2552	. 7448	59
2	. 8532	. 1468	.0605	. 5507	.8016	.1437	. 2566	. 7434	58
3	. 8557	. 1443	.0594	. 5545	.8003	.1439	. 2580	. 7420	57
4	. 8583	. 1417	.0583	. 5583	.7991	.1441	. 2594	. 7405	56
5	.48608	.51392	2.0573	.55621	1.7979	1.1443	.12609	.87391	55
6	. 8633	. 1366	.0562	. 5659	.7966	.1445	. 2623	. 7377	54
7	. 8659	. 1341	.0551	. 5697	.7954	.1446	. 2637	. 7363	53
8	. 8684	. 1316	.0540	. 5735	.7942	.1448	. 2651	. 7349	52
9	. 8710	. 1290	.0530	. 5774	.7930	.1450	. 2665	. 7335	51
10	.48735	.51265	2.0519	.55812	1.7917	1.1452	.12679	.87320	50
11	. 8760	. 1239	.0508	. 5850	.7905	.1454	. 2694	. 7306	49
12	. 8786	. 1214	.0498	. 5888	.7893	.1456	. 2708	. 7292	48
13	. 8811	. 1189	.0487	. 5926	.7881	.1458	. 2722	. 7278	47
14	. 8837	. 1163	.0476	. 5964	.7868	.1459	. 2736	. 7264	46
15	.48862	.51138	2.0466	.56003	1.7856	1.1461	.12750	.87250	45
16	. 8887	. 1112	.0455	. 6041	.7844	.1463	. 2765	. 7235	44
17	. 8913	. 1087	.0444	. 6079	.7832	.1465	. 2779	. 7221	43
18	. 8938	. 1062	.0434	. 6117	.7820	.1467	. 2793	. 7207	42
19	. 8964	. 1036	.0423	. 6156	.7808	.1469	. 2807	. 7193	41
20	.48989	.51011	2.0413	.56194	1.7795	1.1471	.12821	.87178	40
21	. 9014	. 0986	.0402	. 6232	.7783	.1473	. 2836	. 7164	39
22	. 9040	. 0960	.0392	. 6270	.7771	.1474	. 2850	. 7150	38
23	. 9065	. 0935	.0381	. 6309	.7759	.1476	. 2864	. 7136	37
24	. 9090	. 0910	.0370	. 6347	.7747	.1478	. 2879	. 7121	36
25	.49116	.50884	2.0360	.56385	1.7735	1.1480	.12893	.87107	35
26	. 9141	. 0859	.0349	. 6424	.7723	.1482	. 2907	. 7093	34
27	. 9166	. 0834	.0339	. 6462	.7711	.1484	. 2921	. 7078	33
28	. 9192	. 0808	.0329	. 6500	.7699	.1486	. 2936	. 7064	32
29	. 9217	. 0783	.0318	. 6539	.7687	.1488	. 2950	. 7050	31
30	.49242	.50758	2.0308	.56577	1.7675	1.1489	.12964	.87035	30
31	. 9268	. 0732	.0297	. 6616	.7663	.1491	. 2979	. 7021	29
32	. 9293	. 0707	.0287	. 6654	.7651	.1493	. 2993	. 7007	28
33	. 9318	. 0682	.0276	. 6692	.7639	.1495	. 3007	. 6992	27
34	. 9343	. 0656	.0266	. 6731	.7627	.1497	. 3022	. 6978	26
35	.49369	.50631	2.0256	.56769	1.7615	1.1499	.13036	.86964	25
36	. 9394	. 0606	.0245	. 6808	.7603	.1501	. 3050	. 6949	24
37	. 9419	. 0580	.0235	. 6846	.7591	.1503	. 3065	. 6935	23
38	. 9445	. 0555	.0224	. 6885	.7579	.1505	. 3079	. 6921	22
39	. 9470	. 0530	.0214	. 6923	.7567	.1507	. 3094	. 6906	21
40	.49495	.50505	2.0204	.56962	1.7555	1.1508	.13108	.86892	20
41	. 9521	. 0479	.0194	. 7000	.7544	.1510	. 3122	. 6877	19
42	. 9546	. 0454	.0183	. 7039	.7532	.1512	. 3137	. 6863	18
43	. 9571	. 0429	.0173	. 7077	.7520	.1514	. 3151	. 6849	17
44	. 9596	. 0404	.0163	. 7116	.7508	.1516	. 3166	. 6834	16
45	.49622	.50378	2.0152	.57155	1.7496	1.1518	.13180	.86820	15
46	. 9647	. 0353	.0142	. 7193	.7484	.1520	. 3194	. 6805	14
47	. 9672	. 0328	.0132	. 7232	.7473	.1522	. 3209	. 6791	13
48	. 9697	. 0303	.0122	. 7270	.7461	.1524	. 3223	. 6776	12
49	. 9723	. 0277	.0111	. 7309	.7449	.1526	. 3238	. 6762	11
50	.49748	.50252	2.0101	.57348	1.7437	1.1528	.13252	.86748	10
51	. 9773	. 0227	.0091	. 7386	.7426	.1530	. 3267	. 6733	9
52	. 9798	. 0202	.0081	. 7425	.7414	.1531	. 3281	. 6719	8
53	. 9823	. 0176	.0071	. 7464	.7402	.1533	. 3296	. 6704	7
54	. 9849	. 0151	.0061	. 7502	.7390	.1535	. 3310	. 6690	6
55	.49874	.50126	2.0050	.57541	1.7379	1.1537	.13325	.86675	5
56	. 9899	. 0101	.0040	. 7580	.7367	.1539	. 3339	. 6661	4
57	. 9924	. 0076	.0030	. 7619	.7355	.1541	. 3354	. 6646	3
58	. 9950	. 0050	.0020	. 7657	.7344	.1543	. 3368	. 6632	2
59	. 9975	. 0025	.0010	. 7696	.7332	.1545	. 3383	. 6617	1
60	.50000	. 0000	.0000	. 7735	.7320	.1547	. 3397	. 6602	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Sine.	Vrs. cos.	M.

119° 60°

30°			Natural Trigonometrical Functions.					149°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.50000	.50000	2.0000	.57735	1.7320	1.1547	.13397	.86602	60
1	. 0025	.49975	1.9990	. 7774	.7309	.1549	. 3412	. 6588	59
2	. 0050	. 9950	.9980	. 7813	.7297	.1551	. 3426	. 6573	58
3	. 0075	. 9924	.9970	. 7851	.7286	.1553	. 3441	. 6559	57
4	. 0101	. 9899	.9960	. 7890	.7274	.1555	. 3456	. 6544	56
5	.50126	.49874	1.9950	.57929	1.7262	1.1557	.13470	.86530	55
6	. 0151	. 9849	.9940	. 7968	.7251	.1559	. 3485	. 6515	54
7	. 0176	. 9824	.9930	. 8007	.7239	.1561	. 3499	. 6500	53
8	. 0201	. 9799	.9920	. 8046	.7228	.1562	. 3514	. 6486	52
9	. 0226	. 9773	.9910	. 8085	.7216	.1564	. 3529	. 6471	51
10	.50252	.49748	1.9900	.58123	1.7205	1.1566	.13543	.86457	50
11	. 0277	. 9723	.9890	. 8162	.7193	.1568	. 3558	. 6442	49
12	. 0302	. 9698	.9880	. 8201	.7182	.1570	. 3572	. 6427	48
13	. 0327	. 9673	.9870	. 8240	.7170	.1572	. 3587	. 6413	47
14	. 0352	. 9648	.9860	. 8279	.7159	.1574	. 3602	. 6398	46
15	.50377	.49623	1.9850	.58318	1.7147	1.1576	.13616	.86383	45
16	. 0402	. 9597	.9840	. 8357	.7136	.1578	. 3631	. 6369	44
17	. 0428	. 9572	.9830	. 8396	.7124	.1580	. 3646	. 6354	43
18	. 0453	. 9547	.9820	. 8435	.7113	.1582	. 3660	. 6339	42
19	. 0478	. 9522	.9811	. 8474	.7101	.1584	. 3675	. 6325	41
20	.50503	.49497	1.9801	.58513	1.7090	1.1586	.13690	.86310	40
21	. 0528	. 9472	.9791	. 8552	.7079	.1588	. 3704	. 6295	39
22	. 0553	. 9447	.9781	. 8591	.7067	.1590	. 3719	. 6281	38
23	. 0578	. 9422	.9771	. 8630	.7056	.1592	. 3734	. 6266	37
24	. 0603	. 9397	.9761	. 8670	.7044	.1594	. 3749	. 6251	36
25	.50628	.49371	1.9752	.58709	1.7033	1.1596	.13763	.86237	35
26	. 0653	. 9346	.9742	. 8748	.7022	.1598	. 3778	. 6222	34
27	. 0679	. 9321	.9732	. 8787	.7010	.1600	. 3793	. 6207	33
28	. 0704	. 9296	.9722	. 8826	.6999	.1602	. 3807	. 6192	32
29	. 0729	. 9271	.9713	. 8865	.6988	.1604	. 3822	. 6178	31
30	.50754	.49246	1.9703	.58904	1.6977	1.1606	.13837	.86163	30
31	. 0779	. 9221	.9693	. 8944	.6965	.1608	. 3852	. 6148	29
32	. 0804	. 9196	.9683	. 8983	.6954	.1610	. 3867	. 6133	28
33	. 0829	. 9171	.9674	. 9022	.6943	.1612	. 3881	. 6118	27
34	. 0854	. 9146	.9664	. 9061	.6931	.1614	. 3896	. 6104	26
35	.50879	.49121	1.9654	.59100	1.6920	1.1616	.13911	.86089	25
36	. 0904	. 9096	.9645	. 9140	.6909	.1618	. 3926	. 6074	24
37	. 0929	. 9071	.9635	. 9179	.6898	.1620	. 3941	. 6059	23
38	. 0954	. 9046	.9625	. 9218	.6887	.1622	. 3955	. 6044	22
39	. 0979	. 9021	.9616	. 9258	.6875	.1624	. 3970	. 6030	21
40	.51004	.48996	1.9606	.59297	1.6864	1.1626	.13985	.86015	20
41	. 1029	. 8971	.9596	. 9336	.6853	.1628	. 4000	. 6000	19
42	. 1054	. 8946	.9587	. 9376	.6842	.1630	. 4015	. 5985	18
43	. 1079	. 8921	.9577	. 9415	.6831	.1632	. 4030	. 5970	17
44	. 1104	. 8896	.9568	. 9454	.6820	.1634	. 4044	. 5955	16
45	.51129	.48871	1.9558	.59494	1.6808	1.1636	.14059	.85941	15
46	. 1154	. 8846	.9549	. 9533	.6797	.1638	. 4074	. 5926	14
47	. 1179	. 8821	.9539	. 9572	.6786	.1640	. 4089	. 5911	13
48	. 1204	. 8796	.9530	. 9612	.6775	.1642	. 4104	. 5896	12
49	. 1229	. 8771	.9520	. 9651	.6764	.1644	. 4119	. 5881	11
50	.51254	.48746	1.9510	.59691	1.6753	1.1646	.14134	.85866	10
51	. 1279	. 8721	.9501	. 9730	.6742	.1648	. 4149	. 5851	9
52	. 1304	. 8696	.9491	. 9770	.6731	.1650	. 4164	. 5836	8
53	. 1329	. 8671	.9482	. 9809	.6720	.1652	. 4178	. 5821	7
54	. 1354	. 8646	.9473	. 9849	.6709	.1654	. 4193	. 5806	6
55	.51379	.48621	1.9463	.59888	1.6698	1.1656	.14208	.85791	5
56	. 1404	. 8596	.9454	. 9928	.6687	.1658	. 4223	. 5777	4
57	. 1429	. 8571	.9444	. 9967	.6676	.1660	. 4238	. 5762	3
58	. 1454	. 8546	.9435	.60007	.6665	.1662	. 4253	. 5747	2
59	. 1479	. 8521	.9425	. 0046	.6654	.1664	. 4268	. 5732	1
60	. 1504	. 8496	.9416	. 0086	.6643	.1666	. 4283	. 5717	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
120°									59°

31° Natural Trigonometrical Functions. 148°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.51504	.48496	1.9416	.60086	1.6643	1.1666	.14283	.85717	60
1	. 1529	. 8471	.9407	. 0126	.6632	.1668	. 4298	. 5702	59
2	. 1554	. 8446	.9397	. 0165	.6621	.1670	. 4313	. 5687	58
3	. 1578	. 8421	.9388	. 0205	.6610	.1672	. 4328	. 5672	57
4	. 1603	. 8396	.9378	. 0244	.6599	.1674	. 4343	. 5657	56
5	.51628	.48371	1.9369	.60284	1.6588	1.1676	.14358	.85642	55
6	. 1653	. 8347	.9360	. 0324	.6577	.1678	. 4373	. 5627	54
7	. 1678	. 8322	.9350	. 0363	.6566	.1681	. 4388	. 5612	53
8	. 1703	. 8297	.9341	. 0403	.6555	.1683	. 4403	. 5597	52
9	. 1728	. 8272	.9332	. 0443	.6544	.1685	. 4418	. 5582	51
10	.51753	.48247	1.9322	.60483	1.6534	1.1687	.14433	.85566	50
11	. 1778	. 8222	.9313	. 0522	.6523	.1689	. 4448	. 5551	49
12	. 1803	. 8197	.9304	. 0562	.6512	.1691	. 4463	. 5536	48
13	. 1827	. 8172	.9295	. 0602	.6501	.1693	. 4479	. 5521	47
14	. 1852	. 8147	.9285	. 0642	.6490	.1695	. 4494	. 5506	46
15	.51877	.48123	1.9276	.60681	1.6479	1.1697	.14509	.85491	45
16	. 1902	. 8098	.9267	. 0721	.6469	.1699	. 4524	. 5476	44
17	. 1927	. 8073	.9258	. 0761	.6458	.1701	. 4539	. 5461	43
18	. 1952	. 8048	.9248	. 0801	.6447	.1703	. 4554	. 5446	42
19	. 1977	. 8023	.9239	. 0841	.6436	.1705	. 4569	. 5431	41
20	.52002	.47998	1.9230	.60881	1.6425	1.1707	.14584	.85416	40
21	. 2026	. 7973	.9221	. 0920	.6415	.1709	. 4599	. 5400	39
22	. 2051	. 7949	.9212	. 0960	.6404	.1712	. 4615	. 5385	38
23	. 2076	. 7924	.9203	. 1000	.6393	.1714	. 4630	. 5370	37
24	. 2101	. 7899	.9193	. 1040	.6383	.1716	. 4645	. 5355	36
25	.52126	.47874	1.9184	.61080	1.6372	1.1718	.14660	.85340	35
26	. 2151	. 7849	.9175	. 1120	.6361	.1720	. 4675	. 5325	34
27	. 2175	. 7824	.9166	. 1160	.6350	.1722	. 4690	. 5309	33
28	. 2200	. 7800	.9157	. 1200	.6340	.1724	. 4706	. 5294	32
29	. 2225	. 7775	.9148	. 1240	.6329	.1726	. 4721	. 5279	31
30	.52250	.47750	1.9139	.61280	1.6318	1.1728	.14736	.85264	30
31	. 2275	. 7725	.9130	. 1320	.6308	.1730	. 4751	. 5249	29
32	. 2299	. 7700	.9121	. 1360	.6297	.1732	. 4766	. 5234	28
33	. 2324	. 7676	.9112	. 1400	.6286	.1734	. 4782	. 5218	27
34	. 2349	. 7651	.9102	. 1440	.6276	.1737	. 4797	. 5203	26
35	.52374	.47626	1.9093	.61480	1.6265	1.1739	.14812	.85188	25
36	. 2398	. 7601	.9084	. 1520	.6255	.1741	. 4827	. 5173	24
37	. 2423	. 7577	.9075	. 1560	.6244	.1743	. 4842	. 5157	23
38	. 2448	. 7552	.9066	. 1601	.6233	.1745	. 4858	. 5142	22
39	. 2473	. 7527	.9057	. 1641	.6223	.1747	. 4873	. 5127	21
40	.52498	.47502	1.9048	.61681	1.6212	1.1749	.14888	.85112	20
41	. 2522	. 7477	.9039	. 1721	.6202	.1751	. 4904	. 5096	19
42	. 2547	. 7453	.9030	. 1761	.6191	.1753	. 4919	. 5081	18
43	. 2572	. 7428	.9021	. 1801	.6181	.1756	. 4934	. 5066	17
44	. 2597	. 7403	.9013	. 1842	.6170	.1758	. 4949	. 5050	16
45	.52621	.47379	1.9004	.61882	1.6160	1.1760	.14965	.85035	15
46	. 2646	. 7354	.8995	. 1922	.6149	.1762	. 4980	. 5020	14
47	. 2671	. 7329	.8986	. 1962	.6139	.1764	. 4995	. 5004	13
48	. 2695	. 7304	.8977	. 2004	.6128	.1766	. 5011	. 4989	12
49	. 2720	. 7280	.8968	. 2043	.6118	.1768	. 5026	. 4974	11
50	.52745	.47255	1.8959	.62083	1.6107	1.1770	.15041	.84959	10
51	. 2770	. 7230	.8950	. 2123	.6097	.1772	. 5057	. 4943	9
52	. 2794	. 7205	.8941	. 2164	.6086	.1775	. 5072	. 4928	8
53	. 2819	. 7181	.8932	. 2204	.6076	.1777	. 5087	. 4912	7
54	. 2844	. 7156	.8924	. 2244	.6066	.1779	. 5103	. 4897	6
55	.52868	.47131	1.8915	.62285	1.6055	1.1781	.15118	.84882	5
56	. 2893	. 7107	.8906	. 2325	.6045	.1783	. 5133	. 4866	4
57	. 2918	. 7082	.8897	. 2366	.6034	.1785	. 5149	. 4851	3
58	. 2942	. 7057	.8888	. 2406	.6024	.1787	. 5164	. 4836	2
59	. 2967	. 7033	.8879	. 2446	.6014	.1790	. 5180	. 4820	1
60	. 2992	. 7008	.8871	. 2487	.6003	.1792	. 5195	. 4805	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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32° **Natural Trigonometrical Functions.** **147°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.52992	.47008	1.8871	.62487	1.6003	1.1792	.15195	.84805	60
1	. 3016	. 6983	.8862	. 2527	.5993	.1794	. 5211	. 4789	59
2	. 3041	. 6959	.8853	. 2568	.5983	.1796	. 5226	. 4774	58
3	. 3066	. 6934	.8844	. 2608	.5972	.1798	. 5241	. 4758	57
4	. 3090	. 6909	.8836	. 2649	.5962	.1800	. 5257	. 4743	56
5	.53115	.46885	1.8827	.62689	1.5952	1.1802	.15272	.84728	55
6	. 3140	. 6860	.8818	. 2730	.5941	.1805	. 5288	. 4712	54
7	. 3164	. 6835	.8809	. 2770	.5931	.1807	. 5303	. 4697	53
8	. 3189	. 6811	.8801	. 2811	.5921	.1809	. 5319	. 4681	52
9	. 3214	. 6786	.8792	. 2851	.5910	.1811	. 5334	. 4666	51
10	.53238	.46762	1.8783	.62892	1.5900	1.1813	.15350	.84650	50
11	. 3263	. 6737	.8775	. 2933	.5890	.1815	. 5365	. 4635	49
12	. 3288	. 6712	.8766	. 2973	.5880	.1818	. 5381	. 4619	48
13	. 3312	. 6688	.8757	. 3014	.5869	.1820	. 5396	. 4604	47
14	. 3337	. 6663	.8749	. 3055	.5859	.1822	. 5412	. 4588	46
15	.53361	.46638	1.8740	.63095	1.5849	1.1824	.15427	.84573	45
16	. 3386	. 6614	.8731	. 3136	.5839	.1826	. 5443	. 4557	44
17	. 3411	. 6589	.8723	. 3177	.5829	.1828	. 5458	. 4542	43
18	. 3435	. 6565	.8714	. 3217	.5818	.1831	. 5474	. 4526	42
19	. 3460	. 6540	.8706	. 3258	.5808	.1833	. 5489	. 4511	41
20	.53484	.46516	1.8697	.63299	1.5798	1.1835	.15505	.84495	40
21	. 3509	. 6491	.8688	. 3339	.5788	.1837	. 5520	. 4479	39
22	. 3533	. 6466	.8680	. 3380	.5778	.1839	. 5536	. 4464	38
23	. 3558	. 6442	.8671	. 3421	.5768	.1841	. 5552	. 4448	37
24	. 3583	. 6417	.8663	. 3462	.5757	.1844	. 5567	. 4433	36
25	.53607	.46393	1.8654	.63503	1.5747	1.1846	.15583	.84417	35
26	. 3632	. 6368	.8646	. 3543	.5737	.1848	. 5598	. 4402	34
27	. 3656	. 6344	.8637	. 3584	.5727	.1850	. 5614	. 4386	33
28	. 3681	. 6319	.8629	. 3625	.5717	.1852	. 5630	. 4370	32
29	. 3705	. 6294	.8620	. 3666	.5707	.1855	. 5645	. 4355	31
30	.53730	.46270	1.8611	.63707	1.5697	1.1857	.15661	.84339	30
31	. 3754	. 6245	.8603	. 3748	.5687	.1859	. 5676	. 4323	29
32	. 3779	. 6221	.8595	. 3789	.5677	.1861	. 5692	. 4308	28
33	. 3803	. 6196	.8586	. 3830	.5667	.1863	. 5708	. 4292	27
34	. 3828	. 6172	.8578	. 3871	.5657	.1866	. 5723	. 4276	26
35	.53852	.46147	1.8569	.63912	1.5646	1.1868	.15739	.84261	25
36	. 3877	. 6123	.8561	. 3953	.5636	.1870	. 5755	. 4245	24
37	. 3901	. 6098	.8552	. 3994	.5626	.1872	. 5770	. 4229	23
38	. 3926	. 6074	.8544	. 4035	.5616	.1874	. 5786	. 4214	22
39	. 3950	. 6049	.8535	. 4076	.5606	.1877	. 5802	. 4198	21
40	.53975	.46025	1.8527	.64117	1.5596	1.1879	.15817	.84182	20
41	. 3999	. 6000	.8519	. 4158	.5586	.1881	. 5833	. 4167	19
42	. 4024	. 5976	.8510	. 4199	.5577	.1883	. 5849	. 4151	18
43	. 4048	. 5951	.8502	. 4240	.5567	.1886	. 5865	. 4135	17
44	. 4073	. 5927	.8493	. 4281	.5557	.1888	. 5880	. 4120	16
45	.54097	.45902	1.8485	.64322	1.5547	1.1890	.15896	.84104	15
46	. 4122	. 5878	.8477	. 4363	.5537	.1892	. 5912	. 4088	14
47	. 4146	. 5854	.8468	. 4404	.5527	.1894	. 5927	. 4072	13
48	. 4171	. 5829	.8460	. 4446	.5517	.1897	. 5943	. 4057	12
49	. 4195	. 5805	.8452	. 4487	.5507	.1899	. 5959	. 4041	11
50	.54220	.45780	1.8443	.64528	1.5497	1.1901	.15975	.84025	10
51	. 4244	. 5756	.8435	. 4569	.5487	.1903	. 5991	. 4009	9
52	. 4268	. 5731	.8427	. 4610	.5477	.1906	. 6006	. 3993	8
53	. 4293	. 5707	.8418	. 4652	.5467	.1908	. 6022	. 3978	7
54	. 4317	. 5682	.8410	. 4693	.5458	.1910	. 6038	. 3962	6
55	.54342	.45658	1.8402	.64734	1.5448	1.1912	.16054	.83946	5
56	. 4366	. 5634	.8394	. 4775	.5438	.1915	. 6070	. 3930	4
57	. 4391	. 5609	.8385	. 4817	.5428	.1917	. 6085	. 3914	3
58	. 4415	. 5585	.8377	. 4858	.5418	.1919	. 6101	. 3899	2
59	. 4439	. 5560	.8369	. 4899	.5408	.1921	. 6117	. 3883	1
60	. 4464	. 5536	.8361	. 4941	.5399	.1922	. 6133	. 3867	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

122° **57°**

33° Natural Trigonometrical Functions. 146°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.54464	.45536	1.8361	.64941	1.5399	1.1924	.16133	.83867	60
1	. 4488	. 5512	.8352	. 4982	.5389	.1926	. 6149	. 3851	59
2	. 4513	. 5487	.8344	. 5023	.5379	.1928	. 6165	. 3835	58
3	. 4537	. 5463	.8336	. 5065	.5369	.1930	. 6180	. 3819	57
4	. 4561	. 5438	.8328	. 5106	.5359	.1933	. 6196	. 3804	56
5	.54586	.45414	1.8320	.65148	1.5350	1.1935	.16212	.83788	55
6	. 4610	. 5390	.8311	. 5189	.5340	.1937	. 6228	. 3772	54
7	. 4634	. 5365	.8303	. 5231	.5330	.1939	. 6244	. 3756	53
8	. 4659	. 5341	.8295	. 5272	.5320	.1942	. 6260	. 3740	52
9	. 4683	. 5317	.8287	. 5314	.5311	.1944	. 6276	. 3724	51
10	.54708	.45292	1.8279	.65355	1.5301	1.1946	.16292	.83708	50
11	. 4732	. 5268	.8271	. 5397	.5291	.1948	. 6308	. 3692	49
12	. 4756	. 5244	.8263	. 5438	.5282	.1951	. 6323	. 3676	48
13	. 4781	. 5219	.8255	. 5480	.5272	.1953	. 6339	. 3660	47
14	. 4805	. 5195	.8246	. 5521	.5262	.1955	. 6355	. 3644	46
15	.54829	.45171	1.8238	.65563	1.5252	1.1958	.16371	.83629	45
16	. 4854	. 5146	.8230	. 5604	.5243	.1960	. 6387	. 3613	44
17	. 4878	. 5122	.8222	. 5646	.5233	.1962	. 6403	. 3597	43
18	. 4902	. 5098	.8214	. 5688	.5223	.1964	. 6419	. 3581	42
19	. 4926	. 5073	.8206	. 5729	.5214	.1967	. 6435	. 3565	41
20	.54951	.45049	1.8198	.65771	1.5204	1.1969	.16451	.83549	40
21	. 4975	. 5025	.8190	. 5813	.5195	.1971	. 6467	. 3533	39
22	. 4999	. 5000	.8182	. 5854	.5185	.1974	. 6483	. 3517	38
23	. 5024	. 4976	.8174	. 5896	.5175	.1976	. 6499	. 3501	37
24	. 5048	. 4952	.8166	. 5938	.5166	.1978	. 6515	. 3485	36
25	.55072	.44928	1.8158	.65980	1.5156	1.1980	.16531	.83469	35
26	. 5097	. 4903	.8150	. 6021	.5147	.1983	. 6547	. 3453	34
27	. 5121	. 4879	.8142	. 6063	.5137	.1985	. 6563	. 3437	33
28	. 5145	. 4855	.8134	. 6105	.5127	.1987	. 6579	. 3421	32
29	. 5169	. 4830	.8126	. 6147	.5118	.1990	. 6595	. 3405	31
30	.55194	.44806	1.8118	.66188	1.5108	1.1992	.16611	.83388	30
31	. 5218	. 4782	.8110	. 6230	.5099	.1994	. 6627	. 3372	29
32	. 5242	. 4758	.8102	. 6272	.5089	.1997	. 6643	. 3356	28
33	. 5266	. 4733	.8094	. 6314	.5080	.1999	. 6660	. 3340	27
34	. 5291	. 4709	.8086	. 6356	.5070	.2001	. 6676	. 3324	26
35	.55315	.44685	1.8078	.66398	1.5061	1.2004	.16692	.83308	25
36	. 5339	. 4661	.8070	. 6440	.5051	.2006	. 6708	. 3292	24
37	. 5363	. 4637	.8062	. 6482	.5042	.2008	. 6724	. 3276	23
38	. 5388	. 4612	.8054	. 6524	.5032	.2010	. 6740	. 3260	22
39	. 5412	. 4588	.8047	. 6566	.5023	.2013	. 6756	. 3244	21
40	.55436	.44564	1.8039	.66608	1.5013	1.2015	.16772	.83228	20
41	. 5460	. 4540	.8031	. 6650	.5004	.2017	. 6788	. 3211	19
42	. 5484	. 4515	.8023	. 6692	.4994	.2020	. 6804	. 3195	18
43	. 5509	. 4491	.8015	. 6734	.4985	.2022	. 6821	. 3179	17
44	. 5533	. 4467	.8007	. 6776	.4975	.2024	. 6837	. 3163	16
45	.55557	.44443	1.7999	.66818	1.4966	1.2027	.16853	.83147	15
46	. 5581	. 4419	.7992	. 6860	.4957	.2029	. 6869	. 3131	14
47	. 5605	. 4395	.7984	. 6902	.4947	.2031	. 6885	. 3115	13
48	. 5629	. 4370	.7976	. 6944	.4938	.2034	. 6901	. 3098	12
49	. 5654	. 4346	.7968	. 6986	.4928	.2036	. 6918	. 3082	11
50	.55678	.44322	1.7960	.67028	1.4919	1.2039	.16934	.83066	10
51	. 5702	. 4298	.7953	. 7071	.4910	.2041	. 6950	. 3050	9
52	. 5726	. 4274	.7945	. 7113	.4900	.2043	. 6966	. 3034	8
53	. 5750	. 4250	.7937	. 7155	.4891	.2046	. 6982	. 3017	7
54	. 5774	. 4225	.7929	. 7197	.4881	.2048	. 6999	. 3001	6
55	.55799	.44201	1.7921	.67239	1.4872	1.2050	.17015	.82985	5
56	. 5823	. 4177	.7914	. 7282	.4863	.2053	. 7031	. 2969	4
57	. 5847	. 4153	.7906	. 7324	.4853	.2055	. 7047	. 2952	3
58	. 5871	. 4129	.7898	. 7366	.4844	.2057	. 7064	. 2936	2
59	. 5895	. 4105	.7891	. 7408	.4835	.2060	. 7080	. 2920	1
60	. 5919	. 4081	.7883	. 7451	.4826	.2062	. 7096	. 2904	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

123° 56°

34° **Natural Trigonometrical Functions.** 145°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.55919	.44081	1.7883	.67451	1.4826	1.2062	.17096	.82904	60
1	. 5943	. 4057	.7875	. 7493	.4816	.2064	. 7112	. 2887	59
2	. 5967	. 4032	.7867	. 7535	.4807	.2067	. 7129	. 2871	58
3	. 5992	. 4008	.7860	. 7578	.4798	.2069	. 7145	. 2855	57
4	. 6016	. 3984	.7852	. 7620	.4788	.2072	. 7161	. 2839	56
5	.56040	.43960	1.7844	.67663	1.4779	1.2074	.17178	.82822	55
6	. 6064	. 3936	.7837	. 7705	.4770	.2076	. 7194	. 2806	54
7	. 6088	. 3912	.7829	. 7747	.4761	.2079	. 7210	. 2790	53
8	. 6112	. 3888	.7821	. 7790	.4751	.2081	. 7227	. 2773	52
9	. 6136	. 3864	.7814	. 7832	.4742	.2083	. 7243	. 2757	51
10	.56160	.43840	1.7806	.67875	1.4733	1.2086	.17259	.82741	50
11	. 6184	. 3816	.7798	. 7917	.4724	.2088	. 7276	. 2724	49
12	. 6208	. 3792	.7791	. 7960	.4714	.2091	. 7292	. 2708	48
13	. 6232	. 3768	.7783	. 8002	.4705	.2093	. 7308	. 2692	47
14	. 6256	. 3743	.7776	. 8045	.4696	.2095	. 7325	. 2675	46
15	.56280	.43719	1.7768	.68087	1.4687	1.2098	.17341	.82659	45
16	. 6304	. 3695	.7760	. 8130	.4678	.2100	. 7357	. 2643	44
17	. 6328	. 3671	.7753	. 8173	.4669	.2103	. 7374	. 2626	43
18	. 6353	. 3647	.7745	. 8215	.4659	.2105	. 7390	. 2610	42
19	. 6377	. 3623	.7738	. 8258	.4650	.2107	. 7406	. 2593	41
20	.56401	.43599	1.7730	.68301	1.4641	1.2110	.17423	.82577	40
21	. 6425	. 3575	.7723	. 8343	.4632	.2112	. 7439	. 2561	39
22	. 6449	. 3551	.7715	. 8386	.4623	.2115	. 7456	. 2544	38
23	. 6473	. 3527	.7708	. 8429	.4614	.2117	. 7472	. 2528	37
24	. 6497	. 3503	.7700	. 8471	.4605	.2119	. 7489	. 2511	36
25	.56521	.43479	1.7693	.68514	1.4595	1.2122	.17505	.82495	35
26	. 6545	. 3455	.7685	. 8557	.4586	.2124	. 7521	. 2478	34
27	. 6569	. 3431	.7678	. 8600	.4577	.2127	. 7538	. 2462	33
28	. 6593	. 3407	.7670	. 8642	.4568	.2129	. 7554	. 2445	32
29	. 6617	. 3383	.7663	. 8685	.4559	.2132	. 7571	. 2429	31
30	.56641	.43359	1.7655	.68728	1.4550	1.2134	.17587	.82413	30
31	. 6664	. 3335	.7648	. 8771	.4541	.2136	. 7604	. 2396	29
32	. 6688	. 3311	.7640	. 8814	.4532	.2139	. 7620	. 2380	28
33	. 6712	. 3287	.7633	. 8857	.4523	.2141	. 7637	. 2363	27
34	. 6736	. 3263	.7625	. 8899	.4514	.2144	. 7653	. 2347	26
35	.56760	.43239	1.7618	.68942	1.4505	1.2146	.17670	.82330	25
36	. 6784	. 3216	.7610	. 8985	.4496	.2149	. 7686	. 2314	24
37	. 6808	. 3192	.7603	. 9028	.4487	.2151	. 7703	. 2297	23
38	. 6832	. 3168	.7596	. 9071	.4478	.2153	. 7719	. 2280	22
39	. 6856	. 3144	.7588	. 9114	.4469	.2156	. 7736	. 2264	21
40	.56880	.43120	1.7581	.69157	1.4460	1.2158	.17752	.82247	20
41	. 6904	. 3096	.7573	. 9200	.4451	.2161	. 7769	. 2231	19
42	. 6928	. 3072	.7566	. 9243	.4442	.2163	. 7786	. 2214	18
43	. 6952	. 3048	.7559	. 9286	.4433	.2166	. 7802	. 2198	17
44	. 6976	. 3024	.7551	. 9329	.4424	.2168	. 7819	. 2181	16
45	.57000	.43000	1.7544	.69372	1.4415	1.2171	.17835	.82165	15
46	. 7023	. 2976	.7537	. 9415	.4406	.2173	. 7852	. 2148	14
47	. 7047	. 2952	.7529	. 9459	.4397	.2175	. 7868	. 2131	13
48	. 7071	. 2929	.7522	. 9502	.4388	.2178	. 7885	. 2115	12
49	. 7095	. 2905	.7514	. 9545	.4379	.2180	. 7902	. 2098	11
50	.57119	.42881	1.7507	.69588	1.4370	1.2183	.17918	.82082	10
51	. 7143	. 2857	.7500	. 9631	.4361	.2185	. 7935	. 2065	9
52	. 7167	. 2833	.7493	. 9674	.4352	.2188	. 7951	. 2048	8
53	. 7191	. 2809	.7485	. 9718	.4343	.2190	. 7968	. 2032	7
54	. 7214	. 2785	.7478	. 9761	.4335	.2193	. 7985	. 2015	6
55	.57238	.42761	1.7471	.69804	1.4326	1.2195	.18001	.81998	5
56	. 7262	. 2738	.7463	. 9847	.4317	.2198	. 8018	. 1982	4
57	. 7286	. 2714	.7456	. 9891	.4308	.2200	. 8035	. 1965	3
58	. 7310	. 2690	.7449	. 9934	.4299	.2203	. 8051	. 1948	2
59	. 7334	. 2666	.7442	. 9977	.4290	.2205	. 8068	. 1932	1
60	. 7358	. 2642	.7434	.70021	.4281	.2208	. 8085	. 1915	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

124° 55°

35° **Natural Trigonometrical Functions.** 144°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.57358	.42642	1.7434	.70021	1.4281	1.2208	.18085	.81915	60
1	. 7381	. 2618	.7427	. 0064	.4273	.2210	. 8101	. 1898	59
2	. 7405	. 2595	.7420	. 0107	.4264	.2213	. 8118	. 1882	58
3	. 7429	. 2571	.7413	. 0151	.4255	.2215	. 8135	. 1865	57
4	. 7453	. 2547	.7405	. 0194	.4246	.2218	. 8151	. 1848	56
5	.57477	.42523	1.7398	.70238	1.4237	1.2220	.18168	.81832	55
6	. 7500	. 2499	.7391	. 0281	.4228	.2223	. 8185	. 1815	54
7	. 7524	. 2476	.7384	. 0325	.4220	.2225	. 8202	. 1798	53
8	. 7548	. 2452	.7377	. 0368	.4211	.2228	. 8218	. 1781	52
9	. 7572	. 2428	.7369	. 0412	.4202	.2230	. 8235	. 1765	51
10	.57596	.42404	1.7362	.70455	1.4193	1.2233	.18252	.81748	50
11	. 7619	. 2380	.7355	. 0499	.4185	.2235	. 8269	. 1731	49
12	. 7643	. 2357	.7348	. 0542	.4176	.2238	. 8285	. 1714	48
13	. 7667	. 2333	.7341	. 0586	.4167	.2240	. 8302	. 1698	47
14	. 7691	. 2309	.7334	. 0629	.4158	.2243	. 8319	. 1681	46
15	.57714	.42285	1.7327	.70673	1.4150	1.2245	.18336	.81664	45
16	. 7738	. 2262	.7319	. 0717	.4141	.2248	. 8353	. 1647	44
17	. 7762	. 2238	.7312	. 0760	.4132	.2250	. 8369	. 1630	43
18	. 7786	. 2214	.7305	. 0804	.4123	.2253	. 8386	. 1614	42
19	. 7809	. 2190	.7298	. 0848	.4115	.2255	. 8403	. 1597	41
20	.57833	.42167	1.7291	.70891	1.4106	1.2258	.18420	.81580	40
21	. 7857	. 2143	.7284	. 0935	.4097	.2260	. 8437	. 1563	39
22	. 7881	. 2119	.7277	. 0979	.4089	.2263	. 8453	. 1546	38
23	. 7904	. 2096	.7270	. 1022	.4080	.2265	. 8470	. 1530	37
24	. 7928	. 2072	.7263	. 1066	.4071	.2268	. 8487	. 1513	36
25	.57952	.42048	1.7256	.71110	1.4063	1.2270	.18504	.81496	35
26	. 7975	. 2024	.7249	. 1154	.4054	.2273	. 8521	. 1479	34
27	. 7999	. 2001	.7242	. 1198	.4045	.2276	. 8538	. 1462	33
28	. 8023	. 1977	.7234	. 1241	.4037	.2278	. 8555	. 1445	32
29	. 8047	. 1953	.7227	. 1285	.4028	.2281	. 8571	. 1428	31
30	.58070	.41930	1.7220	.71329	1.4019	1.2283	.18588	.81411	30
31	. 8094	. 1906	.7213	. 1373	.4011	.2286	. 8605	. 1395	29
32	. 8118	. 1882	.7206	. 1417	.4002	.2288	. 8622	. 1378	28
33	. 8141	. 1859	.7199	. 1461	.3994	.2291	. 8639	. 1361	27
34	. 8165	. 1835	.7192	. 1505	.3985	.2293	. 8656	. 1344	26
35	.58189	.41811	1.7185	.71549	1.3976	1.2296	.18673	.81327	25
36	. 8212	. 1788	.7178	. 1593	.3968	.2298	. 8690	. 1310	24
37	. 8236	. 1764	.7171	. 1637	.3959	.2301	. 8707	. 1293	23
38	. 8259	. 1740	.7164	. 1681	.3951	.2304	. 8724	. 1276	22
39	. 8283	. 1717	.7157	. 1725	.3942	.2306	. 8741	. 1259	21
40	.58307	.41693	1.7151	.71769	1.3933	1.2309	.18758	.81242	20
41	. 8330	. 1669	.7144	. 1813	.3925	.2311	. 8775	. 1225	19
42	. 8354	. 1646	.7137	. 1857	.3916	.2314	. 8792	. 1208	18
43	. 8378	. 1622	.7130	. 1901	.3908	.2316	. 8809	. 1191	17
44	. 8401	. 1599	.7123	. 1945	.3899	.2319	. 8826	. 1174	16
45	.58425	.41575	1.7116	.71990	1.3891	1.2322	.18843	.81157	15
46	. 8448	. 1551	.7109	. 2034	.3882	.2324	. 8860	. 1140	14
47	. 8472	. 1528	.7102	. 2078	.3874	.2327	. 8877	. 1123	13
48	. 8496	. 1504	.7095	. 2122	.3865	.2329	. 8894	. 1106	12
49	. 8519	. 1481	.7088	. 2166	.3857	.2332	. 8911	. 1089	11
50	.58543	.41457	1.7081	.72211	1.3848	1.2335	.18928	.81072	10
51	. 8566	. 1433	.7075	. 2255	.3840	.2337	. 8945	. 1055	9
52	. 8590	. 1410	.7068	. 2299	.3831	.2340	. 8962	. 1038	8
53	. 8614	. 1386	.7061	. 2344	.3823	.2342	. 8979	. 1021	7
54	. 8637	. 1363	.7054	. 2388	.3814	.2345	. 8996	. 1004	6
55	.58661	.41339	1.7047	.72432	1.3806	1.2348	.19013	.80987	5
56	. 8684	. 1316	.7040	. 2477	.3797	.2350	. 9030	. 0970	4
57	. 8708	. 1292	.7033	. 2521	.3789	.2353	. 9047	. 0953	3
58	. 8731	. 1268	.7027	. 2565	.3781	.2355	. 9064	. 0936	2
59	. 8755	. 1245	.7020	. 2610	.3772	.2358	. 9081	. 0919	1
60	. 8778	. 1221	.7013	. 2654	.3764	.2361	. 9098	. 0902	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

125° 54°

36° Natural Trigonometrical Functions. 143°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.58778	.41221	1.7013	.72654	1.3764	1.2361	.19098	.80902	60
1	. 8802	. 1198	.7006	. 2699	.3755	.2363	. 9115	. 0885	59
2	. 8825	. 1174	.6999	. 2743	.3747	.2366	. 9132	. 0867	58
3	. 8849	. 1151	.6993	. 2788	.3738	.2368	. 9150	. 0850	57
4	. 8873	. 1127	.6986	. 2832	.3730	.2371	. 9167	. 0833	56
5	.58896	.41104	1.6979	.72877	1.3722	1.2374	.19184	.80816	55
6	. 8920	. 1080	.6972	. 2921	.3713	.2376	. 9201	. 0799	54
7	. 8943	. 1057	.6965	. 2966	.3705	.2379	. 9218	. 0782	53
8	. 8967	. 1033	.6959	. 3010	.3697	.2382	. 9235	. 0765	52
9	. 8990	. 1010	.6952	. 3055	.3688	.2384	. 9252	. 0747	51
10	.59014	.40986	1.6945	.73100	1.3680	1.2387	.19270	.80730	50
11	. 9037	. 0963	.6938	. 3144	.3672	.2389	. 9287	. 0713	49
12	. 9060	. 0939	.6932	. 3189	.3663	.2392	. 9304	. 0696	48
13	. 9084	. 0916	.6925	. 3234	.3655	.2395	. 9321	. 0679	47
14	. 9107	. 0892	.6918	. 3278	.3647	.2397	. 9338	. 0662	46
15	.59131	.40869	1.6912	.73323	1.3638	1.2400	.19355	.80644	45
16	. 9154	. 0845	.6905	. 3368	.3630	.2403	. 9373	. 0627	44
17	. 9178	. 0822	.6898	. 3412	.3622	.2405	. 9390	. 0610	43
18	. 9201	. 0799	.6891	. 3457	.3613	.2408	. 9407	. 0593	42
19	. 9225	. 0775	.6885	. 3502	.3605	.2411	. 9424	. 0576	41
20	.59248	.40752	1.6878	.73547	1.3597	1.2413	.19442	.80558	40
21	. 9272	. 0728	.6871	. 3592	.3588	.2416	. 9459	. 0541	39
22	. 9295	. 0705	.6865	. 3637	.3580	.2419	. 9476	. 0524	38
23	. 9318	. 0681	.6858	. 3681	.3572	.2421	. 9493	. 0507	37
24	. 9342	. 0658	.6851	. 3726	.3564	.2424	. 9511	. 0489	36
25	.59365	.40635	1.6845	.73771	1.3555	1.2427	.19528	.80472	35
26	. 9389	. 0611	.6838	. 3816	.3547	.2429	. 9545	. 0455	34
27	. 9412	. 0588	.6831	. 3861	.3539	.2432	. 9562	. 0437	33
28	. 9435	. 0564	.6825	. 3906	.3531	.2435	. 9580	. 0420	32
29	. 9459	. 0541	.6818	. 3951	.3522	.2437	. 9597	. 0403	31
30	.59482	.40518	1.6812	.73996	1.3514	1.2440	.19614	.80386	30
31	. 9506	. 0494	.6805	. 4041	.3506	.2443	. 9632	. 0368	29
32	. 9529	. 0471	.6798	. 4086	.3498	.2445	. 9649	. 0351	28
33	. 9552	. 0447	.6792	. 4131	.3489	.2448	. 9666	. 0334	27
34	. 9576	. 0424	.6785	. 4176	.3481	.2451	. 9683	. 0316	26
35	.59599	.40401	1.6779	.74221	1.3473	1.2453	.19701	.80299	25
36	. 9622	. 0377	.6772	. 4266	.3465	.2456	. 9718	. 0282	24
37	. 9646	. 0354	.6766	. 4312	.3457	.2459	. 9736	. 0264	23
38	. 9669	. 0331	.6759	. 4357	.3449	.2461	. 9753	. 0247	22
39	. 9692	. 0307	.6752	. 4402	.3440	.2464	. 9770	. 0230	21
40	.59716	.40284	1.6746	.74447	1.3432	1.2467	.19788	.80212	20
41	. 9739	. 0261	.6739	. 4492	.3424	.2470	. 9805	. 0195	19
42	. 9762	. 0237	.6733	. 4538	.3416	.2472	. 9822	. 0177	18
43	. 9786	. 0214	.6726	. 4583	.3408	.2475	. 9840	. 0160	17
44	. 9809	. 0191	.6720	. 4628	.3400	.2478	. 9857	. 0143	16
45	.59832	.40167	1.6713	.74673	1.3392	1.2480	.19875	.80125	15
46	. 9856	. 0144	.6707	. 4719	.3383	.2483	. 9892	. 0108	14
47	. 9879	. 0121	.6700	. 4764	.3375	.2486	. 9909	. 0090	13
48	. 9902	. 0098	.6694	. 4809	.3367	.2488	. 9927	. 0073	12
49	. 9926	. 0074	.6687	. 4855	.3359	.2491	. 9944	. 0056	11
50	.59949	.40051	1.6681	.74900	1.3351	1.2494	.19962	.80038	10
51	. 9972	. 0028	.6674	. 4946	.3343	.2497	. 9979	. 0021	9
52	. 9995	. 0004	.6668	. 4991	.3335	.2499	. 9997	. 0003	8
53	.60019	.39981	.6661	. 5037	.3327	.2502	.20014	.79986	7
54	. 0042	. 9958	.6655	. 5082	.3319	.2505	. 0031	. 9968	6
55	.60065	.39935	1.6648	.75128	1.3311	1.2508	.20049	.79951	5
56	. 0088	. 9911	.6642	. 5173	.3303	.2510	. 0066	. 9933	4
57	. 0112	. 9888	.6636	. 5219	.3294	.2513	. 0084	. 9916	3
58	. 0135	. 9865	.6629	. 5264	.3286	.2516	. 0101	. 9898	2
59	. 0158	. 9842	.6623	. 5310	.3278	.2519	. 0119	. 9881	1
60	. 0181	. 9818	.6616	. 5355	.3270	.2521	. 0136	. 9863	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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37° **Natural Trigonometrical Functions.** 142°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.60181	.39813	1.6616	.75355	1.3270	1.2521	.20136	.79863	60
1	. 0205	. 9795	.6610	. 5401	.3262	.2524	. 0154	. 9846	59
2	. 0228	. 9772	.6603	. 5447	.3254	.2527	. 0171	. 9828	58
3	. 0251	. 9749	.6597	. 5492	.3246	.2530	. 0189	. 9811	57
4	. 0274	. 9726	.6591	. 5538	.3238	.2532	. 0206	. 9793	56
5	.60298	.39702	1.6584	.75584	1.3230	1.2535	.20224	.79776	55
6	. 0320	. 9679	.6578	. 5629	.3222	.2538	. 0242	. 9758	54
7	. 0344	. 9656	.6572	. 5675	.3214	.2541	. 0259	. 9741	53
8	. 0367	. 9633	.6565	. 5721	.3206	.2543	. 0277	. 9723	52
9	. 0390	. 9610	.6559	. 5767	.3198	.2546	. 0294	. 9706	51
10	.60413	.39586	1.6552	.75812	1.3190	1.2549	.20312	.79688	50
11	. 0437	. 9563	.6546	. 5858	.3182	.2552	. 0329	. 9670	49
12	. 0460	. 9540	.6540	. 5904	.3174	.2554	. 0347	. 9653	48
13	. 0483	. 9517	.6533	. 5950	.3166	.2557	. 0365	. 9635	47
14	. 0506	. 9494	.6527	. 5996	.3159	.2560	. 0382	. 9618	46
15	.60529	.39471	1.6521	.76042	1.3151	1.2563	.20400	.79600	45
16	. 0552	. 9447	.6514	. 6088	.3143	.2565	. 0417	. 9582	44
17	. 0576	. 9424	.6508	. 6134	.3135	.2568	. 0435	. 9565	43
18	. 0599	. 9401	.6502	. 6179	.3127	.2571	. 0453	. 9547	42
19	. 0622	. 9378	.6496	. 6225	.3119	.2574	. 0470	. 9530	41
20	.60645	.39355	1.6489	.76271	1.3111	1.2577	.20488	.79512	40
21	. 0668	. 9332	.6483	. 6317	.3103	.2579	. 0505	. 9494	39
22	. 0691	. 9309	.6477	. 6364	.3095	.2582	. 0523	. 9477	38
23	. 0714	. 9285	.6470	. 6410	.3087	.2585	. 0541	. 9459	37
24	. 0737	. 9262	.6464	. 6456	.3079	.2588	. 0558	. 9441	36
25	.60761	.39239	1.6458	.76502	1.3071	1.2591	.20576	.79424	35
26	. 0784	. 9216	.6452	. 6548	.3064	.2593	. 0594	. 9406	34
27	. 0807	. 9193	.6445	. 6594	.3056	.2596	. 0611	. 9388	33
28	. 0830	. 9170	.6439	. 6640	.3048	.2599	. 0629	. 9371	32
29	. 0853	. 9147	.6433	. 6686	.3040	.2602	. 0647	. 9353	31
30	.60876	.39124	1.6427	.76733	1.3032	1.2605	.20665	.79335	30
31	. 0899	. 9101	.6420	. 6779	.3024	.2607	. 0682	. 9318	29
32	. 0922	. 9078	.6414	. 6825	.3016	.2610	. 0700	. 9300	28
33	. 0945	. 9055	.6408	. 6871	.3009	.2613	. 0718	. 9282	27
34	. 0963	. 9031	.6402	. 6918	.3001	.2616	. 0735	. 9264	26
35	.60991	.39008	1.6396	.76964	1.2993	1.2619	.20753	.79247	25
36	. 1014	. 8985	.6389	. 7010	.2985	.2622	. 0771	. 9229	24
37	. 1037	. 8962	.6383	. 7057	.2977	.2624	. 0789	. 9211	23
38	. 1061	. 8939	.6377	. 7103	.2970	.2627	. 0806	. 9193	22
39	. 1084	. 8916	.6371	. 7149	.2962	.2630	. 0824	. 9176	21
40	.61107	.38893	1.6365	.77196	1.2954	1.2633	.20842	.79158	20
41	. 1130	. 8870	.6359	. 7242	.2946	.2636	. 0860	. 9140	19
42	. 1153	. 8847	.6352	. 7289	.2938	.2639	. 0878	. 9122	18
43	. 1176	. 8824	.6346	. 7335	.2931	.2641	. 0895	. 9104	17
44	. 1199	. 8801	.6340	. 7382	.2923	.2644	. 0913	. 9087	16
45	.61222	.38778	1.6334	.77428	1.2915	1.2647	.20931	.79069	15
46	. 1245	. 8755	.6328	. 7475	.2907	.2650	. 0949	. 9051	14
47	. 1268	. 8732	.6322	. 7521	.2900	.2653	. 0967	. 9033	13
48	. 1290	. 8709	.6316	. 7568	.2892	.2656	. 0984	. 9015	12
49	. 1314	. 8686	.6309	. 7614	.2884	.2659	. 1002	. 8998	11
50	.61337	.38663	1.6303	.77661	1.2876	1.2661	.21020	.78980	10
51	. 1360	. 8640	.6297	. 7708	.2869	.2664	. 1038	. 8962	9
52	. 1383	. 8617	.6291	. 7754	.2861	.2667	. 1056	. 8944	8
53	. 1405	. 8594	.6285	. 7801	.2853	.2670	. 1074	. 8926	7
54	. 1428	. 8571	.6279	. 7848	.2845	.2673	. 1091	. 8908	6
55	.61451	.38548	1.6273	.77895	1.2838	1.2676	.21109	.78890	5
56	. 1474	. 8525	.6267	. 7941	.2830	.2679	. 1127	. 8873	4
57	. 1497	. 8503	.6261	. 7988	.2822	.2681	. 1145	. 8855	3
58	. 1520	. 8480	.6255	. 8035	.2815	.2684	. 1163	. 8837	2
59	. 1543	. 8457	.6249	. 8082	.2807	.2687	. 1181	. 8819	1
60	. 1566	. 8434	.6243	. 8128	.2799	.2690	. 1199	. 8801	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

127° 52°

38° **Natural Trigonometrical Functions.** 141°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.61566	.38434	1.6243	.78128	1.2799	1.2690	.21199	.78801	60
1	. 1589	. 8411	.6237	. 8175	.2792	.2693	. 1217	. 8783	59
2	. 1612	. 8388	.6231	. 8222	.2784	.2696	. 1235	. 8765	58
3	. 1635	. 8365	.6224	. 8269	.2776	.2699	. 1253	. 8747	57
4	. 1658	. 8342	.6218	. 8316	.2769	.2702	. 1271	. 8729	56
5	.61681	.38319	1.6212	.78363	1.2761	1.2705	.21288	.78711	55
6	. 1703	. 8296	.6206	. 8410	.2753	.2707	. 1306	. 8693	54
7	. 1726	. 8273	.6200	. 8457	.2746	.2710	. 1324	. 8675	53
8	. 1749	. 8251	.6194	. 8504	.2738	.2713	. 1342	. 8657	52
9	. 1772	. 8228	.6188	. 8551	.2730	.2716	. 1360	. 8640	51
10	.61795	.38205	1.6182	.78598	1.2723	1.2719	.21378	.78622	50
11	. 1818	. 8182	.6176	. 8645	.2715	.2722	. 1396	. 8604	49
12	. 1841	. 8159	.6170	. 8692	.2708	.2725	. 1414	. 8586	48
13	. 1864	. 8136	.6164	. 8739	.2700	.2728	. 1432	. 8568	47
14	. 1886	. 8113	.6159	. 8786	.2692	.2731	. 1450	. 8550	46
15	.61909	.38091	1.6153	.78834	1.2685	1.2734	.21468	.78532	45
16	. 1932	. 8068	.6147	. 8881	.2677	.2737	. 1486	. 8514	44
17	. 1955	. 8045	.6141	. 8928	.2670	.2739	. 1504	. 8496	43
18	. 1978	. 8022	.6135	. 8975	.2662	.2742	. 1522	. 8478	42
19	. 2001	. 7999	.6129	. 9022	.2655	.2745	. 1540	. 8460	41
20	.62023	.37976	1.6123	.79070	1.2647	1.2748	.21558	.78441	40
21	. 2046	. 7954	.6117	. 9117	.2639	.2751	. 1576	. 8423	39
22	. 2069	. 7931	.6111	. 9164	.2632	.2754	. 1594	. 8405	38
23	. 2092	. 7908	.6105	. 9212	.2624	.2757	. 1612	. 8387	37
24	. 2115	. 7885	.6099	. 9259	.2617	.2760	. 1631	. 8369	36
25	.62137	.37862	1.6093	.79306	1.2609	1.2763	.21649	.78351	35
26	. 2160	. 7840	.6087	. 9354	.2602	.2766	. 1667	. 8333	34
27	. 2183	. 7817	.6081	. 9401	.2594	.2769	. 1685	. 8315	33
28	. 2206	. 7794	.6077	. 9449	.2587	.2772	. 1703	. 8297	32
29	. 2229	. 7771	.6070	. 9496	.2579	.2775	. 1721	. 8279	31
30	.62251	.37748	1.6064	.79543	1.2572	1.2778	.21739	.78261	30
31	. 2274	. 7726	.6058	. 9591	.2564	.2781	. 1757	. 8243	29
32	. 2297	. 7703	.6052	. 9639	.2557	.2784	. 1775	. 8224	28
33	. 2320	. 7680	.6046	. 9686	.2549	.2787	. 1793	. 8206	27
34	. 2342	. 7657	.6040	. 9734	.2542	.2790	. 1812	. 8188	26
35	.62365	.37635	1.6034	.79781	1.2534	1.2793	.21830	.78170	25
36	. 2388	. 7612	.6029	. 9829	.2527	.2795	. 1848	. 8152	24
37	. 2411	. 7589	.6023	. 9876	.2519	.2798	. 1866	. 8134	23
38	. 2433	. 7566	.6017	. 9924	.2512	.2801	. 1884	. 8116	22
39	. 2456	. 7544	.6011	. 9972	.2504	.2804	. 1902	. 8097	21
40	.62479	.37521	1.6005	.80020	1.2497	1.2807	.21921	.78079	20
41	. 2501	. 7498	.6000	. 0067	.2489	.2810	. 1939	. 8061	19
42	. 2524	. 7476	.5994	. 0115	.2482	.2813	. 1957	. 8043	18
43	. 2547	. 7453	.5988	. 0163	.2475	.2816	. 1975	. 8025	17
44	. 2570	. 7430	.5982	. 0211	.2467	.2819	. 1993	. 8007	16
45	.62592	.37408	1.5976	.80258	1.2460	1.2822	.22011	.77988	15
46	. 2615	. 7385	.5971	. 0306	.2452	.2825	. 2030	. 7970	14
47	. 2638	. 7362	.5965	. 0354	.2445	.2828	. 2048	. 7952	13
48	. 2660	. 7340	.5959	. 0402	.2437	.2831	. 2066	. 7934	12
49	. 2683	. 7317	.5953	. 0450	.2430	.2834	. 2084	. 7915	11
50	.62706	.37294	1.5947	.80498	1.2423	1.2837	.22103	.77897	10
51	. 2728	. 7272	.5942	. 0546	.2415	.2840	. 2121	. 7879	9
52	. 2751	. 7249	.5936	. 0594	.2408	.2843	. 2139	. 7861	8
53	. 2774	. 7226	.5930	. 0642	.2400	.2846	. 2157	. 7842	7
54	. 2796	. 7204	.5924	. 0690	.2393	.2849	. 2176	. 7824	6
55	.62819	.37181	1.5919	.80738	1.2386	1.2852	.22194	.77806	5
56	. 2841	. 7158	.5913	. 0786	.2378	.2855	. 2212	. 7788	4
57	. 2864	. 7136	.5907	. 0834	.2371	.2858	. 2230	. 7769	3
58	. 2887	. 7113	.5901	. 0882	.2364	.2861	. 2249	. 7751	2
59	. 2909	. 7090	.5896	. 0930	.2356	.2864	. 2267	. 7733	1
60	. 2932	. 7068	.5890	. 0978	.2349	.2867	. 2285	. 7715	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

128° 51°

39° **Natural Trigonometrical Functions.** **140°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.62932	.37068	1.5890	.80978	1.2349	1.2867	.22285	.77715	60
1	. 2955	. 7045	.5884	. 1026	.2342	.2871	. 2304	. 7696	59
2	. 2977	. 7023	.5879	. 1075	.2334	.2874	. 2322	. 7678	58
3	. 3000	. 7000	.5873	. 1123	.2327	.2877	. 2340	. 7660	57
4	. 3022	. 6977	.5867	. 1171	.2320	.2880	. 2359	. 7641	56
5	.63045	.36955	1.5862	.81219	1.2312	1.2883	.22377	.77623	55
6	. 3067	. 6932	.5856	. 1268	.2305	.2886	. 2395	. 7605	54
7	. 3090	. 6910	.5850	. 1316	.2297	.2889	. 2414	. 7586	53
8	. 3113	. 6887	.5845	. 1364	.2290	.2892	. 2432	. 7568	52
9	. 3135	. 6865	.5839	. 1413	.2283	.2895	. 2450	. 7549	51
10	.63158	.36842	1.5833	.81461	1.2276	1.2898	.22469	.77531	50
11	. 3180	. 6820	.5828	. 1509	.2268	.2901	. 2487	. 7513	49
12	. 3203	. 6797	.5822	. 1558	.2261	.2904	. 2505	. 7494	48
13	. 3225	. 6774	.5816	. 1606	.2254	.2907	. 2524	. 7476	47
14	. 3248	. 6752	.5811	. 1655	.2247	.2910	. 2542	. 7458	46
15	.63270	.36729	1.5805	.81703	1.2239	1.2913	.22561	.77439	45
16	. 3293	. 6707	.5799	. 1752	.2232	.2916	. 2579	. 7421	44
17	. 3315	. 6684	.5794	. 1800	.2225	.2919	. 2597	. 7402	43
18	. 3338	. 6662	.5788	. 1849	.2218	.2922	. 2616	. 7384	42
19	. 3360	. 6639	.5783	. 1898	.2210	.2926	. 2634	. 7365	41
20	.63383	.36617	1.5777	.81946	1.2203	1.2929	.22653	.77347	40
21	. 3405	. 6594	.5771	. 1995	.2196	.2932	. 2671	. 7329	39
22	. 3428	. 6572	.5766	. 2043	.2189	.2935	. 2690	. 7310	38
23	. 3450	. 6549	.5760	. 2092	.2181	.2938	. 2708	. 7292	37
24	. 3473	. 6527	.5755	. 2141	.2174	.2941	. 2727	. 7273	36
25	.63495	.36504	1.5749	.82190	1.2167	1.2944	.22745	.77255	35
26	. 3518	. 6482	.5743	. 2238	.2160	.2947	. 2763	. 7236	34
27	. 3540	. 6459	.5738	. 2287	.2152	.2950	. 2782	. 7218	33
28	. 3563	. 6437	.5732	. 2336	.2145	.2953	. 2800	. 7199	32
29	. 3585	. 6415	.5727	. 2385	.2138	.2956	. 2819	. 7181	31
30	.63608	.36392	1.5721	.82434	1.2131	1.2960	.22837	.77162	30
31	. 3630	. 6370	.5716	. 2482	.2124	.2963	. 2856	. 7144	29
32	. 3653	. 6347	.5710	. 2531	.2117	.2966	. 2874	. 7125	28
33	. 3675	. 6325	.5705	. 2580	.2109	.2969	. 2893	. 7107	27
34	. 3697	. 6302	.5699	. 2629	.2102	.2972	. 2912	. 7088	26
35	.63720	.36280	1.5694	.82678	1.2095	1.2975	.22930	.77070	25
36	. 3742	. 6258	.5688	. 2727	.2088	.2978	. 2949	. 7051	24
37	. 3765	. 6235	.5683	. 2776	.2081	.2981	. 2967	. 7033	23
38	. 3787	. 6213	.5677	. 2825	.2074	.2985	. 2986	. 7014	22
39	. 3810	. 6190	.5672	. 2874	.2066	.2988	. 3004	. 6996	21
40	.63832	.36168	1.5666	.82923	1.2059	1.2991	.23023	.76977	20
41	. 3854	. 6146	.5661	. 2972	.2052	.2994	. 3041	. 6958	19
42	. 3877	. 6123	.5655	. 3022	.2045	.2997	. 3060	. 6940	18
43	. 3899	. 6101	.5650	. 3071	.2038	.3000	. 3079	. 6921	17
44	. 3921	. 6078	.5644	. 3120	.2031	.3003	. 3097	. 6903	16
45	.63944	.36056	1.5639	.83169	1.2024	1.3006	.23116	.76884	15
46	. 3966	. 6034	.5633	. 3218	.2016	.3010	. 3134	. 6865	14
47	. 3989	. 6011	.5628	. 3267	.2009	.3013	. 3153	. 6847	13
48	. 4011	. 5989	.5622	. 3317	.2002	.3016	. 3172	. 6828	12
49	. 4033	. 5967	.5617	. 3366	.1995	.3019	. 3190	. 6810	11
50	.64056	.35944	1.5611	.83415	1.1988	1.3022	.23209	.76791	10
51	. 4078	. 5922	.5606	. 3465	.1981	.3025	. 3227	. 6772	9
52	. 4100	. 5900	.5600	. 3514	.1974	.3029	. 3246	. 6754	8
53	. 4123	. 5877	.5595	. 3563	.1967	.3032	. 3265	. 6735	7
54	. 4145	. 5855	.5590	. 3613	.1960	.3035	. 3283	. 6716	6
55	.64167	.35833	1.5584	.83662	1.1953	1.3038	.23302	.76698	5
56	. 4189	. 5810	.5579	. 3712	.1946	.3041	. 3321	. 6679	4
57	. 4212	. 5788	.5573	. 3761	.1939	.3044	. 3339	. 6660	3
58	. 4234	. 5766	.5568	. 3811	.1932	.3048	. 3358	. 6642	2
59	. 4256	. 5743	.5563	. 3860	.1924	.3051	. 3377	. 6623	1
60	. 4279	. 5721	.5557	. 3910	.1917	.3054	. 3395	. 6604	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

129° **50°**

40°				Natural Trigonometrical Functions.					139°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.64279	.35721	1.5557	.83910	1.1917	1.3054	.23395	.76604	60
1	. 4301	. 5699	.5552	. 3959	.1910	.3057	. 3414	. 6586	59
2	. 4323	. 5677	.5546	. 4009	.1903	.3060	. 3433	. 6567	58
3	. 4345	. 5654	.5541	. 4059	.1896	.3064	. 3452	. 6548	57
4	. 4368	. 5632	.5536	. 4108	.1889	.3067	. 3470	. 6530	56
5	.64390	.35610	1.5530	.84158	1.1882	1.3070	.23489	.76511	55
6	. 4412	. 5588	.5525	. 4208	.1875	.3073	. 3508	. 6492	54
7	. 4435	. 5565	.5520	. 4257	.1868	.3076	. 3527	. 6473	53
8	. 4457	. 5543	.5514	. 4307	.1861	.3080	. 3545	. 6455	52
9	. 4479	. 5521	.5509	. 4357	.1854	.3083	. 3564	. 6436	51
10	.64501	.35499	1.5503	.84407	1.1847	1.3086	.23583	.76417	50
11	. 4523	. 5476	.5498	. 4457	.1840	.3089	. 3602	. 6398	49
12	. 4546	. 5454	.5493	. 4506	.1833	.3092	. 3620	. 6380	48
13	. 4568	. 5432	.5487	. 4556	.1826	.3096	. 3639	. 6361	47
14	. 4590	. 5410	.5482	. 4606	.1819	.3099	. 3658	. 6342	46
15	.64612	.35388	1.5477	.84656	1.1812	1.3102	.23677	.76323	45
16	. 4635	. 5365	.5471	. 4706	.1805	.3105	. 3695	. 6304	44
17	. 4657	. 5343	.5466	. 4756	.1798	.3109	. 3714	. 6286	43
18	. 4679	. 5321	.5461	. 4806	.1791	.3112	. 3733	. 6267	42
19	. 4701	. 5299	.5456	. 4856	.1785	.3115	. 3752	. 6248	41
20	.64723	.35277	1.5450	.84906	1.1778	1.3118	.23771	.76229	40
21	. 4745	. 5254	.5445	. 4956	.1771	.3121	. 3790	. 6210	39
22	. 4768	. 5232	.5440	. 5006	.1764	.3125	. 3808	. 6191	38
23	. 4790	. 5210	.5434	. 5056	.1757	.3128	. 3827	. 6173	37
24	. 4812	. 5188	.5429	. 5107	.1750	.3131	. 3846	. 6154	36
25	.64834	.35166	1.5424	.85157	1.1743	1.3134	.23865	.76135	35
26	. 4856	. 5144	.5419	. 5207	.1736	.3138	. 3884	. 6116	34
27	. 4878	. 5121	.5413	. 5257	.1729	.3141	. 3903	. 6097	33
28	. 4900	. 5099	.5408	. 5307	.1722	.3144	. 3922	. 6078	32
29	. 4923	. 5077	.5403	. 5358	.1715	.3148	. 3940	. 6059	31
30	.64945	.35055	1.5398	.85408	1.1708	1.3151	.23959	.76041	30
31	. 4967	. 5033	.5392	. 5458	.1702	.3154	. 3978	. 6022	29
32	. 4989	. 5011	.5387	. 5509	.1695	.3157	. 3997	. 6003	28
33	. 5011	. 4989	.5382	. 5559	.1688	.3161	. 4016	. 5984	27
34	. 5033	. 4967	.5377	. 5609	.1681	.3164	. 4035	. 5965	26
35	.65055	.34945	1.5371	.85660	1.1674	1.3167	.24054	.75946	25
36	. 5077	. 4922	.5366	. 5710	.1667	.3170	. 4073	. 5927	24
37	. 5099	. 4900	.5361	. 5761	.1660	.3174	. 4092	. 5908	23
38	. 5121	. 4878	.5356	. 5811	.1653	.3177	. 4111	. 5889	22
39	. 5144	. 4856	.5351	. 5862	.1647	.3180	. 4130	. 5870	21
40	.65166	.34834	1.5345	.85912	1.1640	1.3184	.24149	.75851	20
41	. 5188	. 4812	.5340	. 5963	.1633	.3187	. 4168	. 5832	19
42	. 5210	. 4790	.5335	. 6013	.1626	.3190	. 4186	. 5813	18
43	. 5232	. 4768	.5330	. 6064	.1619	.3193	. 4205	. 5794	17
44	. 5254	. 4746	.5325	. 6115	.1612	.3197	. 4224	. 5775	16
45	.65276	.34724	1.5319	.86165	1.1605	1.3200	.24243	.75756	15
46	. 5298	. 4702	.5314	. 6216	.1599	.3203	. 4262	. 5737	14
47	. 5320	. 4680	.5309	. 6267	.1592	.3207	. 4281	. 5718	13
48	. 5342	. 4658	.5304	. 6318	.1585	.3210	. 4300	. 5699	12
49	. 5364	. 4636	.5299	. 6368	.1578	.3213	. 4319	. 5680	11
50	.65386	.34614	1.5294	.86419	1.1571	1.3217	.24338	.75661	10
51	. 5408	. 4592	.5289	. 6470	.1565	.3220	. 4357	. 5642	9
52	. 5430	. 4570	.5283	. 6521	.1558	.3223	. 4376	. 5623	8
53	. 5452	. 4548	.5278	. 6572	.1551	.3227	. 4396	. 5604	7
54	. 5474	. 4526	.5273	. 6623	.1544	.3230	. 4415	. 5585	6
55	.65496	.34504	1.5268	.86674	1.1537	1.3233	.24434	.75566	5
56	. 5518	. 4482	.5263	. 6725	.1531	.3237	. 4453	. 5547	4
57	. 5540	. 4460	.5258	. 6775	.1524	.3240	. 4472	. 5528	3
58	. 5562	. 4438	.5253	. 6826	.1517	.3243	. 4491	. 5509	2
59	. 5584	. 4416	.5248	. 6878	.1510	.3247	. 4510	. 5490	1
60	. 5606	. 4394	.5242	. 6929	.1504	.3250	. 4529	. 5471	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
130°									49°

41° **Natural Trigonometrical Functions.** 138°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.65606	.34394	1.5242	.86929	1.1504	1.3250	.24529	.75471	60
1	. 5628	. 4372	.5237	. 6980	.1497	.3253	. 4548	. 5452	59
2	. 5650	. 4350	.5232	. 7031	.1490	.3257	. 4567	. 5433	58
3	. 5672	. 4328	.5227	. 7082	.1483	.3260	. 4586	. 5414	57
4	. 5694	. 4306	.5222	. 7133	.1477	.3263	. 4605	. 5394	56
5	.65716	.34284	1.5217	.87184	1.1470	1.3267	.24624	.75375	55
6	. 5737	. 4262	.5212	. 7235	.1463	.3270	. 4644	. 5356	54
7	. 5759	. 4240	.5207	. 7287	.1456	.3274	. 4663	. 5337	53
8	. 5781	. 4219	.5202	. 7338	.1450	.3277	. 4682	. 5318	52
9	. 5803	. 4197	.5197	. 7389	.1443	.3280	. 4701	. 5299	51
10	.65825	.34175	1.5192	.87441	1.1436	1.3284	.24720	.75280	50
11	. 5847	. 4153	.5187	. 7492	.1430	.3287	. 4739	. 5261	49
12	. 5869	. 4131	.5182	. 7543	.1423	.3290	. 4758	. 5241	48
13	. 5891	. 4109	.5177	. 7595	.1416	.3294	. 4778	. 5222	47
14	. 5913	. 4087	.5171	. 7646	.1409	.3297	. 4797	. 5203	46
15	.65934	.34065	1.5166	.87698	1.1403	1.3301	.24816	.75184	45
16	. 5956	. 4043	.5161	. 7749	.1396	.3304	. 4835	. 5165	44
17	. 5978	. 4022	.5156	. 7801	.1389	.3307	. 4854	. 5146	43
18	. 6000	. 4000	.5151	. 7852	.1383	.3311	. 4873	. 5126	42
19	. 6022	. 3978	.5146	. 7904	.1376	.3314	. 4893	. 5107	41
20	.66044	.33956	1.5141	.87955	1.1369	1.3318	.24912	.75088	40
21	. 6066	. 3934	.5136	. 8007	.1363	.3321	. 4931	. 5069	39
22	. 6087	. 3912	.5131	. 8058	.1356	.3324	. 4950	. 5049	38
23	. 6109	. 3891	.5126	. 8110	.1349	.3328	. 4970	. 5030	37
24	. 6131	. 3869	.5121	. 8162	.1343	.3331	. 4989	. 5011	36
25	.66153	.33847	1.5116	.88213	1.1336	1.3335	.25008	.74992	35
26	. 6175	. 3825	.5111	. 8265	.1329	.3338	. 5027	. 4973	34
27	. 6197	. 3803	.5106	. 8317	.1323	.3342	. 5047	. 4953	33
28	. 6218	. 3781	.5101	. 8369	.1316	.3345	. 5066	. 4934	32
29	. 6240	. 3760	.5096	. 8421	.1309	.3348	. 5085	. 4915	31
30	.66262	.33738	1.5092	.88472	1.1303	1.3352	.25104	.74895	30
31	. 6284	. 3716	.5087	. 8524	.1296	.3355	. 5124	. 4876	29
32	. 6305	. 3694	.5082	. 8576	.1290	.3359	. 5143	. 4857	28
33	. 6327	. 3673	.5077	. 8628	.1283	.3362	. 5162	. 4838	27
34	. 6349	. 3651	.5072	. 8680	.1276	.3366	. 5181	. 4818	26
35	.66371	.33629	1.5067	.88732	1.1270	1.3369	.25201	.74799	25
36	. 6393	. 3607	.5062	. 8784	.1263	.3372	. 5220	. 4780	24
37	. 6414	. 3586	.5057	. 8836	.1257	.3376	. 5239	. 4760	23
38	. 6436	. 3564	.5052	. 8888	.1250	.3379	. 5259	. 4741	22
39	. 6458	. 3542	.5047	. 8940	.1243	.3383	. 5278	. 4722	21
40	.66479	.33520	1.5042	.88992	1.1237	1.3386	.25297	.74702	20
41	. 6501	. 3499	.5037	. 9044	.1230	.3390	. 5317	. 4683	19
42	. 6523	. 3477	.5032	. 9097	.1224	.3393	. 5336	. 4664	18
43	. 6545	. 3455	.5027	. 9149	.1217	.3397	. 5355	. 4644	17
44	. 6566	. 3433	.5022	. 9201	.1211	.3400	. 5375	. 4625	16
45	.66588	.33412	1.5018	.89253	1.1204	1.3404	.25394	.74606	15
46	. 6610	. 3390	.5013	. 9306	.1197	.3407	. 5414	. 4586	14
47	. 6631	. 3368	.5008	. 9358	.1191	.3411	. 5433	. 4567	13
48	. 6653	. 3347	.5003	. 9410	.1184	.3414	. 5452	. 4548	12
49	. 6675	. 3325	.4998	. 9463	.1178	.3418	. 5472	. 4528	11
50	.66697	.33303	1.4993	.89515	1.1171	1.3421	.25491	.74509	10
51	. 6718	. 3282	.4988	. 9567	.1165	.3425	. 5510	. 4489	9
52	. 6740	. 3260	.4983	. 9620	.1158	.3428	. 5530	. 4470	8
53	. 6762	. 3238	.4979	. 9672	.1152	.3432	. 5549	. 4450	7
54	. 6783	. 3217	.4974	. 9725	.1145	.3435	. 5569	. 4431	6
55	.66805	.33195	1.4969	.89777	1.1139	1.3439	.25588	.74412	5
56	. 6826	. 3173	.4964	. 9830	.1132	.3442	. 5608	. 4392	4
57	. 6848	. 3152	.4959	. 9882	.1126	.3446	. 5627	. 4373	3
58	. 6870	. 3130	.4954	. 9935	.1119	.3449	. 5647	. 4353	2
59	. 6891	. 3108	.4949	. 9988	.1113	.3453	. 5666	. 4334	1
60	. 6913	. 3087	.4945	.90040	.1106	.3456	. 5685	. 4314	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

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42° **Natural Trigonometrical Functions.** **137°**

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.66913	.33087	1.4945	.90040	1.1106	1.3456	.25685	.74314	60
1	. 6935	. 3065	.4940	. 0093	.1100	.3460	. 5705	. 4295	59
2	. 6956	. 3044	.4935	. 0146	.1093	.3463	. 5724	. 4275	58
3	. 6978	. 3022	.4930	. 0198	.1086	.3467	. 5744	. 4256	57
4	. 6999	. 3000	.4925	. 0251	.1080	.3470	. 5763	. 4236	56
5	.67021	.32979	1.4921	.90304	1.1074	1.3474	.25783	.74217	55
6	. 7043	. 2957	.4916	. 0357	.1067	.3477	. 5802	. 4197	54
7	. 7064	. 2936	.4911	. 0410	.1061	.3481	. 5822	. 4178	53
8	. 7086	. 2914	.4906	. 0463	.1054	.3485	. 5841	. 4158	52
9	. 7107	. 2893	.4901	. 0515	.1048	.3488	. 5861	. 4139	51
10	.67129	.32871	1.4897	.90568	1.1041	1.3492	.25880	.74119	50
11	. 7150	. 2849	.4892	. 0621	.1035	.3495	. 5900	. 4100	49
12	. 7172	. 2828	.4887	. 0674	.1028	.3499	. 5919	. 4080	48
13	. 7194	. 2806	.4882	. 0727	.1022	.3502	. 5939	. 4061	47
14	. 7215	. 2785	.4877	. 0780	.1015	.3506	. 5959	. 4041	46
15	.67237	.32763	1.4873	.90834	1.1009	1.3509	.25978	.74022	45
16	. 7258	. 2742	.4868	. 0887	.1003	.3513	. 5998	. 4002	44
17	. 7280	. 2720	.4863	. 0940	.0996	.3517	. 6017	. 3983	43
18	. 7301	. 2699	.4858	. 0993	.0990	.3520	. 6037	. 3963	42
19	. 7323	. 2677	.4854	. 1046	.0983	.3524	. 6056	. 3943	41
20	.67344	.32656	1.4849	.91099	1.0977	1.3527	.26076	.73924	40
21	. 7366	. 2634	.4844	. 1153	.0971	.3531	. 6096	. 3904	39
22	. 7387	. 2613	.4839	. 1206	.0964	.3534	. 6115	. 3885	38
23	. 7409	. 2591	.4835	. 1259	.0958	.3538	. 6135	. 3865	37
24	. 7430	. 2570	.4830	. 1312	.0951	.3542	. 6154	. 3845	36
25	.67452	.32548	1.4825	.91366	1.0945	1.3545	.26174	.73826	35
26	. 7473	. 2527	.4821	. 1419	.0939	.3549	. 6194	. 3806	34
27	. 7495	. 2505	.4816	. 1473	.0932	.3552	. 6213	. 3787	33
28	. 7516	. 2484	.4811	. 1526	.0926	.3556	. 6233	. 3767	32
29	. 7537	. 2462	.4806	. 1580	.0919	.3560	. 6253	. 3747	31
30	.67559	.32441	1.4802	.91633	1.0913	1.3563	.26272	.73728	30
31	. 7580	. 2419	.4797	. 1687	.0907	.3567	. 6292	. 3708	29
32	. 7602	. 2398	.4792	. 1740	.0900	.3571	. 6311	. 3688	28
33	. 7623	. 2377	.4788	. 1794	.0894	.3574	. 6331	. 3669	27
34	. 7645	. 2355	.4783	. 1847	.0888	.3578	. 6351	. 3649	26
35	.67666	.32334	1.4778	.91901	1.0881	1.3581	.26371	.73629	25
36	. 7688	. 2312	.4774	. 1955	.0875	.3585	. 6390	. 3610	24
37	. 7709	. 2291	.4769	. 2008	.0868	.3589	. 6410	. 3590	23
38	. 7730	. 2269	.4764	. 2062	.0862	.3592	. 6430	. 3570	22
39	. 7752	. 2248	.4760	. 2116	.0856	.3596	. 6449	. 3551	21
40	.67773	.32227	1.4755	.92170	1.0849	1.3600	.26469	.73531	20
41	. 7794	. 2205	.4750	. 2223	.0843	.3603	. 6489	. 3511	19
42	. 7816	. 2184	.4746	. 2277	.0837	.3607	. 6508	. 3491	18
43	. 7837	. 2163	.4741	. 2331	.0830	.3611	. 6528	. 3472	17
44	. 7859	. 2141	.4736	. 2385	.0824	.3614	. 6548	. 3452	16
45	.67880	.32120	1.4732	.92439	1.0818	1.3618	.26568	.73432	15
46	. 7901	. 2098	.4727	. 2493	.0812	.3622	. 6587	. 3412	14
47	. 7923	. 2077	.4723	. 2547	.0805	.3625	. 6607	. 3393	13
48	. 7944	. 2056	.4718	. 2601	.0799	.3629	. 6627	. 3373	12
49	. 7965	. 2034	.4713	. 2655	.0793	.3633	. 6647	. 3353	11
50	.67987	.32013	1.4709	.92709	1.0786	1.3636	.26666	.73333	10
51	. 8008	. 1992	.4704	. 2763	.0780	.3640	. 6686	. 3314	9
52	. 8029	. 1970	.4699	. 2817	.0774	.3644	. 6706	. 3294	8
53	. 8051	. 1949	.4695	. 2871	.0767	.3647	. 6726	. 3274	7
54	. 8072	. 1928	.4690	. 2926	.0761	.3651	. 6746	. 3254	6
55	.68093	.31907	1.4686	.92980	1.0755	1.3655	.26765	.73234	5
56	. 8115	. 1885	.4681	. 3034	.0749	.3658	. 6785	. 3215	4
57	. 8136	. 1864	.4676	. 3088	.0742	.3662	. 6805	. 3195	3
58	. 8157	. 1843	.4672	. 3143	.0736	.3666	. 6825	. 3175	2
59	. 8178	. 1821	.4667	3197	.0730	.3669	. 6845	. 3155	1
60	. 8200	. 1800	.4663	. 3251	.0724	.3673	. 6865	. 3135	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

132° **47°**

43° **Natural Trigonometrical Functions.** 136°

M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.68200	.31800	1.4663	.93251	1.0724	1.3673	.26865	.73135	60
1	. 8221	. 1779	.4658	. 3306	.0717	.3677	. 6884	. 3115	59
2	. 8242	. 1758	.4654	. 3360	.0711	.3681	. 6904	. 3096	58
3	. 8264	. 1736	.4649	. 3415	.0705	.3684	. 6924	. 3076	57
4	. 8285	. 1715	.4644	. 3469	.0699	.3688	. 6944	. 3056	56
5	.68306	.31694	1.4640	.93524	1.0692	1.3692	.26964	.73036	55
6	. 8327	. 1673	.4635	. 3578	.0686	.3695	. 6984	. 3016	54
7	. 8349	. 1651	.4631	. 3633	.0680	.3699	. 7004	. 2996	53
8	. 8370	. 1630	.4626	. 3687	.0674	.3703	. 7023	. 2976	52
9	. 8391	. 1609	.4622	. 3742	.0667	.3707	. 7043	. 2956	51
10	.68412	.31588	1.4617	.93797	1.0661	1.3710	.27063	.72937	50
11	. 8433	. 1566	.4613	. 3851	.0655	.3714	. 7083	. 2917	49
12	. 8455	. 1545	.4608	. 3906	.0649	.3718	. 7103	. 2897	48
13	. 8476	. 1524	.4604	. 3961	.0643	.3722	. 7123	. 2877	47
14	. 8497	. 1503	.4599	. 4016	.0636	.3725	. 7143	. 2857	46
15	.68518	.31482	1.4595	.94071	1.0630	1.3729	.27163	.72837	45
16	. 8539	. 1460	.4590	. 4125	.0624	.3733	. 7183	. 2817	44
17	. 8561	. 1439	.4586	. 4180	.0618	.3737	. 7203	. 2797	43
18	. 8582	. 1418	.4581	. 4235	.0612	.3740	. 7223	. 2777	42
19	. 8603	. 1397	.4577	. 4290	.0605	.3744	. 7243	. 2757	41
20	.68624	.31376	1.4572	.94345	1.0599	1.3748	.27263	.72737	40
21	. 8645	. 1355	.4568	. 4400	.0593	.3752	. 7283	. 2717	39
22	. 8666	. 1333	.4563	. 4455	.0587	.3756	. 7302	. 2697	38
23	. 8688	. 1312	.4559	. 4510	.0581	.3759	. 7322	. 2677	37
24	. 8709	. 1291	.4554	. 4565	.0575	.3763	. 7342	. 2657	36
25	.68730	.31270	1.4550	.94620	1.0568	1.3767	.27362	.72637	35
26	. 8751	. 1249	.4545	. 4675	.0562	.3771	. 7382	. 2617	34
27	. 8772	. 1228	.4541	. 4731	.0556	.3774	. 7402	. 2597	33
28	. 8793	. 1207	.4536	. 4786	.0550	.3778	. 7422	. 2577	32
29	. 8814	. 1186	.4532	. 4841	.0544	.3782	. 7442	. 2557	31
30	.68835	.31164	1.4527	.94896	1.0538	1.3786	.27462	.72537	30
31	. 8856	. 1143	.4523	. 4952	.0532	.3790	. 7482	. 2517	29
32	. 8878	. 1122	.4518	. 5007	.0525	.3794	. 7503	. 2497	28
33	. 8899	. 1101	.4514	. 5062	.0519	.3797	. 7523	. 2477	27
34	. 8920	. 1080	.4510	. 5118	.0513	.3801	. 7543	. 2457	26
35	.68941	.31059	1.4505	.95173	1.0507	1.3805	.27563	.72437	25
36	. 8962	. 1038	.4501	. 5229	.0501	.3809	. 7583	. 2417	24
37	. 8983	. 1017	.4496	. 5284	.0495	.3813	. 7603	. 2397	23
38	. 9004	. 0996	.4492	. 5340	.0489	.3816	. 7623	. 2377	22
39	. 9025	. 0975	.4487	. 5395	.0483	.3820	. 7643	. 2357	21
40	.69046	.30954	1.4483	.95451	1.0476	1.3824	.27663	.72337	20
41	. 9067	. 0933	.4479	. 5506	.0470	.3828	. 7683	. 2317	19
42	. 9088	. 0912	.4474	. 5562	.0464	.3832	. 7703	. 2297	18
43	. 9109	. 0891	.4470	. 5618	.0458	.3836	. 7723	. 2277	17
44	. 9130	. 0870	.4465	. 5673	.0452	.3839	. 7743	. 2256	16
45	.69151	.30849	1.4461	.95729	1.0446	1.3843	.27764	.72236	15
46	. 9172	. 0828	.4457	. 5785	.0440	.3847	. 7784	. 2216	14
47	. 9193	. 0807	.4452	. 5841	.0434	.3851	. 7804	. 2196	13
48	. 9214	. 0786	.4448	. 5896	.0428	.3855	. 7824	. 2176	12
49	. 9235	. 0765	.4443	. 5952	.0422	.3859	. 7844	. 2156	11
50	.69256	.30744	1.4439	.96008	1.0416	1.3863	.27864	.72136	10
51	. 9277	. 0723	.4435	. 6064	.0410	.3867	. 7884	. 2115	9
52	. 9298	. 0702	.4430	. 6120	.0404	.3870	. 7904	. 2095	8
53	. 9319	. 0681	.4426	. 6176	.0397	.3874	. 7925	. 2075	7
54	. 9340	. 0660	.4422	. 6232	.0391	.3878	. 7945	. 2055	6
55	.69361	.30639	1.4417	.96288	1.0385	1.3882	.27965	.72035	5
56	. 9382	. 0618	.4413	. 6344	.0379	.3886	. 7985	. 2015	4
57	. 9403	0597	.4408	. 6400	.0373	.3890	. 8005	. 1994	3
58	. 9424	. 0576	.4404	. 6456	.0367	.3894	. 8026	. 1974	2
59	. 9445	. 0555	.4400	. 6513	.0361	.3898	. 8046	. 1954	1
60	. 9466	. 0534	.4395	. 6569	.0355	.3902	. 8066	. 1934	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.

133° 46°

44°	Natural Trigonometrical Functions.								135°
M.	Sine.	Vrs. cos.	Cosec'nt	Tang.	Cotang.	Secant.	Vrs. sin.	Cosine.	M.
0	.69466	.30534	1.4395	.96569	1.0355	1.3902	.28066	.71934	60
1	. 9487	. 0513	.4391	. 6625	.0349	.3905	. 8086	. 1914	59
2	. 9508	. 0492	.4387	. 6681	.0343	.3909	. 8106	. 1893	58
3	. 9528	. 0471	.4382	. 6738	.0337	.3913	. 8127	. 1873	57
4	. 9549	. 0450	.4378	. 6794	.0331	.3917	. 8147	. 1853	56
5	.69570	.30430	1.4374	.96850	1.0325	1.3921	.28167	.71833	55
6	. 9591	. 0409	.4370	. 6907	.0319	.3925	. 8187	. 1813	54
7	. 9612	. 0388	.4365	. 6963	.0313	.3929	. 8208	. 1792	53
8	. 9633	. 0367	.4361	. 7020	.0307	.3933	. 8228	. 1772	52
9	. 9654	. 0346	.4357	. 7076	.0301	.3937	. 8248	. 1752	51
10	.69675	.30325	1.4352	.97133	1.0295	1.3941	.28268	.71732	50
11	. 9696	. 0304	.4348	. 7189	.0289	.3945	. 8289	. 1711	49
12	. 9716	. 0283	.4344	. 7246	.0283	.3949	. 8309	. 1691	48
13	. 9737	. 0263	.4339	. 7302	.0277	.3953	. 8329	. 1671	47
14	. 9758	. 0242	.4335	. 7359	.0271	.3957	. 8349	. 1650	46
15	.69779	.30221	1.4331	.97416	1.0265	1.3960	.28370	.71630	45
16	. 9800	. 0200	.4327	. 7472	.0259	.3964	. 8390	. 1610	44
17	. 9821	. 0179	.4322	. 7529	.0253	.3968	. 8410	. 1589	43
18	. 9841	. 0158	.4318	. 7586	.0247	.3972	. 8431	. 1569	42
19	. 9862	. 0138	.4314	. 7643	.0241	.3976	. 8451	. 1549	41
20	.69883	.30117	1.4310	.97699	1.0235	1.3980	.28471	.71529	40
21	. 9904	. 0096	.4305	. 7756	.0229	.3984	. 8492	. 1508	39
22	. 9925	. 0075	.4301	. 7813	.0223	.3988	. 8512	. 1488	38
23	. 9945	. 0054	.4297	. 7870	.0218	.3992	. 8532	. 1468	37
24	. 9966	. 0034	.4292	. 7927	.0212	.3996	. 8553	. 1447	36
25	.69987	.30013	1.4288	.97984	1.0206	1.4000	.28573	.71427	35
26	.70008	.29992	.4284	. 8041	.0200	.4004	. 8593	. 1406	34
27	. 0029	. 9971	.4280	. 8098	.0194	.4008	. 8614	. 1386	33
28	. 0049	. 9950	.4276	. 8155	.0188	.4012	. 8634	. 1366	32
29	. 0070	. 9930	.4271	. 8212	.0182	.4016	. 8654	. 1345	31
30	.70091	.29909	1.4267	.98270	1.0176	1.4020	.28675	.71325	30
31	. 0112	. 9888	.4263	. 8327	.0170	.4024	. 8695	. 1305	29
32	. 0132	. 9867	.4259	. 8384	.0164	.4028	. 8716	. 1284	28
33	. 0153	. 9847	.4254	. 8441	.0158	.4032	. 8736	. 1264	27
34	. 0174	. 9826	.4250	. 8499	.0152	.4036	. 8756	. 1243	26
35	.70194	.29805	1.4246	.98556	1.0146	1.4040	.28777	.71223	25
36	. 0215	. 9785	.4242	. 8613	.0141	.4044	. 8797	. 1203	24
37	. 0236	. 9764	.4238	. 8671	.0135	.4048	. 8818	. 1182	23
38	. 0257	. 9743	.4233	. 8728	.0129	.4052	. 8838	. 1162	22
39	. 0277	. 9722	.4229	. 8786	.0123	.4056	. 8859	. 1141	21
40	.70298	.29702	1.4225	.98843	1.0117	1.4060	.28879	.71121	20
41	. 0319	. 9681	.4221	. 8901	.0111	.4065	. 8899	. 1100	19
42	. 0339	. 9660	.4217	. 8958	.0105	.4069	. 8920	. 1080	18
43	. 0360	. 9640	.4212	. 9016	.0099	.4073	. 8940	. 1059	17
44	. 0381	. 9619	.4208	. 9073	.0093	.4077	. 8961	. 1039	16
45	.70401	.29598	1.4204	.99131	1.0088	1.4081	.28981	.71018	15
46	. 0422	. 9578	.4200	. 9189	.0082	.4085	. 9002	. 0998	14
47	. 0443	. 9557	.4196	. 9246	.0076	.4089	. 9022	. 0977	13
48	. 0463	. 9536	.4192	. 9304	.0070	.4093	. 9043	. 0957	12
49	. 0484	. 9516	.4188	. 9362	.0064	.4097	. 9063	. 0936	11
50	.70505	.29495	1.4183	.99420	1.0058	1.4101	.29084	.70916	10
51	. 0525	. 9475	.4179	. 9478	.0052	.4105	. 9104	. 0895	9
52	. 0546	. 9454	.4175	. 9536	.0047	.4109	. 9125	. 0875	8
53	. 0566	. 9433	.4171	. 9593	.0041	.4113	. 9145	. 0854	7
54	. 0587	. 9413	.4167	. 9651	.0035	.4117	. 9166	. 0834	6
55	.70608	.29392	1.4163	.99709	1.0029	1.4122	.29186	.70813	5
56	. 0628	. 9372	.4159	. 9767	.0023	.4126	. 9207	. 0793	4
57	. 0649	. 9351	.4154	. 9826	.0017	.4130	. 9228	. 0772	3
58	. 0669	. 9330	.4150	. 9884	.0012	.4134	. 9248	. 0752	2
59	. 0690	. 9310	.4146	. 9942	.0006	.4138	. 9269	. 0731	1
60	. 0711	. 9289	.4142	1.0000	.0000	.4142	. 9289	. 0711	0
M.	Cosine.	Vrs. sin.	Secant.	Cotang.	Tang.	Cosec'nt	Vrs. cos.	Sine.	M.
134°									45°

0°			Logarithms.				179°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	Inf. Neg.	Infinite.	Inf. Neg.	Infinite.	10.00000	10.00000	60
1	6.46373	13.53627	6.46373	13.53627	00000	00000	59
2	76476	23524	76476	23524	00000	00000	58
3	94085	05915	94085	05915	00000	00000	57
4	7.06579	12.93421	7.06579	12.93421	00000	00000	56
5	7.16270	12.83730	7.16270	12.83730	10.00000	10.00000	55
6	24188	75812	24188	75812	00000	00000	54
7	30882	69118	30882	69118	00000	00000	53
8	36682	63318	36682	63318	00000	00000	52
9	41797	58203	41797	58203	00000	00000	51
10	7.46373	12.53627	7.46373	12.53627	10.00000	10.00000	50
11	50512	49488	50512	49488	00000	00000	49
12	54291	45709	54291	45709	00000	00000	48
13	57767	42233	57767	42233	00000	00000	47
14	60985	39015	60986	39014	00000	00000	46
15	7.63982	12.36018	7.63982	12.36018	10.00000	10.00000	45
16	66784	33216	66785	33215	00000	00000	44
17	69417	30583	69418	30582	00001	9.99999	43
18	71900	28100	71900	28100	00001	99999	42
19	74248	25752	74248	25752	00001	99999	41
20	7.76475	12.23525	7.76476	12.23524	10.00001	9.99999	40
21	78594	21406	78595	21405	00001	99999	39
22	80615	19385	80615	19385	00001	99999	38
23	82545	17455	82546	17454	00001	99999	37
24	84393	15607	84394	15606	00001	99999	36
25	7.86166	12.13834	7.86167	12.13833	10.00001	9.99999	35
26	87870	12130	87871	12129	00001	99999	34
27	89509	10491	89510	10490	00001	99999	33
28	91088	08912	91089	08911	00001	99999	32
29	92612	07388	92613	07387	00002	99998	31
30	7.94084	12.05916	7.94086	12.05914	10.00002	9.99998	30
31	95508	04492	95510	04490	00002	99998	29
32	96887	03113	96889	03111	00002	99998	28
33	98223	01777	98225	01775	00002	99998	27
34	99520	00480	99522	00478	00002	99998	26
35	8.00779	11.99221	8.00781	11.99219	10.00002	9.99998	25
36	02002	97998	02004	97996	00002	99998	24
37	03192	96808	03194	96806	00003	99997	23
38	04350	95650	04353	95647	00003	99997	22
39	05478	94522	05481	94519	00003	99997	21
40	8.06578	11.93422	8.06581	11.93419	10.00003	9.99997	20
41	07650	92350	07653	92347	00003	99997	19
42	08696	91304	08700	91300	00003	99997	18
43	09718	90282	09722	90278	00003	99997	17
44	10717	89283	10720	89280	00004	99996	16
45	8.11693	11.88307	8.11696	11.88304	10.00004	9.99996	15
46	12647	87353	12651	87349	00004	99996	14
47	13581	86419	13585	86415	00004	99996	13
48	14495	85505	14500	85500	00004	99996	12
49	15391	84609	15395	84605	00004	99996	11
50	8.16268	11.83732	8.16273	11.83727	10.00005	9.99995	10
51	17128	82872	17133	82867	00005	99995	9
52	17971	82029	17976	82024	00005	99995	8
53	18798	81202	18804	81196	00005	99995	7
54	19610	80390	19616	80384	00005	99995	6
55	8.20407	11.79593	8.20413	11.79587	10.00006	9.99994	5
56	21189	78811	21195	78805	00006	99994	4
57	21958	78042	21964	78036	00006	99994	3
58	22713	77287	22720	77280	00006	99994	2
59	23456	76544	23462	76538	00006	99994	1
60	24186	75814	24192	75808	00007	99993	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
90°							89°

1° **Logarithms.** **178°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.24186	11.75814	8.24192	11.75808	10.00007	9.99993	60
1	24903	75097	24910	75090	00007	99993	59
2	25609	74391	25616	74384	00007	99993	58
3	26304	73696	26312	73688	00007	99993	57
4	26988	73012	26996	73004	00008	99992	56
5	8.27661	11.72339	8.27669	11.72331	10.00008	9.99992	55
6	28324	71676	28332	71668	00008	99992	54
7	28977	71023	28986	71014	00008	99992	53
8	29621	70379	29629	70371	00008	99992	52
9	30255	69745	30263	69737	00009	99991	51
10	8.30879	11.69121	8.30888	11.69112	10.00009	9.99991	50
11	31495	68505	31505	68495	00009	99991	49
12	32103	67897	32112	67888	00010	99990	48
13	32702	67298	32711	67289	00010	99990	47
14	33292	66708	33302	66698	00010	99990	46
15	8.33875	11.66125	8.33886	11.66114	10.00010	9.99990	45
16	34450	65550	34461	65539	00011	99989	44
17	35018	64982	35029	64971	00011	99989	43
18	35578	64422	35590	64410	00011	99989	42
19	36131	63869	36143	63857	00011	99989	41
20	8.36678	11.63322	8.36689	11.63311	10.00012	9.99988	40
21	37217	62783	37229	62771	00012	99988	39
22	37750	62250	37762	62238	00012	99988	38
23	38276	61724	38289	61711	00013	99987	37
24	38796	61204	38809	61191	00013	99987	36
25	8.39310	11.60690	8.39323	11.60677	10.00013	9.99987	35
26	39818	60182	39832	60168	00014	99986	34
27	40320	59680	40334	59666	00014	99986	33
28	40816	59184	40830	59170	00014	99986	32
29	41307	58693	41321	58679	00015	99985	31
30	8.41792	11.58208	8.41807	11.58193	10.00015	9.99985	30
31	42272	57728	42287	57713	00015	99985	29
32	42746	57254	42762	57238	00016	99984	28
33	43216	56784	43232	56768	00016	99984	27
34	43680	56320	43696	56304	00016	99984	26
35	8.44139	11.55861	8.44156	11.55844	10.00017	9.99983	25
36	44594	55406	44611	55389	00017	99983	24
37	45044	54956	45061	54939	00017	99983	23
38	45489	54511	45507	54493	00018	99982	22
39	45930	54070	45948	54052	00018	99982	21
40	8.46366	11.53634	8.46385	11.53615	10.00018	9.99982	20
41	46799	53201	46817	53183	00019	99981	19
42	47226	52774	47245	52755	00019	99981	18
43	47650	52350	47669	52331	00019	99981	17
44	48069	51931	48089	51911	00020	99980	16
45	8.48485	11.51515	8.48505	11.51495	10.00020	9.99980	15
46	48896	51104	48917	51083	00021	99979	14
47	49304	50696	49325	50675	00021	99979	13
48	49708	50292	49729	50271	00021	99979	12
49	50108	49892	50130	49870	00022	99978	11
50	8.50504	11.49496	8.50527	11.49473	10.00022	9.99978	10
51	50897	49103	50920	49080	00023	99977	9
52	51287	48713	51310	48690	00023	99977	8
53	51673	48327	51696	48304	00023	99977	7
54	52055	47945	52079	47921	00024	99976	6
55	8.52434	11.47566	8.52459	11.47541	10.00024	9.99976	5
56	52810	47190	52835	47165	00025	99975	4
57	53183	46817	53208	46792	00025	99975	3
58	53552	46448	53578	46422	00026	99974	2
59	53919	46081	53945	46055	00026	99974	1
60	54282	45718	54308	45692	00026	99974	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

91° **88°**

2°			Logarithms.				177°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.54282	11.45718	8.54308	11.45692	10.00026	9.99974	60
1	54642	45358	54669	45331	00027	99973	59
2	54999	45001	55027	44973	00027	99973	58
3	55354	44646	55382	44618	00028	99972	57
4	55705	44295	55734	44266	00028	99972	56
5	8.56054	11.43946	8.56083	11.43917	10.00029	9.99971	55
6	56400	43600	56429	43571	00029	99971	54
7	56743	43257	56773	43227	00030	99970	53
8	57084	42916	57114	42886	00030	99970	52
9	57421	42579	57452	42548	00031	99969	51
10	8.57757	11.42243	8.57788	11.42212	10.00031	9.99969	50
11	58089	41911	58121	41879	00032	99968	49
12	58419	41581	58451	41549	00032	99968	48
13	58747	41253	58779	41221	00033	99967	47
14	59072	40928	59105	40895	00033	99967	46
15	8.59395	11.40605	8.59428	11.40572	10.00033	9.99967	45
16	59715	40285	59749	40251	00034	99966	44
17	60033	39967	60068	39932	00034	99966	43
18	60349	39651	60384	39616	00035	99965	42
19	60662	39338	60698	39302	00036	99964	41
20	8.60973	11.39027	8.61009	11.38991	10.00036	9.99964	40
21	61282	38718	61319	38681	00037	99963	39
22	61589	38411	61626	38374	00037	99963	38
23	61894	38106	61931	38069	00038	99962	37
24	62196	37804	62234	37766	00038	99962	36
25	8.62497	11.37503	8.62535	11.37465	10.00039	9.99961	35
26	62795	37205	62834	37166	00039	99961	34
27	63091	36909	63131	36869	00040	99960	33
28	63385	36615	63426	36574	00040	99960	32
29	63678	36322	63718	36282	00041	99959	31
30	8.63968	11.36032	8.64009	11.35991	10.00041	9.99959	30
31	64256	35744	64298	35702	00042	99958	29
32	64543	35457	64585	35415	00042	99958	28
33	64827	35173	64870	35130	00043	99957	27
34	65110	34890	65154	34846	00044	99956	26
35	8.65391	11.34609	8.65435	11.34565	10.00044	9.99956	25
36	65670	34330	65715	34285	00045	99955	24
37	65947	34053	65993	34007	00045	99955	23
38	66223	33777	66269	33731	00046	99954	22
39	66497	33503	66543	33457	00046	99954	21
40	8.66769	11.33231	8.66816	11.33184	10.00047	9.99953	20
41	67039	32961	67087	32913	00048	99952	19
42	67308	32692	67356	32644	00048	99952	18
43	67575	32425	67624	32376	00049	99951	17
44	67841	32159	67890	32110	00049	99951	16
45	8.68104	11.31896	8.68154	11.31846	10.00050	9.99950	15
46	68367	31633	68417	31583	00051	99949	14
47	68627	31373	68678	31322	00051	99949	13
48	68886	31114	68938	31062	00052	99948	12
49	69144	30856	69196	30804	00052	99948	11
50	8.69400	11.30600	8.69453	11.30547	10.00053	9.99947	10
51	69654	30346	69708	30292	00054	99946	9
52	69907	30093	69962	30038	00054	99946	8
53	70159	29841	70214	29786	00055	99945	7
54	70409	29591	70465	29535	00056	99944	6
55	8.70658	11.29342	8.70714	11.29286	10.00056	9.99944	5
56	70905	29095	70962	29038	00057	99943	4
57	71151	28849	71208	28792	00058	99942	3
58	71395	28605	71453	28547	00058	99942	2
59	71638	28362	71697	28303	00059	99941	1
60	71880	28120	71940	28060	00060	99940	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
92°							87°

3° **Logarithms.** **176°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.71880	11.28120	8.71940	11.28060	10.00060	9.99940	60
1	72120	27880	72181	27819	00060	99940	59
2	72359	27641	72420	27580	00061	99939	58
3	72597	27403	72659	27341	00062	99938	57
4	72834	27166	72896	27104	00062	99938	56
5	8.73069	11.26931	8.73132	11.26868	10.00063	9.99937	55
6	73303	26697	73366	26634	00064	99936	54
7	73535	26465	73600	26400	00064	99936	53
8	73767	26233	73832	26168	00065	99935	52
9	73997	26003	74063	25937	00066	99934	51
10	8.74226	11.25774	8.74292	11.25708	10.00066	9.99934	50
11	74454	25546	74521	25479	00067	99933	49
12	74680	25320	74748	25252	00068	99932	48
13	74906	25094	74974	25026	00068	99932	47
14	75130	24870	75199	24801	00069	99931	46
15	8.75353	11.24647	8.75423	11.24577	10.00070	9.99930	45
16	75575	24425	75645	24355	00071	99929	44
17	75795	24205	75867	24133	00071	99929	43
18	76015	23985	76087	23913	00072	99928	42
19	76234	23766	76306	23694	00073	99927	41
20	8.76451	11.23549	8.76525	11.23475	10.00074	9.99926	40
21	76667	23333	76742	23258	00074	99926	39
22	76883	23117	76958	23042	00075	99925	38
23	77097	22903	77173	22827	00076	99924	37
24	77310	22690	77387	22613	00077	99923	36
25	8.77522	11.22478	8.77600	11.22400	10.00077	9.99923	35
26	77733	22267	77811	22189	00078	99922	34
27	77943	22057	78022	21978	00079	99921	33
28	78152	21848	78232	21768	00080	99920	32
29	78360	21640	78441	21559	00080	99920	31
30	8.78568	11.21432	8.78649	11.21351	10.00081	9.99919	30
31	78774	21226	78855	21145	00082	99918	29
32	78979	21021	79061	20939	00083	99917	28
33	79183	20817	79266	20734	00083	99917	27
34	79386	20614	79470	20530	00084	99916	26
35	8.79588	11.20412	8.79673	11.20327	10.00085	9.99915	25
36	79789	20211	79875	20125	00086	99914	24
37	79990	20010	80076	19924	00087	99913	23
38	80189	19811	80277	19723	00087	99913	22
39	80388	19612	80476	19524	00088	99912	21
40	8.80585	11.19415	8.80674	11.19326	10.00089	9.99911	20
41	80782	19218	80872	19128	00090	99910	19
42	80978	19022	81068	18932	00091	99909	18
43	81173	18827	81264	18736	00091	99909	17
44	81367	18633	81459	18541	00092	99908	16
45	8.81560	11.18440	8.81653	11.18347	10.00093	9.99907	15
46	81752	18248	81846	18154	00094	99906	14
47	81944	18056	82038	17962	00095	99905	13
48	82134	17866	82230	17770	00096	99904	12
49	82324	17676	82420	17580	00096	99904	11
50	8.82513	11.17487	8.82610	11.17390	10.00097	9.99903	10
51	82701	17299	82799	17201	00098	99902	9
52	82888	17112	82987	17013	00099	99901	8
53	83075	16925	83175	16825	00100	99900	7
54	83261	16739	83361	16639	00101	99899	6
55	8.83446	11.16554	8.83547	11.16453	10.00102	9.99898	5
56	83630	16370	83732	16268	00102	99898	4
57	83813	16187	83916	16084	00103	99897	3
58	83996	16004	84100	15900	00104	99896	2
59	84177	15823	84282	15718	00105	99895	1
60	84358	15642	84464	15536	00106	99894	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

93° **86°**

4°			Logarithms.				175°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.84358	11.15642	8.84464	11.15536	10.00106	9.99894	60
1	84539	15461	84646	15354	00107	99893	59
2	84718	15282	84826	15174	00108	99892	58
3	84897	15103	85006	14994	00109	99891	57
4	85075	14925	85185	14815	00109	99891	56
5	8.85252	11.14748	8.85363	11.14637	10.00110	9.99890	55
6	85429	14571	85540	14460	00111	99889	54
7	85605	14395	85717	14283	00112	99888	53
8	85780	14220	85893	14107	00113	99887	52
9	85955	14045	86069	13931	00114	99886	51
10	8.86128	11.13872	8.86243	11.13757	10.00115	9.99885	50
11	86301	13699	86417	13583	00116	99884	49
12	86474	13526	86591	13409	00117	99883	48
13	86645	13355	86763	13237	00118	99882	47
14	86816	13184	86935	13065	00119	99881	46
15	8.86987	11.13013	8.87106	11.12894	10.00120	9.99880	45
16	87156	12844	87277	12723	00121	99879	44
17	87325	12675	87447	12553	00121	99879	43
18	87494	12506	87616	12384	00122	99878	42
19	87661	12339	87785	12215	00123	99877	41
20	8.87829	11.12171	8.87953	11.12047	10.00124	9.99876	40
21	87995	12005	88120	11880	00125	99875	39
22	88161	11839	88287	11713	00126	99874	38
23	88326	11674	88453	11547	00127	99873	37
24	88490	11510	88618	11382	00128	99872	36
25	8.88654	11.11346	8.88783	11.11217	10.00129	9.99871	35
26	88817	11183	88948	11052	00130	99870	34
27	88980	11020	89111	10889	00131	99869	33
28	89142	10858	89274	10726	00132	99868	32
29	89304	10696	89437	10563	00133	99867	31
30	8.89464	11.10536	8.89598	11.10402	10.00134	9.99866	30
31	89625	10375	89760	10240	00135	99865	29
32	89784	10216	89920	10080	00136	99864	28
33	89943	10057	90080	09920	00137	99863	27
34	90102	09898	90240	09760	00138	99862	26
35	8.90260	11.09740	8.90399	11.09601	10.00139	9.99861	25
36	90417	09583	90557	09443	00140	99860	24
37	90574	09426	90715	09285	00141	99859	23
38	90730	09270	90872	09128	00142	99858	22
39	90885	09115	91029	08971	00143	99857	21
40	8.91040	11.08960	8.91185	11.08815	10.00144	9.99856	20
41	91195	08805	91340	08660	00145	99855	19
42	91349	08651	91495	08505	00146	99854	18
43	91502	08498	91650	08350	00147	99853	17
44	91655	08345	91803	08197	00148	99852	16
45	8.91807	11.08193	8.91957	11.08043	10.00149	9.99851	15
46	91959	08041	92110	07890	00150	99850	14
47	92110	07890	92262	07738	00152	99848	13
48	92261	07739	92414	07586	00153	99847	12
49	92411	07589	92565	07435	00154	99846	11
50	8.92561	11.07439	8.92716	11.07284	10.00155	9.99845	10
51	92710	07290	92866	07134	00156	99844	9
52	92859	07141	93016	06984	00157	99843	8
53	93007	06993	93165	06835	00158	99842	7
54	93154	06846	93313	06687	00159	99841	6
55	8.93301	11.06699	8.93462	11.06538	10.00160	9.99840	5
56	93448	06552	93609	06391	00161	99839	4
57	93594	06406	93756	06244	00162	99838	3
58	93740	06260	93903	06097	00163	99837	2
59	93885	06115	94049	05951	00164	99836	1
60	94030	05970	94195	05805	00166	99834	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
94°							85°

5°			Logarithms.				174°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	8.94030	11.05970	8.94195	11.05805	10.00166	9.99834	60
1	94174	05826	94340	05660	00167	99833	59
2	94317	05683	94485	05515	00168	99832	58
3	94461	05539	94630	05370	00169	99831	57
4	94603	05397	94773	05227	00170	99830	56
5	8.94746	11.05254	8.94917	11.05083	10.00171	9.99829	55
6	94887	05113	95060	04940	00172	99828	54
7	95029	04971	95202	04798	00173	99827	53
8	95170	04830	95344	04656	00175	99825	52
9	95310	04690	95486	04514	00176	99824	51
10	8.95450	11.04550	8.95627	11.04373	10.00177	9.99823	50
11	95589	04411	95767	04233	00178	99822	49
12	95728	04272	95908	04092	00179	99821	48
13	95867	04133	96047	03953	00180	99820	47
14	96005	03995	96187	03813	00181	99819	46
15	8.96143	11.03857	8.96325	11.03675	10.00183	9.99817	45
16	96280	03720	96464	03536	00184	99816	44
17	96417	03583	96602	03398	00185	99815	43
18	96553	03447	96739	03261	00186	99814	42
19	96689	03311	96877	03123	00187	99813	41
20	8.96825	11.03175	8.97013	11.02987	10.00188	9.99812	40
21	96960	03040	97150	02850	00190	99810	39
22	97095	02905	97285	02715	00191	99809	38
23	97229	02771	97421	02579	00192	99808	37
24	97363	02637	97556	02444	00193	99807	36
25	8.97496	11.02504	8.97691	11.02309	10.00194	9.99806	35
26	97629	02371	97825	02175	00196	99804	34
27	97762	02238	97959	02041	00197	99803	33
28	97894	02106	98092	01908	00198	99802	32
29	98026	01974	98225	01775	00199	99801	31
30	8.98157	11.01843	8.98358	11.01642	10.00200	9.99800	30
31	98288	01712	98490	01510	00202	99798	29
32	98419	01581	98622	01378	00203	99797	28
33	98549	01451	98753	01247	00204	99796	27
34	98679	01321	98884	01116	00205	99795	26
35	8.98808	11.01192	8.99015	11.00985	10.00207	9.99793	25
36	98937	01063	99145	00855	00208	99792	24
37	99066	00934	99275	00725	00209	99791	23
38	99194	00806	99405	00595	00210	99790	22
39	99322	00678	99534	00466	00212	99788	21
40	8.99450	11.00550	8.99662	11.00338	10.00213	9.99787	20
41	99577	00423	99791	00209	00214	99786	19
42	99704	00296	99919	00081	00215	99785	18
43	99830	00170	9.00046	10.99954	00217	99783	17
44	99956	00044	00174	99826	00218	99782	16
45	9.00082	10.99918	9.00301	10.99699	10.00219	9.99781	15
46	00207	99793	00427	99573	00220	99780	14
47	00332	99668	00553	99447	00222	99778	13
48	00456	99544	00679	99321	00223	99777	12
49	00581	99419	00805	99195	00224	99776	11
50	9.00704	10.99296	9.00930	10.99070	10.00225	9.99775	10
51	00828	99172	01055	98945	00227	99773	9
52	00951	99049	01179	98821	00228	99772	8
53	01074	98926	01303	98697	00229	99771	7
54	01196	98804	01427	98573	00231	99769	6
55	9.01318	10.98682	9.01550	10.98450	10.00232	9.99768	5
56	01440	98560	01673	98327	00233	99767	4
57	01561	98439	01796	98204	00235	99765	3
58	01682	98318	01918	98082	00236	99764	2
59	01803	98197	02040	97960	00237	99763	1
60	01923	98077	02162	97838	00239	99761	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
95°							84°

6° **Logarithms.** 173°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.01923	10.98077	9.02162	10.97838	10.00239	9.99761	60
1	02043	97957	02283	97717	00240	99760	59
2	02163	97837	02404	97596	00241	99759	58
3	02283	97717	02525	97475	00243	99757	57
4	02402	97598	02645	97355	00244	99756	56
5	9.02520	10.97480	9.02766	10.97234	10.00245	9.99755	55
6	02639	97361	02885	97115	00247	99753	54
7	02757	97243	03005	96995	00248	99752	53
8	02874	97126	03124	96876	00249	99751	52
9	02992	97008	03242	96758	00251	99749	51
10	9.03109	10.96891	9.03361	10.96639	10.00252	9.99748	50
11	03226	96774	03479	96521	00253	99747	49
12	03342	96658	03597	96403	00255	99745	48
13	03458	96542	03714	96286	00256	99744	47
14	03574	96426	03832	96168	00258	99742	46
15	9.03690	10.96310	9.03948	10.96052	10.00259	9.99741	45
16	03805	96195	04065	95935	00260	99740	44
17	03920	96080	04181	95819	00262	99738	43
18	04034	95966	04297	95703	00263	99737	42
19	04149	95851	04413	95587	00264	99736	41
20	9.04262	10.95738	9.04528	10.95472	10.00266	9.99734	40
21	04376	95624	04643	95357	00267	99733	39
22	04490	95510	04758	95242	00269	99731	38
23	04603	95397	04873	95127	00270	99730	37
24	04715	95285	04987	95013	00272	99728	36
25	9.04828	10.95172	9.05101	10.94899	10.00273	9.99727	35
26	04940	95060	05214	94786	00274	99726	34
27	05052	94948	05328	94672	00276	99724	33
28	05164	94836	05441	94559	00277	99723	32
29	05275	94725	05553	94447	00279	99721	31
30	9.05386	10.94614	9.05666	10.94334	10.00280	9.99720	30
31	05497	94503	05778	94222	00282	99718	29
32	05607	94393	05890	94110	00283	99717	28
33	05717	94283	06002	93998	00284	99716	27
34	05827	94173	06113	93887	00286	99714	26
35	9.05937	10.94063	9.06224	10.93776	10.00287	9.99713	25
36	06046	93954	06335	93665	00289	99711	24
37	06155	93845	06445	93555	00290	99710	23
38	06264	93736	06556	93444	00292	99708	22
39	06372	93628	06666	93334	00293	99707	21
40	9.06481	10.93519	9.06775	10.93225	10.00295	9.99705	20
41	06589	93411	06885	93115	00296	99704	19
42	06696	93304	06994	93006	00298	99702	18
43	06804	93196	07103	92897	00299	99701	17
44	06911	93089	07211	92789	00301	99699	16
45	9.07018	10.92982	9.07320	10.92680	10.00302	9.99698	15
46	07124	92876	07428	92572	00304	99696	14
47	07231	92769	07536	92464	00305	99695	13
48	07337	92663	07643	92357	00307	99693	12
49	07442	92558	07751	92249	00308	99692	11
50	9.07548	10.92452	9.07858	10.92142	10.00310	9.99690	10
51	07653	92347	07964	92036	00311	99689	9
52	07758	92242	08071	91929	00313	99687	8
53	07863	92137	08177	91823	00314	99686	7
54	07968	92032	08283	91717	00316	99684	6
55	9.08072	10.91928	9.08389	10.91611	10.00317	9.99683	5
56	08176	91824	08495	91505	00319	99681	4
57	08280	91720	08600	91400	00320	99680	3
58	08383	91617	08705	91295	00322	99678	2
59	08486	91514	08810	91190	00323	99677	1
60	08589	91411	08914	91086	00325	99675	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

96° 83°

7° **Logarithms.** 172°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.08589	10.91411	9.08914	10.91086	10.00325	9.99675	60
1	08692	91308	09019	90981	00326	99674	59
2	08795	91205	09123	90877	00328	99672	58
3	08897	91103	09227	90773	00330	99670	57
4	08999	91001	09330	90670	00331	99669	56
5	9.09101	10.90899	9.09434	10.90566	10.00333	9.99667	55
6	09202	90798	09537	90463	00334	99666	54
7	09304	90696	09640	90360	00336	99664	53
8	09405	90595	09742	90258	00337	99663	52
9	09506	90494	09845	90155	00339	99661	51
10	9.09606	10.90394	9.09947	10.90053	10.00341	9.99659	50
11	09707	90293	10049	89951	00342	99658	49
12	09807	90193	10150	89850	00344	99656	48
13	09907	90093	10252	89748	00345	99655	47
14	10006	89994	10353	89647	00347	99653	46
15	9.10106	10.89894	9.10454	10.89546	10.00349	9.99651	45
16	10205	89795	10555	89445	00350	99650	44
17	10304	89696	10656	89344	00352	99648	43
18	10402	89598	10756	89244	00353	99647	42
19	10501	89499	10856	89144	00355	99645	41
20	9.10599	10.89401	9.10956	10.89044	10.00357	9.99643	40
21	10697	89303	11056	88944	00358	99642	39
22	10795	89205	11155	88845	00360	99640	38
23	10893	89107	11254	88746	00362	99638	37
24	10990	89010	11353	88647	00363	99637	36
25	9.11087	10.88913	9.11452	10.88548	10.00365	9.99635	35
26	11184	88816	11551	88449	00367	99633	34
27	11281	88719	11649	88351	00368	99632	33
28	11377	88623	11747	88253	00370	99630	32
29	11474	88526	11845	88155	00371	99629	31
30	9.11570	10.88430	9.11943	10.88057	10.00373	9.99627	30
31	11666	88334	12040	87960	00375	99625	29
32	11761	88239	12138	87862	00376	99624	28
33	11857	88143	12235	87765	00378	99622	27
34	11952	88048	12332	87668	00380	99620	26
35	9.12047	10.87953	9.12428	10.87572	10.00382	9.99618	25
36	12142	87858	12525	87475	00383	99617	24
37	12236	87764	12621	87379	00385	99615	23
38	12331	87669	12717	87283	00387	99613	22
39	12425	87575	12813	87187	00388	99612	21
40	9.12519	10.87481	9.12909	10.87091	10.00390	9.99610	20
41	12612	87388	13004	86996	00392	99608	19
42	12706	87294	13099	86901	00393	99607	18
43	12799	87201	13194	86806	00395	99605	17
44	12892	87108	13289	86711	00397	99603	16
45	9.12985	10.87015	9.13384	10.86616	10.00399	9.99601	15
46	13078	86922	13478	86522	00400	99600	14
47	13171	86829	13573	86427	00402	99598	13
48	13263	86737	13667	86333	00404	99596	12
49	13355	86645	13761	86239	00405	99595	11
50	9.13447	10.86553	9.13854	10.86146	10.00407	9.99593	10
51	13539	86461	13948	86052	00409	99591	9
52	13630	86370	14041	85959	00411	99589	8
53	13722	86278	14134	85866	00412	99588	7
54	13813	86187	14227	85773	00414	99586	6
55	9.13904	10.86096	9.14320	10.85680	10.00416	9.99584	5
56	13994	86006	14412	85588	00418	99582	4
57	14085	85915	14504	85496	00419	99581	3
58	14175	85825	14597	85403	00421	99579	2
59	14266	85734	14688	85312	00423	99577	1
60	14356	85644	14780	85220	00425	99575	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

97° 82°

8° **Logarithms.** 171°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.14356	10.85644	9.14780	10.85220	10.00425	9.99575	60
1	14445	85555	14872	85128	00426	99574	59
2	14535	85465	14963	85037	00428	99572	58
3	14624	85376	15054	84946	00430	99570	57
4	14714	85286	15145	84855	00432	99568	56
5	9.14803	10.85197	9.15236	10.84764	10.00434	9.99566	55
6	14891	85109	15327	84673	00435	99565	54
7	14980	85020	15417	84583	00437	99563	53
8	15069	84931	15508	84492	00439	99561	52
9	15157	84843	15598	84402	00441	99559	51
10	9.15245	10.84755	9.15688	10.84312	10.00443	9.99557	50
11	15333	84667	15777	84223	00444	99556	49
12	15421	84579	15867	84133	00446	99554	48
13	15508	84492	15956	84044	00448	99552	47
14	15596	84404	16046	83954	00450	99550	46
15	9.15683	10.84317	9.16135	10.83865	10.00452	9.99548	45
16	15770	84230	16224	83776	00454	99546	44
17	15857	84143	16312	83688	00455	99545	43
18	15944	84056	16401	83599	00457	99543	42
19	16030	83970	16489	83511	00459	99541	41
20	9.16116	10.83884	9.16577	10.83423	10.00461	9.99539	40
21	16203	83797	16665	83335	00463	99537	39
22	16289	83711	16753	83247	00465	99535	38
23	16374	83626	16841	83159	00467	99533	37
24	16460	83540	16928	83072	00468	99532	36
25	9.16545	10.83455	9.17016	10.82984	10.00470	9.99530	35
26	16631	83369	17103	82897	00472	99528	34
27	16716	83284	17190	82810	00474	99526	33
28	16801	83199	17277	82723	00476	99524	32
29	16886	83114	17363	82637	00478	99522	31
30	9.16970	10.83030	9.17450	10.82550	10.00480	9.99520	30
31	17055	82945	17536	82464	00482	99518	29
32	17139	82861	17622	82378	00483	99517	28
33	17223	82777	17708	82292	00485	99515	27
34	17307	82693	17794	82206	00487	99513	26
35	9.17391	10.82609	9.17880	10.82120	10.00489	9.99511	25
36	17474	82526	17965	82035	00491	99509	24
37	17558	82442	18051	81949	00493	99507	23
38	17641	82359	18136	81864	00495	99505	22
39	17724	82276	18221	81779	00497	99503	21
40	9.17807	10.82193	9.18306	10.81694	10.00499	9.99501	20
41	17890	82110	18391	81609	00501	99499	19
42	17973	82027	18475	81525	00503	99497	18
43	18055	81945	18560	81440	00505	99495	17
44	18137	81863	18644	81356	00506	99494	16
45	9.18220	10.81780	9.18728	10.81272	10.00508	9.99492	15
46	18302	81698	18812	81188	00510	99490	14
47	18383	81617	18896	81104	00512	99488	13
48	18465	81535	18979	81021	00514	99486	12
49	18547	81453	19063	80937	00516	99484	11
50	9.18628	10.81372	9.19146	10.80854	10.00518	9.99482	10
51	18709	81291	19229	80771	00520	99480	9
52	18790	81210	19312	80688	00522	99478	8
53	18871	81129	19395	80605	00524	99476	7
54	18952	81048	19478	80522	00526	99474	6
55	9.19033	10.80967	9.19561	10.80439	10.00528	9.99472	5
56	19113	80887	19643	80357	00530	99470	4
57	19193	80807	19725	80275	00532	99468	3
58	19273	80727	19807	80193	00534	99466	2
59	19353	80647	19889	80111	00536	99464	1
60	19433	80567	19971	80029	00538	99462	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

98° 81°

9° **Logarithms.** 170°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.19433	10.80567	9.19971	10.80029	10.00538	9.99462	60
1	19513	80487	20053	79947	00540	99460	59
2	19592	80408	20134	79866	00542	99458	58
3	19672	80328	20216	79784	00544	99456	57
4	19751	80249	20297	79703	00546	99454	56
5	9.19830	10.80170	9.20378	10.79622	10.00548	9.99452	55
6	19909	80091	20459	79541	00550	99450	54
7	19988	80012	20540	79460	00552	99448	53
8	20067	79933	20621	79379	00554	99446	52
9	20145	79855	20701	79299	00556	99444	51
10	9.20223	10.79777	9.20782	10.79218	10.00558	9.99442	50
11	20302	79698	20862	79138	00560	99440	49
12	20380	79620	20942	79058	00562	99438	48
13	20458	79542	21022	78978	00564	99436	47
14	20535	79465	21102	78898	00566	99434	46
15	9.20613	10.79387	9.21182	10.78818	10.00568	9.99432	45
16	20691	79309	21261	78739	00571	99429	44
17	20768	79232	21341	78659	00573	99427	43
18	20845	79155	21420	78580	00575	99425	42
19	20922	79078	21499	78501	00577	99423	41
20	9.20999	10.79001	9.21578	10.78422	10.00579	9.99421	40
21	21076	78924	21657	78343	00581	99419	39
22	21153	78847	21736	78264	00583	99417	38
23	21229	78771	21814	78186	00585	99415	37
24	21306	78694	21893	78107	00587	99413	36
25	9.21382	10.78618	9.21971	10.78029	10.00589	9.99411	35
26	21458	78542	22049	77951	00591	99409	34
27	21534	78466	22127	77873	00593	99407	33
28	21610	78390	22205	77795	00596	99404	32
29	21685	78315	22283	77717	00598	99402	31
30	9.21761	10.78239	9.22361	10.77639	10.00600	9.99400	30
31	21836	78164	22438	77562	00602	99398	29
32	21912	78088	22516	77484	00604	99396	28
33	21987	78013	22593	77407	00606	99394	27
34	22062	77938	22670	77330	00608	99392	26
35	9.22137	10.77863	9.22747	10.77253	10.00610	9.99390	25
36	22211	77789	22824	77176	00612	99388	24
37	22286	77714	22901	77099	00615	99385	23
38	22361	77639	22977	77023	00617	99383	22
39	22435	77565	23054	76946	00619	99381	21
40	9.22509	10.77491	9.23130	10.76870	10.00621	9.99379	20
41	22583	77417	23206	76794	00623	99377	19
42	22657	77343	23283	76717	00625	99375	18
43	22731	77269	23359	76641	00628	99372	17
44	22805	77195	23435	76565	00630	99370	16
45	9.22878	10.77122	9.23510	10.76490	10.00632	9.99368	15
46	22952	77048	23586	76414	00634	99366	14
47	23025	76975	23661	76339	00636	99364	13
48	23098	76902	23737	76263	00638	99362	12
49	23171	76829	23812	76188	00641	99359	11
50	9.23244	10.76756	9.23887	10.76113	10.00643	9.99357	10
51	23317	76683	23962	76038	00645	99355	9
52	23390	76610	24037	75963	00647	99353	8
53	23462	76538	24112	75888	00649	99351	7
54	23535	76465	24186	75814	00652	99348	6
55	9.23607	10.76393	9.24261	10.75739	10.00654	9.99346	5
56	23679	76321	24335	75665	00656	99344	4
57	23752	76248	24410	75590	00658	99342	3
58	23823	76177	24484	75516	00660	99340	2
59	23895	76105	24558	75442	00663	99337	1
60	23967	76033	24632	75368	00665	99335	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

99° 80°

10° **Logarithms.** **169°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.23967	10.76033	9.24632	10.75368	10.00665	9.99335	60
1	24039	75961	24706	75294	00667	99333	59
2	24110	75890	24779	75221	00669	99331	58
3	24181	75819	24853	75147	00672	99328	57
4	24253	75747	24926	75074	00674	99326	56
5	9.24324	10.75676	9.25000	10.75000	10.00676	9.99324	55
6	24395	75605	25073	74927	00678	99322	54
7	24466	75534	25146	74854	00681	99319	53
8	24536	75464	25219	74781	00683	99317	52
9	24607	75393	25292	74708	00685	99315	51
10	9.24677	10.75323	9.25365	10.74635	10.00687	9.99313	50
11	24748	75252	25437	74563	00690	99310	49
12	24818	75182	25510	74490	00692	99308	48
13	24888	75112	25582	74418	00694	99306	47
14	24958	75042	25655	74345	00696	99304	46
15	9.25028	10.74972	9.25727	10.74273	10.00699	9.99301	45
16	25098	74902	25799	74201	00701	99299	44
17	25168	74832	25871	74129	00703	99297	43
18	25237	74763	25943	74057	00706	99294	42
19	25307	74693	26015	73985	00708	99292	41
20	9.25376	10.74624	9.26086	10.73914	10.00710	9.99290	40
21	25445	74555	26158	73842	00712	99288	39
22	25514	74486	26229	73771	00715	99285	38
23	25583	74417	26301	73699	00717	99283	37
24	25652	74348	26372	73628	00719	99281	36
25	9.25721	10.74279	9.26443	10.73557	10.00722	9.99278	35
26	25790	74210	26514	73486	00724	99276	34
27	25858	74142	26585	73415	00726	99274	33
28	25927	74073	26655	73345	00729	99271	32
29	25995	74005	26726	73274	00731	99269	31
30	9.26063	10.73937	9.26797	10.73203	10.00733	9.99267	30
31	26131	73869	26867	73133	00736	99264	29
32	26199	73801	26937	73063	00738	99262	28
33	26267	73733	27008	72992	00740	99260	27
34	26335	73665	27078	72922	00743	99257	26
35	9.26403	10.73597	9.27148	10.72852	10.00745	9.99255	25
36	26470	73530	27218	72782	00748	99252	24
37	26538	73462	27288	72712	00750	99250	23
38	26605	73395	27357	72643	00752	99248	22
39	26672	73328	27427	72573	00755	99245	21
40	9.26739	10.73261	9.27496	10.72504	10.00757	9.99243	20
41	26806	73194	27566	72434	00759	99241	19
42	26873	73127	27635	72365	00762	99238	18
43	26940	73060	27704	72296	00764	99236	17
44	27007	72993	27773	72227	00767	99233	16
45	9.27073	10.72927	9.27842	10.72158	10.00769	9.99231	15
46	27140	72860	27911	72089	00771	99229	14
47	27206	72794	27980	72020	00774	99226	13
48	27273	72727	28049	71951	00776	99224	12
49	27339	72661	28117	71883	00779	99221	11
50	9.27405	10.72595	9.28186	10.71814	10.00781	9.99219	10
51	27471	72529	28254	71746	00783	99217	9
52	27537	72463	28323	71677	00786	99214	8
53	27602	72398	28391	71609	00788	99212	7
54	27668	72332	28459	71541	00791	99209	6
55	9.27734	10.72266	9.28527	10.71473	10.00793	9.99207	5
56	27799	72201	28595	71405	00796	99204	4
57	27864	72136	28662	71338	00798	99202	3
58	27930	72070	28730	71270	00800	99200	2
59	27995	72005	28798	71202	00803	99197	1
60	28060	71940	28865	71135	00805	99195	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

100° **79°**

11° **Logarithms.** **168°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.28060	10.71940	9.28865	10.71135	10.00805	9.99195	60
1	28125	71875	28933	71067	00808	99192	59
2	28190	71810	29000	71000	00810	99190	58
3	28254	71746	29067	70933	00813	99187	57
4	28319	71681	29134	70866	00815	99185	56
5	9.28384	10.71616	9.29201	10.70799	10.00818	9.99182	55
6	28448	71552	29268	70732	00820	99180	54
7	28512	71488	29335	70665	00823	99177	53
8	28577	71423	29402	70598	00825	99175	52
9	28641	71359	29468	70532	00828	99172	51
10	9.28705	10.71295	9.29535	10.70465	10.00830	9.99170	50
11	28769	71231	29601	70399	00833	99167	49
12	28833	71167	29668	70332	00835	99165	48
13	28896	71104	29734	70266	00838	99162	47
14	28960	71040	29800	70200	00840	99160	46
15	9.29024	10.70976	9.29866	10.70134	10.00843	9.99157	45
16	29087	70913	29932	70068	00845	99155	44
17	29150	70850	29998	70002	00848	99152	43
18	29214	70786	30064	69936	00850	99150	42
19	29277	70723	30130	69870	00853	99147	41
20	9.29340	10.70660	9.30195	10.69805	10.00855	9.99145	40
21	29403	70597	30261	69739	00858	99142	39
22	29466	70534	30326	69674	00860	99140	38
23	29529	70471	30391	69609	00863	99137	37
24	29591	70409	30457	69543	00865	99135	36
25	9.29654	10.70346	9.30522	10.69478	10.00868	9.99132	35
26	29716	70284	30587	69413	00870	99130	34
27	29779	70221	30652	69348	00873	99127	33
28	29841	70159	30717	69283	00876	99124	32
29	29903	70097	30782	69218	00878	99122	31
30	9.29966	10.70034	9.30846	10.69154	10.00881	9.99119	30
31	30028	69972	30911	69089	00883	99117	29
32	30090	69910	30975	69025	00886	99114	28
33	30151	69849	31040	68960	00888	99112	27
34	30213	69787	31104	68896	00891	99109	26
35	9.30275	10.69725	9.31168	10.68832	10.00894	9.99106	25
36	30336	69664	31233	68767	00896	99104	24
37	30398	69602	31297	68703	00899	99101	23
38	30459	69541	31361	68639	00901	99099	22
39	30521	69479	31425	68575	00904	99096	21
40	9.30582	10.69418	9.31489	10.68511	10.00907	9.99093	20
41	30643	69357	31552	68448	00909	99091	19
42	30704	69296	31616	68384	00912	99088	18
43	30765	69235	31679	68321	00914	99086	17
44	30826	69174	31743	68257	00917	99083	16
45	9.30887	10.69113	9.31806	10.68194	10.00920	9.99080	15
46	30947	69053	31870	68130	00922	99078	14
47	31008	68992	31933	68067	00925	99075	13
48	31068	68932	31996	68004	00928	99072	12
49	31129	68871	32059	67941	00930	99070	11
50	9.31189	10.68811	9.32122	10.67878	10.00933	9.99067	10
51	31250	68750	32185	67815	00936	99064	9
52	31310	68690	32248	67752	00938	99062	8
53	31370	68630	32311	67689	00941	99059	7
54	31430	68570	32373	67627	00944	99056	6
55	9.31490	10.68510	9.32436	10.67564	10.00946	9.99054	5
56	31549	68451	32498	67502	00949	99051	4
57	31609	68391	32561	67439	00952	99048	3
58	31669	68331	32623	67377	00954	99046	2
59	31728	68272	32685	67315	00957	99043	1
60	31788	68212	32747	67253	00960	99040	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

101° **78°**

12° **Logarithms.** 167°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.31788	10.68212	9.32747	10.67253	10.00960	9.99040	60
1	31847	68153	32810	67190	00962	99038	59
2	31907	68093	32872	67128	00965	99035	58
3	31966	68034	32933	67067	00968	99032	57
4	32025	67975	32995	67005	00970	99030	56
5	9.32084	10.67916	9.33057	10.66943	10.00973	9.99027	55
6	32143	67857	33119	66881	00976	99024	54
7	32202	67798	33180	66820	00978	99022	53
8	32261	67739	33242	66758	00981	99019	52
9	32319	67681	33303	66697	00984	99016	51
10	9.32378	10.67622	9.33365	10.66635	10.00987	9.99013	50
11	32437	67563	33426	66574	00989	99011	49
12	32495	67505	33487	66513	00992	99008	48
13	32553	67447	33548	66452	00995	99005	47
14	32612	67388	33609	66391	00998	99002	46
15	9.32670	10.67330	9.33670	10.66330	10.01000	9.99000	45
16	32728	67272	33731	66269	01003	98997	44
17	32786	67214	33792	66208	01006	98994	43
18	32844	67156	33853	66147	01009	98991	42
19	32902	67098	33913	66087	01011	98989	41
20	9.32960	10.67040	9.33974	10.66026	10.01014	9.98986	40
21	33018	66982	34034	65966	01017	98983	39
22	33075	66925	34095	65905	01020	98980	38
23	33133	66867	34155	65845	01022	98978	37
24	33190	66810	34215	65785	01025	98975	36
25	9.33248	10.66752	9.34276	10.65724	10.01028	9.98972	35
26	33305	66695	34336	65664	01031	98969	34
27	33362	66638	34396	65604	01033	98967	33
28	33420	66580	34456	65544	01036	98964	32
29	33477	66523	34516	65484	01039	98961	31
30	9.33534	10.66466	9.34576	10.65424	10.01042	9.98958	30
31	33591	66409	34635	65365	01045	98955	29
32	33647	66353	34695	65305	01047	98953	28
33	33704	66296	34755	65245	01050	98950	27
34	33761	66239	34814	65186	01053	98947	26
35	9.33818	10.66182	9.34874	10.65126	10.01056	9.98944	25
36	33874	66126	34933	65067	01059	98941	24
37	33931	66069	34992	65008	01062	98938	23
38	33987	66013	35051	64949	01064	98936	22
39	34043	65957	35111	64889	01067	98933	21
40	9.34100	10.65900	9.35170	10.64830	10.01070	9.98930	20
41	34156	65844	35229	64771	01073	98927	19
42	34212	65788	35288	64712	01076	98924	18
43	34268	65732	35347	64653	01079	98921	17
44	34324	65676	35405	64595	01081	98919	16
45	9.34380	10.65620	9.35464	10.64536	10.01084	9.98916	15
46	34436	65564	35523	64477	01087	98913	14
47	34491	65509	35581	64419	01090	98910	13
48	34547	65453	35640	64360	01093	98907	12
49	34602	65398	35698	64302	01096	98904	11
50	9.34658	10.65342	9.35757	10.64243	10.01099	9.98901	10
51	34713	65287	35815	64185	01102	98898	9
52	34769	65231	35873	64127	01104	98896	8
53	34824	65176	35931	64069	01107	98893	7
54	34879	65121	35989	64011	01110	98890	6
55	9.34934	10.65066	9.36047	10.63953	10.01113	9.98887	5
56	34989	65011	36105	63895	01116	98884	4
57	35044	64956	36163	63837	01119	98881	3
58	35099	64901	36221	63779	01122	98878	2
59	35154	64846	36279	63721	01125	98875	1
60	35209	64791	36336	63664	01128	98872	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

102° 77°

13° **Logarithms.** 166°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.35209	10.64791	9.36336	10.63664	10.01128	9.98872	60
1	35263	64737	36394	63606	01131	98869	59
2	35318	64682	36452	63548	01133	98867	58
3	35373	64627	36509	63491	01136	98864	57
4	35427	64573	36566	63434	01139	98861	56
5	9.35481	10.64519	9.36624	10.63376	10.01142	9.98858	55
6	35536	64464	36681	63319	01145	98855	54
7	35590	64410	36738	63262	01148	98852	53
8	35644	64356	36795	63205	01151	98849	52
9	35698	64302	36852	63148	01154	98846	51
10	9.35752	10.64248	9.36909	10.63091	10.01157	9.98843	50
11	35806	64194	36966	63034	01160	98840	49
12	35860	64140	37023	62977	01163	98837	48
13	35914	64086	37080	62920	01166	98834	47
14	35968	64032	37137	62863	01169	98831	46
15	9.36022	10.63978	9.37193	10.62807	10.01172	9.98828	45
16	36075	63925	37250	62750	01175	98825	44
17	36129	63871	37306	62694	01178	98822	43
18	36182	63818	37363	62637	01181	98819	42
19	36236	63764	37419	62581	01184	98816	41
20	9.36289	10.63711	9.37476	10.62524	10.01187	9.98813	40
21	36342	63658	37532	62468	01190	98810	39
22	36395	63605	37588	62412	01193	98807	38
23	36449	63551	37644	62356	01196	98804	37
24	36502	63498	37700	62300	01199	98801	36
25	9.36555	10.63445	9.37756	10.62244	10.01202	9.98798	35
26	36608	63392	37812	62188	01205	98795	34
27	36660	63340	37868	62132	01208	98792	33
28	36713	63287	37924	62076	01211	98789	32
29	36766	63234	37980	62020	01214	98786	31
30	9.36819	10.63181	9.38035	10.61965	10.01217	9.98783	30
31	36871	63129	38091	61909	01220	98780	29
32	36924	63076	38147	61853	01223	98777	28
33	36976	63024	38202	61798	01226	98774	27
34	37028	62972	38257	61743	01229	98771	26
35	9.37081	10.62919	9.38313	10.61687	10.01232	9.98768	25
36	37133	62867	38368	61632	01235	98765	24
37	37185	62815	38423	61577	01238	98762	23
38	37237	62763	38479	61521	01241	98759	22
39	37289	62711	38534	61466	01244	98756	21
40	9.37341	10.62659	9.38589	10.61411	10.01247	9.98753	20
41	37393	62607	38644	61356	01250	98750	19
42	37445	62555	38699	61301	01254	98746	18
43	37497	62503	38754	61246	01257	98743	17
44	37549	62451	38808	61192	01260	98740	16
45	9.37600	10.62400	9.38863	10.61137	10.01263	9.98737	15
46	37652	62348	38918	61082	01266	98734	14
47	37703	62297	38972	61028	01269	98731	13
48	37755	62245	39027	60973	01272	98728	12
49	37806	62194	39082	60918	01275	98725	11
50	9.37858	10.62142	9.39136	10.60864	10.01278	9.98722	10
51	37909	62091	39190	60810	01281	98719	9
52	37960	62040	39245	60755	01285	98715	8
53	38011	61989	39299	60701	01288	98712	7
54	38062	61938	39353	60647	01291	98709	6
55	9.38113	10.61887	9.39407	10.60593	10.01294	9.98706	5
56	38164	61836	39461	60539	01297	98703	4
57	38215	61785	39515	60485	01300	98700	3
58	38266	61734	39569	60431	01303	98697	2
59	38317	61683	39623	60377	01306	98694	1
60	38368	61632	39677	60323	01310	98690	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

103° 76°

14° **Logarithms.** **165°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.38368	10.61632	9.39677	10.60323	10.01310	9.98690	60
1	38418	61582	39731	60269	01313	98687	59
2	38469	61531	39785	60215	01316	98684	58
3	38519	61481	39838	60162	01319	98681	57
4	38570	61430	39892	60108	01322	98678	56
5	9.38620	10.61380	9.39945	10.60055	10.01325	9.98675	55
6	38670	61330	39999	60001	01329	98671	54
7	38721	61279	40052	59948	01332	98668	53
8	38771	61229	40106	59894	01335	98665	52
9	38821	61179	40159	59841	01338	98662	51
10	9.38871	10.61129	9.40212	10.59788	10.01341	9.98659	50
11	38921	61079	40266	59734	01344	98656	49
12	38971	61029	40319	59681	01348	98652	48
13	39021	60979	40372	59628	01351	98649	47
14	39071	60929	40425	59575	01354	98646	46
15	9.39121	10.60879	9.40478	10.59522	10.01357	9.98643	45
16	39170	60830	40531	59469	01360	98640	44
17	39220	60780	40584	59416	01364	98636	43
18	39270	60730	40636	59364	01367	98633	42
19	39319	60681	40689	59311	01370	98630	41
20	9.39369	10.60631	9.40742	10.59258	10.01373	9.98627	40
21	39418	60582	40795	59205	01377	98623	39
22	39467	60533	40847	59153	01380	98620	38
23	39517	60483	40900	59100	01383	98617	37
24	39566	60434	40952	59048	01386	98614	36
25	9.39615	10.60385	9.41005	10.58995	10.01390	9.98610	35
26	39664	60336	41057	58943	01393	98607	34
27	39713	60287	41109	58891	01396	98604	33
28	39762	60238	41161	58839	01399	98601	32
29	39811	60189	41214	58786	01403	98597	31
30	9.39860	10.60140	9.41266	10.58734	10.01406	9.98594	30
31	39909	60091	41318	58682	01409	98591	29
32	39958	60042	41370	58630	01412	98588	28
33	40006	59994	41422	58578	01416	98584	27
34	40055	59945	41474	58526	01419	98581	26
35	9.40103	10.59897	9.41526	10.58474	10.01422	9.98578	25
36	40152	59848	41578	58422	01426	98574	24
37	40200	59800	41629	58371	01429	98571	23
38	40249	59751	41681	58319	01432	98568	22
39	40297	59703	41733	58267	01435	98565	21
40	9.40346	10.59654	9.41784	10.58216	10.01439	9.98561	20
41	40394	59606	41836	58164	01442	98558	19
42	40442	59558	41887	58113	01445	98555	18
43	40490	59510	41939	58061	01449	98551	17
44	40538	59462	41990	58010	01452	98548	16
45	9.40586	10.59414	9.42041	10.57959	10.01455	9.98545	15
46	40634	59366	42093	57907	01459	98541	14
47	40682	59318	42144	57856	01462	98538	13
48	40730	59270	42195	57805	01465	98535	12
49	40778	59222	42246	57754	01469	98531	11
50	9.40825	10.59175	9.42297	10.57703	10.01472	9.98528	10
51	40873	59127	42348	57652	01475	98525	9
52	40921	59079	42399	57601	01479	98521	8
53	40968	59032	42450	57550	01482	98518	7
54	41016	58984	42501	57499	01485	98515	6
55	9.41063	10.58937	9.42552	10.57448	10.01489	9.98511	5
56	41111	58889	42603	57397	01492	98508	4
57	41158	58842	42653	57347	01495	98505	3
58	41205	58795	42704	57296	01499	98501	2
59	41252	58748	42755	57245	01502	98498	1
60	41300	58700	42805	57195	01506	98494	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

104° **75°**

15°			Logarithms.				164°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.41300	10.58700	9.42805	10.57195	10.01506	9.98494	60
1	41347	58653	42856	57144	01509	98491	59
2	41394	58606	42906	57094	01512	98488	58
3	41441	58559	42957	57043	01516	98484	57
4	41488	58512	43007	56993	01519	98481	56
5	9.41535	10.58465	9.43057	10.56943	10.01523	9.98477	55
6	41582	58418	43108	56892	01526	98474	54
7	41628	58372	43158	56842	01529	98471	53
8	41675	58325	43208	56792	01533	98467	52
9	41722	58278	43258	56742	01536	98464	51
10	9.41768	10.58232	9.43308	10.56692	10.01540	9.98460	50
11	41815	58185	43358	56642	01543	98457	49
12	41861	58139	43408	56592	01547	98453	48
13	41908	58092	43458	56542	01550	98450	47
14	41954	58046	43508	56492	01553	98447	46
15	9.42001	10.57999	9.43558	10.56442	10.01557	9.98443	45
16	42047	57953	43607	56393	01560	98440	44
17	42093	57907	43657	56343	01564	98436	43
18	42140	57860	43707	56293	01567	98433	42
19	42186	57814	43756	56244	01571	98429	41
20	9.42232	10.57768	9.43806	10.56194	10.01574	9.98426	40
21	42278	57722	43855	56145	01578	98422	39
22	42324	57676	43905	56095	01581	98419	38
23	42370	57630	43954	56046	01585	98415	37
24	42416	57584	44004	55996	01588	98412	36
25	9.42461	10.57539	9.44053	10.55947	10.01591	9.98409	35
26	42507	57493	44102	55898	01595	98405	34
27	42553	57447	44151	55849	01598	98402	33
28	42599	57401	44201	55799	01602	98398	32
29	42644	57356	44250	55750	01605	98395	31
30	9.42690	10.57310	9.44299	10.55701	10.01609	9.98391	30
31	42735	57265	44348	55652	01612	98388	29
32	42781	57219	44397	55603	01616	98384	28
33	42826	57174	44446	55554	01619	98381	27
34	42872	57128	44495	55505	01623	98377	26
35	9.42917	10.57083	9.44544	10.55456	10.01627	9.98373	25
36	42962	57038	44592	55408	01630	98370	24
37	43008	56992	44641	55359	01634	98366	23
38	43053	56947	44690	55310	01637	98363	22
39	43098	56902	44738	55262	01641	98359	21
40	9.43143	10.56857	9.44787	10.55213	10.01644	9.98356	20
41	43188	56812	44836	55164	01648	98352	19
42	43233	56767	44884	55116	01651	98349	18
43	43278	56722	44933	55067	01655	98345	17
44	43323	56677	44981	55019	01658	98342	16
45	9.43367	10.56633	9.45029	10.54971	10.01662	9.98338	15
46	43412	56588	45078	54922	01666	98334	14
47	43457	56543	45126	54874	01669	98331	13
48	43502	56498	45174	54826	01673	98327	12
49	43546	56454	45222	54778	01676	98324	11
50	9.43591	10.56409	9.45271	10.54729	10.01680	9.98320	10
51	43635	56365	45319	54681	01683	98317	9
52	43680	56320	45367	54633	01687	98313	8
53	43724	56276	45415	54585	01691	98309	7
54	43769	56231	45463	54537	01694	98306	6
55	9.43813	10.56187	9.45511	10.54489	10.01698	9.98302	5
56	43857	56143	45559	54441	01701	98299	4
57	43901	56099	45606	54394	01705	98295	3
58	43946	56054	45654	54346	01709	98291	2
59	43990	56010	45702	54298	01712	98288	1
60	44034	55966	45750	54250	01716	98284	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
105°							74°

16° **Logarithms.** **163°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.44034	10.55966	9.45750	10.54250	10.01716	9.98284	60
1	44078	55922	45797	54203	01719	98281	59
2	44122	55878	45845	54155	01723	98277	58
3	44166	55834	45892	54108	01727	98273	57
4	44210	55790	45940	54060	01730	98270	56
5	9.44253	10.55747	9.45987	10.54013	10.01734	9.98266	55
6	44297	55703	46035	53965	01738	98262	54
7	44341	55659	46082	53918	01741	98259	53
8	44385	55615	46130	53870	01745	98255	52
9	44428	55572	46177	53823	01749	98251	51
10	9.44472	10.55528	9.46224	10.53776	10.01752	9.98248	50
11	44516	55484	46271	53729	01756	98244	49
12	44559	55441	46319	53681	01760	98240	48
13	44602	55398	46366	53634	01763	98237	47
14	44646	55354	46413	53587	01767	98233	46
15	9.44689	10.55311	9.46460	10.53540	10.01771	9.98229	45
16	44733	55267	46507	53493	01774	98226	44
17	44776	55224	46554	53446	01778	98222	43
18	44819	55181	46601	53399	01782	98218	42
19	44862	55138	46648	53352	01785	98215	41
20	9.44905	10.55095	9.46694	10.53306	10.01789	9.98211	40
21	44948	55052	46741	53259	01793	98207	39
22	44992	55008	46788	53212	01796	98204	38
23	45035	54965	46835	53165	01800	98200	37
24	45077	54923	46881	53119	01804	98196	36
25	9.45120	10.54880	9.46928	10.53072	10.01808	9.98192	35
26	45163	54837	46975	53025	01811	98189	34
27	45206	54794	47021	52979	01815	98185	33
28	45249	54751	47068	52932	01819	98181	32
29	45292	54708	47114	52886	01823	98177	31
30	9.45334	10.54666	9.47160	10.52840	10.01826	9.98174	30
31	45377	54623	47207	52793	01830	98170	29
32	45419	54581	47253	52747	01834	98166	28
33	45462	54538	47299	52701	01838	98162	27
34	45504	54496	47346	52654	01841	98159	26
35	9.45547	10.54453	9.47392	10.52608	10.01845	9.98155	25
36	45589	54411	47438	52562	01849	98151	24
37	45632	54368	47484	52516	01853	98147	23
38	45674	54326	47530	52470	01856	98144	22
39	45716	54284	47576	52424	01860	98140	21
40	9.45758	10.54242	9.47622	10.52378	10.01864	9.98136	20
41	45801	54199	47668	52332	01868	98132	19
42	45843	54157	47714	52286	01871	98129	18
43	45885	54115	47760	52240	01875	98125	17
44	45927	54073	47806	52194	01879	98121	16
45	9.45969	10.54031	9.47852	10.52148	10.01883	9.98117	15
46	46011	53989	47897	52103	01887	98113	14
47	46053	53947	47943	52057	01890	98110	13
48	46095	53905	47989	52011	01894	98106	12
49	46136	53864	48035	51965	01898	98102	11
50	9.46178	10.53822	9.48080	10.51920	10.01902	9.98098	10
51	46220	53780	48126	51874	01906	98094	9
52	46262	53738	48171	51829	01910	98090	8
53	46303	53697	48217	51783	01913	98087	7
54	46345	53655	48262	51738	01917	98083	6
55	9.46386	10.53614	9.48307	10.51693	10.01921	9.98079	5
56	46428	53572	48353	51647	01925	98075	4
57	46469	53531	48398	51602	01929	98071	3
58	46511	53489	48443	51557	01933	98067	2
59	46552	53448	48489	51511	01937	98063	1
60	46594	53406	48534	51466	01940	98060	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

106° **73°**

17° **Logarithms.** **162°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.46594	10.53406	9.48534	10.51466	10.01940	9.98060	60
1	46635	53365	48579	51421	01944	98056	59
2	46676	53324	48624	51376	01948	98052	58
3	46717	53283	48669	51331	01952	98048	57
4	46758	53242	48714	51286	01956	98044	56
5	9.46800	10.53200	9.48759	10.51241	10.01960	9.98040	55
6	46841	53159	48804	51196	01964	98036	54
7	46882	53118	48849	51151	01968	98032	53
8	46923	53077	48894	51106	01971	98029	52
9	46964	53036	48939	51061	01975	98025	51
10	9.47005	10.52995	9.48984	10.51016	10.01979	9.98021	50
11	47045	52955	49029	50971	01983	98017	49
12	47086	52914	49073	50927	01987	98013	48
13	47127	52873	49118	50882	01991	98009	47
14	47168	52832	49163	50837	01995	98005	46
15	9.47209	10.52791	9.49207	10.50793	10.01999	9.98001	45
16	47249	52751	49252	50748	02003	97997	44
17	47290	52710	49296	50704	02007	97993	43
18	47330	52670	49341	50659	02011	97989	42
19	47371	52629	49385	50615	02014	97986	41
20	9.47411	10.52589	9.49430	10.50570	10.02018	9.97982	40
21	47452	52548	49474	50526	02022	97978	39
22	47492	52508	49519	50481	02026	97974	38
23	47533	52467	49563	50437	02030	97970	37
24	47573	52427	49607	50393	02034	97966	36
25	9.47613	10.52387	9.49652	10.50348	10.02038	9.97962	35
26	47654	52346	49696	50304	02042	97958	34
27	47694	52306	49740	50260	02046	97954	33
28	47734	52266	49784	50216	02050	97950	32
29	47774	52226	49828	50172	02054	97946	31
30	9.47814	10.52186	9.49872	10.50128	10.02058	9.97942	30
31	47854	52146	49916	50084	02062	97938	29
32	47894	52106	49960	50040	02066	97934	28
33	47934	52066	50004	49996	02070	97930	27
34	47974	52026	50048	49952	02074	97926	26
35	9.48014	10.51986	9.50092	10.49908	10.02078	9.97922	25
36	48054	51946	50136	49864	02082	97918	24
37	48094	51906	50180	49820	02086	97914	23
38	48133	51867	50223	49777	02090	97910	22
39	48173	51827	50267	49733	02094	97906	21
40	9.48213	10.51787	9.50311	10.49689	10.02098	9.97902	20
41	48252	51748	50355	49645	02102	97898	19
42	48292	51708	50398	49602	02106	97894	18
43	48332	51668	50442	49558	02110	97890	17
44	48371	51629	50485	49515	02114	97886	16
45	9.48411	10.51589	9.50529	10.49471	10.02118	9.97882	15
46	48450	51550	50572	49428	02122	97878	14
47	48490	51510	50616	49384	02126	97874	13
48	48529	51471	50659	49341	02130	97870	12
49	48568	51432	50703	49297	02134	97866	11
50	9.48607	10.51393	9.50746	10.49254	10.02139	9.97861	10
51	48647	51353	50789	49211	02143	97857	9
52	48686	51314	50833	49167	02147	97853	8
53	48725	51275	50876	49124	02151	97849	7
54	48764	51236	50919	49081	02155	97845	6
55	9.48803	10.51197	9.50962	10.49038	10.02159	9.97841	5
56	48842	51158	51005	48995	02163	97837	4
57	48881	51119	51048	48952	02167	97833	3
58	48920	51080	51092	48908	02171	97829	2
59	48959	51041	51135	48865	02175	97825	1
60	48998	51002	51178	48822	02179	97821	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

107° **72°**

18° **Logarithms.** 161°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.48998	10.51002	9.51178	10.48822	10.02179	9.97821	60
1	49037	50963	51221	48779	02183	97817	59
2	49076	50924	51264	48736	02188	97812	58
3	49115	50885	51306	48694	02192	97808	57
4	49153	50847	51349	48651	02196	97804	56
5	9.49192	10.50808	9.51392	10.48608	10.02200	9.97800	55
6	49231	50769	51435	48565	02204	97796	54
7	49269	50731	51478	48522	02208	97792	53
8	49308	50692	51520	48480	02212	97788	52
9	49347	50653	51563	48437	02216	97784	51
10	9.49385	10.50615	9.51606	10.48394	10.02221	9.97779	50
11	49424	50576	51648	48352	02225	97775	49
12	49462	50538	51691	48309	02229	97771	48
13	49500	50500	51734	48266	02233	97767	47
14	49539	50461	51776	48224	02237	97763	46
15	9.49577	10.50423	9.51819	10.48181	10.02241	9.97759	45
16	49615	50385	51861	48139	02246	97754	44
17	49654	50346	51903	48097	02250	97750	43
18	49692	50308	51946	48054	02254	97746	42
19	49730	50270	51988	48012	02258	97742	41
20	9.49768	10.50232	9.52031	10.47969	10.02262	9.97738	40
21	49806	50194	52073	47927	02266	97734	39
22	49844	50156	52115	47885	02271	97729	38
23	49882	50118	52157	47843	02275	97725	37
24	49920	50080	52200	47800	02279	97721	36
25	9.49958	10.50042	9.52242	10.47758	10.02283	9.97717	35
26	49996	50004	52284	47716	02287	97713	34
27	50034	49966	52326	47674	02292	97708	33
28	50072	49928	52368	47632	02296	97704	32
29	50110	49890	52410	47590	02300	97700	31
30	9.50148	10.49852	9.52452	10.47548	10.02304	9.97696	30
31	50185	49815	52494	47506	02309	97691	29
32	50223	49777	52536	47464	02313	97687	28
33	50261	49739	52578	47422	02317	97683	27
34	50298	49702	52620	47380	02321	97679	26
35	9.50336	10.49664	9.52661	10.47339	10.02326	9.97674	25
36	50374	49626	52703	47297	02330	97670	24
37	50411	49589	52745	47255	02334	97666	23
38	50449	49551	52787	47213	02338	97662	22
39	50486	49514	52829	47171	02343	97657	21
40	9.50523	10.49477	9.52870	10.47130	10.02347	9.97653	20
41	50561	49439	52912	47088	02351	97649	19
42	50598	49402	52953	47047	02355	97645	18
43	50635	49365	52995	47005	02360	97640	17
44	50673	49327	53037	46963	02364	97636	16
45	9.50710	10.49290	9.53078	10.46922	10.02368	9.97632	15
46	50747	49253	53120	46880	02372	97628	14
47	50784	49216	53161	46839	02377	97623	13
48	50821	49179	53202	46798	02381	97619	12
49	50858	49142	53244	46756	02385	97615	11
50	9.50896	10.49104	9.53285	10.46715	10.02390	9.97610	10
51	50933	49067	53327	46673	02394	97606	9
52	50970	49030	53368	46632	02398	97602	8
53	51007	48993	53409	46591	02403	97597	7
54	51043	48957	53450	46550	02407	97593	6
55	9.51080	10.48920	9.53492	10.46508	10.02411	9.97589	5
56	51117	48883	53533	46467	02416	97584	4
57	51154	48846	53574	46426	02420	97580	3
58	51191	48809	53615	46385	02424	97576	2
59	51227	48773	53656	46344	02429	97571	1
60	51264	48736	53697	46303	02433	97567	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

108° 71°

19°			Logarithms.				160°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.51264	10.48736	9.53697	10.46303	10.02433	9.97567	60
1	51301	48699	53738	46262	02437	97563	59
2	51338	48662	53779	46221	02442	97558	58
3	51374	48626	53820	46180	02446	97554	57
4	51411	48589	53861	46139	02450	97550	56
5	9.51447	10.48553	9.53902	10.46098	10.02455	9.97545	55
6	51484	48516	53943	46057	02459	97541	54
7	51520	48480	53984	46016	02464	97536	53
8	51557	48443	54025	45975	02468	97532	52
9	51593	48407	54065	45935	02472	97528	51
10	9.51629	10.48371	9.54106	10.45894	10.02477	9.97523	50
11	51666	48334	54147	45853	02481	97519	49
12	51702	48298	54187	45813	02485	97515	48
13	51738	48262	54228	45772	02490	97510	47
14	51774	48226	54269	45731	02494	97506	46
15	9.51811	10.48189	9.54309	10.45691	10.02499	9.97501	45
16	51847	48153	54350	45650	02503	97497	44
17	51883	48117	54390	45610	02508	97492	43
18	51919	48081	54431	45569	02512	97488	42
19	51955	48045	54471	45529	02516	97484	41
20	9.51991	10.48009	9.54512	10.45488	10.02521	9.97479	40
21	52027	47973	54552	45448	02525	97475	39
22	52063	47937	54593	45407	02530	97470	38
23	52099	47901	54633	45367	02534	97466	37
24	52135	47865	54673	45327	02539	97461	36
25	9.52171	10.47829	9.54714	10.45286	10.02543	9.97457	35
26	52207	47793	54754	45246	02547	97453	34
27	52242	47758	54794	45206	02552	97448	33
28	52278	47722	54835	45165	02556	97444	32
29	52314	47686	54875	45125	02561	97439	31
30	9.52350	10.47650	9.54915	10.45085	10.02565	9.97435	30
31	52385	47615	54955	45045	02570	97430	29
32	52421	47579	54995	45005	02574	97426	28
33	52456	47544	55035	44965	02579	97421	27
34	52492	47508	55075	44925	02583	97417	26
35	9.52527	10.47473	9.55115	10.44885	10.02588	9.97412	25
36	52563	47437	55155	44845	02592	97408	24
37	52598	47402	55195	44805	02597	97403	23
38	52634	47366	55235	44765	02601	97399	22
39	52669	47331	55275	44725	02606	97394	21
40	9.52705	10.47295	9.55315	10.44685	10.02610	9.97390	20
41	52740	47260	55355	44645	02615	97385	19
42	52775	47225	55395	44605	02619	97381	18
43	52811	47189	55434	44566	02624	97376	17
44	52846	47154	55474	44526	02628	97372	16
45	9.52881	10.47119	9.55514	10.44486	10.02633	9.97367	15
46	52916	47084	55554	44446	02637	97363	14
47	52951	47049	55593	44407	02642	97358	13
48	52986	47014	55633	44367	02647	97353	12
49	53021	46979	55673	44327	02651	97349	11
50	9.53056	10.46944	9.55712	10.44288	10.02656	9.97344	10
51	53092	46908	55752	44248	02660	97340	9
52	53126	46874	55791	44209	02665	97335	8
53	53161	46839	55831	44169	02669	97331	7
54	53196	46804	55870	44130	02674	97326	6
55	9.53231	10.46769	9.55910	10.44090	10.02678	9.97322	5
56	53266	46734	55949	44051	02683	97317	4
57	53301	46699	55989	44011	02688	97312	3
58	53336	46664	56028	43972	02692	97308	2
59	53370	46630	56067	43933	02697	97303	1
60	53405	46595	56107	43893	02701	97299	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
109°							70°

20° **Logarithms.** **159°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.53405	10.46595	9.56107	10.43893	10.02701	9.97299	60
1	53440	46560	56146	43854	02706	97294	59
2	53475	46525	56185	43815	02711	97289	58
3	53509	46491	56224	43776	02715	97285	57
4	53544	46456	56264	43736	02720	97280	56
5	9.53578	10.46422	9.56303	10.43697	10.02724	9.97276	55
6	53613	46387	56342	43658	02729	97271	54
7	53647	46353	56381	43619	02734	97266	53
8	53682	46318	56420	43580	02738	97262	52
9	53716	46284	56459	43541	02743	97257	51
10	9.53751	10.46249	9.56498	10.43502	10.02748	9.97252	50
11	53785	46215	56537	43463	02752	97248	49
12	53819	46181	56576	43424	02757	97243	48
13	53854	46146	56615	43385	02762	97238	47
14	53888	46112	56654	43346	02766	97234	46
15	9.53922	10.46078	9.56693	10.43307	10.02771	9.97229	45
16	53957	46043	56732	43268	02776	97224	44
17	53991	46009	56771	43229	02780	97220	43
18	54025	45975	56810	43190	02785	97215	42
19	54059	45941	56849	43151	02790	97210	41
20	9.54093	10.45907	9.56887	10.43113	10.02794	9.97206	40
21	54127	45873	56926	43074	02799	97201	39
22	54161	45839	56965	43035	02804	97196	38
23	54195	45805	57004	42996	02808	97192	37
24	54229	45771	57042	42958	02813	97187	36
25	9.54263	10.45737	9.57081	10.42919	10.02818	9.97182	35
26	54297	45703	57120	42880	02822	97178	34
27	54331	45669	57158	42842	02827	97173	33
28	54365	45635	57197	42803	02832	97168	32
29	54399	45601	57235	42765	02837	97163	31
30	9.54433	10.45567	9.57274	10.42726	10.02841	9.97159	30
31	54466	45534	57312	42688	02846	97154	29
32	54500	45500	57351	42649	02851	97149	28
33	54534	45466	57389	42611	02855	97145	27
34	54567	45433	57428	42572	02860	97140	26
35	9.54601	10.45399	9.57466	10.42534	10.02865	9.97135	25
36	54635	45365	57504	42496	02870	97130	24
37	54668	45332	57543	42457	02874	97126	23
38	54702	45298	57581	42419	02879	97121	22
39	54735	45265	57619	42381	02884	97116	21
40	9.54769	10.45231	9.57658	10.42342	10.02889	9.97111	20
41	54802	45198	57696	42304	02893	97107	19
42	54836	45164	57734	42266	02898	97102	18
43	54869	45131	57772	42228	02903	97097	17
44	54903	45097	57810	42190	02908	97092	16
45	9.54936	10.45064	9.57849	10.42151	10.02913	9.97087	15
46	54969	45031	57887	42113	02917	97083	14
47	55003	44997	57925	42075	02922	97078	13
48	55036	44964	57963	42037	02927	97073	12
49	55069	44931	58001	41999	02932	97068	11
50	9.55102	10.44898	9.58039	10.41961	10.02937	9.97063	10
51	55136	44864	58077	41923	02941	97059	9
52	55169	44831	58115	41885	02946	97054	8
53	55202	44798	58153	41847	02951	97049	7
54	55235	44765	58191	41809	02956	97044	6
55	9.55268	10.44732	9.58229	10.41771	10.02961	9.97039	5
56	55301	44699	58267	41733	02965	97035	4
57	55334	44666	58304	41696	02970	97030	3
58	55367	44633	58342	41658	02975	97025	2
59	55400	44600	58380	41620	02980	97020	1
60	55433	44567	58418	41582	02985	97015	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

110° **69°**

21° **Logarithms.** 158°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.55433	10.44567	9.58418	10.41582	10.02985	9.97015	60
1	55466	44534	58455	41545	02990	97010	59
2	55499	44501	58493	41507	02995	97005	58
3	55532	44468	58531	41469	02999	97001	57
4	55564	44436	58569	41431	03004	96996	56
5	9.55597	10.44403	9.58606	10.41394	10.03009	9.96991	55
6	55630	44370	58644	41356	03014	96986	54
7	55663	44337	58681	41319	03019	96981	53
8	55695	44305	58719	41281	03024	96976	52
9	55728	44272	58757	41243	03029	96971	51
10	9.55761	10.44239	9.58794	10.41206	10.03034	9.96966	50
11	55793	44207	58832	41168	03038	96962	49
12	55826	44174	58869	41131	03043	96957	48
13	55858	44142	58907	41093	03048	96952	47
14	55891	44109	58944	41056	03053	96947	46
15	9.55923	10.44077	9.58981	10.41019	10.03058	9.96942	45
16	55956	44044	59019	40981	03063	96937	44
17	55988	44012	59056	40944	03068	96932	43
18	56021	43979	59094	40906	03073	96927	42
19	56053	43947	59131	40869	03078	96922	41
20	9.56085	10.43915	9.59168	10.40832	10.03083	9.96917	40
21	56118	43882	59205	40795	03088	96912	39
22	56150	43850	59243	40757	03093	96907	38
23	56182	43818	59280	40720	03097	96903	37
24	56215	43785	59317	40683	03102	96898	36
25	9.56247	10.43753	9.59354	10.40646	10.03107	9.96893	35
26	56279	43721	59391	40609	03112	96888	34
27	56311	43689	59429	40571	03117	96883	33
28	56343	43657	59466	40534	03122	96878	32
29	56375	43625	59503	40497	03127	96873	31
30	9.56408	10.43592	9.59540	10.40460	10.03132	9.96868	30
31	56440	43560	59577	40423	03137	96863	29
32	56472	43528	59614	40386	03142	96858	28
33	56504	43496	59651	40349	03147	96853	27
34	56536	43464	59688	40312	03152	96848	26
35	9.56568	10.43432	9.59725	10.40275	10.03157	9.96843	25
36	56599	43401	59762	40238	03162	96838	24
37	56631	43369	59799	40201	03167	96833	23
38	56663	43337	59835	40165	03172	96828	22
39	56695	43305	59872	40128	03177	96823	21
40	9.56727	10.43273	9.59909	10.40091	10.03182	9.96818	20
41	56759	43241	59946	40054	03187	96813	19
42	56790	43210	59983	40017	03192	96808	18
43	56822	43178	60019	39981	03197	96803	17
44	56854	43146	60056	39944	03202	96798	16
45	9.56886	10.43114	9.60093	10.39907	10.03207	9.96793	15
46	56917	43083	60130	39870	03212	96788	14
47	56949	43051	60166	39834	03217	96783	13
48	56980	43020	60203	39797	03222	96778	12
49	57012	42988	60240	39760	03228	96772	11
50	9.57044	10.42956	9.60276	10.39724	10.03233	9.96767	10
51	57075	42925	60313	39687	03238	96762	9
52	57107	42893	60349	39651	03243	96757	8
53	57138	42862	60386	39614	03248	96752	7
54	57169	42831	60422	39578	03253	96747	6
55	9.57201	10.42799	9.60459	10.39541	10.03258	9.96742	5
56	57232	42768	60495	39505	03263	96737	4
57	57264	42736	60532	39468	03268	96732	3
58	57295	42705	60568	39432	03273	96727	2
59	57326	42674	60605	39395	03278	96722	1
60	57358	42642	60641	39359	03283	96717	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

111° 68°

22° **Logarithms.** 157°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.57358	10.42642	9.60641	10.39359	10.03283	9.96717	60
1	57389	42611	60677	39323	03289	96711	59
2	57420	42580	60714	39286	03294	96706	58
3	57451	42549	60750	39250	03299	96701	57
4	57482	42518	60786	39214	03304	96696	56
5	9.57514	10.42486	9.60823	10.39177	10.03309	9.96691	55
6	57545	42455	60859	39141	03314	96686	54
7	57576	42424	60895	39105	03319	96681	53
8	57607	42393	60931	39069	03324	96676	52
9	57638	42362	60967	39033	03330	96670	51
10	9.57669	10.42331	9.61004	10.38996	10.03335	9.96665	50
11	57700	42300	61040	38960	03340	96660	49
12	57731	42269	61076	38924	03345	96655	48
13	57762	42238	61112	38888	03350	96650	47
14	57793	42207	61148	38852	03355	96645	46
15	9.57824	10.42176	9.61184	10.38816	10.03360	9.96640	45
16	57855	42145	61220	38780	03366	96634	44
17	57885	42115	61256	38744	03371	96629	43
18	57916	42084	61292	38708	03376	96624	42
19	57947	42053	61328	38672	03381	96619	41
20	9.57978	10.42022	9.61364	10.38636	10.03386	9.96614	40
21	58008	41992	61400	38600	03392	96608	39
22	58039	41961	61436	38564	03397	96603	38
23	58070	41930	61472	38528	03402	96598	37
24	58101	41899	61508	38492	03407	96593	36
25	9.58131	10.41869	9.61544	10.38456	10.03412	9.96588	35
26	58162	41838	61579	38421	03418	96582	34
27	58192	41808	61615	38385	03423	96577	33
28	58223	41777	61651	38349	03428	96572	32
29	58253	41747	61687	38313	03433	96567	31
30	9.58284	10.41716	9.61722	10.38278	10.03438	9.96562	30
31	58314	41686	61758	38242	03444	96556	29
32	58345	41655	61794	38206	03449	96551	28
33	58375	41625	61830	38170	03454	96546	27
34	58406	41594	61865	38135	03459	96541	26
35	9.58436	10.41564	9.61901	10.38099	10.03465	9.96535	25
36	58467	41533	61936	38064	03470	96530	24
37	58497	41503	61972	38028	03475	96525	23
38	58527	41473	62008	37992	03480	96520	22
39	58557	41443	62043	37957	03486	96514	21
40	9.58588	10.41412	9.62079	10.37921	10.03491	9.96509	20
41	58618	41382	62114	37886	03496	96504	19
42	58648	41352	62150	37850	03502	96498	18
43	58678	41322	62185	37815	03507	96493	17
44	58709	41291	62221	37779	03512	96488	16
45	9.58739	10.41261	9.62256	10.37744	10.03517	9.96483	15
46	58769	41231	62292	37708	03523	96477	14
47	58799	41201	62327	37673	03528	96472	13
48	58829	41171	62362	37638	03533	96467	12
49	58859	41141	62398	37602	03539	96461	11
50	9.58889	10.41111	9.62433	10.37567	10.03544	9.96456	10
51	58919	41081	62468	37532	03549	96451	9
52	58949	41051	62504	37496	03555	96445	8
53	58979	41021	62539	37461	03560	96440	7
54	59009	40991	62574	37426	03565	96435	6
55	9.59039	10.40961	9.62609	10.37391	10.03571	9.96429	5
56	59069	40931	62645	37355	03576	96424	4
57	59098	40902	62680	37320	03581	96419	3
58	59128	40872	62715	37285	03587	96413	2
59	59158	40842	62750	37250	03592	96408	1
60	59188	40812	62785	37215	03597	96403	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

112° 67°

23° **Logarithms.** **156°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.59188	10.40812	9.62785	10.37215	10.03597	9.96403	60
1	59218	40782	62820	37180	03603	96397	59
2	59247	40753	62855	37145	03608	96392	58
3	59277	40723	62890	37110	03613	96387	57
4	59307	40693	62926	37074	03619	96381	56
5	9.59336	10.40664	9.62961	10.37039	10.03624	9.96376	55
6	59366	40634	62996	37004	03630	96370	54
7	59396	40604	63031	36969	03635	96365	53
8	59425	40575	63066	36934	03640	96360	52
9	59455	40545	63101	36899	03646	96354	51
10	9.59484	10.40516	9.63135	10.36865	10.03651	9.96349	50
11	59514	40486	63170	36830	03657	96343	49
12	59543	40457	63205	36795	03662	96338	48
13	59573	40427	63240	36760	03667	96333	47
14	59602	40398	63275	36725	03673	96327	46
15	9.59632	10.40368	9.63310	10.36690	10.03678	9.96322	45
16	59661	40339	63345	36655	03684	96316	44
17	59690	40310	63379	36621	03689	96311	43
18	59720	40280	63414	36586	03695	96305	42
19	59749	40251	63449	36551	03700	96300	41
20	9.59778	10.40222	9.63484	10.36516	10.03706	9.96294	40
21	59808	40192	63519	36481	03711	96289	39
22	59837	40163	63553	36447	03716	96284	38
23	59866	40134	63588	36412	03722	96278	37
24	59895	40105	63623	36377	03727	96273	36
25	9.59924	10.40076	9.63657	10.36343	10.03733	9.96267	35
26	59954	40046	63692	36308	03738	96262	34
27	59983	40017	63726	36274	03744	96256	33
28	60012	39988	63761	36239	03749	96251	32
29	60041	39959	63796	36204	03755	96245	31
30	9.60070	10.39930	9.63830	10.36170	10.03760	9.96240	30
31	60099	39901	63865	36135	03766	96234	29
32	60128	39872	63899	36101	03771	96229	28
33	60157	39843	63934	36066	03777	96223	27
34	60186	39814	63968	36032	03782	96218	26
35	9.60215	10.39785	9.64003	10.35997	10.03788	9.96212	25
36	60244	39756	64037	35963	03793	96207	24
37	60273	39727	64072	35928	03799	96201	23
38	60302	39698	64106	35894	03804	96196	22
39	60331	39669	64140	35860	03810	96190	21
40	9.60359	10.39641	9.64175	10.35825	10.03815	9.96185	20
41	60388	39612	64209	35791	03821	96179	19
42	60417	39583	64243	35757	03826	96174	18
43	60446	39554	64278	35722	03832	96168	17
44	60474	39526	64312	35688	03838	96162	16
45	9.60503	10.39497	9.64346	10.35654	10.03843	9.96157	15
46	60532	39468	64381	35619	03849	96151	14
47	60561	39439	64415	35585	03854	96146	13
48	60589	39411	64449	35551	03860	96140	12
49	60618	39382	64483	35517	03865	96135	11
50	9.60646	10.39354	9.64517	10.35483	10.03871	9.96129	10
51	60675	39325	64552	35448	03877	96123	9
52	60704	39296	64586	35414	03882	96118	8
53	60732	39268	64620	35380	03888	96112	7
54	60761	39239	64654	35346	03893	96107	6
55	9.60789	10.39211	9.64688	10.35312	10.03899	9.96101	5
56	60818	39182	64722	35278	03905	96095	4
57	60846	39154	64756	35244	03910	96090	3
58	60875	39125	64790	35210	03916	96084	2
59	60903	39097	64824	35176	03921	96079	1
60	60931	39069	64858	35142	03927	96073	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

113° **66°**

24°			Logarithms.				155°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.60931	10.39069	9.64858	10.35142	10.03927	9.96073	60
1	60960	39040	64892	35108	03933	96067	59
2	60988	39012	64926	35074	03938	96062	58
3	61016	38984	64960	35040	03944	96056	57
4	61045	38955	64994	35006	03950	96050	56
5	9.61073	10.38927	9.65028	10.34972	10.03955	9.96045	55
6	61101	38899	65062	34938	03961	96039	54
7	61129	38871	65096	34904	03966	96034	53
8	61158	38842	65130	34870	03972	96028	52
9	61186	38814	65164	34836	03978	96022	51
10	9.61214	10.38786	9.65197	10.34803	10.03983	9.96017	50
11	61242	38758	65231	34769	03989	96011	49
12	61270	38730	65265	34735	03995	96005	48
13	61298	38702	65299	34701	04000	96000	47
14	61326	38674	65333	34667	04006	95994	46
15	9.61354	10.38646	9.65366	10.34634	10.04012	9.95988	45
16	61382	38618	65400	34600	04018	95982	44
17	61411	38589	65434	34566	04023	95977	43
18	61438	38562	65467	34533	04029	95971	42
19	61466	38534	65501	34499	04035	95965	41
20	9.61494	10.38506	9.65535	10.34465	10.04040	9.95960	40
21	61522	38478	65568	34432	04046	95954	39
22	61550	38450	65602	34398	04052	95948	38
23	61578	38422	65636	34364	04058	95942	37
24	61606	38394	65669	34331	04063	95937	36
25	9.61634	10.38366	9.65703	10.34297	10.04069	9.95931	35
26	61662	38338	65736	34264	04075	95925	34
27	61689	38311	65770	34230	04080	95920	33
28	61717	38283	65803	34197	04086	95914	32
29	61745	38255	65837	34163	04092	95908	31
30	9.61773	10.38227	9.65870	10.34130	10.04098	9.95902	30
31	61800	38200	65904	34096	04103	95897	29
32	61828	38172	65937	34063	04109	95891	28
33	61856	38144	65971	34029	04115	95885	27
34	61883	38117	66004	33996	04121	95879	26
35	9.61911	10.38089	9.66038	10.33962	10.04127	9.95873	25
36	61939	38061	66071	33929	04132	95868	24
37	61966	38034	66104	33896	04138	95862	23
38	61994	38006	66138	33862	04144	95856	22
39	62021	37979	66171	33829	04150	95850	21
40	9.62049	10.37951	9.66204	10.33796	10.04156	9.95844	20
41	62076	37924	66238	33762	04161	95839	19
42	62104	37896	66271	33729	04167	95833	18
43	62131	37869	66304	33696	04173	95827	17
44	62159	37841	66337	33663	04179	95821	16
45	9.62186	10.37814	9.66371	10.33629	10.04185	9.95815	15
46	62214	37786	66404	33596	04190	95810	14
47	62241	37759	66437	33563	04196	95804	13
48	62268	37732	66470	33530	04202	95798	12
49	62296	37704	66503	33497	04208	95792	11
50	9.62323	10.37677	9.66537	10.33463	10.04214	9.95786	10
51	62350	37650	66570	33430	04220	95780	9
52	62377	37623	66603	33397	04225	95775	8
53	62405	37595	66636	33364	04231	95769	7
54	62432	37568	66669	33331	04237	95763	6
55	9.62459	10.37541	9.66702	10.33298	10.04243	9.95757	5
56	62486	37514	66735	33265	04249	95751	4
57	62513	37487	66768	33232	04255	95745	3
58	62541	37459	66801	33199	04261	95739	2
59	62568	37432	66834	33166	04267	95733	1
60	62595	37405	66867	33133	04272	95728	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
114°							65°

25° **Logarithms.** 154°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.62595	10.37405	9.66867	10.33133	10.04272	9.95728	60
1	62622	37378	66900	33100	04278	95722	59
2	62649	37351	66933	33067	04284	95716	58
3	62676	37324	66966	33034	04290	95710	57
4	62703	37297	66999	33001	04296	95704	56
5	9.62730	10.37270	9.67032	10.32968	10.04302	9.95698	55
6	62757	37243	67065	32935	04308	95692	54
7	62784	37216	67098	32902	04314	95686	53
8	62811	37189	67131	32869	04320	95680	52
9	62838	37162	67163	32837	04326	95674	51
10	9.62865	10.37135	9.67196	10.32804	10.04332	9.95668	50
11	62892	37108	67229	32771	04337	95663	49
12	62918	37082	67262	32738	04343	95657	48
13	62945	37055	67295	32705	04349	95651	47
14	62972	37028	67327	32673	04355	95645	46
15	9.62999	10.37001	9.67360	10.32640	10.04361	9.95639	45
16	63026	36974	67393	32607	04367	95633	44
17	63052	36948	67426	32574	04373	95627	43
18	63079	36921	67458	32542	04379	95621	42
19	63106	36894	67491	32509	04385	95615	41
20	9.63133	10.36867	9.67524	10.32476	10.04391	9.95609	40
21	63159	36841	67556	32444	04397	95603	39
22	63186	36814	67589	32411	04403	95597	38
23	63213	36787	67622	32378	04409	95591	37
24	63239	36761	67654	32346	04415	95585	36
25	9.63266	10.36734	9.67687	10.32313	10.04421	9.95579	35
26	63292	36708	67719	32281	04427	95573	34
27	63319	36681	67752	32248	04433	95567	33
28	63345	36655	67785	32215	04439	95561	32
29	63372	36628	67817	32183	04445	95555	31
30	9.63398	10.36602	9.67850	10.32150	10.04451	9.95549	30
31	63425	36575	67882	32118	04457	95543	29
32	63451	36549	67915	32085	04463	95537	28
33	63478	36522	67947	32053	04469	95531	27
34	63504	36496	67980	32020	04475	95525	26
35	9.63531	10.36469	9.68012	10.31988	10.04481	9.95519	25
36	63557	36443	68044	31956	04487	95513	24
37	63583	36417	68077	31923	04493	95507	23
38	63610	36390	68109	31891	04500	95500	22
39	63636	36364	68142	31858	04506	95494	21
40	9.63662	10.36338	9.68174	10.31826	10.04512	9.95488	20
41	63689	36311	68206	31794	04518	95482	19
42	63715	36285	68239	31761	04524	95476	18
43	63741	36259	68271	31729	04530	95470	17
44	63767	36233	68303	31697	04536	95464	16
45	9.63794	10.36206	9.68336	10.31664	10.04542	9.95458	15
46	63820	36180	68368	31632	04548	95452	14
47	63846	36154	68400	31600	04554	95446	13
48	63872	36128	68432	31568	04560	95440	12
49	63898	36102	68465	31535	04566	95434	11
50	9.63924	10.36076	9.68497	10.31503	10.04573	9.95427	10
51	63950	36050	68529	31471	04579	95421	9
52	63976	36024	68561	31439	04585	95415	8
53	64002	35998	68593	31407	04591	95409	7
54	64028	35972	68626	31374	04597	95403	6
55	9.64054	10.35946	9.68658	10.31342	10.04603	9.95397	5
56	64080	35920	68690	31310	04609	95391	4
57	64106	35894	68722	31278	04616	95384	3
58	64132	35868	68754	31246	04622	95378	2
59	64158	35842	68786	31214	04628	95372	1
60	64184	35816	68818	31182	04634	95366	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

115° 64°

26° **Logarithms.** 153°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.64184	10.35816	9.68818	10.31182	10.04634	9.95366	60
1	64210	35790	68850	31150	04640	95360	59
2	64236	35764	68882	31118	04646	95354	58
3	64262	35738	68914	31086	04652	95348	57
4	64288	35712	68946	31054	04659	95341	56
5	9.64313	10.35687	9.68978	10.31022	10.04665	9.95335	55
6	64339	35661	69010	30990	04671	95329	54
7	64365	35635	69042	30958	04677	95323	53
8	64391	35609	69074	30926	04683	95317	52
9	64417	35583	69106	30894	04690	95310	51
10	9.64442	10.35558	9.69138	10.30862	10.04696	9.95304	50
11	64468	35532	69170	30830	04702	95298	49
12	64494	35506	69202	30798	04708	95292	48
13	64519	35481	69234	30766	04714	95286	47
14	64545	35455	69266	30734	04721	95279	46
15	9.64571	10.35429	9.69298	10.30702	10.04727	9.95273	45
16	64596	35404	69329	30671	04733	95267	44
17	64622	35378	69361	30639	04739	95261	43
18	64647	35353	69393	30607	04746	95254	42
19	64673	35327	69425	30575	04752	95248	41
20	9.64698	10.35302	9.69457	10.30543	10.04758	9.95242	40
21	64724	35276	69488	30512	04764	95236	39
22	64749	35251	69520	30480	04771	95229	38
23	64775	35225	69552	30448	04777	95223	37
24	64800	35200	69584	30416	04783	95217	36
25	9.64826	10.35174	9.69615	10.30385	10.04789	9.95211	35
26	64851	35149	69647	30353	04796	95204	34
27	64877	35123	69679	30321	04802	95198	33
28	64902	35098	69710	30290	04808	95192	32
29	64927	35073	69742	30258	04815	95185	31
30	9.64953	10.35047	9.69774	10.30226	10.04821	9.95179	30
31	64978	35022	69805	30195	04827	95173	29
32	65003	34997	69837	30163	04833	95167	28
33	65029	34971	69868	30132	04840	95160	27
34	65054	34946	69900	30100	04846	95154	26
35	9.65079	10.34921	9.69932	10.30068	10.04852	9.95148	25
36	65104	34896	69963	30037	04859	95141	24
37	65130	34870	69995	30005	04865	95135	23
38	65155	34845	70026	29974	04871	95129	22
39	65180	34820	70058	29942	04878	95122	21
40	9.65205	10.34795	9.70089	10.29911	10.04884	9.95116	20
41	65230	34770	70121	29879	04890	95110	19
42	65255	34745	70152	29848	04897	95103	18
43	65281	34719	70184	29816	04903	95097	17
44	65306	34694	70215	29785	04910	95090	16
45	9.65331	10.34669	9.70247	10.29753	10.04916	9.95084	15
46	65356	34644	70278	29722	04922	95078	14
47	65381	34619	70309	29691	04929	95071	13
48	65406	34594	70341	29659	04935	95065	12
49	65431	34569	70372	29628	04941	95059	11
50	9.65456	10.34544	9.70404	10.29596	10.04948	9.95052	10
51	65481	34519	70435	29565	04954	95046	9
52	65506	34494	70466	29534	04961	95039	8
53	65531	34469	70498	29502	04967	95033	7
54	65556	34444	70529	29471	04973	95027	6
55	9.65580	10.34420	9.70560	10.29440	10.04980	9.95020	5
56	65605	34395	70592	29408	04986	95014	4
57	65630	34370	70623	29377	04993	95007	3
58	65665	34345	70654	29346	04999	95001	2
59	65680	34320	70685	29315	05005	94995	1
60	65705	34295	70717	29283	05012	94988	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

116° 63°

27° **Logarithms.** **152°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.65705	10.34295	9.70717	10.29283	10.05012	9.94988	60
1	65729	34271	70748	29252	05018	94982	59
2	65754	34246	70779	29221	05025	94975	58
3	65779	34221	70810	29190	05031	94969	57
4	65804	34196	70841	29159	05038	94962	56
5	9.65828	10.34172	9.70873	10.29127	10.05044	9.94956	55
6	65853	34147	70904	29096	05051	94949	54
7	65878	34122	70935	29065	05057	94943	53
8	65902	34098	70966	29034	05064	94936	52
9	65927	34073	70997	29003	05070	94930	51
10	9.65952	10.34048	9.71028	10.28972	10.05077	9.94923	50
11	65976	34024	71059	28941	05083	94917	49
12	66001	33999	71090	28910	05089	94911	48
13	66025	33975	71121	28879	05096	94904	47
14	66050	33950	71153	28847	05102	94898	46
15	9.66075	10.33925	9.71184	10.28816	10.05109	9.94891	45
16	66099	33901	71215	28785	05115	94885	44
17	66124	33876	71246	28754	05122	94878	43
18	66148	33852	71277	28723	05129	94871	42
19	66173	33827	71308	28692	05135	94865	41
20	9.66197	10.33803	9.71339	10.28661	10.05142	9.94858	40
21	66221	33779	71370	28630	05148	94852	39
22	66246	33754	71401	28599	05155	94845	38
23	66270	33730	71431	28569	05161	94839	37
24	66295	33705	71462	28538	05168	94832	36
25	9.66319	10.33681	9.71493	10.28507	10.05174	9.94826	35
26	66343	33657	71524	28476	05181	94819	34
27	66368	33632	71555	28445	05187	94813	33
28	66392	33608	71586	28414	05194	94806	32
29	66416	33584	71617	28383	05201	94799	31
30	9.66441	10.33559	9.71648	10.28352	10.05207	9.94793	30
31	66465	33535	71679	28321	05214	94786	29
32	66489	33511	71709	28291	05220	94780	28
33	66513	33487	71740	28260	05227	94773	27
34	66537	33463	71771	28229	05233	94767	26
35	9.66562	10.33438	9.71802	10.28198	10.05240	9.94760	25
36	66586	33414	71833	28167	05247	94753	24
37	66610	33390	71863	28137	05253	94747	23
38	66634	33366	71894	28106	05260	94740	22
39	66658	33342	71925	28075	05266	94734	21
40	9.66682	10.33318	9.71955	10.28045	10.05273	9.94727	20
41	66706	33294	71986	28014	05280	94720	19
42	66731	33269	72017	27983	05286	94714	18
43	66755	33245	72048	27952	05293	94707	17
44	66779	33221	72078	27922	05300	94700	16
45	9.66803	10.33197	9.72109	10.27891	10.05306	9.94694	15
46	66827	33173	72140	27860	05313	94687	14
47	66851	33149	72170	27830	05320	94680	13
48	66875	33125	72201	27799	05326	94674	12
49	66899	33101	72231	27769	05333	94667	11
50	9.66922	10.33078	9.72262	10.27738	10.05340	9.94660	10
51	66946	33054	72293	27707	05346	94654	9
52	66970	33030	72323	27677	05353	94647	8
53	66994	33006	72354	27646	05360	94640	7
54	67018	32982	72384	27616	05366	94634	6
55	9.67042	10.32958	9.72415	10.27585	10.05373	9.94627	5
56	67066	32934	72445	27555	05380	94620	4
57	67090	32910	72476	27524	05386	94614	3
58	67113	32887	72506	27494	05393	94607	2
59	67137	32863	72537	27463	05400	94600	1
60	67161	32839	72567	27433	05407	94593	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

117° **62°**

28° **Logarithms.** **151°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.67161	10.32839	9.72567	10.27433	10.05407	9.94593	60
1	67185	32815	72598	27402	05413	94587	59
2	67208	32792	72628	27372	05420	94580	58
3	67232	32768	72659	27341	05427	94573	57
4	67256	32744	72689	27311	05433	94567	56
5	9.67280	10.32720	9.72720	10.27280	10.05440	9.94560	55
6	67303	32697	72750	27250	05447	94553	54
7	67327	32673	72780	27220	05454	94546	53
8	67350	32650	72811	27189	05460	94540	52
9	67374	32626	72841	27159	05467	94533	51
10	9.67398	10.32602	9.72872	10.27128	10.05474	9.94526	50
11	67421	32579	72902	27098	05481	94519	49
12	67445	32555	72932	27068	05487	94513	48
13	67468	32532	72963	27037	05494	94506	47
14	67492	32508	72993	27007	05501	94499	46
15	9.67515	10.32485	9.73023	10.26977	10.05508	9.94492	45
16	67539	32461	73054	26946	05515	94485	44
17	67562	32438	73084	26916	05521	94479	43
18	67586	32414	73114	26886	05528	94472	42
19	67609	32391	73144	26856	05535	94465	41
20	9.67633	10.32367	9.73175	10.26825	10.05542	9.94458	40
21	67656	32344	73205	26795	05549	94451	39
22	67680	32320	73235	26765	05555	94445	38
23	67703	32297	73265	26735	05562	94438	37
24	67726	32274	73295	26705	05569	94431	36
25	9.67750	10.32250	9.73326	10.26674	10.05576	9.94424	35
26	67773	32227	73356	26644	05583	94417	34
27	67796	32204	73386	26614	05590	94410	33
28	67820	32180	73416	26584	05596	94404	32
29	67843	32157	73446	26554	05603	94397	31
30	9.67866	10.32134	9.73476	10.26524	10.05610	9.94390	30
31	67890	32110	73507	26493	05617	94383	29
32	67913	32087	73537	26463	05624	94376	28
33	67936	32064	73567	26433	05631	94369	27
34	67959	32041	73597	26403	05638	94362	26
35	9.67982	10.32018	9.73627	10.26373	10.05645	9.94355	25
36	68006	31994	73657	26343	05651	94349	24
37	68029	31971	73687	26313	05658	94342	23
38	68052	31948	73717	26283	05665	94335	22
39	68075	31925	73747	26253	05672	94328	21
40	9.68098	10.31902	9.73777	10.26223	10.05679	9.94321	20
41	68121	31879	73807	26193	05686	94314	19
42	68144	31856	73837	26163	05693	94307	18
43	68167	31833	73867	26133	05700	94300	17
44	68190	31810	73897	26103	05707	94293	16
45	9.68213	10.31787	9.73927	10.26073	10.05714	9.94286	15
46	68237	31763	73957	26043	05721	94279	14
47	68260	31740	73987	26013	05727	94273	13
48	68283	31717	74017	25983	05734	94266	12
49	68305	31695	74047	25953	05741	94259	11
50	9.68328	10.31672	9.74077	10.25923	10.05748	9.94252	10
51	68351	31649	74107	25893	05755	94245	9
52	68374	31626	74137	25863	05762	94238	8
53	68397	31603	74166	25834	05769	94231	7
54	68420	31580	74196	25804	05776	94224	6
55	9.68443	10.31557	9.74226	10.25774	10.05783	9.94217	5
56	68466	31534	74256	25744	05790	94210	4
57	68489	31511	74286	25714	05797	94203	3
58	68512	31488	74316	25684	05804	94196	2
59	68534	31466	74345	25655	05811	94189	1
60	68557	31443	74375	25625	05818	94182	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

118° **61°**

29° **Logarithms.** **150°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.68557	10.31443	9.74375	10.25625	10.05818	9.94182	60
1	68580	31420	74405	25595	05825	94175	59
2	68603	31397	74435	25565	05832	94168	58
3	68625	31375	74465	25535	05839	94161	57
4	68648	31352	74494	25506	05846	94154	56
5	9.68671	10.31329	9.74524	10.25476	10.05853	9.94147	55
6	68694	31306	74554	25446	05860	94140	54
7	68716	31284	74583	25417	05867	94133	53
8	68739	31261	74613	25387	05874	94126	52
9	68762	31238	74643	25357	05881	94119	51
10	9.68784	10.31216	9.74673	10.25327	10.05888	9.94112	50
11	68807	31193	74702	25298	05895	94105	49
12	68829	31171	74732	25268	05902	94098	48
13	68852	31148	74762	25238	05910	94090	47
14	68875	31125	74791	25209	05917	94083	46
15	9.68897	10.31103	9.74821	10.25179	10.05924	9.94076	45
16	68920	31080	74851	25149	05931	94069	44
17	68942	31058	74880	25120	05938	94062	43
18	68965	31035	74910	25090	05945	94055	42
19	68987	31013	74939	25061	05952	94048	41
20	9.69010	10.30990	9.74969	10.25031	10.05959	9.94041	40
21	69032	30968	74998	25002	05966	94034	39
22	69055	30945	75028	24972	05973	94027	38
23	69077	30923	75058	24942	05980	94020	37
24	69100	30900	75087	24913	05988	94012	36
25	9.69122	10.30878	9.75117	10.24883	10.05995	9.94005	35
26	69144	30856	75146	24854	06002	93998	34
27	69167	30833	75176	24824	06009	93991	33
28	69189	30811	75205	24795	06016	93984	32
29	69212	30788	75235	24765	06023	93977	31
30	9.69234	10.30766	9.75264	10.24736	10.06030	9.93970	30
31	69256	30744	75294	24706	06037	93963	29
32	69279	30721	75323	24677	06045	93955	28
33	69301	30699	75353	24647	06052	93948	27
34	69323	30677	75382	24618	06059	93941	26
35	9.69345	10.30655	9.75411	10.24589	10.06066	9.93934	25
36	69368	30632	75441	24559	06073	93927	24
37	69390	30610	75470	24530	06080	93920	23
38	69412	30588	75500	24500	06088	93912	22
39	69434	30566	75529	24471	06095	93905	21
40	9.69456	10.30544	9.75558	10.24442	10.06102	9.93898	20
41	69479	30521	75588	24412	06109	93891	19
42	69501	30499	75617	24383	06116	93884	18
43	69523	30477	75647	24353	06124	93876	17
44	69545	30455	75676	24324	06131	93869	16
45	9.69567	10.30433	9.75705	10.24295	10.06138	9.93862	15
46	69589	30411	75735	24265	06145	93855	14
47	69611	30389	75764	24236	06153	93847	13
48	69633	30367	75793	24207	06160	93840	12
49	69655	30345	75822	24178	06167	93833	11
50	9.69677	10.30323	9.75852	10.24148	10.06174	9.93826	10
51	69699	30301	75881	24119	06181	93819	9
52	69721	30279	75910	24090	06189	93811	8
53	69743	30257	75939	24061	06196	93804	7
54	69765	30235	75969	24031	06203	93797	6
55	9.69787	10.30213	9.75998	10.24002	10.06211	9.93789	5
56	69809	30191	76027	23973	06218	93782	4
57	69831	30169	76056	23944	06225	93775	3
58	69853	30147	76086	23914	06232	93768	2
59	69875	30125	76115	23885	06240	93760	1
60	69897	30103	76144	23856	06247	93753	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

119° **60°**

30° **Logarithms.** 149°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.69897	10.30103	9.76144	10.23856	10.06247	9.93753	60
1	69919	30081	76173	23827	06254	93746	59
2	69941	30059	76202	23798	06262	93738	58
3	69963	30037	76231	23769	06269	93731	57
4	69984	30016	76261	23739	06276	93724	56
5	9.70006	10.29994	9.76290	10.23710	10.06283	9.93717	55
6	70028	29972	76319	23681	06291	93709	54
7	70050	29950	76348	23652	06298	93702	53
8	70072	29928	76377	23623	06305	93695	52
9	70093	29907	76406	23594	06313	93687	51
10	9.70115	10.29885	9.76435	10.23565	10.06320	9.93680	50
11	70137	29863	76464	23536	06327	93673	49
12	70159	29841	76493	23507	06335	93665	48
13	70180	29820	76522	23478	06342	93658	47
14	70202	29798	76551	23449	06350	93650	46
15	9.70224	10.29776	9.76580	10.23420	10.06357	9.93643	45
16	70245	29755	76609	23391	06364	93636	44
17	70267	29733	76639	23361	06372	93628	43
18	70288	29712	76668	23332	06379	93621	42
19	70310	29690	76697	23303	06386	93614	41
20	9.70332	10.29668	9.76725	10.23275	10.06394	9.93606	40
21	70353	29647	76754	23246	06401	93599	39
22	70375	29625	76783	23217	06409	93591	38
23	70396	29604	76812	23188	06416	93584	37
24	70418	29582	76841	23159	06423	93577	36
25	9.70439	10.29561	9.76870	10.23130	10.06431	9.93569	35
26	70461	29539	76899	23101	06438	93562	34
27	70482	29518	76928	23072	06446	93554	33
28	70504	29496	76957	23043	06453	93547	32
29	70525	29475	76986	23014	06461	93539	31
30	9.70547	10.29453	9.77015	10.22985	10.06468	9.93532	30
31	70568	29432	77044	22956	06475	93525	29
32	70590	29410	77073	22927	06483	93517	28
33	70611	29389	77101	22899	06490	93510	27
34	70633	29367	77130	22870	06498	93502	26
35	9.70654	10.29346	9.77159	10.22841	10.06505	9.93495	25
36	70675	29325	77188	22812	06513	93487	24
37	70697	29303	77217	22783	06520	93480	23
38	70718	29282	77246	22754	06528	93472	22
39	70739	29261	77274	22726	06535	93465	21
40	9.70761	10.29239	9.77303	10.22697	10.06543	9.93457	20
41	70782	29218	77332	22668	06550	93450	19
42	70803	29197	77361	22639	06558	93442	18
43	70824	29176	77390	22610	06565	93435	17
44	70846	29154	77418	22582	06573	93427	16
45	9.70867	10.29133	9.77447	10.22553	10.06580	9.93420	15
46	70888	29112	77476	22524	06588	93412	14
47	70909	29091	77505	22495	06595	93405	13
48	70931	29069	77533	22467	06603	93397	12
49	70952	29048	77562	22438	06610	93390	11
50	9.70973	10.29027	9.77591	10.22409	10.06618	9.93382	10
51	70994	29006	77619	22381	06625	93375	9
52	71015	28985	77648	22352	06633	93367	8
53	71036	28964	77677	22323	06640	93360	7
54	71058	28942	77706	22294	06648	93352	6
55	9.71079	10.28921	9.77734	10.22266	10.06656	9.93344	5
56	71100	28900	77763	22237	06663	93337	4
57	71121	28879	77791	22209	06671	93329	3
58	71142	28858	77820	22180	06678	93322	2
59	71163	28837	77849	22151	06686	93314	1
60	71184	28816	77877	22123	06693	93307	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

120° 59°

31°			Logarithms.				148°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.71184	10.28816	9.77877	10.22123	10.06693	9.93307	60
1	71205	28795	77906	22094	06701	93299	59
2	71226	28774	77935	22065	06709	93291	58
3	71247	28753	77963	22037	06716	93284	57
4	71268	28732	77992	22008	06724	93276	56
5	9.71289	10.28711	9.78020	10.21980	10.06731	9.93269	55
6	71310	28690	78049	21951	06739	93261	54
7	71331	28669	78077	21923	06747	93253	53
8	71352	28648	78106	21894	06754	93246	52
9	71373	28627	78135	21865	06762	93238	51
10	9.71393	10.28607	9.78163	10.21837	10.06770	9.93230	50
11	71414	28586	78192	21808	06777	93223	49
12	71435	28565	78220	21780	06785	93215	48
13	71456	28544	78249	21751	06793	93207	47
14	71477	28523	78277	21723	06800	93200	46
15	9.71498	10.28502	9.78306	10.21694	10.06808	9.93192	45
16	71519	28481	78334	21666	06816	93184	44
17	71539	28461	78363	21637	06823	93177	43
18	71560	28440	78391	21609	06831	93169	42
19	71581	28419	78419	21581	06839	93161	41
20	9.71602	10.28398	9.78448	10.21552	10.06846	9.93154	40
21	71622	28378	78476	21524	06854	93146	39
22	71643	28357	78505	21495	06862	93138	38
23	71664	28336	78533	21467	06869	93131	37
24	71685	28315	78562	21438	06877	93123	36
25	9.71705	10.28295	9.78590	10.21410	10.06885	9.93115	35
26	71726	28274	78618	21382	06892	93108	34
27	71747	28253	78647	21353	06900	93100	33
28	71767	28233	78675	21325	06908	93092	32
29	71788	28212	78704	21296	06916	93084	31
30	9.71809	10.28191	9.78732	10.21268	10.06923	9.93077	30
31	71829	28171	78760	21240	06931	93069	29
32	71850	28150	78789	21211	06939	93061	28
33	71870	28130	78817	21183	06947	93053	27
34	71891	28109	78845	21155	06954	93046	26
35	9.71911	10.28089	9.78874	10.21126	10.06962	9.93038	25
36	71932	28068	78902	21098	06970	93030	24
37	71952	28048	78930	21070	06978	93022	23
38	71973	28027	78959	21041	06986	93014	22
39	71994	28006	78987	21013	06993	93007	21
40	9.72014	10.27986	9.79015	10.20985	10.07001	9.92999	20
41	72034	27966	79043	20957	07009	92991	19
42	72055	27945	79072	20928	07017	92983	18
43	72075	27925	79100	20900	07024	92976	17
44	72096	27904	79128	20872	07032	92968	16
45	9.72116	10.27884	9.79156	10.20844	10.07040	9.92960	15
46	72137	27863	79185	20815	07048	92952	14
47	72157	27843	79213	20787	07056	92944	13
48	72177	27823	79241	20759	07064	92936	12
49	72198	27802	79269	20731	07071	92929	11
50	9.72218	10.27782	9.79297	10.20703	10.07079	9.92921	10
51	72238	27762	79326	20674	07087	92913	9
52	72259	27741	79354	20646	07095	92905	8
53	72279	27721	79382	20618	07103	92897	7
54	72299	27701	79410	20590	07111	92889	6
55	9.72320	10.27680	9.79438	10.20562	10.07119	9.92881	5
56	72340	27660	79466	20534	07126	92874	4
57	72360	27640	79495	20505	07134	92866	3
58	72381	27619	79523	20477	07142	92858	2
59	72401	27599	79551	20449	07150	92850	1
60	72421	27579	79579	20421	07158	92842	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
121°							58°

32° **Logarithms.** 147°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.72421	10.27579	9.79579	10.20421	10.07158	9.92842	60
1	72441	27559	79607	20393	07166	92834	59
2	72461	27539	79635	20365	07174	92826	58
3	72482	27518	79663	20337	07182	92818	57
4	72502	27498	79691	20309	07190	92810	56
5	9.72522	10.27478	9.79719	10.20281	10.07197	9.92803	55
6	72542	27458	79747	20253	07205	92795	54
7	72562	27438	79776	20224	07213	92787	53
8	72582	27418	79804	20196	07221	92779	52
9	72602	27398	79832	20168	07229	92771	51
10	9.72622	10.27378	9.79860	10.20140	10.07237	9.92763	50
11	72643	27357	79888	20112	07245	92755	49
12	72663	27337	79916	20084	07253	92747	48
13	72683	27317	79944	20056	07261	92739	47
14	72703	27297	79972	20028	07269	92731	46
15	9.72723	10.27277	9.80000	10.20000	10.07277	9.92723	45
16	72743	27257	80028	19972	07285	92715	44
17	72763	27237	80056	19944	07293	92707	43
18	72783	27217	80084	19916	07301	92699	42
19	72803	27197	80112	19888	07309	92691	41
20	9.72823	10.27177	9.80140	10.19860	10.07317	9.92683	40
21	72843	27157	80168	19832	07325	92675	39
22	72863	27137	80195	19805	07333	92667	38
23	72883	27117	80223	19777	07341	92659	37
24	72902	27098	80251	19749	07349	92651	36
25	9.72922	10.27078	9.80279	10.19721	10.07357	9.92643	35
26	72942	27058	80307	19693	07365	92635	34
27	72962	27038	80335	19665	07373	92627	33
28	72982	27018	80363	19637	07381	92619	32
29	73002	26998	80391	19609	07389	92611	31
30	9.73022	10.26978	9.80419	10.19581	10.07397	9.92603	30
31	73041	26959	80447	19553	07405	92595	29
32	73061	26939	80474	19526	07413	92587	28
33	73081	26919	80502	19498	07421	92579	27
34	73101	26899	80530	19470	07429	92571	26
35	9.73121	10.26879	9.80558	10.19442	10.07437	9.92563	25
36	73140	26860	80586	19414	07445	92555	24
37	73160	26840	80614	19386	07454	92546	23
38	73180	26820	80642	19358	07462	92538	22
39	73200	26800	80669	19331	07470	92530	21
40	9.73219	10.26781	9.80697	10.19303	10.07478	9.92522	20
41	73239	26761	80725	19275	07486	92514	19
42	73259	26741	80753	19247	07494	92506	18
43	73278	26722	80781	19219	07502	92498	17
44	73298	26702	80808	19192	07510	92490	16
45	9.73318	10.26682	9.80836	10.19164	10.07518	9.92482	15
46	73337	26663	80864	19136	07527	92473	14
47	73357	26643	80892	19108	07535	92465	13
48	73377	26623	80919	19081	07543	92457	12
49	73396	26604	80947	19053	07551	92449	11
50	9.73416	10.26584	9.80975	10.19025	10.07559	9.92441	10
51	73435	26565	81003	18997	07567	92433	9
52	73455	26545	81030	18970	07575	92425	8
53	73474	26526	81058	18942	07584	92416	7
54	73494	26506	81086	18914	07592	92408	6
55	9.73513	10.26487	9.81113	10.18887	10.07600	9.92400	5
56	73533	26467	81141	18859	07608	92392	4
57	73552	26448	81169	18831	07616	92384	3
58	73572	26428	81196	18804	07624	92376	2
59	73591	26409	81224	18776	07633	92367	1
60	73611	26389	81252	18748	07641	92359	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

122° 57°

33° **Logarithms.** **146°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.73611	10.26389	9.81252	10.18748	10.07641	9.92359	60
1	73630	26370	81279	18721	07649	92351	59
2	73650	26350	81307	18693	07657	92343	58
3	73669	26331	81335	18665	07665	92335	57
4	73689	26311	81362	18638	07674	92326	56
5	9.73708	10.26292	9.81390	10.18610	10.07682	9.92318	55
6	73727	26273	81418	18582	07690	92310	54
7	73747	26253	81445	18555	07698	92302	53
8	73766	26234	81473	18527	07707	92293	52
9	73785	26215	81500	18500	07715	92285	51
10	9.73805	10.26195	9.81528	10.18472	10.07723	9.92277	50
11	73824	26176	81556	18444	07731	92269	49
12	73843	26157	81583	18417	07740	92260	48
13	73863	26137	81611	18389	07748	92252	47
14	73882	26118	81638	18362	07756	92244	46
15	9.73901	10.26099	9.81666	10.18334	10.07765	9.92235	45
16	73921	26079	81693	18307	07773	92227	44
17	73940	26060	81721	18279	07781	92219	43
18	73959	26041	81748	18252	07789	92211	42
19	73978	26022	81776	18224	07798	92202	41
20	9.73997	10.26003	9.81803	10.18197	10.07806	9.92194	40
21	74017	25983	81831	18169	07814	92186	39
22	74036	25964	81858	18142	07823	92177	38
23	74055	25945	81886	18114	07831	92169	37
24	74074	25926	81913	18087	07839	92161	36
25	9.74093	10.25907	9.81941	10.18059	10.07848	9.92152	35
26	74113	25887	81968	18032	07856	92144	34
27	74132	25868	81996	18004	07864	92136	33
28	74151	25849	82023	17977	07873	92127	32
29	74170	25830	82051	17949	07881	92119	31
30	9.74189	10.25811	9.82078	10.17922	10.07889	9.92111	30
31	74208	25792	82106	17894	07898	92102	29
32	74227	25773	82133	17867	07906	92094	28
33	74246	25754	82161	17839	07914	92086	27
34	74265	25735	82188	17812	07923	92077	26
35	9.74284	10.25716	9.82215	10.17785	10.07931	9.92069	25
36	74303	25697	82243	17757	07940	92060	24
37	74322	25678	82270	17730	07948	92052	23
38	74341	25659	82298	17702	07956	92044	22
39	74360	25640	82325	17675	07965	92035	21
40	9.74379	10.25621	9.82352	10.17648	10.07973	9.92027	20
41	74398	25602	82380	17620	07982	92018	19
42	74417	25583	82407	17593	07990	92010	18
43	74436	25564	82435	17565	07998	92002	17
44	74455	25545	82462	17538	08007	91993	16
45	9.74474	10.25526	9.82489	10.17511	10.08015	9.91985	15
46	74493	25507	82517	17483	08024	91976	14
47	74512	25488	82544	17456	08032	91968	13
48	74531	25469	82571	17429	08041	91959	12
49	74549	25451	82599	17401	08049	91951	11
50	9.74568	10.25432	9.82626	10.17374	10.08058	9.91942	10
51	74587	25413	82653	17347	08066	91934	9
52	74606	25394	82681	17319	08075	91925	8
53	74625	25375	82708	17292	08083	91917	7
54	74644	25356	82735	17265	08092	91908	6
55	9.74662	10.25338	9.82762	10.17238	10.08100	9.91900	5
56	74681	25319	82790	17210	08109	91891	4
57	74700	25300	82817	17183	08117	91883	3
58	74719	25281	82844	17156	08126	91874	2
59	74737	25263	82871	17129	08134	91866	1
60	74756	25244	82899	17101	08143	91857	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

123° **56°**

34°			Logarithms.				145°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.74756	10.25244	9.82899	10.17101	10.08143	9.91857	60
1	74775	25225	82926	17074	08151	91849	59
2	74794	25206	82953	17047	08160	91840	58
3	74812	25188	82980	17020	08168	91832	57
4	74831	25169	83008	16992	08177	91823	56
5	9.74850	10.25150	9.83035	10.16965	10.08185	9.91815	55
6	74868	25132	83062	16938	08194	91806	54
7	74887	25113	83089	16911	08202	91798	53
8	74906	25094	83117	16883	08211	91789	52
9	74924	25076	83144	16856	08219	91781	51
10	9.74943	10.25057	9.83171	10.16829	10.08228	9.91772	50
11	74961	25039	83198	16802	08237	91763	49
12	74980	25020	83225	16775	08245	91755	48
13	74999	25001	83252	16748	08254	91746	47
14	75017	24983	83280	16720	08262	91738	46
15	9.75036	10.24964	9.83307	10.16693	10.08271	9.91729	45
16	75054	24946	83334	16666	08280	91720	44
17	75073	24927	83361	16639	08288	91712	43
18	75091	24909	83388	16612	08297	91703	42
19	75110	24890	83415	16585	08305	91695	41
20	9.75128	10.24872	9.83442	10.16558	10.08314	9.91686	40
21	75147	24853	83470	16530	08323	91677	39
22	75165	24835	83497	16503	08331	91669	38
23	75184	24816	83524	16476	08340	91660	37
24	75202	24798	83551	16449	08349	91651	36
25	9.75221	10.24779	9.83578	10.16422	10.08357	9.91643	35
26	75239	24761	83605	16395	08366	91634	34
27	75258	24742	83632	16368	08375	91625	33
28	75276	24724	83659	16341	08383	91617	32
29	75294	24706	83686	16314	08392	91608	31
30	9.75313	10.24687	9.83713	10.16287	10.08401	9.91599	30
31	75331	24669	83740	16260	08409	91591	29
32	75350	24650	83768	16232	08418	91582	28
33	75368	24632	83795	16205	08427	91573	27
34	75386	24614	83822	16178	08435	91565	26
35	9.75405	10.24595	9.83849	10.16151	10.08444	9.91556	25
36	75423	24577	83876	16124	08453	91547	24
37	75441	24559	83903	16097	08462	91538	23
38	75459	24541	83930	16070	08470	91530	22
39	75478	24522	83957	16043	08479	91521	21
40	9.75496	10.24504	9.83984	10.16016	10.08488	9.91512	20
41	75514	24486	84011	15989	08496	91504	19
42	75533	24467	84038	15962	08505	91495	18
43	75551	24449	84065	15935	08514	91486	17
44	75569	24431	84092	15908	08523	91477	16
45	9.75587	10.24413	9.84119	10.15881	10.08531	9.91469	15
46	75605	24395	84146	15854	08540	91460	14
47	75624	24376	84173	15827	08549	91451	13
48	75642	24358	84200	15800	08558	91442	12
49	75660	24340	84227	15773	08567	91433	11
50	9.75678	10.24322	9.84254	10.15746	10.08575	9.91425	10
51	75696	24304	84280	15720	08584	91416	9
52	75714	24286	84307	15693	08593	91407	8
53	75733	24267	84334	15666	08602	91398	7
54	75751	24249	84361	15639	08611	91389	6
55	9.75769	10.24231	9.84388	10.15612	10.08619	9.91381	5
56	75787	24213	84415	15585	08628	91372	4
57	75805	24195	84442	15558	08637	91363	3
58	75823	24177	84469	15531	08646	91354	2
59	75841	24159	84496	15504	08655	91345	1
60	75859	24141	84523	15477	08664	91336	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
124°							55°

35° **Logarithms.** **144°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.75859	10.24141	9.84523	10.15477	10.08664	9.91336	60
1	75877	24123	84550	15450	08672	91328	59
2	75895	24105	84576	15424	08681	91319	58
3	75913	24087	84603	15397	08690	91310	57
4	75931	24069	84630	15370	08699	91301	56
5	9.75949	10.24051	9.84657	10.15343	10.08708	9.91292	55
6	75967	24033	84684	15316	08717	91283	54
7	75985	24015	84711	15289	08726	91274	53
8	76003	23997	84738	15262	08734	91266	52
9	76021	23979	84764	15236	08743	91257	51
10	9.76039	10.23961	9.84791	10.15209	10.08752	9.91248	50
11	76057	23943	84818	15182	08761	91239	49
12	76075	23925	84845	15155	08770	91230	48
13	76093	23907	84872	15128	08779	91221	47
14	76111	23889	84899	15101	08788	91212	46
15	9.76129	10.23871	9.84925	10.15075	10.08797	9.91203	45
16	76146	23854	84952	15048	08806	91194	44
17	76164	23836	84979	15021	08815	91185	43
18	76182	23818	85006	14994	08824	91176	42
19	76200	23800	85033	14967	08833	91167	41
20	9.76218	10.23782	9.85059	10.14941	10.08842	9.91158	40
21	76236	23764	85086	14914	08851	91149	39
22	76253	23747	85113	14887	08859	91141	38
23	76271	23729	85140	14860	08868	91132	37
24	76289	23711	85166	14834	08877	91123	36
25	9.76307	10.23693	9.85193	10.14807	10.08886	9.91114	35
26	76324	23676	85220	14780	08895	91105	34
27	76342	23658	85247	14753	08904	91096	33
28	76360	23640	85273	14727	08913	91087	32
29	76378	23622	85300	14700	08922	91078	31
30	9.76395	10.23605	9.85327	10.14673	10.08931	9.91069	30
31	76413	23587	85354	14646	08940	91060	29
32	76431	23569	85380	14620	08949	91051	28
33	76448	23552	85407	14593	08958	91042	27
34	76466	23534	85434	14566	08967	91033	26
35	9.76484	10.23516	9.85460	10.14540	10.08977	9.91023	25
36	76501	23499	85487	14513	08986	91014	24
37	76519	23481	85514	14486	08995	91005	23
38	76537	23463	85540	14460	09004	90996	22
39	76554	23446	85567	14433	09013	90987	21
40	9.76572	10.23428	9.85594	10.14406	10.09022	9.90978	20
41	76590	23410	85620	14380	09031	90969	19
42	76607	23393	85647	14353	09040	90960	18
43	76625	23375	85674	14326	09049	90951	17
44	76642	23358	85700	14300	09058	90942	16
45	9.76660	10.23340	9.85727	10.14273	10.09067	9.90933	15
46	76677	23323	85754	14246	09076	90924	14
47	76695	23305	85780	14220	09085	90915	13
48	76712	23288	85807	14193	09094	90906	12
49	76730	23270	85834	14166	09104	90896	11
50	9.76747	10.23253	9.85860	10.14140	10.09113	9.90887	10
51	76765	23235	85887	14113	09122	90878	9
52	76782	23218	85913	14087	09131	90869	8
53	76800	23200	85940	14060	09140	90860	7
54	76817	23183	85967	14033	09149	90851	6
55	9.76835	10.23165	9.85993	10.14007	10.09158	9.90842	5
56	76852	23148	86020	13980	09168	90832	4
57	76870	23130	86046	13954	09177	90823	3
58	76887	23113	86073	13927	09186	90814	2
59	76904	23096	86100	13900	09195	90805	1
60	76922	23078	86126	13874	09204	90796	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

125° **54°**

36° **Logarithms.** 143°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.76922	10.23078	9.86126	10.13874	10.09204	9.90796	60
1	76939	23061	86153	13847	09213	90787	59
2	76957	23043	86179	13821	09223	90777	58
3	76974	23026	86206	13794	09232	90768	57
4	76991	23009	86232	13768	09241	90759	56
5	9.77009	10.22991	9.86259	10.13741	10.09250	9.90750	55
6	77026	22974	86285	13715	09259	90741	54
7	77043	22957	86312	13688	09269	90731	53
8	77061	22939	86338	13662	09278	90722	52
9	77078	22922	86365	13635	09287	90713	51
10	9.77095	10.22905	9.86392	10.13608	10.09296	9.90704	50
11	77112	22888	86418	13582	09306	90694	49
12	77130	22870	86445	13555	09315	90685	48
13	77147	22853	86471	13529	09324	90676	47
14	77164	22836	86498	13502	09333	90667	46
15	9.77181	10.22819	9.86524	10.13476	10.09343	9.90657	45
16	77199	22801	86551	13449	09352	90648	44
17	77216	22784	86577	13423	09361	90639	43
18	77233	22767	86603	13397	09370	90630	42
19	77250	22750	86630	13370	09380	90620	41
20	9.77268	10.22732	9.86656	10.13344	10.09389	9.90611	40
21	77285	22715	86683	13317	09398	90602	39
22	77302	22698	86709	13291	09408	90592	38
23	77319	22681	86736	13264	09417	90583	37
24	77336	22664	86762	13238	09426	90574	36
25	9.77353	10.22647	9.86789	10.13211	10.09435	9.90565	35
26	77370	22630	86815	13185	09445	90555	34
27	77387	22613	86842	13158	09454	90546	33
28	77405	22595	86868	13132	09463	90537	32
29	77422	22578	86894	13106	09473	90527	31
30	9.77439	10.22561	9.86921	10.13079	10.09482	9.90518	30
31	77456	22544	86947	13053	09491	90509	29
32	77473	22527	86974	13026	09501	90499	28
33	77490	22510	87000	13000	09510	90490	27
34	77507	22493	87027	12973	09520	90480	26
35	9.77524	10.22476	9.87053	10.12947	10.09529	9.90471	25
36	77541	22459	87079	12921	09538	90462	24
37	77558	22442	87106	12894	09548	90452	23
38	77575	22425	87132	12868	09557	90443	22
39	77592	22408	87158	12842	09566	90434	21
40	9.77609	10.22391	9.87185	10.12815	10.09576	9.90424	20
41	77626	22374	87211	12789	09585	90415	19
42	77643	22357	87238	12762	09595	90405	18
43	77660	22340	87264	12736	09604	90396	17
44	77677	22323	87290	12710	09614	90386	16
45	9.77694	10.22306	9.87317	10.12683	10.09623	9.90377	15
46	77711	22289	87343	12657	09632	90368	14
47	77728	22272	87369	12631	09642	90358	13
48	77744	22256	87396	12604	09651	90349	12
49	77761	22239	87422	12578	09661	90339	11
50	9.77778	10.22222	9.87448	10.12552	10.09670	9.90330	10
51	77795	22205	87475	12525	09680	90320	9
52	77812	22188	87501	12499	09689	90311	8
53	77829	22171	87527	12473	09699	90301	7
54	77846	22154	87554	12446	09708	90292	6
55	9.77862	10.22138	9.87580	10.12420	10.09718	9.90282	5
56	77879	22121	87606	12394	09727	90273	4
57	77896	22104	87633	12367	09737	90263	3
58	77913	22087	87659	12341	09746	90254	2
59	77930	22070	87685	12315	09756	90244	1
60	77946	22054	87711	12289	09765	90235	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

126° 53°

37°			Logarithms.				142°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.77946	10.22054	9.87711	10.12289	10.09765	9.90235	60
1	77963	22037	87738	12262	09775	90225	59
2	77980	22020	87764	12236	09784	90216	58
3	77997	22003	87790	12210	09794	90206	57
4	78013	21987	87817	12183	09803	90197	56
5	9.78030	10.21970	9.87843	10.12157	10.09813	9.90187	55
6	78047	21953	87869	12131	09822	90178	54
7	78063	21937	87895	12105	09832	90168	53
8	78080	21920	87922	12078	09841	90159	52
9	78097	21903	87948	12052	09851	90149	51
10	9.78113	10.21887	9.87974	10.12026	10.09861	9.90139	50
11	78130	21870	88000	12000	09870	90130	49
12	78147	21853	88027	11973	09880	90120	48
13	78163	21837	88053	11947	09889	90111	47
14	78180	21820	88079	11921	09899	90101	46
15	9.78197	10.21803	9.88105	10.11895	10.09909	9.90091	45
16	78213	21787	88131	11869	09918	90082	44
17	78230	21770	88158	11842	09928	90072	43
18	78246	21754	88184	11816	09937	90063	42
19	78263	21737	88210	11790	09947	90053	41
20	9.78280	10.21720	9.88236	10.11764	10.09957	9.90043	40
21	78296	21704	88262	11738	09966	90034	39
22	78313	21687	88289	11711	09976	90024	38
23	78329	21671	88315	11685	09986	90014	37
24	78346	21654	88341	11659	09995	90005	36
25	9.78362	10.21638	9.88367	10.11633	10.10005	9.89995	35
26	78379	21621	88393	11607	10015	89985	34
27	78395	21605	88420	11580	10024	89976	33
28	78412	21588	88446	11554	10034	89966	32
29	78428	21572	88472	11528	10044	89956	31
30	9.78445	10.21555	9.88498	10.11502	10.10053	9.89947	30
31	78461	21539	88524	11476	10063	89937	29
32	78478	21522	88550	11450	10073	89927	28
33	78494	21506	88577	11423	10082	89918	27
34	78510	21490	88603	11397	10092	89908	26
35	9.78527	10.21473	9.88629	10.11371	10.10102	9.89898	25
36	78543	21457	88655	11345	10112	89888	24
37	78560	21440	88681	11319	10121	89879	23
38	78576	21424	88707	11293	10131	89869	22
39	78592	21408	88733	11267	10141	89859	21
40	9.78609	10.21391	9.88759	10.11241	10.10151	9.89849	20
41	78625	21375	88780	11214	10160	89840	19
42	78642	21358	88812	11188	10170	89830	18
43	78658	21342	88838	11162	10180	89820	17
44	78674	21326	88864	11136	10190	89810	16
45	9.78691	10.21309	9.88890	10.11110	10.10199	9.89801	15
46	78707	21293	88916	11084	10209	89791	14
47	78723	21277	88942	11058	10219	89781	13
48	78739	21261	88968	11032	10229	89771	12
49	78756	21244	88994	11006	10239	89761	11
50	9.78772	10.21228	9.89020	10.10980	10.10248	9.89752	10
51	78788	21212	89046	10954	10258	89742	9
52	78805	21195	89073	10927	10268	89732	8
53	78821	21179	89099	10901	10278	89722	7
54	78837	21163	89125	10875	10288	89712	6
55	9.78853	10.21147	9.89151	10.10849	10.10298	9.89702	5
56	78869	21131	89177	10823	10307	89693	4
57	78886	21114	89203	10797	10317	89683	3
58	78902	21098	89229	10771	10327	89673	2
59	78918	21082	89255	10745	10337	89663	1
60	78934	21066	89281	10719	10347	89653	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
127°							52°

38° **Logarithms.** 141°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.78934	10.21066	9.89281	10.10719	10.10347	9.89653	60
1	78950	21050	89307	10693	10357	89643	59
2	78967	21033	89333	10667	10367	89633	58
3	78983	21017	89359	10641	10376	89624	57
4	78999	21001	89385	10615	10386	89614	56
5	9.79015	10.20985	9.89411	10.10589	10.10396	9.89604	55
6	79031	20969	89437	10563	10406	89594	54
7	79047	20953	89463	10537	10416	89584	53
8	79063	20937	89489	10511	10426	89574	52
9	79079	20921	89515	10485	10436	89564	51
10	9.79095	10.20905	9.89541	10.10459	10.10446	9.89554	50
11	79111	20889	89567	10433	10456	89544	49
12	79128	20872	89593	10407	10466	89534	48
13	79144	20856	89619	10381	10476	89524	47
14	79160	20840	89645	10355	10486	89514	46
15	9.79176	10.20824	9.89671	10.10329	10.10496	9.89504	45
16	79192	20808	89697	10303	10505	89495	44
17	79208	20792	89723	10277	10515	89485	43
18	79224	20776	89749	10251	10525	89475	42
19	79240	20760	89775	10225	10535	89465	41
20	9.79256	10.20744	9.89801	10.10199	10.10545	9.89455	40
21	79272	20728	89827	10173	10555	89445	39
22	79288	20712	89853	10147	10565	89435	38
23	79304	20696	89879	10121	10575	89425	37
24	79319	20681	89905	10095	10585	89415	36
25	9.79335	10.20665	9.89931	10.10069	10.10595	9.89405	35
26	79351	20649	89957	10043	10605	89395	34
27	79367	20633	89983	10017	10615	89385	33
28	79383	20617	90009	09991	10625	89375	32
29	79399	20601	90035	09965	10636	89364	31
30	9.79415	10.20585	9.90061	10.09939	10.10646	9.89354	30
31	79431	20569	90086	09914	10656	89344	29
32	79447	20553	90112	09888	10666	89334	28
33	79463	20537	90138	09862	10676	89324	27
34	79478	20522	90164	09836	10686	89314	26
35	9.79494	10.20506	9.90190	10.09810	10.10696	9.89304	25
36	79510	20490	90216	09784	10706	89294	24
37	79526	20474	90242	09758	10716	89284	23
38	79542	20458	90268	09732	10726	89274	22
39	79558	20442	90294	09706	10736	89264	21
40	9.79573	10.20427	9.90320	10.09680	10.10746	9.89254	20
41	79589	20411	90346	09654	10756	89244	19
42	79605	20395	90371	09629	10767	89233	18
43	79621	20379	90397	09603	10777	89223	17
44	79636	20364	90423	09577	10787	89213	16
45	9.79652	10.20348	9.90449	10.09551	10.10797	9.89203	15
46	79668	20332	90475	09525	10807	89193	14
47	79684	20316	90501	09499	10817	89183	13
48	79699	20301	90527	09473	10827	89173	12
49	79715	20285	90553	09447	10838	89162	11
50	9.79731	10.20269	9.90578	10.09422	10.10848	9.89152	10
51	79746	20254	90604	09396	10858	89142	9
52	79762	20238	90630	09370	10868	89132	8
53	79778	20222	90656	09344	10878	89122	7
54	79793	20207	90682	09318	10888	89112	6
55	9.79809	10.20191	9.90708	10.09292	10.10899	9.89101	5
56	79825	20175	90734	09266	10909	89091	4
57	79840	20160	90759	09241	10919	89081	3
58	79856	20144	90785	09215	10929	89071	2
59	79872	20128	90811	09189	10940	89060	1
60	79887	20113	90837	09163	10950	89050	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

128° 51°

39°			Logarithms.				140°
M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.79887	10.20113	9.90837	10.09163	10.10950	9.89050	60
1	79903	20097	90863	09137	10960	89040	59
2	79918	20082	90889	09111	10970	89030	58
3	79934	20066	90914	09086	10980	89020	57
4	79950	20050	90940	09060	10991	89009	56
5	9.79965	10.20035	9.90966	10.09034	10.11001	9.88999	55
6	79981	20019	90992	09008	11011	88989	54
7	79996	20004	91018	08982	11022	88978	53
8	80012	19988	91043	08957	11032	88968	52
9	80027	19973	91069	08931	11042	88958	51
10	9.80043	10.19957	9.91095	10.08905	10.11052	9.88948	50
11	80058	19942	91121	08879	11063	88937	49
12	80074	19926	91147	08853	11073	88927	48
13	80089	19911	91172	08828	11083	88917	47
14	80105	19895	91198	08802	11094	88906	46
15	9.80120	10.19880	9.91224	10.08776	10.11104	9.88896	45
16	80136	19864	91250	08750	11114	88886	44
17	80151	19849	91276	08724	11125	88875	43
18	80166	19834	91301	08699	11135	88865	42
19	80182	19818	91327	08673	11145	88855	41
20	9.80197	10.19803	9.91353	10.08647	10.11156	9.88844	40
21	80213	19787	91379	08621	11166	88834	39
22	80228	19772	91404	08596	11176	88824	38
23	80244	19756	91430	08570	11187	88813	37
24	80259	19741	91456	08544	11197	88803	36
25	9.80274	10.19726	9.91482	10.08518	10.11207	9.88793	35
26	80290	19710	91507	08493	11218	88782	34
27	80305	19695	91533	08467	11228	88772	33
28	80320	19680	91559	08441	11239	88761	32
29	80336	19664	91585	08415	11249	88751	31
30	9.80351	10.19649	9.91610	10.08390	10.11259	9.88741	30
31	80366	19634	91636	08364	11270	88730	29
32	80382	19618	91662	08338	11280	88720	28
33	80397	19603	91688	08312	11291	88709	27
34	80412	19588	91713	08287	11301	88699	26
35	9.80428	10.19572	9.91739	10.08261	10.11312	9.88688	25
36	80443	19557	91765	08235	11322	88678	24
37	80458	19542	91791	08209	11332	88668	23
38	80473	19527	91816	08184	11343	88657	22
39	80489	19511	91842	08158	11353	88647	21
40	9.80504	10.19496	9.91868	10.08132	10.11364	9.88636	20
41	80519	19481	91893	08107	11374	88626	19
42	80534	19466	91919	08081	11385	88615	18
43	80550	19450	91945	08055	11395	88605	17
44	80565	19435	91971	08029	11406	88594	16
45	9.80580	10.19420	9.91996	10.08004	10.11416	9.88584	15
46	80595	19405	92022	07978	11427	88573	14
47	80610	19390	92048	07952	11437	88563	13
48	80625	19375	92073	07927	11448	88552	12
49	80641	19359	92099	07901	11458	88542	11
50	9.80656	10.19344	9.92125	10.07875	10.11469	9.88531	10
51	80671	19329	92150	07850	11479	88521	9
52	80686	19314	92176	07824	11490	88510	8
53	80701	19299	92202	07798	11501	88499	7
54	80716	19284	92227	07773	11511	88489	6
55	9.80731	10.19269	9.92253	10.07747	10.11522	9.88478	5
56	80746	19254	92279	07721	11532	88468	4
57	80762	19238	92304	07696	11543	88457	3
58	80777	19223	92330	07670	11553	88447	2
59	80792	19208	92356	07644	11564	88436	1
60	80807	19193	92381	07619	11575	88425	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.
129°							50°

40° **Logarithms.** 139°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.80807	10.19193	9.92381	10.07619	10.11575	9.88425	60
1	80822	19178	92407	07593	11585	88415	59
2	80837	19163	92433	07567	11596	88404	58
3	80852	19148	92458	07542	11606	88394	57
4	80867	19133	92484	07516	11617	88383	56
5	9.80882	10.19118	9.92510	10.07490	10.11628	9.88372	55
6	80897	19103	92535	07465	11638	88362	54
7	80912	19088	92561	07439	11649	88351	53
8	80927	19073	92587	07413	11660	88340	52
9	80942	19058	92612	07388	11670	88330	51
10	9.80957	10.19043	9.92638	10.07362	10.11681	9.88319	50
11	80972	19028	92663	07337	11692	88308	49
12	80987	19013	92689	07311	11702	88298	48
13	81002	18998	92715	07285	11713	88287	47
14	81017	18983	92740	07260	11724	88276	46
15	9.81032	10.18968	9.92766	10.07234	10.11734	9.88266	45
16	81047	18953	92792	07208	11745	88255	44
17	81061	18939	92817	07183	11756	88244	43
18	81076	18924	92843	07157	11766	88234	42
19	81091	18909	92868	07132	11777	88223	41
20	9.81106	10.18894	9.92894	10.07106	10.11788	9.88212	40
21	81121	18879	92920	07080	11799	88201	39
22	81136	18864	92945	07055	11809	88191	38
23	81151	18849	92971	07029	11820	88180	37
24	81166	18834	92996	07004	11831	88169	36
25	9.81180	10.18820	9.93022	10.06978	10.11842	9.88158	35
26	81195	18805	93048	06952	11852	88148	34
27	81210	18790	93073	06927	11863	88137	33
28	81225	18775	93099	06901	11874	88126	32
29	81240	18760	93124	06876	11885	88115	31
30	9.81254	10.18746	9.93150	10.06850	10.11895	9.88105	30
31	81269	18731	93175	06825	11906	88094	29
32	81284	18716	93201	06799	11917	88083	28
33	81299	18701	93227	06773	11928	88072	27
34	81314	18686	93252	06748	11939	88061	26
35	9.81328	10.18672	9.93278	10.06722	10.11949	9.88051	25
36	81343	18657	93303	06697	11960	88040	24
37	81358	18642	93329	06671	11971	88029	23
38	81372	18628	93354	06646	11982	88018	22
39	81387	18613	93380	06620	11993	88007	21
40	9.81402	10.18598	9.93406	10.06594	10.12004	9.87996	20
41	81417	18583	93431	06569	12015	87985	19
42	81431	18569	93457	06543	12025	87975	18
43	81446	18554	93482	06518	12036	87964	17
44	81461	18539	93508	06492	12047	87953	16
45	9.81475	10.18525	9.93533	10.06467	10.12058	9.87942	15
46	81490	18510	93559	06441	12069	87931	14
47	81505	18495	93584	06416	12080	87920	13
48	81519	18481	93610	06390	12091	87909	12
49	81534	18466	93636	06364	12102	87898	11
50	9.81549	10.18451	9.93661	10.06339	10.12113	9.87887	10
51	81563	18437	93687	06313	12123	87877	9
52	81578	18422	93712	06288	12134	87866	8
53	81592	18408	93738	06262	12145	87855	7
54	81607	18393	93763	06237	12156	87844	6
55	9.81622	10.18378	9.93789	10.06211	10.12167	9.87833	5
56	81636	18364	93814	06186	12178	87822	4
57	81651	18349	93840	06160	12189	87811	3
58	81665	18335	93865	06135	12200	87800	2
59	81680	18320	93891	06109	12211	87789	1
60	81694	18306	93916	06084	12222	87778	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

130° 49°

41° **Logarithms.** **138°**

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.81694	10.18306	9.93916	10.06084	10.12222	9.87778	60
1	81709	18291	93942	06058	12233	87767	59
2	81723	18277	93967	06033	12244	87756	58
3	81738	18262	93993	06007	12255	87745	57
4	81752	18248	94018	05982	12266	87734	56
5	9.81767	10.18233	9.94044	10.05956	10.12277	9.87723	55
6	81781	18219	94069	05931	12288	87712	54
7	81796	18204	94095	05905	12299	87701	53
8	81810	18190	94120	05880	12310	87690	52
9	81825	18175	94146	05854	12321	87679	51
10	9.81839	10.18161	9.94171	10.05829	10.12332	9.87668	50
11	81854	18146	94197	05803	12343	87657	49
12	81868	18132	94222	05778	12354	87646	48
13	81882	18118	94248	05752	12365	87635	47
14	81897	18103	94273	05727	12376	87624	46
15	9.81911	10.18089	9.94299	10.05701	10.12387	9.87613	45
16	81926	18074	94324	05676	12399	87601	44
17	81940	18060	94350	05650	12410	87590	43
18	81955	18045	94375	05625	12421	87579	42
19	81969	18031	94401	05599	12432	87568	41
20	9.81983	10.18017	9.94426	10.05574	10.12443	9.87557	40
21	81998	18002	94452	05548	12454	87546	39
22	82012	17988	94477	05523	12465	87535	38
23	82026	17974	94503	05497	12476	87524	37
24	82041	17959	94528	05472	12487	87513	36
25	9.82055	10.17945	9.94554	10.05446	10.12499	9.87501	35
26	82069	17931	94579	05421	12510	87490	34
27	82084	17916	94604	05396	12521	87479	33
28	82098	17902	94630	05370	12532	87468	32
29	82112	17888	94655	05345	12543	87457	31
30	9.82126	10.17874	9.94681	10.05319	10.12554	9.87446	30
31	82141	17859	94706	05294	12566	87434	29
32	82155	17845	94732	05268	12577	87423	28
33	82169	17831	94757	05243	12588	87412	27
34	82184	17816	94783	05217	12599	87401	26
35	9.82198	10.17802	9.94808	10.05192	10.12610	9.87390	25
36	82212	17788	94834	05166	12622	87378	24
37	82226	17774	94859	05141	12633	87367	23
38	82240	17760	94884	05116	12644	87356	22
39	82255	17745	94910	05090	12655	87345	21
40	9.82269	10.17731	9.94935	10.05065	10.12666	9.87334	20
41	82283	17717	94961	05039	12678	87322	19
42	82297	17703	94986	05014	12689	87311	18
43	82311	17689	95012	04988	12700	87300	17
44	82326	17674	95037	04963	12712	87288	16
45	9.82340	10.17660	9.95062	10.04938	10.12723	9.87277	15
46	82354	17646	95088	04912	12734	87266	14
47	82368	17632	95113	04887	12745	87255	13
48	82382	17618	95139	04861	12757	87243	12
49	82396	17604	95164	04836	12768	87232	11
50	9.82410	10.17590	9.95190	10.04810	10.12779	9.87221	10
51	82424	17576	95215	04785	12791	87209	9
52	82439	17561	95240	04760	12802	87198	8
53	82453	17547	95266	04734	12813	87187	7
54	82467	17533	95291	04709	12825	87175	6
55	9.82481	10.17519	9.95317	10.04683	10.12836	9.87164	5
56	82495	17505	95342	04658	12847	87153	4
57	82509	17491	95368	04632	12859	87141	3
58	82523	17477	95393	04607	12870	87130	2
59	82537	17463	95418	04582	12881	87119	1
60	82551	17449	95444	04556	12893	87107	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

131° **48°**

42° **Logarithms.** 137°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.82551	10.17449	9.95444	10.04556	10.12893	9.87107	60
1	82565	17435	95469	04531	12904	87096	59
2	82579	17421	95495	04505	12915	87085	58
3	82593	17407	95520	04480	12927	87073	57
4	82607	17393	95545	04455	12938	87062	56
5	9.82621	10.17379	9.95571	10.04429	10.12950	9 87050	55
6	82635	17365	95596	04404	12961	87039	54
7	82649	17351	95622	04378	12972	87028	53
8	82663	17337	95647	04353	12984	87016	52
9	82677	17323	95672	04328	12995	87005	51
10	9.82691	10.17309	9.95698	10.04302	10.13007	9.86993	50
11	82705	17295	95723	04277	13018	86982	49
12	82719	17281	95748	04252	13030	86970	48
13	82733	17267	95774	04226	13041	86959	47
14	82747	17253	95799	04201	13053	86947	46
15	9.82761	10.17239	9.95825	10.04175	10.13064	9.86936	45
16	82775	17225	95850	04150	13076	86924	44
17	82788	17212	95875	04125	13087	86913	43
18	82802	17198	95901	04099	13098	86902	42
19	82816	17184	95926	04074	13110	86890	41
20	9.82830	10.17170	9.95952	10.04048	10.13121	9.86879	40
21	82844	17156	95977	04023	13133	86867	39
22	82858	17142	96002	03998	13145	86855	38
23	82872	17128	96028	03972	13156	86844	37
24	82885	17115	96053	03947	13168	86832	36
25	9.82899	10.17101	9.96078	10.03922	10.13179	9.86821	35
26	82913	17087	96104	03896	13191	86809	34
27	82927	17073	96129	03871	13202	86798	33
28	82941	17059	96155	03845	13214	86786	32
29	82955	17045	96180	03820	13225	86775	31
30	9.82968	10.17032	9.96205	10.03795	10.13237	9.86763	30
31	82982	17018	96231	03769	13248	86752	29
32	82996	17004	96256	03744	13260	86740	28
33	83010	16990	96281	03719	13272	86728	27
34	83023	16977	96307	03693	13283	86717	26
35	9.83037	10.16963	9.96332	10.03668	10.13295	9.86705	25
36	83051	16949	96357	03643	13306	86694	24
37	83065	16935	96383	03617	13318	86682	23
38	83078	16922	96408	03592	13330	86670	22
39	83092	16908	96433	03567	13341	86659	21
40	9.83106	10.16894	9.96459	10.03541	10.13353	9.86647	20
41	83120	16880	96484	03516	13365	86635	19
42	83133	16867	96510	03490	13376	86624	18
43	83147	16853	96535	03465	13388	86612	17
44	83161	16839	96560	03440	13400	86600	16
45	9.83174	10.16826	9.96586	10.03414	10.13411	9.86589	15
46	83188	16812	96611	03389	13423	86577	14
47	83202	16798	96636	03364	13435	86565	13
48	83215	16785	96662	03338	13446	86554	12
49	83229	16771	96687	03313	13458	86542	11
50	9.83242	10.16758	9.96712	10.03288	10.13470	9.86530	10
51	83256	16744	96738	03262	13482	86518	9
52	83270	16730	96763	03237	13493	86507	8
53	83283	16717	96788	03212	13505	86495	7
54	83297	16703	96814	03186	13517	86483	6
55	9.83310	10.16690	9.96839	10.03161	10.13528	9.86472	5
56	83324	16676	96864	03136	13540	86460	4
57	83338	16662	96890	03110	13552	86448	3
58	83351	16649	96915	03085	13564	86436	2
59	83365	16635	96940	03060	13575	86425	1
60	83378	16622	96966	03034	13587	86413	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

132° 47°

43° **Logarithms.** 136°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.83378	10.16622	9.96966	10.03034	10.13587	9.86413	60
1	83392	16608	96991	03009	13599	86401	59
2	83405	16595	97016	02984	13611	86389	58
3	83419	16581	97042	02958	13623	86377	57
4	83432	16568	97067	02933	13634	86366	56
5	9.83446	10.16554	9.97092	10.02908	10.13646	9.86354	55
6	83459	16541	97118	02882	13658	86342	54
7	83473	16527	97143	02857	13670	86330	53
8	83486	16514	97168	02832	13682	86318	52
9	83500	16500	97193	02807	13694	86306	51
10	9.83513	10.16487	9.97219	10.02781	10.13705	9.86295	50
11	83527	16473	97244	02756	13717	86283	49
12	83540	16460	97269	02731	13729	86271	48
13	83554	16446	97295	02705	13741	86259	47
14	83567	16433	97320	02680	13753	86247	46
15	9.83581	10.16419	9.97345	10.02655	10.13765	9.86235	45
16	83594	16406	97371	02629	13777	86223	44
17	83608	16392	97396	02604	13789	86211	43
18	83621	16379	97421	02579	13800	86200	42
19	83634	16366	97447	02553	13812	86188	41
20	9.83648	10.16352	9.97472	10.02528	10.13824	9.86176	40
21	83661	16339	97497	02503	13836	86164	39
22	83674	16326	97523	02477	13848	86152	38
23	83688	16312	97548	02452	13860	86140	37
24	83701	16299	97573	02427	13872	86128	36
25	9.83715	10.16285	9.97598	10.02402	10.13884	9.86116	35
26	83728	16272	97624	02376	13896	86104	34
27	83741	16259	97649	02351	13908	86092	33
28	83755	16245	97674	02326	13920	86080	32
29	83768	16232	97700	02300	13932	86068	31
30	9.83781	10.16219	9.97725	10.02275	10.13944	9.86056	30
31	83795	16205	97750	02250	13956	86044	29
32	83808	16192	97776	02224	13968	86032	28
33	83821	16179	97801	02199	13980	86020	27
34	83834	16166	97826	02174	13992	86008	26
35	9.83848	10.16152	9.97851	10.02149	10.14004	9.85996	25
36	83861	16139	97877	02123	14016	85984	24
37	83874	16126	97902	02098	14028	85972	23
38	83887	16113	97927	02073	14040	85960	22
39	83901	16099	97953	02047	14052	85948	21
40	9.83914	10.16086	9.97978	10.02022	10.14064	9.85936	20
41	83927	16073	98003	01997	14076	85924	19
42	83940	16060	98029	01971	14088	85912	18
43	83954	16046	98054	01946	14100	85900	17
44	83967	16033	98079	01921	14112	85888	16
45	9.83980	10.16020	9.98104	10.01896	10.14124	9.85876	15
46	83993	16007	98130	01870	14136	85864	14
47	84006	15994	98155	01845	14149	85851	13
48	84020	15980	98180	01820	14161	85839	12
49	84033	15967	98206	01794	14173	85827	11
50	9.84046	10.15954	9.98231	10.01769	10.14185	9.85815	10
51	84059	15941	98256	01744	14197	85803	9
52	84072	15928	98281	01719	14209	85791	8
53	84085	15915	98307	01693	14221	85779	7
54	84098	15902	98332	01668	14234	85766	6
55	9.84112	10.15888	9.98357	10.01643	10.14246	9.85754	5
56	84125	15875	98383	01617	14258	85742	4
57	84138	15862	98408	01592	14270	85730	3
58	84151	15849	98433	01567	14282	85718	2
59	84164	15836	98458	01542	14294	85706	1
60	84177	15823	98484	01516	14307	85693	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

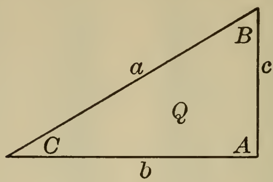
133° 46°

44° **Logarithms.** 135°

M.	Sine.	Cosecant.	Tangent.	Cotangent.	Secant.	Cosine.	M.
0	9.84177	10.15823	9.98484	10.01516	10.14307	9.85693	60
1	84190	15810	98509	01491	14319	85681	59
2	84203	15797	98534	01466	14331	85669	58
3	84216	15784	98560	01440	14343	85657	57
4	84229	15771	98585	01415	14355	85645	56
5	9.84242	10.15758	9.98610	10.01390	10.14368	9.85632	55
6	84255	15745	98635	01365	14380	85620	54
7	84269	15731	98661	01339	14392	85608	53
8	84282	15718	98686	01314	14404	85596	52
9	84295	15705	98711	01289	14417	85583	51
10	9.84308	10.15692	9.98737	10.01263	10.14429	9.85571	50
11	84321	15679	98762	01238	14441	85559	49
12	84334	15666	98787	01213	14453	85547	48
13	84347	15653	98812	01188	14466	85534	47
14	84360	15640	98838	01162	14478	85522	46
15	9.84373	10.15627	9.98863	10.01137	10.14490	9.85510	45
16	84385	15615	98888	01112	14503	85497	44
17	84398	15602	98913	01087	14515	85485	43
18	84411	15589	98939	01061	14527	85473	42
19	84424	15576	98964	01036	14540	85460	41
20	9.84437	10.15563	9.98989	10.01011	10.14552	9.85448	40
21	84450	15550	99015	00985	14564	85436	39
22	84463	15537	99040	00960	14577	85423	38
23	84476	15524	99065	00935	14589	85411	37
24	84489	15511	99090	00910	14601	85399	36
25	9.84502	10.15498	9.99116	10.00884	10.14614	9.85386	35
26	84515	15485	99141	00859	14626	85374	34
27	84528	15472	99166	00834	14639	85361	33
28	84540	15460	99191	00809	14651	85349	32
29	84553	15447	99217	00783	14663	85337	31
30	9.84566	10.15434	9.99242	10.00758	10.14676	9.85324	30
31	84579	15421	99267	00733	14688	85312	29
32	84592	15408	99293	00707	14701	85299	28
33	84605	15395	99318	00682	14713	85287	27
34	84618	15382	99343	00657	14726	85274	26
35	9.84630	10.15370	9.99368	10.00632	10.14738	9.85262	25
36	84643	15357	99394	00606	14750	85250	24
37	84656	15344	99419	00581	14763	85237	23
38	84669	15331	99444	00556	14775	85225	22
39	84682	15318	99469	00531	14788	85212	21
40	9.84694	10.15306	9.99495	10.00505	10.14800	9.85200	20
41	84707	15293	99520	00480	14813	85187	19
42	84720	15280	99545	00455	14825	85175	18
43	84733	15267	99570	00430	14838	85162	17
44	84745	15255	99596	00404	14850	85150	16
45	9.84758	10.15242	9.99621	10.00379	10.14863	9.85137	15
46	84771	15229	99646	00354	14875	85125	14
47	84784	15216	99672	00328	14888	85112	13
48	84796	15204	99697	00303	14900	85100	12
49	84809	15191	99722	00278	14913	85087	11
50	9.84822	10.15178	9.99747	10.00253	10.14926	9.85074	10
51	84835	15165	99773	00227	14938	85062	9
52	84847	15153	99798	00202	14951	85049	8
53	84860	15140	99823	00177	14963	85037	7
54	84873	15127	99848	00152	14976	85024	6
55	9.84885	10.15115	9.99874	10.00126	10.14988	9.85012	5
56	84898	15102	99899	00101	15001	84999	4
57	84911	15089	99924	00076	15014	84986	3
58	84923	15077	99949	00051	15026	84974	2
59	84936	15064	99975	00025	15039	84961	1
60	84949	15051	10.00000	00000	15051	84949	0
M.	Cosine.	Secant.	Cotangent.	Tangent.	Cosecant.	Sine.	M.

134° 45°

Formulas for Right-Angled Triangles.



1. $a = \sqrt{b^2 + c^2}$.

2. $a = \dfrac{c}{\sin C}$.

3. $a = \dfrac{b}{\cos C}$.

4. $a = 2\sqrt{\dfrac{Q}{\sin 2C}}$.

5. $b = a \cos C$.

6. $b = c \cot C$.

7. $b = a \sin B$.

8. $b = c \tan B$.

9. $b = \sqrt{\dfrac{2Q}{\tan C}}$.

10. $Q = \dfrac{a^2 \sin 2C}{4}$.

11. $Q = \frac{1}{2} b^2 \tan C$.

12. $Q = \frac{1}{2} c^2 \cot C$.

13. $Q = \frac{1}{2} c \sqrt{(a + c)(a - c)}$.

14. $\sin C = \dfrac{c}{a}$.

15. $\cos C = \dfrac{b}{a}$.

16. $\tan C = \dfrac{c}{b}$.

17. $\sin 2C = \dfrac{4Q}{a^2}$.

18. $\tan C = \dfrac{2Q}{b^2}$.

Say the angle to be $C = 60°$. In the first column of the table of sines, 60° corresponds with 0.86602 in the next column, which is the length of sin 60°, when the radius of the circle is one, or the unit, and the expression $\sin 60° \times 36$ means $0.86602 \times 36 = 31.17672$, and likewise with all the other trigonometrical expressions.

In a triangle the functions of an angle have a certain relation to the opposite side; it is this relationship which enables us to solve the triangle by the application of simple arithmetic.

In triangles the sides are denoted by the letters a, b, and c; their respective opposite angles are denoted by A, B, and C, and the area by Q.

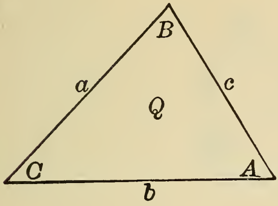
Example. The side c in a right-angled triangle being 365 feet, and the angle $C = 39° 20'$, how long is the side $a = ?$

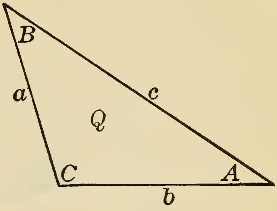
Formula 2. $a = \dfrac{c}{\sin C} = \dfrac{365}{\sin 39° 20'} = \dfrac{365}{0.63383} = 575.86$ feet, the answer.

Or, by logarithms,

$$\begin{aligned} \log. a &= \log. 365 - \log. \sin 39° 20' \\ &= 2.56229 - 9.80197 \\ &= 2.76032, \text{ num.} = 575.86. \end{aligned}$$

Formulas for Oblique-Angled Triangles.





$a : b = \sin A : \sin B$, and $b : c = \sin B : \sin C$.
$a : c = \sin A : \sin C$, and $Q : ab = \sin C : 2$.

1. $a = \dfrac{c \sin A}{\sin C}$.

2. $a = \dfrac{c \sin A}{\sin (A + B)}$.

3. $a = \dfrac{2Q}{b \sin C}$.

4. $b = \dfrac{c \sin B}{\sin C}$.

5. $b = \dfrac{2Q}{c \sin A}$.

6. $\sin C = \dfrac{c \sin B}{b}$.

7. $\sin C = \dfrac{c \sin A}{a}$.

8. $\sin A = \dfrac{2Q}{bc}$.

9. $\sin A = \dfrac{a \sin C}{c}$.

10. $a = \sqrt{b^2 + c^2 - 2bc \cos A}$.

11. $a = \sqrt{\dfrac{2Q \sin A}{\sin B \sin (A + B)}}$.

12. $S = \frac{1}{2}(a + b + c)$.

13. $\sin \frac{1}{2}A = \sqrt{\dfrac{(s - b)(s - c)}{bc}}$.

14. $\sin \frac{1}{2}B = \sqrt{\dfrac{(s - a)(s - c)}{ac}}$.

15. $\cos \frac{1}{2}A = \sqrt{\dfrac{s(s - a)}{bc}}$.

16. $\cos \frac{1}{2}B = \sqrt{\dfrac{s(s - b)}{ac}}$.

17. $Q = \dfrac{bc \sin A}{2}$.

18. $Q = \dfrac{ab \sin C}{2}$.

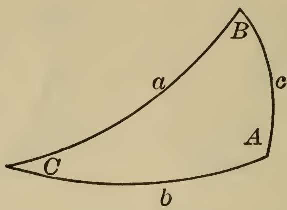
19. $Q = \dfrac{c^2 \sin A \sin B}{2 \sin (A + B)}$.

20. $Q = \sqrt{(S - a)(S - b)(S - c)S}$.

21. $b = \sqrt{\dfrac{2Q \sin (A + C)}{\sin A \sin C}}$.

22. $c = \sqrt{\dfrac{2Q \sin C}{\sin A \sin (A + C)}}$.

Right-Angled Spherical Triangle.



1. $\sin\ b = \sin a \sin B.$
2. $\tan\ c = \tan a \cos B.$
3. $\cot\ C = \cos a \tan B.$
4. $\tan\ c = \sin b \tan C.$
5. $\cos\ a = \cos b \cos c.$
6. $\cos B = \cos b \sin C.$
7. $\tan a = \dfrac{\tan b}{\cos C}.$
8. $\sin\ c = \dfrac{\tan b}{\tan B}.$
9. $\sin\ a = \dfrac{\sin b}{\sin B}.$
10. $\sin\ C = \dfrac{\cos B}{\cos b}.$
11. $\cos\ c = \dfrac{\cos a}{\cos b}.$
12. $\sin\ B = \dfrac{\sin b}{\sin a}.$
13. $\cos\ C = \dfrac{\tan b}{\tan a}.$
14. $\tan\ C = \dfrac{\tan c}{\sin b}.$
15. $\tan\ B = \dfrac{\tan b}{\sin c}.$
16. $\cos\ c = \dfrac{\cos C}{\sin B}.$
17. $\cos\ b = \dfrac{\cos B}{\sin C}.$
18. $\cos\ a = \dfrac{\cot C}{\tan B}.$

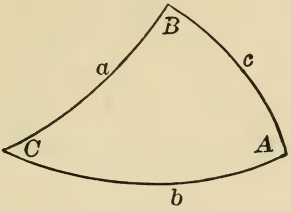
The sum of the three angles in a spherical triangle is greater than two right angles and less than six right angles.

By spherical trigonometry we ascertain distances and courses on the surface of the earth, positions and motions of the heavenly bodies, etc., etc. Examples will be furnished in geography and astronomy.

Example. In a right-angled spherical triangle the side or hypothenuse $a = 36° 20'$, the angle $B = 68° 50'$. How long is the side $b = ?$

Formula 1.	$\sin b = \sin a \sin B = \sin 36° 20' \times \sin 68° 60'$.	
a	log. sin $36° 20' = 9{:}77267$	
B	log. sin $68° 50' = 9{:}96966$	
The answer,	log. sin $33° 32' = 9{:}74233$	or, $b = 33° 32'$.

Oblique-Angled Spherical Triangle.



19. $\sin a : \sin b = \sin A : \sin B.$ $\qquad \sin a = \dfrac{\sin b \sin A}{\sin B}.$

20. $\sin b : \sin c = \sin B : \sin C.$ $\qquad \sin b = \dfrac{\sin c \sin B}{\sin C}.$

21. $$\tan \tfrac{1}{2}(a+b) = \tan \tfrac{1}{2}c \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)}.$$

22. $$\tan \tfrac{1}{2}(a-b) = \tan \tfrac{1}{2}c \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)}.$$

23. $$\tan \tfrac{1}{2}(B+C) = \cot \tfrac{1}{2}A \frac{\cos \frac{1}{2}(b-c)}{\cos \frac{1}{2}(b+c)}.$$

24. $$\tan \tfrac{1}{2}(B-C) = \cot \tfrac{1}{2}A \frac{\sin \frac{1}{2}(b-c)}{\sin \frac{1}{2}(b+c)}.$$

25. $$\cot \tfrac{1}{2}A = \tan \tfrac{1}{2}(B-C) \frac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)}.$$

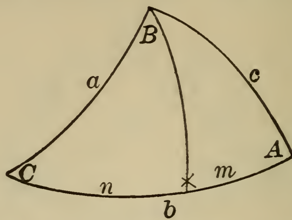
26. $$\tan \tfrac{1}{2}c = \tan \tfrac{1}{2}(a-b) \frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)}.$$

Example. Oblique-angled spherical triangle. $c = 72° 30'$, $B = 17° 30'$, $C = 79° 50'$. How long is the side $b = ?$

Formula 20. $$\sin b = \frac{\sin c \sin B}{\sin C} = \frac{\sin 72° 30' \times \sin 17° 30'}{\sin 79° 50'}.$$

c	$+$ log. sin $72° 30' = 9{:}97942$	
B	$+$ log. sin $17° 30' = 9{:}47812$	
	$+ \qquad = 1{:}45754$	
C	$+$ log. sin $79° 50' = 9{:}99312$	
The answer,	log. sin $16° 56' = 9{:}46442$	or, $b = 16° 56'$.

Oblique-Angled Spherical Triangle.



[B refers to the whole angle between a and c, and b to the whole line opposite B.]

27. $\tan \frac{1}{2}(m+n) \tan \frac{1}{2}(m-n) = \tan \frac{1}{2}(a+c) \tan \frac{1}{2}(a \sim c) \tan m = \tan c \cos A.$

28. $$\tan C = \frac{\sin m \tan A}{\sin (b-m)}.$$

29. $$\cos a = \frac{\cos c \cos (b-m)}{\cos m}.$$

30. $$\cos n = \frac{\cos a \cos m}{\cos c}.$$

$$b = m \pm n.$$

31. $$\cot m = \frac{\cos c \tan A}{\tan a}.$$

$$s = \frac{a+b+c}{2}, \quad S = \frac{A+B+C}{2}.$$

32. $$\sin \tfrac{1}{2}A = \sqrt{\frac{\sin (s-c) \sin (s-b)}{\sin b \sin c}}.$$

33. $$\sin \tfrac{1}{2}a = \sqrt{\frac{\cos S \cos (S-A)}{\sin B \sin C}}.$$

To find the area of a spherical triangle:

Let Q be the area of the triangle in square degrees. If $R =$ radius of the sphere, the length of one degree will

$$= \frac{2\pi R}{360}, \text{ or one square degree} = \frac{R^2}{3285.58}.$$

1. $$\cot \tfrac{1}{2}Q = \frac{\cot \frac{1}{2}c \cot \frac{1}{2}a + \cos B}{\sin B}.$$

2. $$\sin \tfrac{1}{2}Q = \frac{\sin \frac{1}{2}c \sin \frac{1}{2}a \sin B}{\cos \frac{1}{2}b}.$$

Trigonometrical Formulas.

1. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta.$

2. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta.$

3. $\sin 2\alpha = 2\sin\alpha\cos\alpha.$

4. $\sin 3\alpha = 3\sin\alpha - 4\sin\alpha^3 = \sin\alpha(4\cos\alpha^2 - 1).$

5. $\cos 2\alpha = \cos\alpha^2 - \sin\alpha^2 = 2\cos\alpha^2 - 1 = 1 - 2\sin\alpha^2.$

6. $\cos 3\alpha = 4\cos\alpha^3 - 3\cos\alpha = \cos\alpha(1 - 4\sin\alpha^2).$

7. $\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}.$

8. $\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}.$

9. $\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}.$

10. $\cos\alpha - \cos\beta = 2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\beta-\alpha}{2}.$

11. $\sin\alpha^2 = \tfrac{1}{2}(1 - \cos 2\alpha).$

12. $\cos\alpha^2 = \tfrac{1}{2}(1 + \cos 2\alpha).$

13. $\sin\alpha^3 = \tfrac{1}{4}(3\sin\alpha - \sin 3\alpha).$

14. $\cos\alpha^3 = \tfrac{1}{4}(3\cos\alpha + \cos 3\alpha).$

15. $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}.$

16. $\cot(\alpha \pm \beta) = \dfrac{\cot\alpha\cot\beta \mp 1}{\pm\cot\alpha + \cot\beta}.$

17. $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan\alpha^2}.$

18. $\cot 2\alpha = \dfrac{\cot\alpha^2 - 1}{2\cot\alpha}.$

19. $\tan\alpha = \sqrt{\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \dfrac{\sin 2\alpha}{1 + 2\cos\alpha} = \dfrac{2\tan\frac{1}{2}\alpha}{1 - \tan\frac{1}{2}\alpha^2}.$

20. $\cot\alpha = \sqrt{\dfrac{1+\cos 2\alpha}{1-\cos 2\alpha}} = \dfrac{\sin 2\alpha}{1 - \cos 2\alpha} = \dfrac{\cot\frac{1}{2}\alpha^2 - 1}{2\cot\frac{1}{2}\alpha}.$

21. $\tan\alpha \pm \tan\beta = \dfrac{\sin(\alpha \pm \beta)}{\cos\alpha\cos\beta}.$

22. $\cot\alpha \pm \cot\beta = \dfrac{\sin(\beta \pm \alpha)}{\sin\alpha\sin\beta}.$

23. $\dfrac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \dfrac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}.$

Differential and Integral Calculus.

When one quantity depends upon another, so that the variations of one produce certain variations in the other, the one quantity is said to be a *function* of the other. There are many such functional relations occurring in mechanics and engineering; as, for example, those between time and distance in falling bodies, the expansion of steam in a cylinder, etc.

Thus, in the case of a falling body, we know that the motion begins slowly and grows quicker and quicker, so that after a body has been falling for several seconds it passes over a much greater distance in each later second than it did in the first second. By observing the spaces passed over by a falling body in several consecutive seconds we will find, as did Galileo, that the distances increase proportionally to the squares of the times, or, in modern notation,

$$s = \tfrac{1}{2}gt^2.$$

The closer together the successive observations are taken, the more nearly the truth will the deduction be. Thus, if we platted the values of s for a number of values of t,—taking the time intervals as one second,—and joined the points by straight lines, we should have a broken line, indicating roughly the curve. By taking the time intervals closer, the broken character of the line becomes less apparent. When the time intervals are taken so very close to each other that the broken character of the line can no longer be distinguished, it appears as a smooth curve, the equation of which gives the law connecting the two interdependent variables.

The object of the calculus is to discuss the immediately consecutive values of variables, in order that their relations may be reduced to expressions suitable for use in computation.

The method used is to take any single relation between the variable quantities under consideration, make a small increase in one of them, and compute what the corresponding increase will become in the other. Then, by deducing the ratio of the two increases in value, we get an algebraic expression corresponding to the geometrical construction giving the broken line instead of the curve, as described above. By a simple transposition in the equation the actual value of the increment may be made equal to zero, and, at the same time, permit the ratio of the variations at that instant to be determined.

Thus, let $y = ax^2$; then let x be increased by a quantity, $\triangle x$ or h, and y will have a corresponding increase, $\triangle y$, and we have

$$y + \triangle y = a(x + h)^2$$
$$= ax^2 + 2axh + ah^2$$

Subtracting $$y = ax^2$$

$$\triangle y = 2axh + ah^2$$

Dividing by $$\triangle x = h$$

we get $$\frac{\triangle y}{\triangle x} = 2ax + ah$$

Now, when $\triangle x$ is equal to zero, $\triangle y$ is also equal to zero; and thus, when the increment, $\triangle x = h$, is zero, we have

$$\frac{0}{0} = 2ax.$$

That is, the ratio between the increments of x and of y at their zero values is equal to $2ax$. It is usually stated in this demonstration that the value, $2ax$, is reached when the increment is made infinitely small, so that its square, h^2, may be considered still smaller, and hence negligible; but this manœuvring is altogether unnecessary, as there is no reason to object to the determination of the value of $\frac{0}{0}$ as the true and exact ratio of the two increments.

This is seen by an example in falling bodies. In the equation $y = ax^2$, let $a = 16$, and substitute s for y and t for x,—s representing space, and t, time,—and we have $s = 16t^2$, and $2ax$ becomes $32t$, the well-known formula for the velocity at the end of t seconds.

The integral calculus is the reverse of the differential calculus, being the study of the methods of finding the quantities and expressions which correspond to given differentials. The differential of a quantity is indi-

cated by prefixing the letter d, as dx (read differential of x), and the integral of an expression is indicated by the symbol $\int$, an old-fashioned form of the letter s (signifying summation).

The usual working method of applying the calculus to a technical problem is first to state, in the form of an equation, the relation existing between two immediately consecutive states of the functions under consideration, these being found from observation or from their known relations by differentiation. This statement being made, both sides of the equation are integrated simultaneously, the result being a general algebraic statement of the relation between the varying quantities within the limits for which the integration is made.

As an example, we may give the determination of the law of barometric pressure, or the formula for computing differences of altitude by observing differences in atmospheric pressure by the barometer.

Taking a column of air with a base equal to a unit of area (say 1 square metre) and an unknown height, x, and calling the pressure at the bottom of the column $= p$, and letting the weight of a unit volume of air (say 1 cubic metre) at the bottom $= q$, we have the following:

Let the height of the column, x, be increased by a very small quantity, dx, so that it becomes $x + dx$. We then have the pressure on the base increased by some small quantity, dp. But this is also equal to qdx, or the weight per cubic metre times the portion of a cubic metre which has been added; hence, we have

$$dp = qdx.$$

According to Mariotte's law, the weights of given volumes of air are proportional to the pressures; or, for another pair of pressures and volumes, p' and q', we have

$$\frac{q}{q'} = \frac{p}{p'}, \text{ and } q = \frac{q'}{p'}p,$$

which, in the above equation, gives

$$dp = \frac{q'}{p'}pdx;$$

or, calling the constant quantity $\frac{q'}{p'} = c$, we have

$$dp = cpdx.$$

Dividing both sides by p, we get

$$\frac{dp}{p} = cdx;$$

and integrating $\int \frac{dp}{p} = \int cdx$, or $\log. p = cx + C$,

when $x = 0$, $p = p'$, and $\log. p' = C$.

Subtracting, we have $\log. p - \log. p' = cx;$

but $\log. p - \log. p' = \log. \frac{p}{p'}$, and hence $cx = \log. \frac{p}{p'}$,

or

$$x = \frac{1}{c} \log. \frac{p}{p'},$$

and we have thus derived a formula giving the value of x for any two pressures,—that is, for the vertical height between two points at which the air pressures are p and p'.

The whole art of using the calculus in engineering or applied science lies in the framing of the equations and the use of the observed relations of the quantities, the processes of differentiation and integration being almost as much matters of routine as the use of logarithms.

For those desiring to refresh their recollection, reference may be made to Professor Perry's "Calculus for Engineers," Professor R. H. Smith's "Calculus for Engineers and Physicists," and Autenheimer's "Elementarbuch der differential und integral Rechnung," the latter being especially rich in numerical applications to mechanics, physics, and practical science.

Formulas.	Differentials.
1. $y = x$	$dy = dx.$
2. $y = ax^2$	$dy = 2axdx.$
3. $y = x^n$	$dy = nx^{n-1}dx.$
4. $3abx^3$	$= 9abx^2dx.$
5. $4ab^2x^n$	$= 4nab^2x^{n-1}dx.$
6. $a + x^3$	$= 3x^2dx.$
7. $(a+b)x^2$	$= 2x(a+b)dx.$
8. $6ab^4x^3 - c$	$= 18ab^4x^2dx.$
9. $x + 3z^2 - v$	$= dx + 6zdz - dv.$
10. $6x^3 + 4ax^2 - 3ax$	$= (18x^2 + 8ax - 3)dx.$
11. xv^2	$= vdx + 2xvdv.$
12. xvz	$= xvz\left(\frac{dx}{x} + \frac{dv}{v} + \frac{dz}{z}\right).$
13. $x(x^2 - bx)$	$= (3x^2 - 2b^2x)dx.$
14. $\frac{x^2}{v}$	$= \frac{2xvdx - x^2dv}{v^2}.$
15. $\frac{a}{x}$	$= \frac{adx}{x^3}.$
16. $\frac{a}{x^n}$	$= -\frac{nax^{n-1}dx}{x^{2n}}.$
17. $(a + \sqrt{x})^3$	$= \frac{3(a+\sqrt{x})^2dx}{2\sqrt{x}}.$
18. $(a + \sqrt[n]{x})^m$	$= m(a + \sqrt[n]{x})^{m-1} \frac{1}{n}x^{\frac{1}{n}-1}dx.$
19. $\frac{1}{n(a-x)^n}$	$= \frac{dx}{(a-x)^{n-1}}.$
20. $\frac{2\sqrt{2ax - x^2}}{x}$	$= -\frac{2adx}{\sqrt{2ax - x^2}}.$

Formulas.	Differentials.
21. a^x	$= a^x l{\cdot}adx.$
22. $d{\cdot}l{\cdot}x$	$= \frac{dx}{x}.$
23. $xl{\cdot}x$	$= (1 + l{\cdot}x)dx.$
24. $\frac{l{\cdot}x}{x^n}$	$= \frac{(1 - l{\cdot}x)dx}{x^{n+1}}.$
25. $\frac{x}{l{\cdot}x}$	$= \frac{(l{\cdot}x - 1)dx}{(l{\cdot}x)^2}.$
26. $\frac{ay}{\sqrt{x^2+y^2}}$	$= \frac{ayxdx - ax^2dy}{\sqrt{(x^2+y^2)^3}}.$
27. $\frac{a - 2bx}{(a+bx)^2}$	$= \frac{2b^2xdx}{(a+bx)^3}.$
28. $\sqrt{x} = x^{\frac{1}{2}}$	$= \frac{dx}{2\sqrt{x}}.$
29. $(ax + x^2)^n$	$= n(ax + x^2)^{n-1}(a + 2x)dx.$
30. $\sqrt{a^2 + bx^2}$	$= \frac{bxdx}{\sqrt{a^2 + bx^2}}.$
31. $d{\cdot}^2(ax^3)$	$= 6axdx^2.$
32. $d^3(ax^3)$	$= 6adx^3.$
33. $d^4(ax^3)$	$= 6ax^{o-1}dx = o.$
34. $\sin v$	$= +\cos vdv.$
35. $\cos v$	$= -\sin vdv.$
36. $\tan v$	$= +\frac{dv}{\cos^2 v}.$
37. $\cot v$	$= -\frac{dv}{\sin^2 v}.$
38. $\sec v$	$= +\frac{\cos vdv}{\cos^2 v}.$
39. $\operatorname{cosec} v$	$= -\frac{\cos vdv}{\sin^2 v}.$
40. Tan for any curve,	$t = y\sqrt{1 + \frac{dx^2}{dy^2}}.$

Differentials. Integrals.

1. $\int dx = x + c \qquad \int x dx = \frac{x^3}{2} + C.$

2. $\int 4ax^3 dx = 4a\int x^3 dx = ax^4 + C.$

3. $\int x^n dx = \frac{x^{n+1}}{n+1} + C.$

4. $\int \sqrt{xdx\int x}^{-\frac{1}{2}} dx = \int x^{\frac{1}{2}} dx.$

5. $\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}} dx = 2\sqrt{x} + C.$

6. $\int \frac{dx}{x^2} = \int x^{-1} dx = \int x^{-1} dx.$

7. $\int \frac{dx}{x^2} = \int x^{-2} dx = \frac{1}{x} + C.$

8. $\int \frac{dx}{x^3} = \int x^{-3} dx = -\frac{1}{2x^2} + C.$

9. $\int \left(ax^3 + \frac{b}{2\sqrt{x}}\right) dx = \frac{ax^4}{4} + b\sqrt{x} + C.$

10. $\int \frac{adx}{x} = al\cdot x + C.$

11. $\int \frac{bdx}{a+x} = bl\cdot(a+x) + C.$

12. $\int \frac{3ax^2dx}{b+ax^3} = l\cdot(b+ax^3) + C.$

13. $\int axdx + 3x^2dx - b^2dx = \frac{ax^2}{2} + x^3 - b^2x + C.$

14. $\int (a^2+b^2) = \int (a^2+b^2)dx.$

15. $\int (ax-2x^2)^2 dx = x^3\left(\frac{a^2}{3} - ax + \frac{4x^2}{5}\right) + C.$

16. $\int 3(ax-x^2)^2 (a-2x)dx = (ax-x^2)^3 + C.$

17. $\int \frac{n(x^{n-1}dx)}{\sqrt{a+x^n}} = 2\sqrt{a^2+x^n} + C.$

18. $\int \frac{2adx}{a^2-x^2} = l\cdot\frac{a+x}{a-x} + C.$

19. $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}l\cdot(x+\sqrt{a^2+x^2}).$

20. $\int \sqrt{a+bx}\cdot dx = \frac{2}{3b}(a+bx)^3 + C.$

Differentials. Integrals.

21. $\int \frac{dx}{\sqrt{a^2+x^2}} = l\cdot(x+\sqrt{a^2+x^2}).$

22. $\int_a^b 3mx^2dx = mb^3 - ma^3.$

23. $\int_a^b mxdx = \frac{m}{2}(b^2-a^2).$

24. $\int_0^\infty \frac{dx}{a^2+x^2} = \frac{\pi}{2a}.$

25. $\int_0^a \frac{dx}{\sqrt{a^2-x^2}} = \frac{\pi}{2}.$

26. $\int_a^b dx = -\int_b^a dx = \int_a^c = \int_a^b + \int_b^c.$

27. $\int \sin xdx = -\cos x + C.$

28. $\int \cos xdx = \sin x + C.$

29. $\int \tan xdx = -l\cdot\cos x + C.$

30. $\int \cot xdx = -l\cdot\sin x + C.$

31. $\int \frac{dx}{\sin x} = l\cdot\tan\frac{x}{2} + C.$

32. $\int \frac{dx}{\cos x} = l\cdot\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + C.$

33. $\int \sin x\cos xdx = \frac{1}{2}\sin{}^2x + C.$

34. $\int^{\infty} \frac{\sin bx}{x}dx = \frac{\pi}{2}.$

35. $\int_0^\infty \frac{\cos bx}{x}dx = \infty.$

36. $\int \frac{dt}{1+t^2}$ = circle arc, of which t = tan.

37. $\int \frac{dx}{\sqrt{2x-x^2}}$ = circle arc, of which x = sin versus.

38. $\iiint 6adx^3 = \iint 6axdx^2 = \int 3ax^2dx = ax^3 + C.$

39. $\iint 2(a+b)dx^2 = (a+b)x^2 + C_0x + C_1.$

40. $\iint 2v^2dx^2 + 8vxdxdv + 2x^2dv^2 = x^2v^2.$

MECHANICS.

In considering the action of force upon matter, it is important to have a clear understanding of the terms as used in the following pages.

Without going deeply into the theoretical considerations of analytical mechanics, we will discuss briefly those relations commonly used by the engineer in daily practice, leaving the profounder questions for the elaborate theoretical treatises, such as those of Rankine, Hertz, Lagrange, Du Bois, and others.

There are three elementary quantities used in mechanics, from which numerous compound quantities are derived:

1. **Force,** usually expressed in units of weight, as pounds, tons, kilogrammes, etc.

2. **Distance,** expressed in linear units, feet, yards, metres, etc.

3. **Time,** expressed in hours, minutes, or seconds.

From these we derive a number of compound expressions, some of which are given here, others will be used as occasion requires.

Thus, we have

Work, which is the product of force by distance, and expressed by a combination of units of weight and distance, as foot-pounds, kilogrammetres, etc.

Power, which is the product of force by distance, divided by time, or the performance of a given amount of work in a given time, expressed as foot-pounds per minute, kilogrammetres per second, etc.

Velocity is distance divided by time, as feet per minute, metres per second, miles per hour.

Acceleration is the time-rate of change of the velocity of a body, expressed as a velocity divided by time, feet per second per second, miles per hour per second, etc.

Forces may be conveniently represented by straight lines, the position of the line showing the direction of action of the force, and the length of the line indicating the magnitude of the force on some convenient scale. The convenience of the graphical method of solving problems in statics and mechanics renders it most useful, and in the following pages it will be extensively employed.

So far as precision is concerned, it is quite as practicable to construct force diagrams with a high degree of precision as it is to make the drawings of the structures to which they are to be applied, while the accuracy of the work is materially increased by the possibility of examining the relations of all the forces at once.

STATICS.

Statical problems are those which deal with the equilibrium of forces acting upon bodies at rest.

It is customary to consider the bodies upon which the forces act as being rigid, although it is well understood that all substances are more or less elastic; it being found more practicable to determine the relations of the forces first, and then to modify these, when necessary, for the influence of the elasticity of the material under consideration.

In order that a body or a structure shall remain at rest, it is necessary that all the forces acting upon it should balance each other. If this were not the case, the body would move in a direction dominated by the preponderating force. This fact is used to aid in the determination of statical problems. The influence of the combined action of all the *known* forces acting on a body enables the magnitude and direction of the remaining force which holds them in equilibrium to be determined.

The most convenient, rapid, and accurate method of combining and resolving the action of forces is the *graphical* method.

A *single force* may be indicated by a straight line, the length of which, on any convenient scale, shows the magnitude of the force.

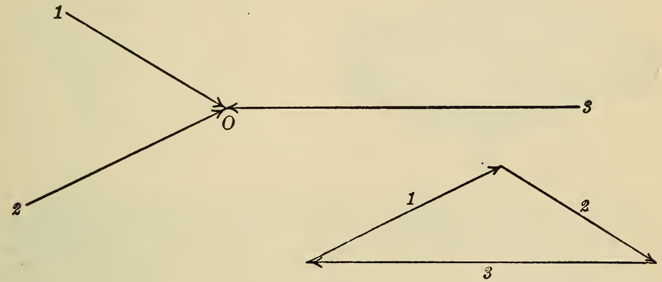
Thus, a force of 10 pounds may be represented as a straight line 10 inches long, in which case the scale is 1 inch to the pound. The direction of the action of the force is shown by the direction of the line, and,

if unopposed, the body upon which the force acts will move in the direction of the line of action of the force.

If the body *does not move*, equilibrium must be maintained by reason of the action of a force of equal magnitude to the first force, acting in the opposite direction.

Thus, a weight of 10 pounds, suspended from a cord, hangs stationary. There must, therefore, be produced in the cord a reacting force of 10 pounds, acting upward, otherwise the weight would fall. The upward reaction in the cord cannot be greater than 10 pounds, or the weight would move upward; hence, we know that the reaction in the cord is exactly equal to the force of the weight, but acts in the opposite direction.

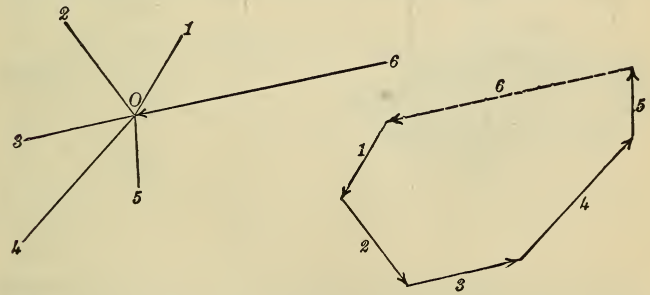
When more than one force is to be considered, the question becomes more complicated, but the principle is the same.



Thus, if we have two forces, 1*O*, 2*O*, acting upon the point, *O*, we find the magnitude and direction of the opposing force, which just balances and holds them in equilibrium, as follows:

At any convenient place on the paper draw a line, 1, parallel to 1*O*, and of a length corresponding, on any convenient scale, to the force, 1*O*. Thus, if 1*O* is 5 pounds, the line, 1, may be 5 inches, or 5 centimetres, or 5 feet long. From the extremity of 1 draw 2, parallel to 2*O*, and of a length equal to the force, 2*O*, on the same scale as used for 1. Then join the extremities of 1 and 2 by a line, 3. This last line will then be equal in length to the desired force, which holds 1*O* and 2*O* in equilibrium, and it will also be parallel to it in direction. By drawing *O*3 parallel to 3 we have the balancing force fully determined.

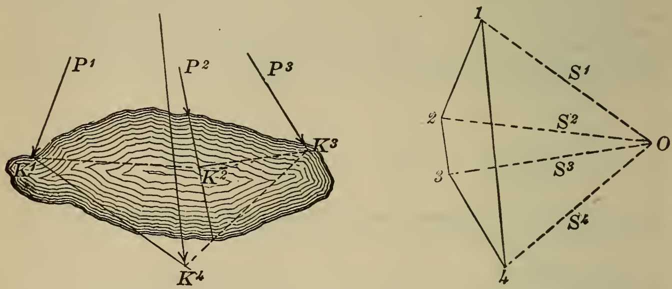
For more than two forces we may proceed in a similar manner.



Thus, if we have five forces acting upon the point, *O*, we draw the polygon, having the sides 1, 2, 3, 4, and 5, respectively, parallel to the forces and, proportional to their magnitude, upon the same scale, and then close the polygon by the dotted line, 6, which gives the magnitude and direction of the resultant, *O*6, which will hold the other forces in equilibrium.

If the polygon closes of itself, the system of forces is already in equilibrium; if it does not close of itself, the length and direction of the side necessary to close it will give the required result.

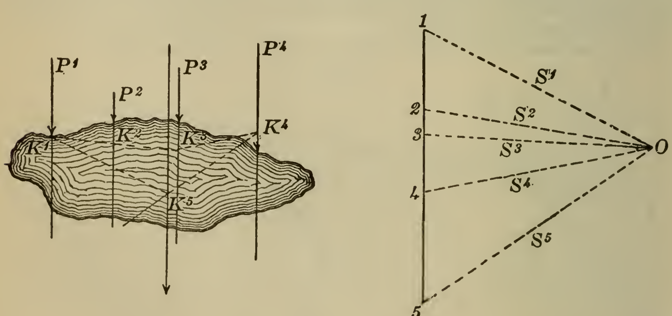
The foregoing discussion has assumed that the forces under consideration all act at the same point. When, however, the forces act at various points in a body which may be assumed as rigid, the resultant may be found as follows:



Suppose we have any rigid body, upon which several forces, P^1, P^2, P^3, are acting at several points. Construct the polygon as in the figure at the right, the line, 1–2, corresponding to P^1; 2–3, to P^2; and 3–4, to P^3. The resultant will then be equal to 1–4. Then choose any point, O, as a *pole*, and draw the rays, S^1, S^2, S^3, S^4. Now, as in the figure on the left, draw a line, K^1K^2, parallel to S^2, intersecting P^2 prolonged. From K^2 to K^3 draw a line parallel to S^3; then draw K^1 to K^4, parallel to S^1; and K^3–K^4, parallel to S^4, and the intersection will give the point, K^4, through which the resultant must pass.

The position of the pole, O, does not affect the result, as will be found by choosing several poles and observing that the position of the resultant is not affected thereby.

If we have a number of parallel forces acting upon a rigid body, the same method may be used, but the diagram becomes simplified.



Thus, if we have the vertical forces, P^1, P^2, P^3, P^4, we draw a vertical line, as in the diagram on the right, and lay off $1–2 = P^1$, $2–3 = P^2$, $3–4 = P^3$, $4–5 = P^4$. Taking any pole, O, and drawing the rays, S^1, S^2, S^3, S^4, S^5, we draw, as in the figure on the left, the line, K^1K^2, parallel to S^2; K^2K^3, parallel to S^3; K^3K^4, parallel to S^4. Then draw K^1K^5, parallel to S^1, and K^4K^5, parallel to S^5, these two lines intersecting at K^5. The resultant, equal to 1–5, will then pass through K^5.

Funicular Polygons.

If we have a flexible cord, secured at the ends and having weights suspended from it at various points, we may use the polar force diagram to determine the various forces acting in the combination.

Thus, if we have a cord suspended from two points, K^1, K^5, and to the points K^2, K^3, K^4, suspend weights, P^1, P^2, P^3, the cord will assume a shape similar to that shown in the figure. The combination will be in equilibrium, since the flexibility of the cord permits the weights to draw it into a position in which the forces balance each other. The various parts of the cord will then be subjected to tensions which are to be determined. There will also be vertical and horizontal forces at the points K^1 and K^5 which are to be found. All of these questions are solved by the

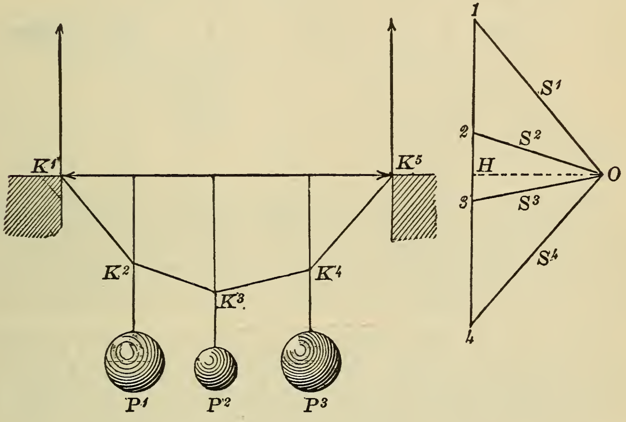


diagram on the right. First draw the vertical line, 1–4, making 1–2 = P^1, 2–3 = P^2, and 3–4 = P^3. Then from 1 draw S^1, parallel to K^1K^2, and from 4 draw S^4, parallel to K^4K^5, and the intersection of these two rays determines the position of pole, O. The rays S^2 and S^3 are parallel to K^2K^3 and K^3K^4.

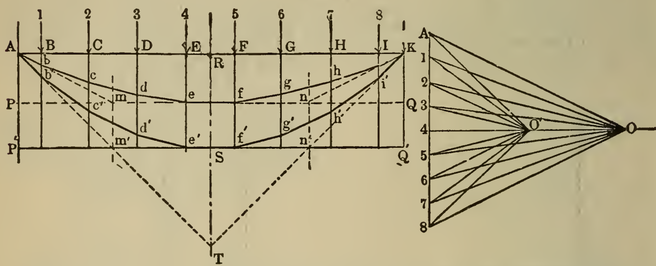
The lengths of the various rays, S^1, S^2, S^3, S^4, measured on the same scale on which the vertical forces were laid off, will then give the magnitude of the tensions in the parts of the cord to which they are parallel. By drawing a horizontal line, OH, through the pole, O, the vertical will be divided at a point, H, and $1H$ and $H4$ will be the vertical reactions at K^1 and K^5, while the length, OH, will give the horizontal force acting to draw the two ends of the cord together. If the butments were removed, and a rod extending from K^1 to K^5 substituted, the length, OH, would give the compression on the rod. If the whole diagram be imagined as inverted, and the parts of the cord be replaced by rigid struts, the figure will represent a framework which will sustain the same weights without distortion, the tensions in the various parts of the cord being converted into thrusts in the corresponding members of the framework.

In the preceding example the form taken by the cord is assumed to be given, and the only requirement is the determination of the forces. In some important cases, to be discussed hereafter, it is desirable to determine the form of the curve under various conditions of loading, as well as the stresses. Thus, the data given may be the span and the depth of the lowest point in the curve; also, the position and magnitude of the loads; and it may be required to find the form which these conditions give to the cord. The importance of these questions will be seen when it is understood that the flexibility of the cord permits it to assume a position of equilibrium

under any loading; and hence from it can be deduced the stresses which are produced in rigid bodies, such as beams and similar constructions.

It is well known that a cord suspended from two points on the same horizontal line, and uniformly loaded, will assume the form of a parabola; but instead of acting on this assumption we may proceed just as if we had only to depend upon the methods of graphical statics, and then apply the same methods to the cases of unequal loading and unequal distribution.*

FIG. 1.



Suppose, Fig. 1, that we have the forces, 1, 2, 3, 4, 5, 6, 7, 8, equal in magnitude and at equal distances apart, acting vertically, as under the action of gravity, and that it is desired to determine the shape of the curve assumed by a cord sustaining these forces. The points of suspension are given at A and K, the forces are given in position, and the sag, RS, of the curve is given. The horizontal tension and the vertical reactions at the points, A and K, are required, and also the tension in each portion of the curve.

Referring to the force diagram at the right, we draw the horizontal line, $4O$, and draw a perpendicular through 4. We then lay off the spaces, A, 1, 2, 3, 4, 5, 6, 7, 8, equal, on any convenient scale, to the forces, and choose any point, O, as a pole, and draw the rays, $1O$, $2O$, $3O$, ---$8O$. We have taken the point, O, on the horizontal, because we know that the curve is symmetrical, being uniformly loaded, and for reasons which will appear hereafter. Now, starting at A, in the diagram to the left, we draw Ab, parallel to AO; bc, parallel to $1O$; cd, parallel to $2O$; and so on until we come to iK, parallel to $8O$. This gives us a curve, A, b, c, d, e, f, g, h, i, K, which is a force polygon corresponding to the forces given. The horizontal tension at A and at K will then be equal to the distance, $4O$, measured on the same scale as was used for the given forces in making the diagram, and the vertical reactions will be equal to $A4$ and 4–8.

Now this diagram, while undoubtedly correct, is not the one we want, as the sag is too small; and this is due to the fact that we have taken our pole, O, too far from the point, 4, or, in other words, we have assumed the tension too great. Having once obtained one equilibrium curve, however, it is easy to transform it into any other one of any desired sag, in the following manner:

Draw the horizontal line, PQ, through the lowest point of the curve, which we have already obtained. Then prolong the line, Ab, until it intersects this horizontal at m; also prolong Ki until it intersects the horizontal at n. Draw $P'Q'$ through S, the point of the desired sag, and drop perpendiculars from m and n until they intersect $P'Q'$ at m' and n'; join Am' and Kn'.

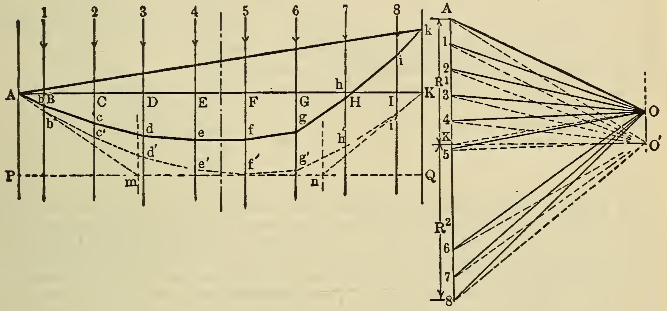
Now, in the force diagram at the right of the figure, draw from A a line, AO', parallel to Am', and a line, $8O'$, parallel to Kn'. They will intersect on the horizontal at O', and this will be a new pole. Using this pole by drawing rays to O' from A, 1, 2, ----8, we have a new force diagram. Now, starting again at A, we draw Ab', parallel to AO'; $b'c'$,

* The following treatment of the subject of the catenary is substantially the same as that given in an article by the author in *Engineering-Mechanics*, June, 1896.

parallel to $1O'$, etc., and we get a new catenary, A', b', c', ---- $i'K$, which will have just the sag required. The distance, $4O'$, will then be the correct horizontal tension, which corresponds to the sag, RS; and the tension in any portion of the curve is equal to the length of the corresponding parallel ray.

All this is very clear; but this is the simple case of uniform loading, and might just as well have been solved by drawing a parabola of the required span and sag. Suppose now, however, that the loads are not uniform. Such an example is shown in Fig. 2. Here the loads are all the same, except that at 6, which is as much greater as is indicated by the length of the arrow. As before, we know only the magnitude and direction of the forces and the span and sag of the curve, and desire to find the horizontal tension and other forces. Referring to the force diagram on the right, we draw a vertical line, $A8$, making the distances,

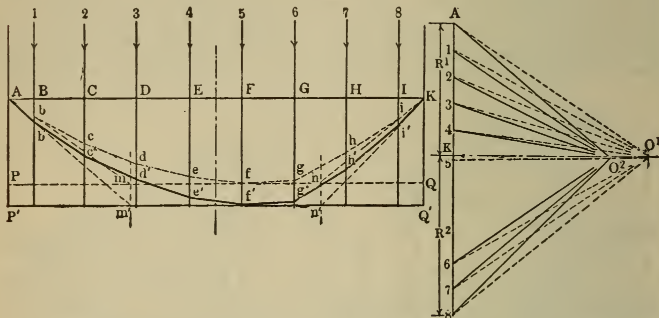
FIG. 2.



$A1$, 1–2, 2–3, ---- 7–8, proportional to the various forces, and it will be noticed that 5–6 is the large force, the others being equal to each other. We now choose any point, O, for a pole, and draw the rays, AO, $1O$, $2O$, ---- $8O$. Then, starting at A in the diagram to the left, we draw Ab, parallel to AO; bc, parallel to $1O$; cd, parallel to $2O$, etc., and get the curve, A, b, c, d, e, f, g, h, i, k. We then join k back to A by drawing the inclined line, Ak. This gives us a complete polygon, but it is not the one we want, for two reasons: first, it has not the right sag; and second, the points of suspension are not on a horizontal line. We can readily bring the points of suspension on a horizontal line, in the following manner: In the force diagram, draw from the line, O, a line, OK, parallel to kA; then will the distances, R' and R, be the vertical reactions at the points of suspension, and OK will be the tension at A and at k, in the direction of the line, Ak. If, now, we draw in the force diagram a line, KO', horizontally through K, and place a new pole, O', on this horizontal line vertically under O, we can draw a new force diagram, as shown in the dotted lines, and the polygon drawn from A, with its sides parallel to the rays of this new diagram, will give us the dotted curve, which has the same sag as the first curve, but has its points of suspension on a horizontal line. We thus see that even if the first curve—obtained by choosing any pole—does not give us a curve with the required points of suspension, that it can readily be transformed into the desired form. If, instead of having the points of suspension on a horizontal line, it is desired to have them at different elevations, it is only necessary to draw a line through K, on the force diagram, parallel to a line joining the desired points of suspension, and place the pole on the line so obtained, and the desired curve will be found. Now, to obtain the sag which is wanted, we have only to proceed as in the first case, Fig. 3. In this figure the dotted curve corresponds to the dotted curve of Fig. 2. Draw the horizontal line, PQ, through the lowest point, f, of the curve already obtained; prolong Ab to m, and Ki to n. Draw, also, $P'Q'$ horizontally through the desired point of lowest sag, and drop perpendiculars from m and n to it at m' and

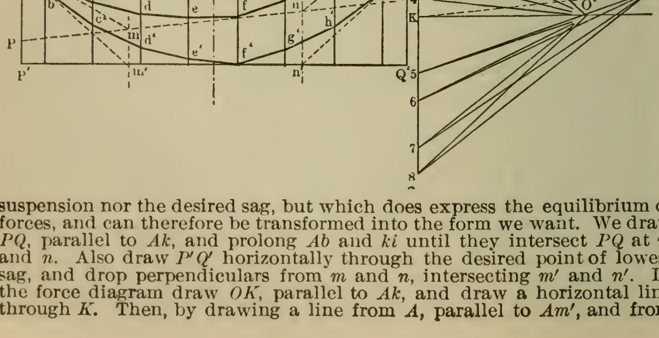
n'. Join Am' and Kn', and draw from A, in the force diagram, a line parallel to Am', and from 8, a line parallel to Kn'. These two lines will intersect at the point, O^2, which will then be the correct pole for the curve of the desired sag. Drawing a new set of rays, we have only to draw the new polygon, $A, b', c', d', e', f', g', h', i', K$, with its sides parallel to the corresponding rays, and the problem is solved. The vertical reactions at the point of support are R' and R, and the horizontal tension is equal to KO^2.

FIG. 3.



In actual practice the two operations of bringing the points of support to the horizontal (or to any desired inclination), and the adjustment of the tension to produce any required sag, may be combined so as to give the proper pole at one operation, as shown in Fig. 4, in which, also, the forces are all shown as different, so as to show the general nature of the solution. We first draw the vertical line of the force diagram on the left, making the spaces from A, downward, proportional to the forces of the corresponding numbers, and then choose a trial pole, O. Drawing the rays, and constructing the polygon, $A, b, c, d, e, f, g, h, i, k$, and joining kA, we have a polygon which has neither the proper position of the points of

FIG. 4.



suspension nor the desired sag, but which does express the equilibrium of forces, and can therefore be transformed into the form we want. We draw PQ, parallel to Ak, and prolong Ab and ki until they intersect PQ at m and n. Also draw $P'Q'$ horizontally through the desired point of lowest sag, and drop perpendiculars from m and n, intersecting m' and n'. In the force diagram draw OK, parallel to Ak, and draw a horizontal line through K. Then, by drawing a line from A, parallel to Am', and from

8, parallel to Kn', we find that they intersect at O', on the horizontal line, KO', and O' will at once be the new pole for the final curve, A, b', c', d', e', f', g', h', i', K.

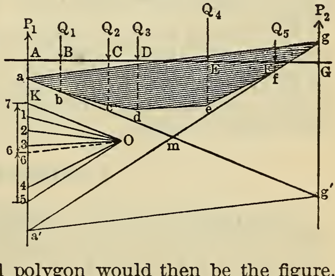
As a general idea of the process we may imagine the pole to be connected to the points, A, 1, 2, 3, 4, 5, 6, 7, 8, by elastic cords, so that they will remain taut and straight as O is moved about. Then, if we move the pole up and down anywhere, always keeping it at a constant distance from the line, $A8$, we shall obtain diagrams which will give correct polygons for the forces under consideration, and of any desired inclination. The horizontal tension being unchanged, the sag will remain constant in all these curves. If we move the pole, O, to and from the line, $A8$, we shall obtain curves of varying sag and correspondingly varying horizontal tensions, and, as we have shown how to obtain the position of the pole for any desired sag, we have only to place it there and proceed with the construction of the curve. If the forces, 1, 2, 3, – –, etc., are not spaced equally, it is only necessary to draw verticals through their points of application and use them in the construction of the curve, instead of the lines as given in the figures. By this simple graphical process, therefore, all the problems involved in the construction of such curves may be rapidly and accurately solved.

The space which has been given to the variably-loaded catenary in the preceding pages will be understood when it is seen that the construction of such curves enables the distribution of stresses in a great variety of structures to be readily and accurately determined. The flexible cord, being at liberty to assume a position of equilibrium, is free from any bending stresses, every portion of the curve being, in fact, a resultant of the forces acting upon it, the tension in the various portions of the cord being measured by the length of the corresponding ray in the force diagram. If, now, we invert the catenary, we have the proper curve for an arch subjected to similar forces, the only difference being that the arch is in compression, while the catenary is in tension. This will be discussed more fully when treating of the arch.

If, instead of a cord, we have a horizontal beam resting upon two supports and loaded in any given manner, we may use the catenary to determine the stresses. The beam, unless loaded excessively, will not have an appreciable deflection, and so will not place itself in the line of the catenary. In consequence, it is subjected to internal stresses of a kind differing from the simple tension of the catenary. By drawing the catenary and the force diagram we get the data to determine these internal forces, and thus are able to proportion the beam to resist the loads properly.

If we have a beam, AG, loaded with parallel forces, Q_1 to Q_5 (Fig. 5), whose load is to be opposed by reactions, P_1 and P_2, at A and G, we may first determine a resultant, Q, of all the forces, and then decompose this into values for P_1 and P_2. We also omit the determination of Q altogether, and proceed to determine P_1 and P_2 directly, as follows:

FIG. 5.



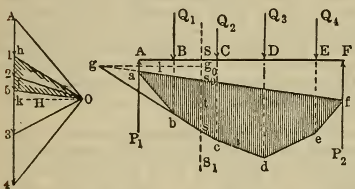
Choose any pole, O, and form the force polygon, $K1.2....5O$, and construct the cord polygon, making its sides parallel to their respective rays, and draw ba, parallel to KO, and fg, parallel to $O5$, their intersections with the lines of the forces, P_1 and P_2, being a and g. Join ag, which will be the closing line of the polygon, and its parallel, $O6$, in the force polygon gives $P_2 = 5.6$ and $P_1 = 6.7$. If the sides, ab and fg, of the cord polygon are prolonged in the other direction we obtain a' and g', giving, however, the same result, since $a'g'$ is parallel to ag. The cord polygon would then be the figure, a', g', m, b, d, c, e, f, m, a', and m indicates the position of the resultant of the forces, Q_1 to Q_5, or of P_1 and P_2.

The cord polygon, or catenary, therefore, gives the proportion of load borne by each of the supports. But it does more, it enables the determination of both the shear and the statical moment at any point.

A **Statical Moment** is the product of a force by the normal distance from the point of resistance against which it acts. Thus, a force of 10 pounds, hanging from the end of a rod projecting 36 inches from a wall, has a statical moment of $36 \times 10 = 360$ inch-pounds,—*moment* being merely the technical term for *leverage.*

A statical moment is a compound quantity, expressed in terms of force and distance, as inch-pounds, foot-tons, kilogramme-metres, etc. In the case of a beam resting upon two supports, and having various loads upon it, the statical moment at any point is the product of the resultant of all the forces acting upon the beam on either side of the point into the distance of the line of action of the resultant from the given point for which the statical moment is determined.

FIG. 6.



In order to show how the statical moments in any loaded beam may be determined graphically, take the example shown in Fig. 6.

After constructing the force polygon, *A O*4, and cord polygon, *a, b, c, d, e, f,* let it be required to find the statical moment for any point, *S*, upon the beam. This moment is the product of the resultant of all the forces upon one side or the other of the line, SS_1, into the lever arm, *l*, of this resultant from SS_1.

The magnitude of this resultant is obtained from the distance, $hi = 1.5$, in the force polygon, cut off by the rays, *O*1 and *O*5, which are parallel to *bc* and *fa*, and its point of application is determined by prolonging these sides until they intersect at *g*. By drawing the perpendicular, gg_0, the lever arm, *l*, of the resultant, $P = hi$, is determined for the force acting at the point, *S;* and hence we have $M = Pl$.

This multiplication may also be performed graphically. By drawing the perpendicular, *Ok*, in the force polygon, we obtain the altitude of the triangle, *Ohi*, from the base, *hi*, and this triangle is similar to the triangle, gss_0, whose altitude is *l*. Call in $Ok = H$, and $ss_0 = t$, we have

$$P : H = t : l,$$
$$\text{or} \qquad M = Pl = Ht.$$

This proves that the statical moment at any point in a beam is proportional to the corresponding ordinate of the cord polygon, parallel to the direction of the forces, since *H* is a constant. By making *H* equal to unity the moment, *M*, becomes equal to the ordinate, *t*. It is not necessary to determine the position of the point of application, *g*, of the resultant, since it is the relation between the statical moments which is most desirable, whether *H* be chosen as a unit or not. This property of the cord polygon for parallel forces is most useful, and an example may be found in the case of axles.

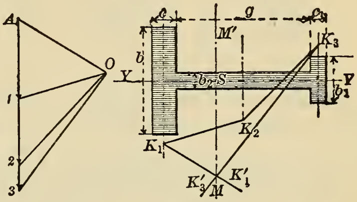
The shearing force in a beam at either support is evidently equal to the entire reaction at that support. Thus, in Fig. 6, the shearing force at *A* is equal to *A*5 on the force diagram. The shearing force at any other point in the beam is equal to the distance from 5 to the point on *A*4 corresponding to that point in the force diagram. Thus, the shearing force at *B* is equal to 1–5, etc.

Centre of Gravity.

Every particle of a body is attracted by the force of gravitation to the earth, and the sum of all these forces upon the particles constitutes the weight of the body. In accordance with the methods already given for determining the resultant of a number of parallel forces, the point of application of the resultant of the force of gravity may be found. This point is known as the *Centre of Gravity* of the body. For homogeneous bodies the position of the centre of gravity may generally be computed from the form of the body. For bodies which are entirely symmetrical and homogeneous, the centre of gravity is situated at the centre of figure.

For bodies which are symmetrical about a given axis, such as a cone, etc., the centre of gravity is situated in the axis. Various methods are used for determining the position of the centre of gravity of non-symmetrical figures, most of them based upon the subdivision of the figure into parts, of which the centres of gravity are known.

The most convenient of these is the graphical method.



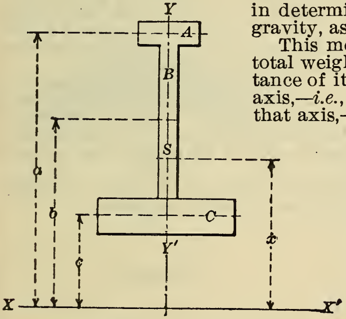
This may be done by dividing the figure into a number of strips of uniform width such that their area may be considered as proportional to their middle ordinate, constructing the force and cord polygons, and taking the line of the resultant as a line of gravity. If the figure is not symmetrical, it will be necessary to divide the figure again in another direction and determine another line of gravity, when the position of the centre of gravity will be found at the intersection of the two lines. For figures of simple form larger determinate sections may be taken instead of strips, their area determined in any convenient manner, and the diagram constructed accordingly.

Suppose, for example, that it is required to determine the position of the centre of gravity of the T-shaped section shown in the above cut. The figure is symmetrical about the axis, *YY*, so that the centre of gravity must lie somewhere in that line. We may divide the figure into the rectangular portions $b \times c$, $b_1 \times c_1$, and $b_2 \times g$, which we will call respectively the areas 1, 2, and 3.

These three forces are laid off at *A*, 1, 2, 3, a pole, *O*, selected, and $K_1'K_1$ drawn parallel to OA; K_1K_2, parallel to $O1$; K_2K_3, parallel to $O2$; K_3K_3', parallel to $O3$, when the intersection of the sides, K_1K_1' and K_3K_3', at *M* gives a point on the line of gravity, MM', whose intersection, *S*, with the axis, *YY*, is the centre of gravity of the figure.

The method of moments may also be used in determining the position of the centre of gravity, as follows:

This method is based on the fact that the total weight of a body, multiplied by the distance of its centre of gravity from any given axis,—*i.e.*, its statical moment with regard to that axis,—is equal to the sum of the statical moments of its various parts.



Thus, if we have the section here shown, we see that its figure is symmetrical about the axis, YY', so the centre of gravity must lie in that axis. Taking any convenient axis, XX', we divide the section into the three rectangles, *A*, *B*, and *C*, of which the positions of the centres of gravity are known, we have their distances from the axis, XX', equal respectively to *a*, *b*, and *c*; and their statical moments with reference to the axis, XX', will be

$$Aa,\ Bb, \text{ and } Cc.$$

The area of the whole figure is equal to $A + B + C$, which we will call *M*, and the distance of its centre of gravity, *x*, from the axis, XX', is unknown and sought.

We have

$$Aa + Bb + Cc = Mx;$$

or

$$\frac{Aa + Bb + Cc}{M} = x.$$

Thus, if $A = 4$ square inches, $B = 5$ square inches, $C = 9$ square inches, and $a = 12$ inches, $b = 9$ inches, $c = 4$ inches, we have

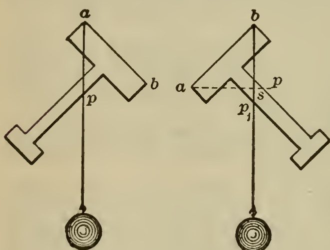
$$\begin{aligned} Aa &= 4 \times 12 = 48 \\ Bb &= 5 \times \ \ 9 = 45 \\ Cc &= 9 \times \ \ 4 = 36 \\ & \qquad\qquad \overline{129} \end{aligned}$$

and this, divided by the area of the whole figure, or 18 square inches, gives $\frac{129}{18} = 7.166$ for x, the distance of the centre of gravity, S, from the axis, XX' The position of the axis, XX', is immaterial, so long as all the moments are taken with reference to the same axis.

When a figure is not symmetrical, the moments must be taken first with reference to a vertical axis and then with reference to a horizontal axis, and the centre of gravity will be found at the intersection of the two lines thus determined.

In practical work the position of the centre of gravity is often most conveniently found by experiment.

Thus, if a scale drawing of the section be cut out of stiff card-board, or better, thin sheet metal, it may be hung up by one corner, a plumb-bob made of a fine thread and weight being suspended from the same point, a, as in the figure. By marking the point where the thread intersects the edge of the section, as at p, the path of the vertical across the section may be drawn from the supporting point. The section is then suspended from another point, b, and the point p' marked; the intersection of the lines ap and bp' gives the position of the centre of gravity, s. Care must be taken to have the section perfectly free to oscillate about the point of suspension, usually a pin, and errors due to friction against the wall must be avoided.

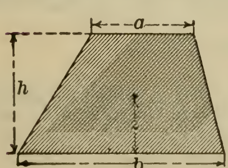


Another convenient method is to balance the section across a horizontal knife-edge, in two successive positions, marking the intersection of the two positions of the knife-edge. This latter method may be conveniently applied by using a draftsman's triangular scale as a knife-edge.

The position of the centre of gravity for some of the more generally occurring figures may be obtained from the following diagrams:

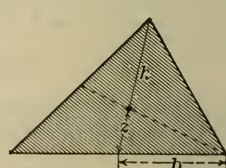
Quadrangle.

a and b parallel.



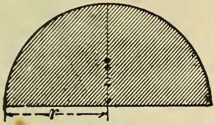
$$z = \frac{h}{2} - \frac{h}{6}\left(\frac{b-a}{b+a}\right).$$

Triangle.



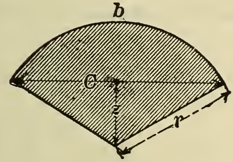
$$z = \frac{h}{3}.$$

Half a Circle Plane, or Elliptic Plane.



$z = 0.4244r.$

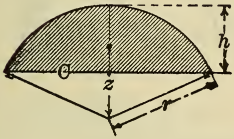
Circle Sector.



$$z = \frac{2cr}{3b}.$$

Circle Segment

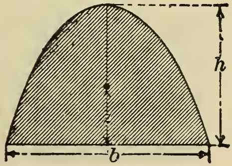
$\mathbf{a}$ = area.



$$z = \frac{c^3}{12\mathbf{a}},$$

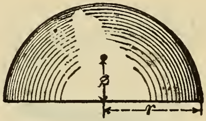
$$x = h + z - r.$$

Parabola.



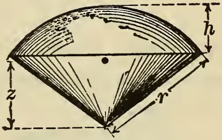
$$z = \frac{2h}{5}.$$

Half Sphere.



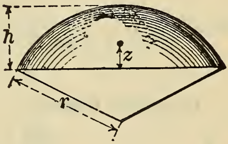
Convex surface, $z = \frac{1}{2}r.$
Solid, $z = \frac{3}{8}r.$

Spherical Sector.



Solid, $z = \frac{3}{4}\left(r - \frac{h}{2}\right).$

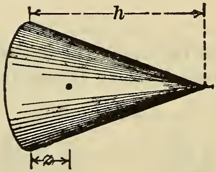
Spherical Segment.



Convex surface, $z = \frac{h}{2}.$

Solid, $z = \frac{h}{4} \cdot \left[\frac{4r - h}{3r - h}\right].$

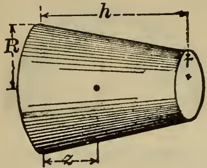
Cone.



Convex surface, $z = \frac{h}{3}.$

Solid, $z = \frac{h}{4}.$

Conic Frustum.

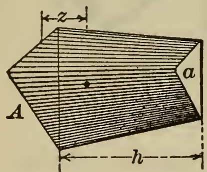


Convex surf'e, $z = \frac{h}{2} - \frac{h}{6}\left[\frac{R-r}{R+r}\right].$

Solid, $z = \frac{h}{4} \cdot \left[\frac{R^2 + r(2R+3r)}{R^2 + r(R+r)}\right].$

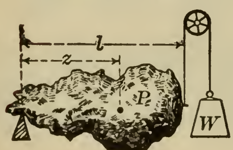
Pyramidic Frustum.

A and a = area of the two bases.



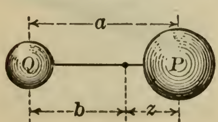
Solid, $z = \frac{h}{4}\left[\frac{A + 3a + 2\sqrt{Aa}}{A + a + \sqrt{Aa}}\right].$

Irregular Figure.



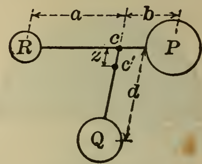
$P : W = l : z,$ $\quad z = \frac{Wl}{P}.$

To find the Centre of Gravity of Two Bodies, *P* and *Q*.



$z = \frac{Qa}{P+Q},$ $\quad b = \frac{Pa}{P+Q}.$

To find the Centre of Gravity of a System of Bodies.

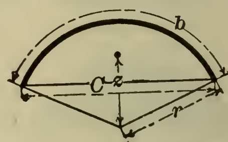


$b = \frac{Ra}{P+R},$ $\quad z = \frac{Qd}{P+R+Q}.$

Half a Circumference of a Circle or Ellipse.

$z = 0.4244r.$

Circle Arc, or Elliptic Arc.

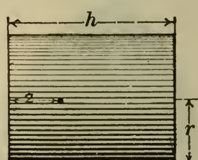


$z = \frac{cr}{b} = \frac{c(c^2 + 4h^2)}{8hb}.$

For semicircular line,

$z = \frac{2r}{\pi} = 0.6366r.$

Cylindric Surface, with a bottom in one end.



$z = \frac{h^2}{2(h + r^2)}.$

Statics of Framed Structures.

As the distribution of stresses in simple beams or in suspended cords may be determined graphically, so may the stresses in the various members of framed structures be investigated.

Framed structures are of very general application wherever loads are to be supported, and their discussion may be classified as a system by itself, while their use extends from the simple trussed beam to the bridge and roof truss; also for walking beams and many other uses.

The tensile and compressive stresses in these various forms may readily be examined by means of the force plan, which consists of both the force and cord polygons and their modifications. The subsequent examples will serve to illustrate the principal cases. In all of these cases it is assumed that at the knots—*i.e.*, at the points where several members meet, —a joint is supposed to exist; or at least no account is taken of the resistance to bending at the knots.

In order to form such a plan for any given construction, it is necessary first to determine the division and direction of the forces, and then, beginning at one of the external forces and laying off its direction and magnitude to the next knot, combining it there with the external forces at that point, laying off the resultant to the next bend, etc. Upon such combinations of force triangles or quadrangles the force plan is constructed.

If it is desired to determine the directions of the components of a given or determined force, the principles laid down in the following rules must be borne in mind.

If one force is to be separated into two or more forces, its direction is to be reversed and it is to be made the closing line, S', in the paths of the other forces.

If two or more given forces are to be combined with two or more other forces, the force polygon will consist of the given forces and their closing line, S.

The first rule is only a special case under the second or general rule, since the single force may be considered as an unclosed force polygon whose closing line passes backward over the same path to the starting point.

In the investigation of each member in a frame without error, it is best to assume the member to be cut, and to determine the external forces at each section which oppose the internal forces; the direction of the forces may then also be determined with precision.

I. **Simple-Trussed Beams.**—The beam, ABC, is supposed to carry at B a load equal to $2P$, acting in a direction normal to AC, and to be supported at A and C. Since $AB = BC$, the reaction at each support is equal to P. It is then required to determine the stresses upon the various members from 1 to 5, as marked in the figure.

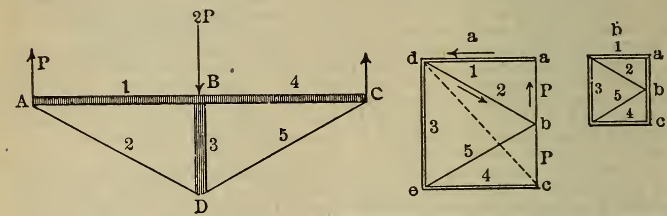
Referring to the diagram marked a, let ab be the reaction, P, which acts upward at A. We now have to construct a diagram of the internal forces acting in AB and AD. To simplify matters we will give these forces the same numbers as their corresponding members, drawing 1 parallel to AB, and 2 parallel to AD. The direction of the force, P, in the closing line of the force triangle determines the direction in the other two sides, as shown by the arrows and by the lines 1 and 2. In this case there will be compression in AB and tension in AD.

In order to show this clearly, in all the following strain diagrams the forces acting compressively in struts or posts will be indicated by double lines, while all tension members, links, or rods will be shown by single lines.

Following out this idea, we shall, in the following illustrations, show all struts or compression members in the construction drawings as having a measurable thickness, as if made of wood, while the tension members

will be represented by simple lines, although this is not intended to indicate any limit as to the choice of materials.

For the knot at B we make $abc = 2P$, and, following in the direction dac (because the thrust is from A towards B), join the closing lines 3 and 4, both of which represent compression. The combination of 2 and 3 determines 5, which is tension. This gives an entirely symmetrical plan,

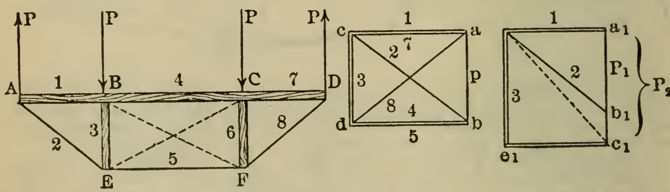


Simple-Trussed Beam.—I.

which was to be expected from the symmetrical form of the structure, and an investigation of one-half is practically sufficient.

If the load, $2P$, is taken as uniformly distributed over the entire distance, ABC, instead of being concentrated at B, the reactions at A and B will each be equal to $\frac{P}{2}$, and the load at $B = P$, so that one-half of the load on AB and BC is referred to the knots A, B, and C. From these conditions we obtain the force plan b, which is geometrically similar to the other, but only half as large.

II. **Double-Trussed Beam** (much used for constructions of all sizes).—In this case take vertical forces, P, at B and C, and corresponding vertical reactions at A and D. In the first force plan a is drawn equal to P, and 1 and 2 parallel respectively to AB and AE, thus determining the forces 1 and 2,—1 being compression, and 2 tension. Lines now drawn parallel to BE and EF determine the compression in 3 and the tension in 5, while the compression at 4 is the closing line of 3, 1, and P; and the other half of



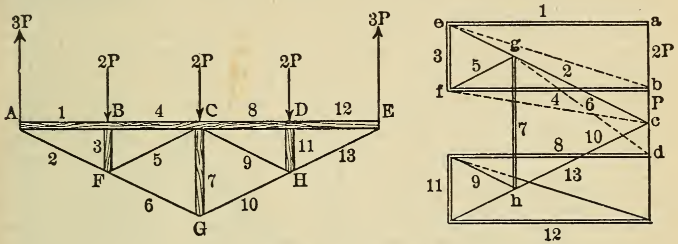
Double-Trussed Beam.—II.

the diagram is similar. If the vertical forces at A and B are not of the same magnitude, which is often the case in practice, the structure should be strengthened by the introduction of the diagonals, EC and BF.

The second diagram shows the construction in this case. Let $P_1 = a_1b_1$ be the force acting at A, and $P_2 = a_2c_2$ be the force acting at B. Draw a vertical line from 1 to a horizontal through C_1, which gives the length, 3, of the vertical force at B, and by drawing the dotted diagonal line their resultant is found. If any of the tension members are omitted the framework will tend to take an inclined position until the various parts are at such an angle with each other that both constructions will give the same value for 3. For this reason it is best in nearly every case to use the diagonal counterbraces.

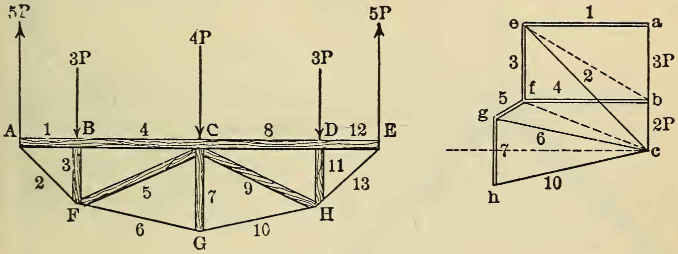
III. **Triple-Trussed Beam.**—The uniformly distributed load upon the framework gives the following distribution of forces. The force, $3P = abc$, is first decomposed in 2 and 1, or ce and ea; then 1 is connected to $ab = 2P$

by the line be, and this latter decomposed into 3 and 4, or ef and fb; 2 and 3 are now joined by fc, and the components at 5 and 6, or fg and gc, found. Since 6 and 10 are equal to each other, we may draw ch parallel to GH, and equal to cg, which gives $gh = 7$; the rest of the force plan is similar to the first half.



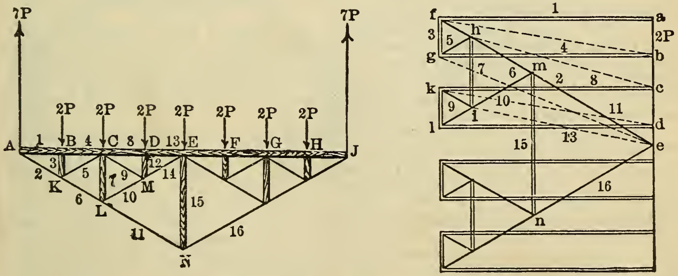
Triple-Trussed Beam.—III.

IV. Another form of Triple-Trussed Beam is shown below.—The space between B and C is twice as great as between A and B, and the uniformly distributed load is equal to $12P$, acting at the various knots, as shown in the figure.



Triple-Trussed Beam.—IV.

In the force plan make $abc = 5P$, and draw parallel to 1 and 2 the lines ae and ec; then join 1 with $3p$ (for the knot at B), and decompose into 3 and 4, or ef and fb. Now combine 2 with 3, giving cf, and draw 5 and 6 parallel to FC and FG, respectively. This case differs from the preceding, in that 5 is now compression instead of tension. The equality of the forces 6 and 10 gives $gh = 7$, and the similar half of the diagram need not be drawn.

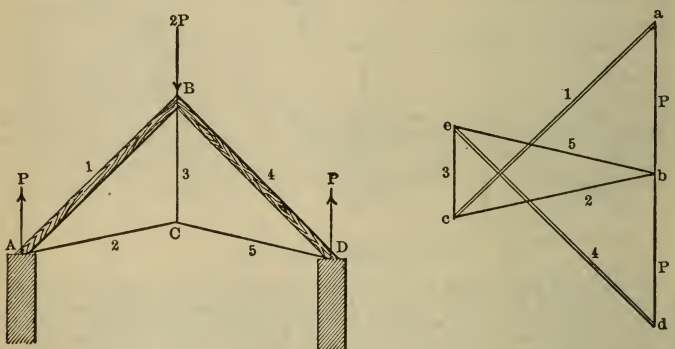


Multiple-Trussed Beam.—V.

V. **Multiple-Trussed Beam.**—The beam, AJ, is divided into eight equal parts, which are represented as being uniformly loaded, the load at each knot being shown in the figure. In constructing the force plan we make $ae = 7P$, and by drawing the lines parallel to 1 and 2 we obtain af and fe; then lay off $ab = 2P$, and join the resultant, bf. This decomposes into 3 and 4, or fg and gb. The forces 2 and 3 combine to give the resultant, ge, which, by drawing lines parallel to KC and KL, gives gh and he for the values of 5 and 6. We now find that to proceed further we have three forces of given direction only, and, since this is indeterminate, we must obtain one magnitude as well. This, for example, may be done for the force 7, as follows: the strut, CL, sustains the vertical components of 5 and 9, as well as its own direct load, $2P$. Now 5 and 9 are equal to each other, since they are placed symmetrically, and carry equal loads from the struts, BK and KM; hence, in the force plan, we may make hi, which represents the force 7, equal to twice the projection of 5 upon the vertical $+ 2P$. This we can now combine with $6 = he$, giving ie, which in turn decomposes into im and me, or 10 and 11. Returning to the knot, C, we may now take the line, hi, and by drawing parallels to CL, CM, and CD, obtain the figure, $hikc$, which determines the forces 8 and 9. In the same manner proceed from 12 to 15, which will complete the half plan. It may be noted that the principal beam, AJ, is subjected to a uniform compression throughout its entire length.

The force plan will, of course, be modified by various distributions of the load, as in the case of simple beams.

Roof trusses furnish many and varied examples of framework. In the following examples a uniformly distributed vertical load is assumed, so



Simple Roof.—I.

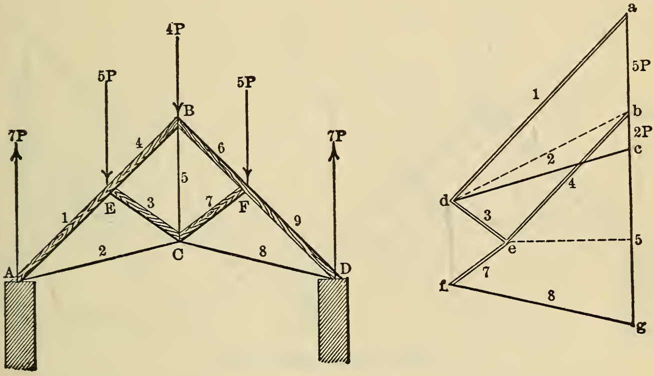
that the burden upon any portion of a rafter may be considered as proportional to the length of that portion.

I. **Roof with Simple Principals.**—A uniform load, $2P$, upon each half gives as the external forces P, $2P$, and P at A, B, and C. Lay off in the force plan $ab = P$, and draw ac and bc parallel to AB and AC, determining the forces 1 and 2,—1 being compression, and 2 tension. Then draw the vertical, ce, and also draw be parallel to CD, thus giving both 3 and 5, and the diagram is completed by drawing de.

II. **Roof with Single-Trussed Principals.**—This form is similar to the preceding, with the addition of the struts, CE and CF. The distance, AE, is to EB as 3 is to 2; and the loads upon the respective portions are $6P$ and $4P$, which give the forces at the various knots, as shown in the figure. Make ac in the force plan equal to $7P$, and by drawing lines parallel to AE and AC obtain the forces 1 and 2, or ad and dc; then combine 1 with $5P = ab$, and decompose the dotted resultant into de and eb, respectively parallel to EC and EB, giving the forces 3 and 4, both being compression. By repeating 2 and 3, in drawing 7 and 8, we obtain the figure, $cdefg$, in which cg gives 5. This latter force might also have been

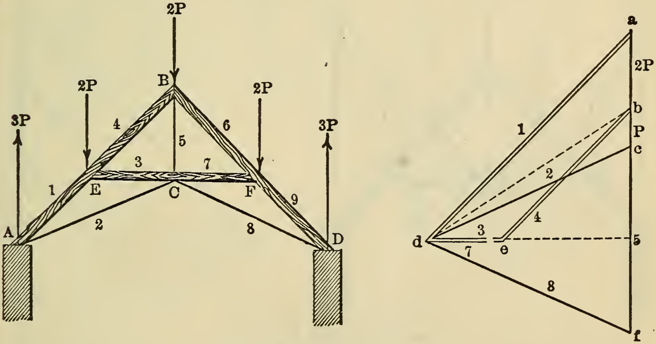
found by combining 4 and $4P$, and decomposing the resultant by lines parallel to BC and BF, an illustration of the various methods in which the force plan may be used.

III. Another form, with Single-Trussed Principals.—This roof is similar to the preceding, except that the struts, EC and CF, are placed horizontally. In this case $AE = EB$, and the external forces at A and D are both



Single-Trussed Roof.—II.

equal to $3P$. The forces from a to c in the force plan are determined as before, giving da and cd for the forces 1 and 2, and the combination of 1 with $2P$ gives the resultant, db, from which the thrusts 3 and 4, or de and eb, are obtained. The value of 1 is the same as 3, and 8 is the same as 2, while 5 is the closing line of $cdedf$ or of cdf. The force 5 must also be the combination of the equal forces 4 and 6 with $2P$, which the diagram shows to be the case. If the rod, CB, is omitted, as is frequently done, the strut, ECF, if there is no joint at C, will oppose its resistance to bending to the

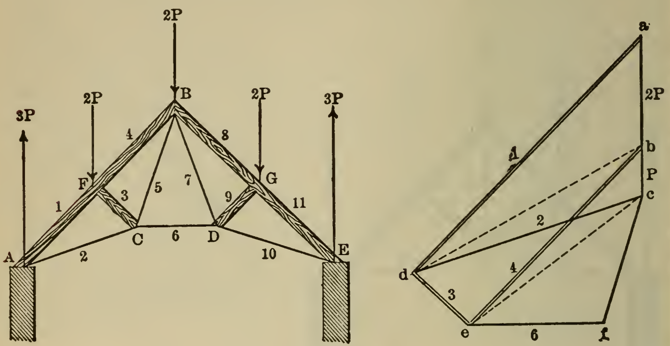


Single-Trussed Roof.—III.

force 5; but there will be a tendency to rise at the apex, B, if the fastening be not made sufficiently strong.

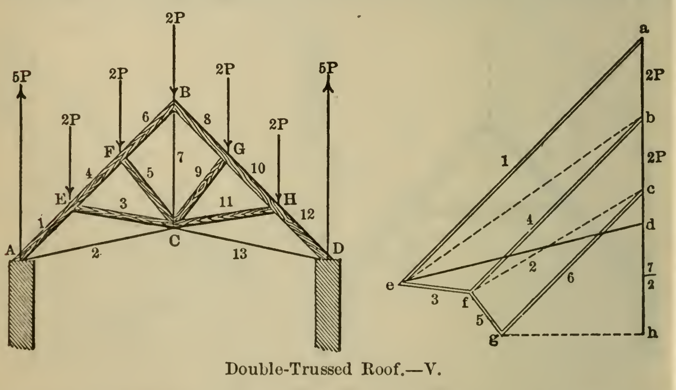
IV. Third Roof with Single-Trussed Principals.—In this form of truss, frequently known as the Belgian or French truss, the single vertical rod of the preceding form is replaced by a triangle, BCD. The struts are placed

in the middle of the rafters and the external forces are distributed as shown in the figure. In the force plan $abc = 3P$, and 1 and 2 are determined as before. By the decomposition of the resultant of 1 and $2P$ we obtain the forces 3 and 4, or *de* and *be*, and from the resultant, *ec*, of the forces 2 and 3 we get the tensions 5 and 6, in *cf* and *ef*. The second half of the diagram is the symmetrical counterpart of the first.



Single-Trussed Roof.—IV.

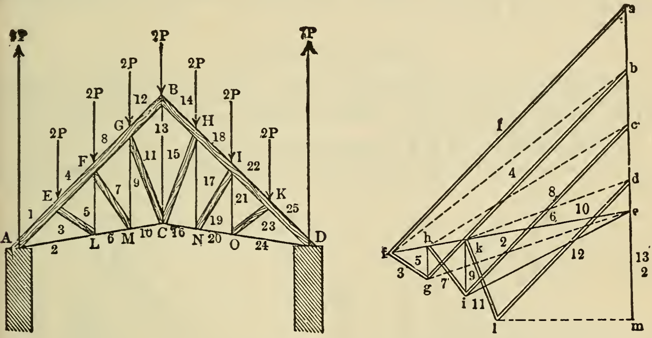
V. **Roof Truss with Double-Trussed Principals.**—This construction does not differ greatly from the preceding, except that the struts employed to strengthen the rafters are divided into two. The spaces are equal to each other and the load uniformly distributed. As shown in the figure this gives a reaction of $5P$, or A and D. In the force plan $ad = 5P$, and lines parallel to AE and AC drawn, determining the forces 1 and 2, or *de* and *ea*. We then combine *ea* with $ab = 2P$, and decompose the dotted resultant, *eb*, into the thrusts, *ef* and *fb*, or 3 and 4, by drawing these lines



Double-Trussed Roof.—V.

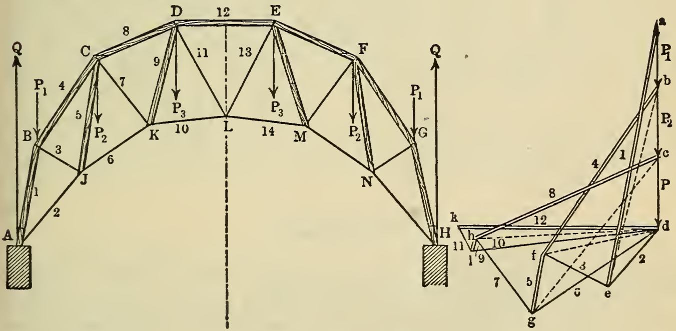
parallel to EC and EF. Again, we take the resultant of the forces 4 and $2P$ and decompose it into 5 and 6, or *fg* and *gc*, which brings us to the middle of the symmetrical figure. The force 7 is the resultant of 6 and its counterpart, 8, and the load $2P$, and the half of this force is therefore equal to the projection of 6 upon the vertical, less P, or, in the diagram, to *dh*.

VI. **English Roof Truss, with Multiple-Trussed Principals.**—Here we have inclined struts, with vertical tie-rods. The load is again uniformly distributed, each space bearing the load of $2P$. The reactions at A and D are each $= 7P$. In the force plan we have $ab + bc + cd + de = 3 \times 2P + P = 7P$, which gives the length of ae. The forces 1 and 2 are found by drawing fa and ef parallel to AE and AL. Now consider 1 as combined



Multiple-Trussed Roof.—VI.

with $ab = 2P$, and the resultant, fb, decomposed into fg and gb, giving the forces 3 and 4. Again, combine 2 and 3, and then decompose the resultant, ge, into 5 and 6, or gh and he, by drawing these latter parallel to LF and LM. In this manner we continue until we reach 12, or ld, which we then project upon the vertical. Now, taking from dm one-half the load $P = de$, we have me for one-half the stress on the middle rod, BC. The remaining half of the force plan is similar.



Polygonal Roof Truss.—VII.

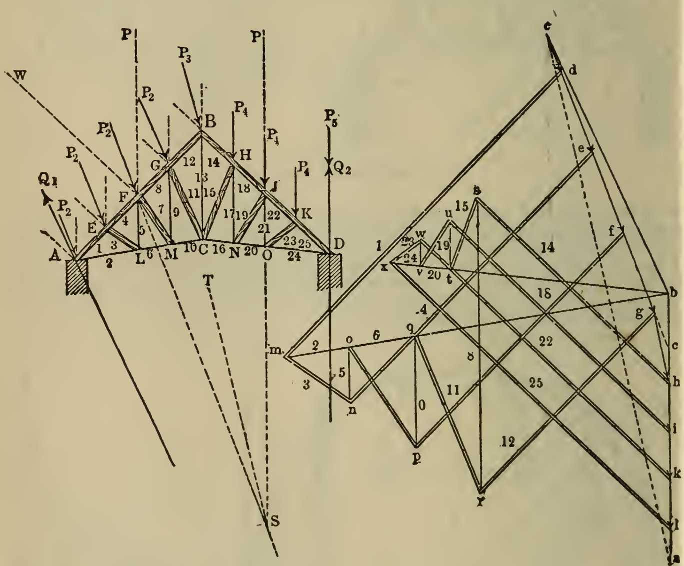
VII. **Polygonal- or Sickel-Shaped Roof Truss.**—This roof may be considered as a modification of the preceding form, and is used for higher and wider spans. It is hardly proper to assume that the load is here uniformly distributed, even if the spaces are equal, for in the case of snow much less weight would be carried by the steep portions, AB or GH, than by the flatter surfaces, CD or DE. We must therefore estimate the forces P_1, P_2, P_3, acting as B, C, D, E, F, G, and make the reactions at A and B equal to $Q = P_1 + P_2 + P_3$.

In the force plan $ab = P_1$, $bc = P_2$, $cd = P_3$, and $ad = Q$, which is first decomposed into 1 and 2 by drawing *ea* and *de* parallel to *AB* and *AJ;* then, combining 1 with P_1, and decomposing the resultant, as before, we get 3 and 4, or *ef* and *fb*. Having 2 and 3, we get in like manner 5 and 6, or *gf* and *dg;* then combining 4 and 5 with P_2, and decomposing with parallels to *CK* and *CD*, we obtain the forces 8 and 9, and so proceed until we reach 12, which is the middle of the symmetrical figure. The members *KL*, *DL*, *EL*, and *ML* are all subject to tension.

WIND STRESSES.

In designing large and important roof trusses it is important to investigate the stresses due to wind pressure, as well as those due to the weight of the roof and of snow; and, indeed, in some cases the resistance to wind is the most important of all.

As an illustration of the applicability of the graphical method to the determination of wind stresses, we will take the English roof truss, whose



Wind Stresses.

conditions under a vertical load have already been examined, and consider it as also subjected to a wind stress, *W*.

We have first to determine the forces, Q_1 and Q_2, acting at the points, *A* and *D*. The wind pressure will be taken as acting on the surface of the roof from *A* to *B*. Let *W* be the resultant of the entire wind pressure acting normal to *AB*, and let *P* be the total vertical load upon that half of the truss. By combining these two forces we obtain the direction, *FS*, of their resultant, and also its magnitude, which we then lay off on the force plan at cc_1. Upon the other half of the truss we have only the vertical load, which may be considered as acting at *J*, and equal in magnitude to *P*. By prolonging its direction until it intersects the previously determined line at *S*, we have at *S* a point in the resultant of the entire load upon the roof, including wind pressure. By making c_1a_2 in the force plan equal to *P*, we have *ac* for the direction of this resultant, which may then be laid off at *ST* in the drawing. In order to determine the forces

Q_1 and Q_2 we must recollect that when we have two closing forces to determine we must also have at least two conditions given. In this case, then, we must first find the direction of Q_1 and Q_2.

The wind pressure produces a horizontal thrust which must be met by the stability of the walls or columns upon which the roof rests. In each case it must be determined whether this horizontal thrust is borne equally or unequally by both supports, and in what proportion it is divided. To this end we first find the proportion of the vertical component of the force *ac*, which comes upon each support (as found by the intersection of *ST*, prolonged with *AD*), and then combine these vertical forces with their respective horizontal components. It often happens that all the horizontal thrust is borne by one of the supports, which it must of course be prepared to resist. This often occurs in the case of railway stations, and under such circumstances the direction of each force must be determined separately. First prolong the vertical at *D* downward until it intersects *ST*, and join the intersection with *A* (the lines are only indicated in the figure). This gives the direction of the force at *A*. We have now both the direction of the reaction at *D* and the direction of that at *A*. We must also consider the distribution of the forces at the various knots between *A* and *B* and between *B* and *D*. We have for the points between *A* and *B* the resultants between the proportional parts of *P* and *W*, while from *B* to *D* we have simply the proportional parts of *P*. This gives at *A* the force P_1; at *E*, *F*, and *G*, the force P_2; at the peak, the force P_3; at *H*, *J*, and *K*, the force $P_4 = \frac{P}{4}$; and at *D*, the vertical force $P_5 = \frac{P}{8}$.

Returning now to the force plan, we make $cd = P_1$, $de = ef = fg = P_2$, $gh = P_3$, $hi = ik = kl = P_4$, and $la = P_5$. We now have finally the length, *bl*, for the value of the reaction, Q_2, at the point, *D*, and a line (not shown) from *b* to *d* gives the magnitude of the force, Q_1, acting at *A*.

The determination of the stresses in the various members can now readily be made. The decomposition of *bd* by drawing *bm* and *md* parallel respectively to *AE* and *AL* gives the forces 1 and 2. We thus proceed until we reach the rod, *BC*, or No. 13, for which we get the tension, $rs = 13$, by drawing the vertical, *rs* from *r*, until it intersects the line, *ns*, drawn parallel to *BD*. We then continue to determine the forces from 15 to 25, as already shown. The force plan shows that under these conditions similarly placed struts are subjected to dissimilar stresses. The determination of the stresses might have been made in the reverse order, beginning with the triangle, *xbl*, which should give the same results, and which may be used to prove the accuracy of the work. A proof is also made by the accuracy with which the line, *wx*, drawn from *w*, parallel to *KO*, intersects the point, *x*, which was first determined by the intersection of *bx* and *lx*. As a matter of fact, it will be found to require careful drawing in order to insure the closing of the diagram.

By comparing the last force plan with that found for the same roof truss, under vertical loads only, it will be seen how greatly the wind stresses affect the structure. In order to complete the calculation, a second plan should be drawn, assuming the wind to act also upon *BD*.

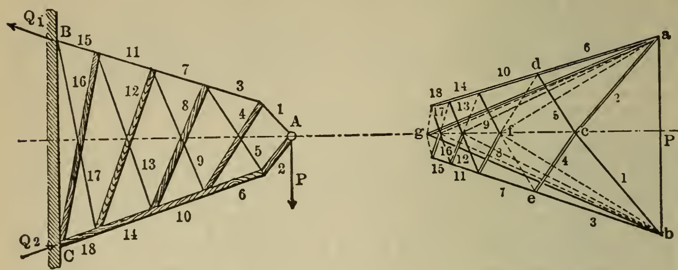
FRAMED BEAMS.

Beams of various forms are often framed in various shapes and made both of wrought and cast iron, and have many applications, such as walking beams for steam engines, for cranes, arms, etc. A few examples will show the method of investigation for such cases.

I. **Cantilevers with Straight Members.**—The load, *P*, acts at *A* in a direction normal to the axis of the frame, which is supported at *B* and *C*. The force plan is constructed as follows: Draw $ab = P$, and from its extremities draw *ac* and *bc* parallel to 1 and 2, which gives the forces in those members. Each of these is then decomposed into two other forces, —1 into 3 and 4, 2 into 5 and 6, giving the triangles, *bec* and *adc*.

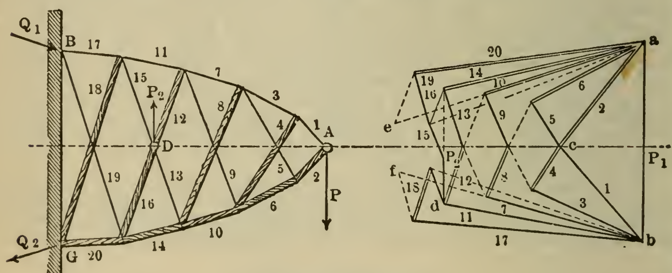
The forces 3 and 5 are then combined and the resultant decomposed into 7 and 8. To do this we transfer $5 = dc$ to *fe*, and join the resultant, *fb*, which can readily be separated into 7 and 8. We proceed in this manner for the remaining members, and as the frame is symmetrical about the axis, *gc*, only one-half of the diagram need be completed. The lines, *ga* and *bg*, which are the final resultants of 15 with 17, and 16 with 18, are

also the external forces at B and C, the points of attachment, provided that their direction be permitted to remain the same.



Cantilever.—I.

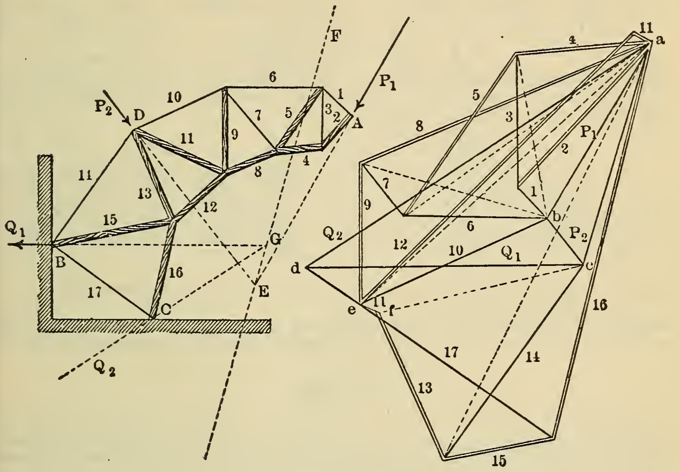
II. **Double-Loaded Frame.**—In this case we have the force, P_1, acting downwards at A, and a force, P_2, acting upwards at D, while the points of attachment remain at B and C, as before. The members, AB and AC, are polygonal-formed. The force plan is drawn just as before, until the force 13 is reached. At D the members are attached to each other at their intersection, so that the force, P_2, acts upon both 15 and 16. At this same point we have the action of the forces 12 and 13. Now join the extremities of 12 and 13 by the dotted line shown, and mark off the length of the force, P_2, which is subtracted, because its action is upward, thus obtaining the resultant of the three forces. We can then draw 15 and 16 and proceed without interruption to 20. Finally, we draw *bf* and *ea*, the external forces at Q_1 and Q_2, which hold the entire frame in equilibrium.



Cantilever.—II.

III. **Framed Boom.**—This figure is a portion of a framed arch which may be used for the projecting boom of a large crane. At A and D we have the forces, P_1 and P_2, and at B and C the external forces, Q_1 and Q_2. The force plan is now required to determine the internal forces acting on the various members of the structure. Before this can be done we must first determine the as yet unknown direction of the force, Q_2. Prolong P_1 and P_2 to their intersection at E, and by drawing in the force plan the triangle, *abc*, determine the direction, FE, of their resultant; then prolong Q_1 until it intersects EF at G, and join CG, which will be the required direction of the force, Q_2. Completing the figure in the force plan, we have $cd = Q_1$ and $da = Q_2$. We now proceed from $P_1 = ab$ and lay off the forces 1 and 2, decomposing 2 into 3 and 4; combine 3 and 1 and decompose their resultant, obtaining 5 and 6. We thus proceed until we reach 12, which we obtain by combining 9 and 8 and decomposing the resultant into 11 and 12. We now have to combine 10 and 11 with P_2, and decompose the

resultant into 13 and 14. We first transfer the force 11 to e, making it equal to ef, in order to avoid the confusion of lines, which would occur if the construction were made at a. Now, drawing the path 11, 10, P_2, we have the closing line, cf, which decomposes into 13 and 14. We then have 15



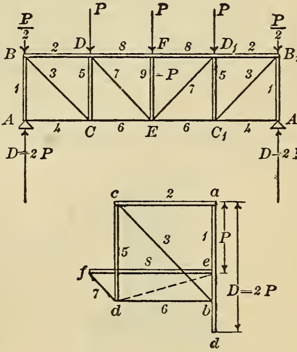
Framed Boom.—III.

and 16 from the resultant of 13 and 12, and finally, 17, as the line joining 15 and 16 with d, since 16 and 17 must have the resultant, $ad = Q_2$. If the work is correctly done, we will find 17 falls parallel to BC, which affords a convenient and valuable proof for the whole work.

BRIDGE TRUSSES

may be examined in a similar manner to roof trusses.

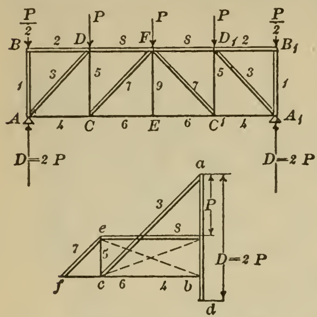
I. **Simple Truss.**—In the case of a truss of four panels, with vertical struts and diagonal tie-rods, as in the figure, we have on each pillar a load, P, except at the ends, where it is equal to $\frac{P}{2}$, this giving a total load of $4P$, or a vertical reaction of $2P$ on each pier. The diagram shown is constructed for one-half of the truss, the forces in the other half being identical. In the diagram we make $ad = 2P$. Since $\frac{1}{2}P$ is supported directly upon the pier at A, we make $ab = \frac{3}{2}P$. Then draw 2, parallel to BD, and 3, parallel to BC, the lengths of these lines giving the stresses in the corresponding members. From c draw 5, parallel to CD, and from b draw 6, parallel to CE. Combine 5 and 2 with P for a resultant, ed, and draw 7, parallel to DE, and 8, parallel to DF. Each member will then have its load given by the lengths of the lines in the diagram, the double lines representing compression and the single lines tension, as before. The middle strut, FE, bears a compression equal to its top load, P.



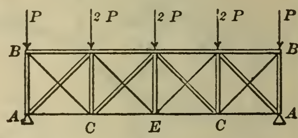
Simple Truss.—I.

II. **Simple Truss.**—In the case of a truss with diagonal struts and vertical tie-rods, as in the figure, we have similar loading, and the diagram is given below. The tension on EF is zero, and there is no compression on BD.

III. **Combined Truss.**—By combining the two simple trusses the combination is formed in which all the loads may be doubled for the same stresses as shown in the previous diagrams, *except for those members which coincide.* We thus have loads of $2P$ on the vertical struts, except the middle one, while the loads on the diagonals remain unchanged.



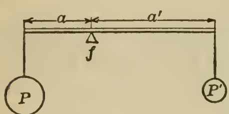
Simple Truss.—II.



Combined Truss.—III.

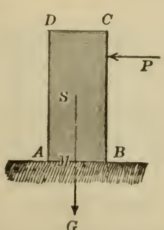
Leverage.

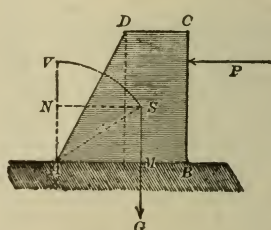
The *statical moment* of a force, as already explained, is the *leverage* of that force,—that is, the magnitude of the force, multiplied by the perpendicular distance from the centre about which it acts. If two or more forces are in equilibrium, so that motion does not take place, their statical moments must be equal. This is only a general statement of what may be called the *principle of the lever.*



Thus, the statical moment of the force, P, is the force multiplied by the distance, a, from the fulcrum, f, or $= Pa$. In like manner the statical moment of P' is equal to $P'a'$, and, if the beam remains stationary, $Pa = P'a'$. This is true no matter how the lever arms may be disguised by the form or material which may include them. Thus, the force may act at the perimeter of a wheel, the radius of the wheel then becoming the lever arm, or it may be included in some other form; but the forces themselves must always be considered as acting upon lever arms of a length equal to the *perpendicular* distance from the lines of action of the forces to the fulcrum.

In the case of a force acting at any point to overturn a mass, the resistance must be considered as the weight of the body acting at the *centre of gravity.*





Thus, in the case of either wall shown in the illustration, the force, P, acting to overturn the wall about the corner, A, is opposed by a force, G, equal to the weight of the wall, acting at a lever arm, AM, equal to the

distance of the corner, A, from the centre of gravity, S, measured at right angles to the line of the force, G. This gives for the *moment* of stability of the wall

$$AM \times S.$$

MOTION.

Falling Bodies.

According to the law of gravitation enunciated by Sir Isaac Newton, *every particle of matter in the universe attracts every other particle of matter with a force which varies directly as the mass, and inversely as the square of the distance.*

In accordance with this law any body above the surface of the earth, when permitted to fall freely, does so with an accelerated velocity.

The unit or measure of force of gravity is assumed to be the velocity a falling body has attained at the end of the first second of descent. This unit is commonly denoted by the letter g; its value at the level of the sea in New York is $g = 32.17$ feet per second, in vacuum. g is called *the acceleration of gravity.* The space fallen through in the first second is $\frac{1}{2}g = 16.085$ feet.

This value increases with the latitude, and decreases with the elevation above the level of the sea.

$l =$ latitude, $h =$ height in feet above the level of the sea, and $r =$ radius of the earth in feet, at the given latitude, l.

$$r = 208\,87510\ (1 + 0.00164 \cos 2l),$$

$$g = 32.16954\ (1 - 0.00284 \cos 2l) \left(1 - \frac{2h}{r}\right).$$

Notation.

$S =$ the space in feet which the falling body passes through in the time T.
$u =$ the space in feet which the body falls in the Tth second.
$V =$ velocity in feet per second of the falling body at the end of the time T.
$T =$ time in seconds the body is falling.

In the metric system the value of g is given in metres per second, and is taken as equal to 9.81 metres at latitude 45° and at the level of the sea.

Formulas for Accelerated Motion.

Velocity, V, in Feet per Second.

1. $V = gT.$
2. $V = \frac{2S}{T}.$
3. $V = \sqrt{2gS}.$
4. $V = 8.02\sqrt{S}.$

4a. $V = 4.429\sqrt{S}.$ (*Metric.*)

Space, S, Fallen through in Feet.

5. $S = \frac{gT^2}{2}.$
6. $S = \frac{VT}{2}.$
7. $S = \frac{V^2}{2g}.$
8. $S = \frac{V^2}{64.33}.$

8a. $S = \frac{V^2}{19.62}.$ (*Metric.*)

Time of Fall in Seconds.

9. $T = \frac{V}{g}.$
10. $T = \frac{2S}{V}.$
11. $T = \sqrt{\frac{2S}{g}}.$
12. $T = \frac{\sqrt{S}}{4.01}.$

12a. $T = \frac{\sqrt{S}}{2.04}$ (*Metric*)

Space Fallen through in the Tth Second.

13. $u = g\left(T - \frac{1}{2}\right).$
14. $T = \frac{u}{g} + \frac{1}{2}.$

Example 1. What velocity has a body attained after having fallen freely for a time of $T = 2\frac{1}{2}$ seconds?

Velocity, $V = 32.17 \times 2.5 = 80.2$ feet per second.

Example 4. A body is dropped from a height of $S = 98$ feet. What velocity will it have on reaching the ground, and what time is required for its fall?

Formula 4. Velocity, $V = 8.02\sqrt{98} = 79.3939$ feet per second.

Formula 12. Time, $T = \dfrac{\sqrt{S}}{4.01} = \dfrac{\sqrt{98}}{4.01} = 2.46$ seconds.

Example 5. A body was dropped at the opening of a hole in the rock, and reached the bottom in $T = 3.5$ seconds. Required the depth of the hole?

Formula 5. Depth, $S = g\dfrac{T^2}{2} = \dfrac{32.17 \times 3.5^2}{2} = 196.98$ feet.

Example 8. What space must a body fall through in order to acquire a velocity $V = 369$ feet per second?

$$\text{Space, } S = \frac{V^2}{64.33} = \frac{369^2}{64.33} = 2116.6 \text{ feet.}$$

Example 10. What time is required for a body to fall $S = 2116.6$ feet, when the final velocity $V = 369$ feet per second?

$$\text{Time, } T = \frac{2S}{V} = \frac{2 \times 2116.6}{369} = 11.472 \text{ seconds.}$$

Example 13. A body falls freely for a time of $T = 4\frac{1}{2}$ seconds. How much will it fall in the last second?

Formula 13. $u = g(T - \frac{1}{2}) = 32.17\,(4.5 - 0.5) = 128.68$ feet.

Retarded Motion.

A body thrown up vertically will obtain inversely the same motion as when it falls down, because it is the same force that acts upon it, causing *retarded motion* when it ascends, and *accelerated motion* when it descends.

V = the velocity at which the body starts to ascend.
v = velocity at the end of the time t.
T = time in seconds in which the body will ascend.
t = any time less than T.
S = height in feet to which the body will ascend.
s = the space it ascends in the time t.

Velocity in Feet per Second at the End of the Time *t*.

15. $v = V - gt.$ | 16. $v = \dfrac{s}{t} - \dfrac{gt}{2}.$

Height of Ascension in the Time *t*.

17. $s = t\left(V - g\dfrac{t}{2}\right).$ | 18. $s = t\left(v + g\dfrac{t}{2}\right).$

Starting Velocity in Feet per Second.

19. $V = v + gt.$ | 20. $V = \dfrac{s}{t} + g\dfrac{t}{2}.$

Time of Ascension in Seconds.

21. $t = \dfrac{V - v}{g}.$ | 22. $t = \dfrac{V}{g} - \sqrt{\dfrac{V^2}{g^2} - \dfrac{2s}{g}}.$

Starting and Ending Velocities.

23. $v = \sqrt{V^2 - 2gs}.$ | 24. $V = \sqrt{v^2 + 2gs}.$

Formulas for T and S are the same as for accelerated motion.

Example 22. A ball starts to ascend with a velocity of 135 feet per second. At what velocity will it strike an object 60 feet above?

Find the time t by the Formula 22.

$$t = \frac{135}{32.16} - \sqrt{\frac{135^2}{32.16} - \frac{2 \times 60}{32.16}} = 0.41 \text{ seconds,}$$

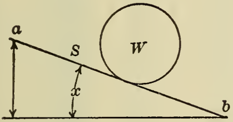
until it strikes; and from Formula 15 we have

$$v = 135 - 32.16 \times 0.41 = 121.83 \text{ feet per second.}$$

Example 24. With what velocity must a body start to ascend in order to strike an object $s = 15$ feet above with a velocity $v = 10$ feet per second?

$$\text{Velocity, } V = \sqrt{10^2 + 2 \times 32.17 \times 15} = 32.63 \text{ feet per second.}$$

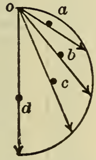
Force of Gravity.



$$V = gT \sin x = \sqrt{2gS \sin x},$$

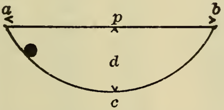
$$S = \frac{gT^2}{2 \sin x} = \frac{V^2}{2g \sin x},$$

$$T = \frac{V}{g \sin x} = \sqrt{\frac{2S \sin x}{g}}.$$



A body will fall from o the distances a, b, c, and d, in equal times.

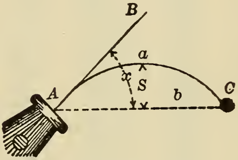
$$T = \sqrt{\frac{2d}{g}}.$$



A body will fall from a to b, via c, in the shortest time, if the curve is cycloid.

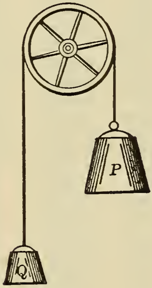
$S = 4d$, the length of the cycloid,

$$T = \pi\sqrt{\frac{d}{2g}} = \pi\sqrt{\frac{p}{2\pi g}}.$$



$$b = \frac{2V^2 \sin x \cos x}{g},$$

$$T = \frac{V \sin x}{g}, \qquad S = \frac{V^2 \sin^2 x}{2g}.$$



$$S = g\frac{T^2 F}{2W} = \frac{V^2 W}{2gF},$$

$$V = gT\frac{F}{W} = \sqrt{\frac{2gSF}{W}},$$

$$T = \frac{VW}{gF} = \sqrt{\frac{2SW}{gF}},$$

$$F = \frac{VW}{gT} = \frac{2SW}{gT^2},$$

$W = P + Q$, and $F = P - Q$.

Falling Bodies.

English Units.

V = velocity in feet per second at the end of fall.

T = time in seconds of the fall.

S = space fallen through in feet.

V	T	S	V	T	S	V	T	S
0.1	0.0031	.00015	5.1	0.1585	0.4042	11	0.3419	1.8804
0.2	0.0062	.00031	5.2	0.1616	0.4202	12	0.3730	2.2380
0.3	0.0093	0.0014	5.3	0.1647	0.4364	13	0.4041	2.6266
0.4	0.0124	0.0025	5.4	0.1678	0.4530	14	0.4352	3.0464
0.5	0.0155	0.0039	5.5	0.1709	0.4700	15	0.4663	3.4975
0.6	0.0186	0.0055	5.6	0.1740	0.4872	16	0.4973	3.9784
0.7	0.0217	0.0076	5.7	0.1771	0.5047	17	0.5284	4.4914
0.8	0.0248	0.0099	5.8	0.1802	0.5226	18	0.5595	5.0355
0.9	0.0279	0.0125	5.9	0.1833	0.5407	19	0.5906	5.6107
1.	0.0311	0.0155	6.	0.1865	0.5595	20	0.6217	6.2170
1.1	0.0342	0.0188	6.1	0.1896	0.5782	21	0.6527	6.8502
1.2	0.0373	0.0224	6.2	0.1927	0.5973	22	0.6838	7.5218
1.3	0.0404	0.0262	6.3	0.1958	0.6168	23	0.7149	8.2213
1.4	0.0435	0.0304	6.4	0.1989	0.6365	24	0.7460	8.9520
1.5	0.0466	0.0335	6.5	0.2020	0.6565	25	0.7771	9.7125
1.6	0.0497	0.0381	6.6	0.2051	0.6768	26	0.8082	10.566
1.7	0.0528	0.0432	6.7	0.2082	0.6975	27	0.8393	11.330
1.8	0.0559	0.0485	6.8	0.2113	0.7184	28	0.8704	12.185
1.9	0.0590	0.0551	6.9	0.2144	0.7397	29	0.9015	13.072
2.	0.0622	0.0622	7.	0.2176	0.7616	30	0.9325	13.987
2.1	0.0653	0.0685	7.1	0.2207	0.7835	31	0.9636	14.936
2.2	0.0684	0.0756	7.2	0.2238	0.8057	32	0.9947	15.915
2.3	0.0715	0.0822	7.3	0.2269	0.8282	33	1.0258	16.926
2.4	0.0746	0.0895	7.4	0.2300	0.8510	34	1.0569	17.967
2.5	0.0777	0.0971	7.5	0.2331	0.8741	35	1.0879	19.038
2.6	0.0808	0.1050	7.6	0.2362	0.8975	36	1.1190	20.142
2.7	0.0839	0.1135	7.7	0.2393	0.9213	37	1.1501	21.277
2.8	0.0870	0.1218	7.8	0.2424	0.9453	38	1.1812	22.443
2.9	0.0901	0.1305	7.9	0.2455	0.9697	39	1.2123	23.640
3.	0.0932	0.1398	8.	0.2487	0.9948	40	1.2434	24.868
3.1	0.0963	0.1492	8.1	0.2518	1.0168	41	1.2745	26.127
3.2	0.0994	0.1590	8.2	0.2549	1.0451	42	1.3056	27.417
3.3	0.1025	0.1691	8.3	0.2580	1.0707	43	1.3367	28.739
3.4	0.1056	0.1795	8.4	0.2611	1.0966	44	1.3678	29.407
3.5	0.1087	0.1886	8.5	0.2642	1.1228	45	1.3989	31.475
3.6	0.1118	0.2012	8.6	0.2673	1.1494	46	1.4300	32.890
3.7	0.1149	0.2125	8.7	0.2704	1.1762	47	1.4611	34.336
3.8	0.1170	0.2223	8.8	0.2735	1.2034	48	1.4922	35.813
3.9	0.1201	0.2355	8.9	0.2766	1.2259	49	1.5233	37.321
4.	0.1243	0.2486	9.	0.2797	1.2586	50	1.5544	38.830
4.1	0.1274	0.2611	9.1	0.2828	1.2867	51	1.5854	40.413
4.2	0.1305	0.2740	9.2	0.2859	1.3151	52	1.6165	42.029
4.3	0.1336	0.2872	9.3	0.2890	1.3438	53	1.6475	43.659
4.4	0.1367	0.2939	9.4	0.2921	1.3729	54	1.6786	45.322
4.5	0.1398	0.3145	9.5	0.2952	1.4022	55	1.7097	47.017
4.6	0.1429	0.3286	9.6	0.2983	1.4318	56	1.7407	48.740
4.7	0.1460	[illegible]431	9.7	0.3014	1.4618	57	1.7718	50.396
4.8	0.1491	0.3578	9.8	0.3045	1.4920	58	1.8029	52.284
4.9	0.1522	0.3729	9.9	0.3076	1.5226	59	1.8340	54.103
5.	0.1554	0.3885	10.	0.3108	1.5540	60	1.8651	55.953

Falling Bodies.

English Units.

$$V = \frac{2S}{T}. \qquad T = \sqrt{\frac{2S}{g}}. \qquad S = \frac{gT^2}{2}.$$

V	T	S	V	T	S	V	T	S
65	2.0206	65.669	530	16.478	4366.6	1030	32.027	16494
70	2.1769	76.260	540	16.788	4452.8	1040	32.338	16815
75	2.3314	87.427	550	17.099	4701.7	1050	32.649	17141
80	2.4868	97.472	560	17.409	4874.5	1060	32.950	17463
85	2.6422	112.29	570	17.720	5050.2	1070	33.261	17794
90	2.7976	125.89	580	18.030	5228.7	1080	33.572	18129
95	2.9530	140.27	590	18.341	5410.6	1090	33.883	18446
100	3.1085	155.42	600	18.651	5595.3	1100	34.194	18806
110	3.4194	188.07	610	18.961	5783.1	1110	34.504	19149
120	3.7302	223.81	620	19.271	5974.0	1120	34.815	19496
130	4.0411	262.67	630	19.582	6168.3	1130	35.126	19846
140	4.3519	304.63	640	19.893	6365.7	1140	35.436	20198
150	4.6627	349.70	650	20.204	6566.3	1150	35.747	20504
160	4.9736	397.88	660	20.515	6770.0	1160	36.058	20913
170	5.2844	449.18	670	20.826	6976.7	1170	36.369	21275
180	5.5953	503.36	680	21.137	7186.6	1180	36.680	21641
190	5.9061	561.08	690	21.448	7399.5	1190	36.991	22009
200	6.2170	621.70	700	21.759	7615.6	1200	37.302	22381
210	6.5279	689.43	710	22.070	7834.8	1210	37.613	22755
220	6.8387	752.26	720	22.380	8056.8	1220	37.924	23133
230	7.1496	822.20	730	22.691	8282.2	1230	38.235	23514
240	7.4604	895.25	740	23.002	8510.7	1240	38.546	23898
250	7.7713	971.41	750	23.313	8742.4	1250	38.857	24285
260	8.0821	1050.6	760	23.623	8976.7	1260	39.168	24676
270	8.3930	1133.1	770	23.934	9214.6	1270	39.479	25069
280	8.7038	1218.5	780	24.245	9455.5	1280	39.780	25459
290	9.0147	1308.2	790	24.556	9699.6	1290	40.090	25855
300	9.3255	1398.8	800	24.868	9947.2	1300	40.411	26267
310	9.6363	1493.7	810	25.179	10197	1310	40.722	26673
320	9.9472	1591.6	820	25.490	10451	1320	41.033	27081
330	10.258	1690.6	830	25.801	10707	1330	41.343	27493
340	10.569	1791.7	840	26.112	10967	1340	41.654	27908
350	10.879	1903.8	850	26.423	11230	1350	41.965	28326
360	11.190	2014.2	860	26.733	11495	1360	42.276	28747
370	11.501	2127.7	870	27.044	11764	1370	42.587	29172
380	11.812	2244.3	880	27.354	12035	1380	42.897	29599
390	12.123	2364.0	890	27.665	12311	1390	43.208	30029
400	12.434	2486.8	900	27.976	12589	1400	43.519	30463
410	12.745	2612.7	910	28.287	12871	1410	43.820	30893
420	13.055	2741.5	920	28.598	13155	1420	44.131	31333
430	13.366	2873.7	930	28.908	13442	1430	44.442	31776
440	13.677	3008.9	940	29.219	13733	1440	44.753	32222
450	13.989	3144.8	950	29.530	14027	1450	45.064	32671
460	14.300	3289.0	960	29.841	14323	1460	45.375	33123
470	14.611	3433.6	970	30.152	14623	1470	45.686	33579
480	14.922	3581.3	980	30.463	14927	1480	45.997	34037
490	15.233	3732.1	990	30.774	15233	1490	46.308	34499
500	15.545	3886.2	1000	31.085	15542	1500	46.619	34964
510	15.856	4043.3	1010	31.396	15855	1510	46.930	35432
520	16.167	4203.4	1020	31.707	16179	1520	47.241	35853

Falling Bodies.

Metric System.

Space, s, for terminal velocity, v, in metres.

$$s = \frac{v^2}{2g}.$$

v	s	v	s	v	s
0.0	0.0000	4.0	0.8157	8.0	3.2627
1	0.0005	1	0.8570	1	3.3447
2	0.0020	2	0.8993	2	3.4278
3	0.0046	3	0.9426	3	3.5120
4	0.0082	4	0.9869	4	3.5971
5	0.0127	5	1.0323	5	3.6832
6	0.0184	6	1.0787	6	3.7704
7	0.0250	7	1.1261	7	3.8586
8	0.0326	8	1.1746	8	3.9478
9	0.0413	9	1.2240	9	4.0381
1.0	0.0510	5.0	1.2745	9.0	4.1293
1	0.0617	1	1.3260	1	4.2216
2	0.0734	2	1.3785	2	4.3149
3	0.0862	3	1.4320	3	4.4092
4	0.0999	4	1.4866	4	4.5045
5	0.1147	5	1.5421	5	4.6009
6	0.1305	6	1.5987	6	4.6982
7	0.1473	7	1.6563	7	4.7966
8	0.1652	8	1.7149	8	4.8960
9	0.1840	9	1.7746	9	4.9965
2.0	0.2039	6.0	1.8352	10.0	5.0979
1	0.2248	1	1.8969	1	5.2004
2	0.2467	2	1.9596	2	5.3039
3	0.2697	3	2.0234	3	5.4084
4	0.2936	4	2.0881	4	5.5139
5	0.3186	5	2.1539	5	5.6204
6	0.3446	6	2.2207	6	5.7280
7	0.3716	7	2.2885	7	5.8366
8	0.3997	8	2.3573	8	5.9462
9	0.4287	9	2.4271	9	6.0568
3.0	0.4588	7.0	2.4980	11.0	6.1685
1	0.4899	1	2.5699	12.0	7.3410
2	0.5220	2	2.6428	13.0	8.6155
3	0.5552	3	2.7167	14.0	9.9919
4	0.5893	4	2.7916	15.0	11.4703
5	0.6245	5	2.8676	16.0	13.0507
6	0.6607	6	2.9446	17.0	14.7330
7	0.6979	7	3.0226	18.0	16.5172
8	0.7361	8	3.1016	19.0	18.4035
9	0.7754	9	3.1816	20.0	20.3916

Falling Bodies.

Metric System.

Terminal velocity, v, for space, s, in metres.

$$v = \sqrt{2gs}.$$

s	v	s	v	s	v
0.0	0.0000	4.0	8.8580	8.0	12.5271
1	1.4006	1	8.9681	1	12.6052
2	1.9807	2	9.0767	2	12.6827
3	2.4259	3	9.1842	3	12.7598
4	2.8012	4	9.2904	4	12.8365
5	3.1318	5	9.3953	5	12.9127
6	3.4307	6	9.4991	6	12.9884
7	3.7056	7	9.6019	7	13.0637
8	3.9614	8	9.7035	8	13.1385
9	4.2017	9	9.8040	9	13.2130
1.0	4.4290	5.0	9.9036	9.0	13.2870
1	4.6452	1	10.0021	1	13.3606
2	4.8517	2	10.0997	2	13.4338
3	5.0499	3	10.1963	3	13.5066
4	5.2405	4	10.2921	4	13.5790
5	5.4244	5	10.3869	5	13.6511
6	5.6023	6	10.4809	6	13.7228
7	5.7747	7	10.5740	7	13.7940
8	5.9421	8	10.6664	8	13.8650
9	6.1049	9	10.7580	9	13.9355
2.0	6.2635	6.0	10.8488	10.0	14.0057
1	6.4182	1	10.9388	1	14.0756
2	6.5693	2	11.0281	2	14.1451
3	6.7169	3	11.1167	3	14.2143
4	6.8614	4	11.2046	4	14.2831
5	7.0029	5	11.2918	5	14.3516
6	7.1415	6	11.3783	6	14.4198
7	7.2776	7	11.4642	7	14.4877
8	7.4111	8	11.5495	8	14.5552
9	7.5423	9	11.6340	9	14.6224
3.0	7.6712	7.0	11.7180	11.0	14.6893
1	7.7981	1	11.8014	12.0	15.3425
2	7.9228	2	11.8842	13.0	15.9692
3	8.0457	3	11.9665	14.0	16.5720
4	8.1667	4	12.0482	15.0	17.1535
5	8.2859	5	12.1293	16.0	17.7160
6	8.4035	6	12.2099	17.0	18.2612
7	8.5194	7	12.2900	18.0	18.7907
8	8.6337	8	12.3695	19.0	19.3056
9	8.7466	9	12.4485	20.0	19.8071

Falling Bodies.

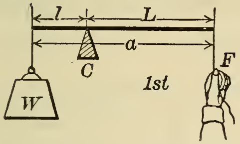
Metric System.

Space, s, in metres for time, t, from 0.1 to 10 seconds.

$$s = \tfrac{1}{2}gt^2.$$

t	s	t	s	t	s
0.0	0.0000	3.0	44.1362	6.0	176.5446
1	0.0490	1	47.1276	1	182.4785
2	0.1962	2	50.2171	2	188.5104
3	0.4414	3	53.4047	3	194.6404
4	0.7846	4	56.6904	4	200.8685
5	1.2260	5	60.0742	5	207.1947
6	1.7654	6	63.5561	6	213.6190
7	2.4030	7	67.1360	7	220.1413
8	3.1386	8	70.8140	8	226.7618
9	3.9723	9	74.5901	9	233.4802
1.0	4.9040	4.0	78.4643	7.0	240.2968
1	5.9339	1	82.4365	1	247.2115
2	7.0618	2	86.5069	2	254.2243
3	8.2878	3	90.6753	3	261.3351
4	9.6119	4	94.9418	4	268.5440
5	11.0340	5	99.3063	5	275.8510
6	12.5543	6	103.7690	6	283.2560
7	14.1726	7	108.3297	7	290.7592
8	15.8890	8	112.9886	8	298.3604
9	17.7035	9	117.7455	9	306.0597
2.0	19.6161	5.0	122.6004	8.0	313.8571
1	21.6267	1	127.5535	1	321.7526
2	23.7354	2	132.6046	2	329.7461
3	25.9422	3	137.7538	3	337.8377
4	28.2471	4	143.0011	4	346.0274
5	30.6501	5	148.3465	8.5	354.3153
6	33.1512	6	153.7900	9.0	397.2254
7	35.7503	7	159.3315	9.5	442.5875
8	38.4475	8	164.9711	10.0	490.4017
9	41.2428	9	170.7088	11.0	593.3911

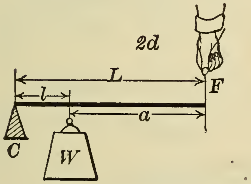
Leverage.



$F : W = l : L,\qquad FL = Wl.$

1. $F = \dfrac{Wl}{L}.$ 3. $l = \dfrac{Fa}{W+F}.$

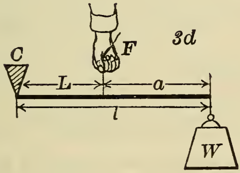
2. $W = \dfrac{FL}{l}.$ 4. $L = \dfrac{Wa}{W+F}.$



$F : W = l : L,\qquad FL = Wl.$

5. $F = \dfrac{Wl}{L}.$ 7. $L = \dfrac{Wa}{W-F}.$

6. $W = \dfrac{FL}{l}.$ 8. $l = \dfrac{Fa}{W-F}.$



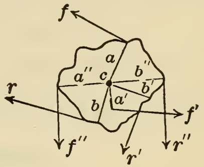
$F : W = l : L,\qquad FL = Wl.$

9. $F = \dfrac{Wl}{L}.$ 11. $L = \dfrac{Wa}{F-W}.$

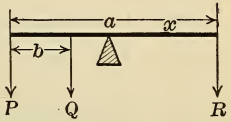
10. $W = \dfrac{FL}{l}.$ 12. $l = \dfrac{Fa}{F-W}.$

Static Moments.

$$af + a'f' + a''f'' = br + b'r' + b''r''.$$



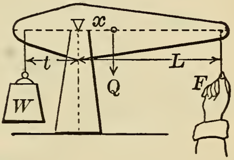
If the sum of the moments that act to move the body in one direction are equal to the sum of the moments that act opposite, the acting forces will be in equilibrium; c being the centre or fulcrum.



To find the fulcrum, c, when three forces act on the lever,

$$Rx = Q(a-b-x) + P(a-x),$$

$$x = \frac{Qa + Pa - Qb}{R + Q + P}.$$



Q = weight of the lever, x = distance from the centre of gravity of the lever to the fulcrum. Balance the lever over a sharp edge, and the centre of gravity is found.

$$F = \frac{Wl - Qx}{L},\qquad W = \frac{FL + Qx}{l}.$$

DYNAMICS.

We have already referred to the fact that in practical engineering work there are but three elementary quantities: **Force**, expressed as a *weight;* **Space**, expressed as a *lineal distance;* and **Time**, expressed in *hours, minutes*, or *seconds*.

In the problems of Dynamics, or the study of force and motion, we have the following relations, in which

$$F = \text{force},\ S = \text{space},\ T = \text{time},\ M = \text{mass},$$

$$V = \frac{S}{T} = \text{velocity}, \qquad P = \frac{FS}{T} = FV = \text{power},$$

$$K = FS = \text{work}, \qquad K = \tfrac{1}{2}MV^2 = \text{work}.$$

Dynamical Formulas.

Force or Pressure, in Pounds.

1. $F = \frac{P}{V}$.	2. $F = \frac{550\,HP}{V}$.	3. $F = \frac{K}{S}$.	4. $F = \frac{K}{VT}$.

Velocity, in Feet, per Second. For Uniform Motion.

5. $V = \frac{S}{T}$.	6. $V = \frac{P}{F}$.	7. $V = \frac{550\,HP}{F}$.	8. $V = \frac{K}{FT}$.

Time of Action, in Seconds. For Uniform Motion.

9. $T = \frac{S}{V}$.	10. $T = \frac{FS}{P}$.	11. $T = \frac{FS}{550\,HP}$.	12. $T = \frac{K}{FV}$.

Power, in Foot-Pounds, per Second.

13. $P = FV$.	14. $P = \frac{FS}{T}$.	15. $P = 550\,HP$.	16. $P = \frac{K}{T}$.

Space passed through in the Time *T*.

17. $S = VT$.	18. $S = \frac{PT}{F}$.	19. $S = \frac{550\,T\,HP}{F}$.	20. $S = \frac{K}{F}$.

Horse-power.

21. $HP = \frac{P}{550}$.	22. $HP = \frac{FV}{550}$.	23. $HP = \frac{FS}{550\,T}$.	24. $HP = \frac{K}{550\,T}$.

Work, in Foot-Pounds.

25. $K = FVT$ in time T.	27. $K = FS$.
26. $K = PT$ in time T.	28. $K = 550\,HP\,T$ in time T.

It will be observed in the preceding formulas that an element is never divided by an element; but a function is divided by an element only when that function contains that element.

Power divided by velocity gives force, because power contains the elements force and velocity; but power cannot be divided by time, because time is not a constituent element of power.

Work can be divided by either one or two of its three constituent factors. When work is divided by either two of its elements, the product will be the third element.

Different elements or functions cannot be added to or subtracted from one another. Power or space cannot be added to or subtracted from work. Force, velocity, or time cannot be added to or subtracted from space.

In the metric system, force is given in Kilogrammes and space in Metres. Work is given in *Kilogrammetres;* Power in Kilogrammetres per second.

The metric Horse-power is 75 kilogrammetres per second, = 32,547 foot-pounds per minute, or about 1.4 per cent. less than the British horse-power of 33,000 foot-pounds per minute.

Work ($K = FS$).

Work is the product obtained by multiplying together the elements *force*, F, and *space*, S.

Work may also be expressed by $K = PT$, or the product of power and time.

The work of a steam-engine operating with a constant power will be directly as the time of operation, and so with all labor, whether it be mechanical or manual.

Moment of a Force (Fl).

The moment of a force is its lever arm at right angles to its direction of action multiplied by its intensity in pounds or tons.

Momentum (MV).

The momentum of a moving body is the intensity of that *constant force* which, resisting its movement, will bring it to rest in *one second.*

$$M = \frac{weight}{32.2}.$$

V = velocity in feet per second.

Moment of Inertia (MVr).

The moment of inertia of a rotating body is the moment of its momentum, and is equal to its momentum, MV, multiplied by its radius of oscillation, r.

Radius of Oscillation.

The radius of oscillation is the mean lever-arm of the momentum of a revolving body. It is equal to the moment of inertia divided by the momentum of the revolving body.

Radius of Gyration.

The square of the radius of gyration of an oscillating body is equal to the product of the radius of oscillation and of the distance of the centre of gravity of the suspended body from its point of suspension.

The intensity of the force of momentum is proportional to the distance of the centre of gravity from the axis of suspension, and the mean leverage of the momentum is the radius of oscillation. The square of the "radius of gyration," then, is a convenient product of these two quantities, as including both, and therefore giving them in a convenient mathematical form. If a straight rod be balanced at its middle, we are obliged to consider each half separately and add them together.

Units of Work.

The usual unit of work in the British notation is the *foot-pound*, equal to one pound raised through a space of one foot. For large measurements, where this unit is too small, the *foot-ton* is used, this being the usual unit in ordnance computations.

Units of Power.

The unit of power most generally used in England and America is the *horse-power*. In rating the early steam-engines Watt made a number of experiments with powerful draught horses, and arrived at the value of

22,000 foot-pounds per minute as the horse-power. In order to allow liberal measure in proportioning his steam-engines, he increased this by 50 per cent., and called the steam horse-power 33,000 foot-pounds per minute, or 550 foot-pounds per second.

The unit of power generally used in connection with electrical work is the *watt*, and for most mechanical purposes the kilowatt = 1000 watts is used. One English horse-power = 746 watts, or one kilowatt = 1.34 horse-power. The metric horse-power = 736 watts. Since electric generators are usually rated in kilowatts, and are frequently coupled directly to steam-engines, or even built into combined generating sets with them, the power of steam-engines is sometimes rated in kilowatts, and this practice is probably destined to become more and more general as electric driving is introduced. For all practical purposes the kilowatt may be taken as three-quarters of a horse-power.

Formulas for Rotary Motion.

F = force.
P = power, in foot-pounds, per second.
V = velocity, in feet, per second.
S = space, in feet.
T = time, in seconds.
K = work, in foot-pounds.
R = radius from centre of rotation.
n = revolutions per minute.
N = total number of revolutions in a time T.

Force, F, acting in the Direction of the Tangent.

29. $F = \frac{60\,P}{2\pi Rn}$.	30. $F = \frac{9.55\,P}{Rn}$.	31. $F = \frac{9.55\,K}{RnT}$.	32. $F = \frac{5252\,IP}{Rn}$.

Circumferential Velocity and Revolutions per Minute.

33. $V = \frac{2\pi Rn}{60}$.	34. $V = 0.10472 Rn$.	35. $n = \frac{9.55\,V}{R}$.	36. $n = \frac{5252\,IP}{FR}$.

Time of Operation, in Seconds.

37. $T = \frac{9.55\,S}{Rn}$.	38. $T = \frac{9.55\,K}{FRn}$.	39. $T = \frac{FRn}{9.55\,P}$.	40. $T = \frac{FRN}{87.5\,IP}$.

Radius of Revolution.

41. $R = \frac{9.55\,V}{n}$.	42. $R = \frac{9.55\,P}{Fn}$.	43. $R = \frac{5252\,IP}{Fn}$.	44. $R = \frac{9.55\,K}{FnT}$.

Power Generated, in Foot-pounds, per Second.

45. $P = \frac{2\pi RnF}{60}$.	46. $P = \frac{FRn}{9.55}$.	47. $P = \frac{FRN}{9.55\,T}$.	48. $N = \frac{9.55\,PT}{FR}$.

Space Generated, in Feet.

49. $S = \frac{2\pi RnT}{60}$.	50. $S = \frac{RnT}{9.55}$.	51. $S = \frac{FnN}{755.625\,IP}$.	52. $S = N2\pi R$.

Horse-power Generated.

53. $IP = \frac{FRn}{5252}$.	54. $IP = \frac{FRN}{87.5\,T}$.	55. $N = \frac{87.5\,IPT}{FR}$.	56. $N = \frac{S}{2\pi R}$.

Work Accomplished, in Foot-pounds, in Time T.

57. $K = \frac{FRnT}{9.55}$.	58. $K = F2\pi RN$.	59. $N = \frac{K}{F2\pi R}$.	60. $R = \frac{K}{F2\pi N}$.

Force, Power, and Work in Moving Bodies.

It requires force, power, and work to change the state of motion or rest of a body.

In the dynamic expression $MV = FT$ we have

1.	Force, $F = \frac{MV}{T}$.	3.	$M = \frac{FT}{V}$.
2.	$T = \frac{MV}{F}$.	4.	$V = \frac{FT}{M}$.

The force, F, required to set a mass, M, in motion with velocity, V, depends inversely on the time, T, of action. The more time the less need the force be for a certain velocity, and therefore it cannot be determined what force has set a mass in motion without knowing its time of action; but when the mass and its velocity are given, then we can determine the exact amount of work bestowed on the motion.

Multiply the dynamic momentum by the velocity, V, and we have

$$MV^2 = FVT.$$

Here we recognize the work, $\frac{V}{2}FT$, which is that bestowed on the mass, M, in giving it the velocity, V, or the mass multiplied by one-half the square of its velocity is the work stored in it.

Vis-viva.—The term MV^2 has formerly been called *vis-viva*, but that term is now seldom used.

The real work in foot-pounds is $\frac{1}{2}MV^2 = \frac{1}{2}FVT$. The space, S, in which the mass was set in motion is $S = \frac{1}{2}VT$, which inserted in the formula gives the

$$\text{Work, } K = \tfrac{1}{2}MV^2 = FS.$$

Dynamical Formulas for Accelerated or Retarded Motion.

Constant Force, in Pounds, acting on a Body free to move.

$$F = \frac{GW}{g} = \frac{WV}{gT} = \frac{2WS}{gT^2} = \frac{WV^2}{2gS} = \frac{PT}{S} = \sqrt{\frac{2PW}{gT}} = \frac{2K}{GT^2} = \frac{K}{S}.$$

Final Velocity in the Time, T, or Uniform Velocity of a moving Body.

$$V = GT = \frac{gFT}{W} = \frac{2S}{T} = \sqrt{\frac{2gSF}{W}} = \sqrt{2GS} = \frac{PT}{K} = \sqrt{\frac{2gPT}{W}} = \sqrt{\frac{2gK}{W}}.$$

Time, in Seconds, in which the Force acts on the Body free to move.

$$T = \frac{V}{G} = \frac{WV}{gF} = \sqrt{\frac{2WS}{gF}} = \sqrt{\frac{2S}{G}} = \frac{2FS^2}{VK} = \frac{K}{P} = \frac{2SW}{gTF} = \sqrt{\frac{2WK}{gF^2}}.$$

Constant Acceleration of the Force, F, in Feet per Second.

$$G = \frac{gF}{W} = \frac{2S}{T^2} = \frac{V}{T} = \frac{V^2}{2S} = \frac{gPT}{WS} = \frac{FV^2}{PT} = \frac{gK}{WS} = \frac{2K}{FT^2}.$$

Space, in Feet, in which the Force acts on the Body free to move.

$$S = \frac{GT^2}{2} = \frac{VT}{2} = \frac{V^2}{2G} = \frac{gFT^2}{2W} = \frac{PT}{F} = \frac{gPT^2}{WV} = \frac{gK}{GW} = \frac{K}{F}.$$

Weight, in Pounds, of the moving Body.

$$W = \frac{gF}{G} = \frac{gFT^2}{2S} = \frac{2gFS}{V^2} = \frac{gFT}{V} = \frac{gPT^3}{2S^2} = \frac{gF^2T}{2P} = \frac{2gK}{V^2} = \frac{gT^2K}{2S^2}.$$

Mean Power in Effects during the Time, T, or in the Space, S.

$$P=\frac{FS}{T}=\frac{gF^2T}{2W}=\frac{2WS^2}{gT^3}=\frac{WV^2}{2gT}=\frac{2K}{T}=\frac{TK}{2S}=\frac{VK}{S}=\frac{FV^2}{GT}.$$

Work, in Foot-pounds, concentrated in a moving Body.

$$K=FS=\frac{WV^2}{2g}=\frac{FVT}{2}=\frac{GWVT}{2g}=\frac{FGT^2}{2}=\frac{gF^2T^2}{2W}=\frac{2SP}{T}=PT.$$

REVOLVING BODIES.

Centre of Gyration.

The **Centre of Gyration** is a point in a revolving body in which, if all the revolving matter were there collected, it would obtain equal *angular velocity* from and sustain equal resistance to the force that gives it the rotary motion. The distance of the centre of gyration from the axis of rotation for different shapes in practical work will be found in the diagrams on pages 273 and 274.

Formulas for Accelerated Circular Motion.

Force, F, in Pounds, acting on the Lever or Radius, r, to rotate the Body.

$$F=\frac{Wx^2n}{307.49\,Tr}=\frac{Wx^2N}{2.562\,T^2r}=\frac{60\,K}{\pi rnT}=\frac{K}{2\pi rN}.$$

Final Revolutions per Minute in the Time T.

$$n=\frac{120\,N}{T}=\frac{307.49\,FTr}{Wx^2}=\frac{60\,K}{\pi rTF}=\sqrt{\frac{5872.2\,K}{Wx^2}}.$$

Total Number of Revolutions in the Time T.

$$N=\frac{Tn}{120}=\frac{2.562\,FT^2r}{Wx^2}=\frac{K}{2\pi rF}=\frac{T}{1.565x}\sqrt{\frac{K}{W}}.$$

Time of Acceleration, in Seconds, from the Start of Change of Motion.

$$T=\frac{Wx^2n}{307.49\,Fr}=\sqrt{\frac{Wx^2N}{2.562\,Fr}}=\frac{60\,K}{\pi rnF}=\frac{x\sqrt{WK}}{4.09\,Fr}.$$

Radius of Gyration, in Feet, of the revolving Body.

$$x=\sqrt{\frac{307.49\,FrT}{Wn}}=\sqrt{\frac{2.562\,FrT^2}{WN}}=\frac{KT}{3.9\,N\sqrt{WNFr}}=\frac{334.9\,K}{n\sqrt{WnTFr}}.$$

Weight, in Pounds, of the revolving Body.

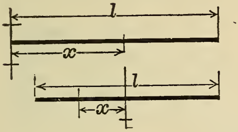
$$W=\frac{307.49\,TFr}{x^2n}=\frac{2.562\,T^2Fr}{x^2N}=\frac{5872.2\,K}{n^2x^2}=\frac{KT^2}{2.452\,x^2N^2}.$$

Work, in Foot-pounds, concentrated in a revolving Body.

$$K=\frac{Wx^2n^2}{5872.2}=\frac{2.452\,Wx^2N^2}{T^2}=\frac{\pi rnFT}{60}=2\pi rNF.$$

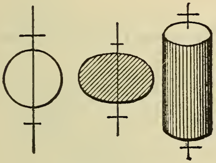
Radius of Gyration.

A Line or Bar.



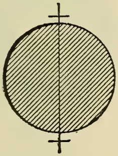
$x = 0.5773l$, $\qquad x = 0.2887l$.

A Circumference around its Diameter.
A Disk around its Centre.
A Cylinder around its Axis.



$x = 0.7072r$.

A Disk around its Diameter.

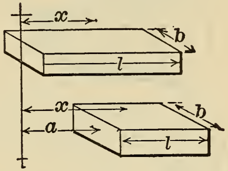


$x = 0.5r$.

A Sphere around its Diameter.

Spherical shell, $x = 0.8165r$,
Solid, $x = 0.6324r$.

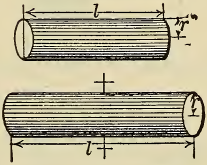
Parallelopipedon.



$$x = \sqrt{\frac{4l^2 + b^2}{12}},$$

$$x = \sqrt{\frac{4l^2 + b^2}{12} + a^2 + al}.$$

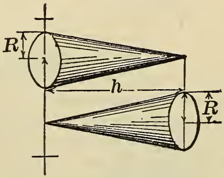
Cylinder.



$$x = \sqrt{\frac{4l^2 + 3r^2}{12}},$$

$$x = \sqrt{\frac{l^2 + 3r^2}{12}}.$$

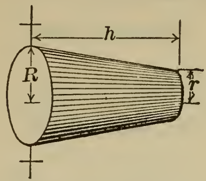
Cone.



$$x = \sqrt{\frac{2h^2 + 3R^2}{20}},$$

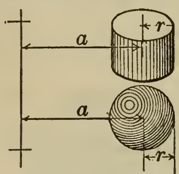
$$x = \sqrt{\frac{12h^2 + 3R^2}{20}}.$$

Conic Frustum.



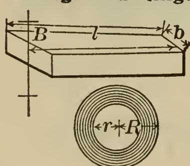
$$x=\sqrt{\frac{h}{10}\left(\frac{R^2+3Rr+Rr^2}{R^2+Rr+r^2}\right)+\frac{3}{20}\left(\frac{R^5-r^5}{R^3-r^3}\right)}.$$

Cylinder and Sphere.



$x=\sqrt{a^2+\frac{1}{2}r^2}$, $\quad x=\sqrt{a^2+\frac{2}{5}r^2}$.

Wedge and Ring.

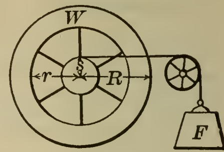


r = internal radius of ring, R = external radius of ring.

$$x=0.204\sqrt{12l^2+B^2+b^2},$$

$$x=\sqrt{\frac{R^2+r^2}{2}}.$$

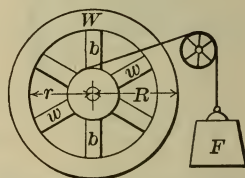
Fly-wheel.



$$x=\sqrt{\frac{R^2+r^2}{2}},$$

$$FG : Wg = x^2 : s^2.$$

Fly-wheel with Arms.



$$x^2(W+w)=W\frac{R^2+r^2}{2}+w\frac{4r^2+b^2}{12},$$

$$x=\sqrt{\frac{6W(R^2+r^2)+w(4r^2+b^2)}{12(W+w)}}.$$

CENTRAL FORCES.

Central Forces are of two kinds, *centrifugal* and *centripetal.*

Centrifugal Force is the resistance which a revolving body offers to being moved in the arc of a circle.

Centripetal Force is that by which a revolving body is attracted or attached to its centre of motion.

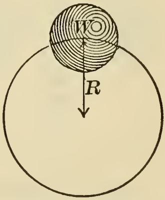
The centrifugal and centripetal forces are opposites to each other, and when equal the body revolves in a circle; but when they differ the body will revolve in other curved lines, as the ellipse, the parabola, etc., according to the nature of the difference in the forces. If the centrifugal force is *o* while the other is acting, the body will move straight to the centre of motion; and if the centripetal force is *o* while the other is act-

ing, the body will depart from the circle in a straight line, tangent to the circle in the point where the centripetal force ceased to act. The central forces are distinct from the force that has set the body in motion.

If the centrifugal force be made use of to produce an effect, such effect will be at the expense of the one producing the rotary motion.

Notation.

F = centrifugal force, in pounds.
W = the weight of the revolving body, in pounds.
v = velocity of the revolving body, in feet, per second.
R = radius of the circle in which the body revolves, in feet.
n = number of revolutions per minute.



$$F = \frac{Wv^2}{gR} = \frac{Wv^2}{32.2R},$$

$$F = \frac{4\,WR\pi^2n^2}{60^2g} = \frac{WRn^2}{2933} = 0.00034\,WRn^2,$$

$$W = \frac{FgR}{v^2} = \frac{2933F}{Rn^2}, \qquad R = \frac{Wv^2}{Fg} = \frac{2933F}{Wn^2},$$

$$n = \sqrt{\frac{2933F}{WR}}, \qquad v = \sqrt{\frac{FRg}{W}}.$$

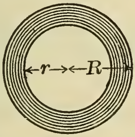
Thus, if we have a weight of 63 pounds at a radius of 4 feet 4 inches, making 163 revolutions per minute, we have

$$W = 63,\ R = 4.333,\ n = 163,$$

and the centrifugal force, or tension produced on the radial arm, will be

$$0.00034\,WRn^2 = 0.00034 \times 63 \times 4.333 \times 163^2 = 2466 \text{ pounds.}$$

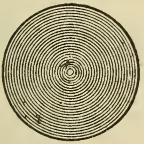
Centrifugal Force of a Ring.



r = internal radius, R = external radius.

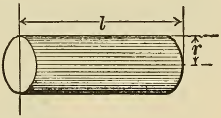
$$F = \frac{Wn_2(R-r)}{\pi 4150}.$$

Centrifugal Force of a Grinding Stone, Thin Disk, or Cylinder rotating around its centre.



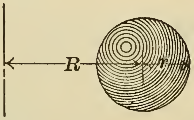
$$F = \frac{WRn^2}{\pi 4150}.$$

Centrifugal Force of a Cylinder rotating around the diameter of its base.

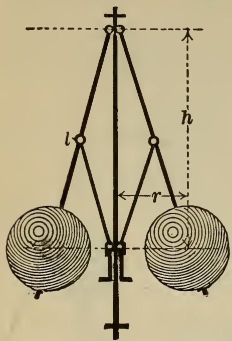


$$F = \frac{Wn^2l}{5867}.$$

Centrifugal Force of a Ball.



$$F = \frac{Wn^2R}{2933}.$$



Governor.

$$n = \frac{60}{2\pi}\sqrt{\frac{g}{h}} = \frac{54.16}{\sqrt{h}} = \frac{54.16}{\sqrt{l \cos x}},$$

$$h = \frac{2933}{n^2}, \qquad l = \frac{2933}{n^2 \cos x} = \frac{h}{\cos x},$$

$$\cos x = \frac{2933}{n^2 l} = \frac{h}{l}, \qquad r = \sqrt{l^2 - h^2},$$

x = angle made by arm with the vertical axis.

For a weighted governor of the Porter type, in which W = the axial weight and B = the weight of the ball, the height, h, will be equal to the height of a simple governor multiplied by $\left(1 + \frac{W}{B}\right)$.

PENDULUM.

Simple Pendulum is a material point under the action of gravitation, and suspended at a fixed point by a line of no weight.

Compound Pendulum is a suspended rod and body of sensible magnitude, fixed as the simple pendulum.

Centre of Oscillation is a point at which if all the matter in the compound pendulum were there collected, it would make a simple pendulum oscillate in the same periods.

Angle of Oscillation is the space a pendulum describes when in motion.

The velocity of an oscillating body through the vertical position is equal to the velocity a body would obtain by falling vertically the distance *versed sine* of half the angle of oscillation.

Notation.

l = length of the simple pendulum, or the distance between the centre of suspension and centre of oscillation, in inches.
t = time, in seconds, for n oscillations.
n = number of single oscillations in the time t.

Example. Required the length of a pendulum that will vibrate seconds? Here $n = 1$ and $t = 1''$.

$$l = 39.10\frac{t^2}{n^2} = 39.10 \text{ inches, the length of a pendulum for seconds.}$$

Example. Required the length of a pendulum that will make 180 vibrations per minute? Here $t = 60''$ and $n = 180$.

$$l = \frac{39.10t^2}{n^2} = \frac{39.10 \times 60^2}{180^2} = 4.344 \text{ inches.}$$

Example. How many vibrations will a pendulum of 25 inches length make in 8 seconds?

$$n = \frac{6.254t}{\sqrt{l}} = \frac{6.254 \times 8}{\sqrt{25}} = 10 \text{ vibrations.}$$

Example. A pendulum is 137.67 inches long and makes 8 vibrations in 15 seconds. Required the acceleration of gravity, g?

$$g = \frac{0.8225ln^2}{t^2} = \frac{0.8225 \times 137.67 \times 8^2}{15^2} = 32.209.$$

Example. A compound pendulum of two iron balls, P and Q, having the centre of suspension between themselves, as shown in the illustrations on the opposite page. $P = 38$ pounds, $Q = 12$ pounds, $a = 25$ inches, and $b = 18$ inches. How long is the simple pendulum, and how many vibrations will the pendulum make in 10 seconds?

$$x = \frac{aP - bQ}{P + Q} = \frac{25 \times 38 - 18 \times 12}{38 + 12} = 14.68 \text{ inches.}$$

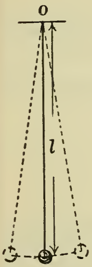
$$l = \frac{a^2P + b^2Q}{x(P + Q)} = \frac{25^2 \times 38 + 18^2 \times 12}{14.68(38 + 12)} = 37.68 \text{ inches,}$$

the length of the single pendulum.

$$n = \frac{6.254t}{\sqrt{l}} = \frac{6.254 \times 10}{\sqrt{37.68}} = 10.193 \text{ vibrations in 10 seconds.}$$

If a compound pendulum is hung up at its centre of oscillation, the former centre of suspension will be the centre of oscillation and the pendulum will oscillate the same time.

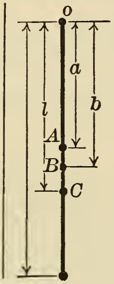
Simple Pendulum.



$$l = \frac{12gt^2}{\pi^2 n^2} = \frac{39.1t^2}{n^2},$$

$$t = \frac{n\sqrt{l}}{6.25},$$

$$n = \frac{6.254t}{\sqrt{l}}.$$



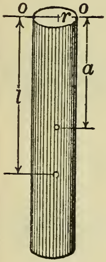
A = centre of gravity,
B = centre of gyration,
C = centre of oscillation.

$$a : b = b : l,$$

$$b = \sqrt{al} = 1.414a,$$

$$l = 1\tfrac{1}{3}a.$$

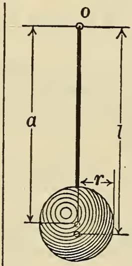
Compound Pendulum.



r = radius of cylinder,

$$l = \frac{16a^2 + 3r^2}{12a},$$

$$= \frac{4a}{3} + \frac{r^2}{4a}.$$

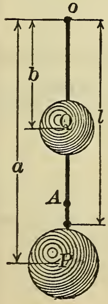


$$g = \frac{l\pi^2 n^2}{12t^2},$$

$$g = \frac{0.8225ln^2}{t^2},$$

o = centre of suspension,

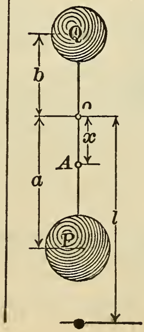
$$l = a + \frac{2r^2}{5a}.$$



$$l = \frac{a^2P + b^2Q}{aP + bQ}.$$

P and Q expressed in pounds or cubic contents.

Connecting wire neglected.



$$x = \frac{aP - bQ}{P + Q},$$

$$l = \frac{a^2P + b^2Q}{x(P + Q)}.$$

Connecting wire neglected.

Length of Pendulum Vibrating Seconds at Sea-level.*

	Latitude.	Metres.	Inches.
At Equator	0° 00′	0.99092	39.012
At Washington, D. C.	38° 53′	0.99299	39.094
At New York	40° 43′	0.99316	39.101
At Latitude 45°	45° 00′	0.99355	39.116
At London, Eng.	51° 31′	0.99414	39.139
At Stockholm	59° 21′	0.99481	39.166

IMPACT.

Impact of Moving Bodies.

When bodies in motion come in collision with each other, the sum of their concentrated momentum will be the same after the collision as before, but their velocities and sometimes their directions of motion will differ.

In the illustrations on page 279 the bodies are supposed to move in the same straight line, and the formula illustrates the consequences after collision.

Notation.

M and m = weight of the bodies, in pounds.
V and v = their respective velocities, in feet, per second.
V' and v' = respective velocities of the bodies after impact.
K and k = coefficient of elasticity, which for perfectly hard bodies $k = 0$ and for perfectly elastic bodies $k = 1$; therefore the elastic coefficient will always be between 0 and 1. When the bodies are perfectly hard their velocities after impact will be common.

$$\text{For } M, \quad K = \frac{MV}{M(V - V')}; \qquad \text{for } m, \quad k = \frac{mv}{m(v - V')}.$$

Example 1. The non-elastic body weighs $M = 25$ pounds, and moves at a velocity $V = 12$ feet per second; $m = 16$ pounds and $v = 9$ feet per second. Required the bodies' common velocities $v = ?$ after impact, both bodies moving in the same direction.

$$v' = \frac{MV + mv}{M + m} = \frac{25 \times 12 + 16 \times 9}{25 + 16} = 10.83 \text{ feet per second.}$$

Example 2. The perfect elastic body $M = 84$ pounds, $V = 18$ feet per second, $m = 48$ pounds, and $v = 27$ feet per second. Required the velocity $V' = ?$ after impact with the body m, the bodies moving in opposite directions.

$$V = \frac{18(84 - 48) - 2 \times 48 \times 27}{84 + 48} = -23.64.$$

The negative sign denotes that the body will return after the collision with a velocity of 23.63 feet per second.

Example 3. The partly elastic body $M = 38$ pounds and $V = 79$ feet per second will strike the body in rest $m = 24$ pounds. What will be the velocity $v = ?$ of the body m, its elasticity being $k' = 0.6$.

$$v' = \frac{79 \times 38(1 + 0.6)}{38 + 24} = 70.6 \text{ feet per second.}$$

When a moving body strikes a stationary elastic plane its course of departure from the plane will be equal to its course of incidence.

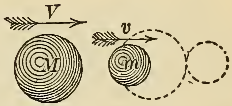
* Authority of T. C. Mendenhall, Superintendent United States Coast and Geodetic Survey, January, 1894.

The Bodies Perfectly Hard.

The bodies move in the same direction.

$$v'(M+m) = MV + mv,$$

$$v' = \frac{MV + mv}{M+m}.$$



The bodies move in opposite directions.

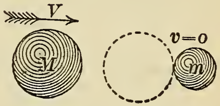
$$v'(M+m) = MV - mv,$$

$$v' = \frac{MV - mv}{M+m}.$$

Only one body in motion.

$$v'(M+m) = MV,$$

$$v' = \frac{MV}{M+m}.$$

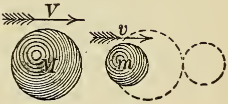


The Bodies Elastic.

The bodies move in the same direction.

$$V' = \frac{V(M - Km) + vm(1+K)}{M+m},$$

$$v' = \frac{MV(1+k) + v(m + kM)}{M+m}.$$



The bodies move in opposite directions.

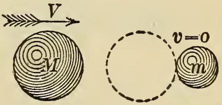
$$V' = \frac{V(M - Km) - vm(1+K)}{M+m},$$

$$v' = \frac{MV(1+k) - v(m - kM)}{M+m}.$$

Only one body in motion.

$$V' = \frac{V(M - Km)}{M+m},$$

$$v' = \frac{VM(1+k)}{M+m}.$$



FRICTION.

The resistance to motion which is experienced when one body is moved upon another is expressed by the general term Friction. Theoretically, it is assumed to be due to the interlocking of the roughness and inequalities which exist to a greater or less degree in the surfaces of all solids. The term friction, however, is applied to the resistance encountered by air or gases flowing through pipes or flues, or by water in pipes and channels, and in all cases of motion it is an element to be considered.

The first modern study of the subject of friction was that made in France by General Morin about 1831, and for a long time the laws enunciated by him as the result of his experiments, and the coefficients of friction given by him, were generally accepted and extensively reprinted. It is now generally understood, however, that these laws and results were true only for the conditions under which they were made, and that modern operative conditions require them to be modified. At the same time, the general results of Morin's experiments may here be referred to as forming a basis for the more recent data.

Morin's experiments were made by measuring the force required to cause one body to slide upon another. The ratio between this force and the pressure upon the sliding body is called the coefficient of friction, so that the coefficient of friction is the proportion which the resistance of friction bears to the pressure upon the sliding body. The pressure upon the sliding body is always taken as acting normal to the sliding surfaces. As a result of his experiments, Morin announced:

1. Friction is directly proportional to the pressure; the coefficient being thus constant at all pressures.
2. Friction, both in amount and coefficient, is independent of the areas in contact; the pressure remaining the same.
3. The coefficient of friction is independent of the velocity of the rubbing surfaces. This is understood to refer to the friction of motion, since it takes a greater force to overcome the friction of rest than to maintain the surfaces in motion thereafter.

The second law is a natural consequence of the first, since any increase in area for the same total pressure reduces the pressure per unit of area in the same proportion. If the area be doubled, the pressure per square inch will be halved, but there will be twice as many square inches, and the frictional resistance will be unchanged.

The principal modifications which have to be made in these laws, in the light of modern practice, are in the expansion of an expression made by Morin himself in connection with the experiments,—namely, that the *condition of the surfaces* must be taken into consideration. It is now possible to produce surfaces, both plane and cylindrical, so far superior to those with which Morin experimented that the coefficients deduced by him, and tabulated in many reference books since, are now considered far too great in nearly every case. The improvement in lubricants and the influence of temperature also enter as factors, and the number of variables thus introduced make it impossible to do more than furnish general data for preliminary use; and in all undertakings of importance experimental determinations should be made with the given materials, as nearly under the actual conditions as possible.

For plane-sliding friction, in which the speed of the movement is moderate and the pressures not excessive, Morin's laws and coefficients are fairly correct, although the latter are somewhat higher than are found with highly-polished surfaces, well lubricated.

Morin's coefficients of friction, given by him in detail under numerous varying conditions, may be taken in general as follows:

Material.	Coefficient.
Wood on wood, dry	0.25
Metal on metal, dry	0.15
" " well lubricated	0.07 to 0.08

Any attempt to use closer refinements when the exact conditions are not known is both useless and deceptive.

Journal Friction.

Recent experiments have shown that Morin's laws do not hold for revolving journals at high speeds and under heavy pressures.

The experiments of Mr. Beauchamp Tower* showed that the coefficient of friction, f, increased as the square root of the linear velocity, and diminished directly with the increase in pressure.

If $v =$ the linear velocity in feet per second, and $p =$ the pressure in pounds per square inch, the coefficient $f = c\frac{\sqrt{v}}{p}$, in which c is a constant, dependent upon the lubricant.

The following values of c may be used with pressures of 100 to 600 pounds per square inch:

Lubricant.	c	Lubricant.	c
Olive oil	0.289	Sperm oil	0.194
Lard oil	0.281	Rape oil	0.212
Mineral grease	0.431	Mineral oil	0.276

With olive oil lubrication, at a velocity of 3½ feet per second, and pressures ranging from 520 pounds down to 100 pounds per square inch, the coefficient of friction, f, varied from 0.001 to 0.0055.

In Mr. Tower's experiments the pressure at which the journal seized varied from 520 to 625 pounds per square inch of projected area,—that is, the length multiplied by the diameter.

On collar bearings, such as are used for the thrust bearings of screw-propeller shafts, the coefficient of friction is found to be independent of the speed, and for pressures between 45 and 75 pounds per square inch it ranges between 0.040 and 0.035. Good practice allows a pressure of 50 pounds per square inch, at which a coefficient of 0.036 may be used.

In computing the resistance of friction a clear distinction must be made between the frictional resistance itself and the *work* of friction. The work is measured by the product of the frictional resistance and the lineal speed of the rubbing surfaces, this giving the power absorbed in foot-pounds per minute.

The question of temperature is often an important element in frictional resistance, the work of friction appearing as heat, which, if not carried away, produces a rapid rise in temperature. The increased temperature reduces the viscosity of the lubricant, which is then forced out by the pressure, and the bearing runs dry and seizes.

Pillow-blocks and similar bearings should contain sufficient mass of metal to permit the heat to be conducted away freely, and attempts to economize in metal by coring out to the limit of mere strength may reduce the thermal conductivity of a bearing to such an extent as to render it liable to heat.

The thrust bearings of marine engines, and other bearings in which it is of great importance that motion should not be interrupted, are made with passages for the circulation of cooling water, which is to be turned on promptly in case of an abnormal increase in temperature.

The maximum pressures which are permitted depend upon the nature of the motion. When the pressure is exerted continuously in the same direction, as in the case of heavy shafting, etc., there is not the same opportunity for the lubricant to flow in, as in the case of alternating or intermittent pressure. In the first case the pressure should not exceed 450 pounds per square inch at a maximum, and should be kept down to 250 or 300 pounds, when practicable. For the second case, such as crank pins, in which the pressure acts in alternate directions, pressures from 500 to 900 pounds per square inch are used in stationary practice and 1200 to 1800 pounds per square inch in locomotive engines.

* Proceedings of the Institution of Mechanical Engineers, 1883.

MATERIALS OF ENGINEERING.

Martens divides the materials of engineering into two main classes:

1. **Materials of Construction.** Being those which constitute the completed structures. To this class belong the metals, woods, stone, cement, etc.

2. **Materials of Consumption.** Being those which are consumed or transformed while being used. These include such substances as coal, water, oil, etc.

While these distinctions are not rigid or absolute, they may serve as a convenient classification.

Materials of Construction may be considered according to their physical or their chemical properties, or both.

For engineering purposes the chemical properties are not so generally considered as are the physical properties, although in some respects the ultimate composition, as well as the manner of combination, must be taken into account. Materials must generally be defined according to their chemical nomenclature, after which their physical properties demand the most attention. No attempt will be made here to discuss any but the materials in general use, the rarer elements and their combinations forming properly the subjects for treatises on chemistry and physics.

Apart from their chemical composition, the principal properties of importance to the engineer are:

Density, represented by *specific gravity.*

Resistance, or capacity to oppose stresses.

Hardness, or opposition to penetration.

Toughness, or capacity for elongation under tension.

Brittleness, the opposite of toughness.

Besides these there are many other physical properties, such as behavior during heating or cooling, fusing, or working in innumerable ways, but these must be considered in connection with the operations in which they appear.

The Specific Gravity, or relative density of substances, is the ratio of the weight of a given volume of the substance to the same volume of water. For gases, the unit of comparison is an equal volume of air. The water unit in specific gravity determinations is assumed to be pure and at its temperature of greatest density.

Since, in the metric system, the units of weight are derived from the units of volume in terms of the weight of water, the specific gravity of any substance is also its weight in metric units. Thus, if the specific gravity of a certain iron is 7, a cubic centimetre of it will weigh 7 grammes, or a cubic decimetre will weigh 7 kilogrammes. In English units there is no such integral relation between the units of weight and volume of water, and hence the weight of a cubic inch or cubic foot of any substance must be given in pounds in addition to the specific gravity, or it can be computed from the latter by multiplying it by the weight of the given volume of water.

Since a submerged body is buoyed up by a force equal to the weight of an equal volume of water, the specific gravity of any solid substance may be found by the following methods:

To Find the Specific Gravity.

$W =$ weight of a body in the air.
$w =$ weight of the body (heavier than water) immersed in water.
$S =$ specific gravity of the body. Then

1. $$W - w : W = 1 : S. \qquad S = \frac{W}{W - w}.$$

Required the specific gravity of a piece of iron ore weighing 6.345 pounds in the air and 4.935 pounds in water, $S = ?$

$$S = \frac{6.345}{6.345 - 4.935} = 4.5, \text{ the specific gravity.}$$

When the body is lighter than water, attach to it a heavier body that is able to sink the lighter one.

$S =$ specific gravity of the heavier attached body.
$s =$ specific gravity of the lighter body.
$W =$ weight of the two bodies in air.
$w =$ weight of the two bodies in water.
$V =$ weight of the heavier body in air.
$v =$ weight of the lighter body in air.

2. $$s = \frac{v}{W - w - \frac{V}{S}}.$$

To a piece of wood, which weighs $v = 14$ pounds in the air, is fastened a piece of cast-iron, $V = 28$ pounds; the two bodies together weigh $w = 11.7$ pounds in water. Required the specific gravity of the wood?

$W = V + v = 28 + 14 = 42$ pounds.
$S = 7.2$, the specific gravity of cast-iron.

Formula 2. $$S = \frac{14}{42 - 11.7 - \frac{28}{7.2}} = 0.529,$$ the specific gravity of the wood (Spanish white poplar).

A simple way to obtain the specific gravity of wood is to make it into a rod and place it vertically in water; then, when in equilibrium, the immersed end is to the whole rod as the specific gravity is to 1.

A cylinder of wood is 6 feet 3 inches long. When immersed vertically in water it will sink 3 feet 9 inches by its own weight. Required its specific gravity?

$$3.75 : 6.25 = S : 1. \qquad S = \frac{3.75}{6.25} = 0.600.$$

To Find the Percentage of Alloy in Metals, or to Find the Proportions of Two Ingredients in a Compound.

3. $$V = \frac{W - s(W - w)}{1 - \frac{s}{S}}.$$

A metal compounded of silver and gold weighs $W = 6$ pounds in the air, and in water $w = 5.636$ pounds. Required the proportions of silver and gold?

$S = 19.36$, the specific gravity of gold.
$s = 10.51$, the specific gravity of silver.

Weight $$V = \frac{6 - 10.51(6 - 5.636)}{1 - \frac{10.51}{19.36}} =$$ 4.755 pounds of gold and 1.245 pounds of silver.

Names of substances.	Specific gravity.	Weight per cubic inch.
Metals.		Lb.
Platinum, rolled	22.669	.798
" wire	21.042	.761
" hammered	20.337	.736
" purified	19.500	.706
" crude, grs.	15.602	.565
Gold, hammered	19.361	.700
" pure cast	19.258	.697
" 22 carats' fine	17.486	.733
" 20 " "	15.702	.568
Mercury, solid at —40°	15.632	.566
" at +32° F.	13.619	.493
" " 60° F.	13.580	.491
" " 212° F.	13.375	.484
Lead, pure	11.330	.410
" hammered	11.388	.412
Silver, hammered	10.511	.381
" pure	10.474	.379
Bismuth	9.823	.355
Red lead	8.940	.324
Cinnabar	8.098	.293
Manganese	8.030	.290
Copper, wire & rolled	8.878	.321
" pure	8.788	.318
Bronze, gun metal	8.700	.315
Brass, common	7.820	.282
Steel, cast-steel	7.919	.286
" common soft	7.833	.283
" hard'ed & temp.	7.818	.283
Iron, pure	7.768	.281
" wrought & rol'd.	7.780	.282
" hammered	7.789	.282
" cast-iron	7.207	.261
Tin, from Bohmen	7.312	.265
" English	7.291	.264
Zinc, rolled	7.191	.260
" cast	6.861	.248
Antimony	6.712	.244
Aluminium	2.500	.090
Arsenic	5.763	.208
Stones and Earths.		
Topaz, Oriental	4.011	.145
Emery	4.000	.144
Diamond	3.521	.127
Limestone, green	3.180	.115
" white	3.156	.114
Asbestos, starry	3.073	.111
Glass, flint	2.933	.106
" white	2.892	.104
" bottle	2.732	.0987
" green	2.642	.0954
Marble, Parian	2.838	.1030
" African	2.708	.0978
" Egyptian	2.668	.0964
Mica	2.800	.1000
Hone, white razor	2.838	.1040
Chalk	2.784	.1000
Porphyry	2.765	.0999
Spar, green	2.704	.0976
" blue	2.693	.0971
Alabaster, white	2.730	.0987

Names of substances.	Specific gravity.	Weight per cubic inch.
Stones.—*Continued.*		Lb.
Alabaster, yellow	2.699	.0974
Coral, red	2.700	.0974
Granite, Susquehanna	2.704	.0976
" Quincy	2.652	.0958
" Patapsco	2.640	.0954
" Scotch	2.625	.0948
Marble, white Italian	2.708	.0978
" common	2.686	.0968
Talc, black	2.900	.0105
Quartz	2.660	.0962
Slate	2.672	.0965
Pearl, Oriental	2.650	.0957
Shale	2.600	.0940
Flint, white	2.594	.0936
" black	2.582	.0933
Stone, common	2.520	.0910
" Bristol	2.510	.0906
" mill	2.484	.0897
" paving	2.416	.0873
Gypsum, opaque	2.168	.0783
Grindstone	2.143	.0775
Salt, common	2.130	.0770
Saltpetre	2.090	.0755
Sulphur, native	2.033	.0735
Common soil	1.984	.0717
Rotten stone	1.981	.0416
Clay	1.930	.0698
Brick	1.900	.0686
Nitre	1.900	.0636
Plaster of Paris	1.872 2.473	.0677 .0894
Ivory	1.822	.0659
Sand	1.800	.0651
Phosphorus	1.770	.0640
Borax	1.714	.0620
Coal, anthracite	1.640 1.436	.0593 .0592
" Maryland	1.355	.0490
" Scotch	1.300	.0470
" New Castle	1.270	.0460
" bituminous	1.270	.0460
Charcoal, triturated	1.380	.0500
Earth, loose	1.500	.0542
Amber	1.078	.0387
Pimstone	1.647	.0596
Lime, quick	.804	.0291
Charcoal	.441	.0160
Woods (Dry).		
Alder	.800	.0289
Apple-tree	.793	.0287
Ash, the trunk	.845	.0306
Bay-tree	.822	.0297
Beech	.852	.0308
Box, French	.912	.0330
" Dutch	1.328	.0480
" Brazilian red	1.031	.0373
Cedar, wild	.596	.0219
" Palestine	.613	.0222
" Indian	1.315	.0476
" American	.561	.0203

Names of substances.	Specific gravity.	Weight per cubic inch.
Woods.—*Continued.*		Lb.
Citron	.726	.0263
Cocoa-wood	1.040	.0376
Cherry-tree	.715	.0259
Cork	.240	.0087
Cypress, Spanish	.644	.0233
Ebony, American	1.331	.0481
" Indian	1.209	.0437
Elder-tree	.695	.0252
Elm, trunk of	.671	.0243
Filbert-tree	.600	.0217
Fir, male	.550	.0199
" female	.498	.0180
Hazel	.600	.0217
Jasmine, Spanish	.770	.0279
Juniper-tree	.556	.0201
Lemon-tree	.703	.0254
Lignum-vitæ	1.333	.0482
Linden-tree	.604	.0219
Log-wood	.913	.0331
Mastic-tree	.849	.0307
Mahogany	1.063	.0385
Maple	.750	.0271
Medlar	.944	.0342
Mulberry	.897	.0324
Oak, heart of, 60 old	1.170	.0423
Orange-tree	.705	.0255
Pear-tree	.661	.0239
Pomegranate-tree	1.354	.0490
Poplar	.383	.0138
" white Spanish	.529	.0191
Plum-tree	.785	.0284
Quince-tree	.705	.0255
Sassafras	.482	.0174
Spruce	.500	.0181
" old	.460	.0166
Pine, yellow	.660	.0239
" white	.554	.0200
Vine	1.327	.0480
Walnut	.671	.0243
Yew, Dutch	.788	.0285
" Spanish	.807	.0292
Liquids.		
Acid, acetic	1.062	.0384
" nitric	1.217	.0440
" sulphuric	1.841	.0666
" muriatic	1.200	.0434
" fluoric	1.500	.0542
" phosphoric	1.558	.0563
Alcohol, commercial	.833	.0301
" pure	.792	.0287
Ammoniac, liquid	.897	.0324
Beer, lager	1.034	.0374
Champagne	9.970	.0360
Cider	1.018	.0361
Egg	1.090	.0394
Ether, sulphuric	.739	.0267
Honey	1.450	.0524
Human blood	1.054	.0381
Milk	1.032	.0373
Oil, linseed	.940	.0340

Names of substances.	Specific gravity.	Weight per cubic inch.
Liquids.—*Continued.*		Lb.
Oil, olive	.915	.0331
" turpentine	.870	.0314
" whale	.932	.0337
Proof spirit	.925	.0334
Vinegar	1.080	.0390
Water, distilled	1.000	.0361
" sea	1.030	.0371
" Dead Sea	1.240	.0448
Wine	.992	.0359
" port	.997	.0361
Miscellaneous.		
Asphaltum	.905 1.650	.0327 .0597
Atmospheric air	.0012	...43
Beeswax	.965	.0349
Butter	.942	.0341
Camphor	.988	.0357
India rubber	.933	.0338
Fat of beef	.923	.0334
" hogs	.936	.0338
" mutton	.923	.0334
Gamboge	1.222	.0442
Gunpowder, loose	.900	.0325
" shaken	1.000	.0361
" solid	1.550 1.800	.0561 .0650
Gum Arabic	1.452	.0525
Indigo	1.009	.0365
Lard	.947	.0343
Mastic	1.074	.0388
Spermaceti	.943	.0341
Sugar	1.605	.0580
Tallow, sheep	.924	.0334
" calf	.934	.0338
" ox	.923	.0334
Gases, Vapors.		Weight cubic ft. Grains.
Acetylene	.910	480.00
Ammonia gas	.590	311.00
Atmospheric air	1.000	527.00
Carbonic acid	1.527	805.30
Carbonic oxid	.972	512.70
Carburetted hydrog	.972	512.70
Chlorine	2.500	1316.00
Ethylene	.984	519.00
Hydrogen	.069	36.33
Methane	.566	297.00
Nitrogen	.972	512.00
Oxygen	1.104	581.80
Smoke of bitum. coal	.102	53.80
" " wood	.900	474.00
Steam at 212°	.488	257.30
Sulphuretted hydrog	1.777	9370.00
Sulphurous acid	2.222	1171.00
Vapor of alcohol	1.613	851.00
" " turp. spir	5.013	2642.00
" " water	.623	328.00

Weight of Flat Rolled Iron per Lineal Foot.

For Thicknesses from 1/16 inch to 2 inches, and Widths from 1 inch to 3¾ inches.

Iron weighing 480 pounds per cubic foot.

Thickness, in inches.	Width, in inches.											
	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼	3½	3¾
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1/16	.208	.260	.313	.365	.417	.469	.521	.573	.625	.677	.729	.781
1/8	.417	.521	.625	.729	.833	.938	1.04	1.15	1.25	1.35	1.46	1.56
3/16	.625	.781	.938	1.09	1.25	1.41	1.56	1.72	1.88	2.03	2.19	2.34
1/4	.833	1.04	1.25	1.46	1.67	1.88	2.08	2.29	2.50	2.71	2.92	3.13
5/16	1.04	1.30	1.56	1.82	2.08	2.34	2.60	2.86	3.13	3.39	3.65	3.91
3/8	1.25	1.56	1.88	2.19	2.50	2.81	3.13	3.44	3.75	4.06	4.38	4.69
7/16	1.46	1.82	2.19	2.55	2.92	3.28	3.65	4.01	4.38	4.74	5.10	5.47
1/2	1.67	2.08	2.50	2.92	3.33	3.75	4.17	4.58	5.00	5.42	5.83	6.25
9/16	1.88	2.34	2.81	3.28	3.75	4.22	4.69	5.16	5.63	6.09	6.56	7.03
5/8	2.08	2.60	3.13	3.65	4.17	4.69	5.21	5.73	6.25	6.77	7.29	7.81
11/16	2.29	2.86	3.44	4.01	4.58	5.16	5.73	6.30	6.88	7.45	8.02	8.59
3/4	2.50	3.13	3.75	4.38	5.00	5.63	6.25	6.88	7.50	8.13	8.75	9.38
13/16	2.71	3.39	4.06	4.74	5.42	6.09	6.77	7.45	8.13	8.80	9.48	10.16
7/8	2.92	3.65	4.38	5.10	5.83	6.56	7.29	8.02	8.75	9.48	10.21	10.94
15/16	3.13	3.91	4.69	5.47	6.25	7.03	7.81	8.59	9.38	10.16	10.94	11.72
1	3.33	4.17	5.00	5.83	6.67	7.50	8.33	9.17	10.00	10.83	11.67	12.50
1/16	3.54	4.43	5.31	6.20	7.08	7.97	8.85	9.74	10.63	11.51	12.40	13.28
1/8	3.75	4.69	5.63	6.56	7.50	8.44	9.38	10.31	11.25	12.19	13.13	14.06
3/16	3.96	4.95	5.94	6.93	7.92	8.91	9.90	10.89	11.88	12.86	13.85	14.84
1/4	4.17	5.21	6.25	7.29	8.33	9.38	10.42	11.46	12.50	13.54	14.58	15.63
5/16	4.37	5.47	6.56	7.66	8.75	9.84	10.94	12.03	13.13	14.22	15.31	16.41
3/8	4.58	5.73	6.88	8.02	9.17	10.31	11.46	12.60	13.75	14.90	16.04	17.19
7/16	4.79	5.99	7.19	8.39	9.58	10.78	11.98	13.18	14.38	15.57	16.77	17.97
1/2	5.00	6.25	7.50	8.75	10.00	11.25	12.50	13.75	15.00	16.25	17.50	18.75
9/16	5.21	6.51	7.81	9.11	10.42	11.72	13.02	14.32	15.63	16.93	18.23	19.53
5/8	5.42	6.77	8.13	9.48	10.83	12.19	13.54	14.90	16.25	17.60	18.96	20.31
11/16	5.63	7.03	8.44	9.84	11.25	12.66	14.06	15.47	16.88	18.28	19.69	21.09
3/4	5.83	7.29	8.75	10.21	11.67	13.13	14.58	16.04	17.50	18.96	20.42	21.88
13/16	6.04	7.55	9.06	10.57	12.08	13.59	15.10	16.61	18.13	19.64	21.15	22.66
7/8	6.25	7.81	9.38	10.94	12.50	14.06	15.63	17.19	18.75	20.31	21.88	23.44
15/16	6.46	8.07	9.69	11.30	12.92	14.53	16.15	17.76	19.38	20.99	22.60	24.22
2	6.67	8.33	10.00	11.67	13.33	15.00	16.67	18.33	20.00	21.67	23.33	25.00

For Steel add 2 per cent.

Weight of Flat Rolled Iron per Lineal Foot.

For Thicknesses from $\frac{1}{16}$ inch to 2 inches, and Widths from 4 inches to $6\frac{3}{4}$ inches.

Iron weighing 480 pounds per cubic foot.

Thickness, in inches.	Width, in inches.											
	4	4¼	4½	4¾	5	5¼	5½	5¾	6	6¼	6½	6¾
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
$\frac{1}{16}$	.833	.885	.938	.990	1.04	1.09	1.15	1.20	1.25	1.30	1.35	1.41
⅛	1.67	1.77	1.88	1.98	2.08	2.19	2.29	2.40	2.50	2.60	2.71	2.81
$\frac{3}{16}$	2.50	2.66	2.81	2.97	3.13	3.28	3.44	3.59	3.75	3.91	4.06	4.22
¼	3.33	3.54	3.75	3.96	4.17	4.38	4.58	4.79	5.00	5.21	5.42	5.63
$\frac{5}{16}$	4.17	4.43	4.69	4.95	5.21	5.47	5.73	5.99	6.25	6.51	6.77	7.03
⅜	5.00	5.31	5.63	5.94	6.25	6.56	6.88	7.19	7.50	7.81	8.13	8.44
$\frac{7}{16}$	5.83	6.20	6.56	6.93	7.29	7.66	8.02	8.39	8.75	9.11	9.48	9.84
½	6.67	7.08	7.50	7.92	8.33	8.75	9.17	9.58	10.00	10.42	10.83	11.25
$\frac{9}{16}$	7.50	7.97	8.44	8.91	9.38	9.84	10.31	10.78	11.25	11.72	12.19	12.66
⅝	8.33	8.85	9.38	9.90	10.42	10.94	11.46	11.98	12.50	13.02	13.54	14.06
$\frac{11}{16}$	9.17	9.74	10.31	10.89	11.46	12.03	12.60	13.18	13.75	14.32	14.90	15.47
¾	10.00	10.63	11.25	11.88	12.50	13.13	13.75	14.38	15.00	15.63	16.25	16.88
$\frac{13}{16}$	10.83	11.51	12.19	12.86	13.54	14.22	14.90	15.57	16.25	16.93	17.60	18.28
⅞	11.67	12.40	13.13	13.85	14.58	15.31	16.04	16.77	17.50	18.23	18.96	19.69
$\frac{15}{16}$	12.50	13.28	14.06	14.84	15.63	16.41	17.19	17.97	18.75	19.53	20.31	21.09
1	13.33	14.17	15.00	15.83	16.67	17.50	18.33	19.17	20.00	20.83	21.67	22.50
$\frac{1}{16}$	14.17	15.05	15.94	16.82	17.71	18.59	19.48	20.36	21.25	22.14	23.02	23.91
⅛	15.00	15.94	16.88	17.81	18.75	19.69	20.63	21.56	22.50	23.44	24.38	25.31
$\frac{3}{16}$	15.83	16.82	17.81	18.80	19.79	20.78	21.77	22.76	23.75	24.74	25.73	26.72
¼	16.67	17.71	18.75	19.79	20.83	21.88	22.92	23.96	25.00	26.04	27.08	28.13
$\frac{5}{16}$	17.50	18.59	19.69	20.78	21.88	22.97	24.06	25.16	26.25	27.34	28.44	29.53
⅜	18.33	19.48	20.63	21.77	22.92	24.06	25.21	26.35	27.50	28.65	29.79	30.94
$\frac{7}{16}$	19.17	20.36	21.56	22.76	23.96	25.16	26.35	27.55	28.75	29.95	31.15	32.34
½	20.00	21.25	22.50	23.75	25.00	26.25	27.50	28.75	30.00	31.25	32.50	33.75
$\frac{9}{16}$	20.83	22.14	23.44	24.74	26.04	27.34	28.65	29.95	31.25	32.55	33.85	35.16
⅝	21.67	23.02	24.38	25.73	27.08	28.44	29.79	31.15	32.50	33.85	35.21	36.56
$\frac{11}{16}$	22.50	23.91	25.31	26.72	28.13	29.53	30.94	32.34	33.75	35.16	36.56	37.97
¾	23.33	24.79	26.25	27.71	29.17	30.63	32.08	33.54	35.00	36.46	37.92	39.38
$\frac{13}{16}$	24.17	25.68	27.19	28.70	30.21	31.72	33.23	34.74	36.25	37.76	39.27	40.78
⅞	25.00	26.56	28.13	29.69	31.25	32.81	34.38	35.94	37.50	39.06	40.63	42.19
$\frac{15}{16}$	25.83	27.45	29.06	30.68	32.29	33.91	35.52	37.14	38.75	40.36	41.98	43.59
2	26.67	28.33	30.00	31.67	33.33	35.00	36.67	38.33	40.00	41.67	43.33	45.00

For Steel add 2 per cent.

Weight of Flat Rolled Iron per Lineal Foot.

For Thicknesses from $\frac{1}{16}$ inch to 2 inches, and Widths from 7 inches to $9\frac{3}{4}$ inches.

Iron weighing 480 pounds per cubic foot.

Thickness, in inches.	Width, in inches.											
	7	$7\frac{1}{4}$	$7\frac{1}{2}$	$7\frac{3}{4}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$	$8\frac{3}{4}$	9	$9\frac{1}{4}$	$9\frac{1}{2}$	$9\frac{3}{4}$
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
$\frac{1}{16}$	1.46	1.51	1.56	1.61	1.67	1.72	1.77	1.82	1.88	1.93	1.98	2.03
$\frac{1}{8}$	2.92	3.02	3.13	3.23	3.33	3.44	3.54	3.65	3.75	3.85	3.96	4.06
$\frac{3}{16}$	4.38	4.53	4.69	4.84	5.00	5.16	5.31	5.47	5.63	5.78	5.94	6.09
$\frac{1}{4}$	5.83	6.04	6.25	6.46	6.67	6.88	7.08	7.29	7.50	7.71	7.92	8.13
$\frac{5}{16}$	7.29	7.55	7.81	8.07	8.33	8.59	8.85	9.11	9.38	9.64	9.90	10.16
$\frac{3}{8}$	8.75	9.06	9.38	9.69	10.00	10.31	10.63	10.94	11.25	11.56	11.88	12.19
$\frac{7}{16}$	10.21	10.57	10.94	11.30	11.67	12.03	12.40	12.76	13.13	13.49	13.85	14.22
$\frac{1}{2}$	11.67	12.08	12.50	12.92	13.33	13.75	14.17	14.58	15.00	15.42	15.83	16.25
$\frac{9}{16}$	13.13	13.59	14.06	14.53	15.00	15.47	15.94	16.41	16.88	17.34	17.81	18.28
$\frac{5}{8}$	14.58	15.10	15.63	16.15	16.67	17.19	17.71	18.23	18.75	19.27	19.79	20.31
$\frac{11}{16}$	16.04	16.61	17.19	17.76	18.33	18.91	19.48	20.05	20.63	21.20	21.77	22.34
$\frac{3}{4}$	17.50	18.13	18.75	19.38	20.00	20.63	21.25	21.88	22.50	23.13	23.75	24.38
$\frac{13}{16}$	18.96	19.64	20.31	20.99	21.67	22.34	23.02	23.70	24.38	25.05	25.73	26.41
$\frac{7}{8}$	20.42	21.15	21.88	22.60	23.33	24.06	24.79	25.52	26.25	26.98	27.71	28.44
$\frac{15}{16}$	21.88	22.66	23.44	24.22	25.00	25.78	26.56	27.34	28.13	28.91	29.69	30.47
1	23.33	24.17	25.00	25.83	26.67	27.50	28.33	29.17	30.00	30.83	31.67	32.50
$\frac{1}{16}$	24.79	25.68	26.56	27.45	28.33	29.22	30.10	30.99	31.88	32.76	33.65	34.53
$\frac{1}{8}$	26.25	27.19	28.13	29.06	30.00	30.94	31.88	32.81	33.75	34.69	35.63	36.56
$\frac{3}{16}$	27.71	28.70	29.69	30.68	31.67	32.66	33.65	34.64	35.63	36.61	37.60	38.59
$\frac{1}{4}$	29.17	30.21	31.25	32.29	33.33	34.38	35.42	36.46	37.50	38.54	39.58	40.63
$\frac{5}{16}$	30.62	31.72	32.81	33.91	35.00	36.09	37.19	38.28	39.38	40.47	41.56	42.66
$\frac{3}{8}$	32.08	33.23	34.38	35.52	36.67	37.81	38.96	40.10	41.25	42.40	43.54	44.69
$\frac{7}{16}$	33.54	34.74	35.94	37.14	38.33	39.53	40.73	41.93	43.13	44.32	45.52	46.72
$\frac{1}{2}$	35.00	36.25	37.50	38.75	40.00	41.25	42.50	43.75	45.00	46.25	47.50	48.75
$\frac{9}{16}$	36.46	37.76	39.06	40.36	41.67	42.97	44.27	45.57	46.88	48.18	49.48	50.78
$\frac{5}{8}$	37.92	39.27	40.63	41.98	43.33	44.69	46.04	47.40	48.75	50.10	51.46	52.81
$\frac{11}{16}$	39.38	40.78	42.19	43.59	45.00	46.41	47.81	49.22	50.63	52.03	53.44	54.84
$\frac{3}{4}$	40.83	42.29	43.75	45.21	46.67	48.13	49.58	51.04	52.50	53.96	55.42	56.88
$\frac{13}{16}$	42.29	43.80	45.31	46.82	48.33	49.84	51.35	52.86	54.38	55.89	57.40	58.91
$\frac{7}{8}$	43.75	45.31	46.88	48.44	50.00	51.56	53.13	54.69	56.25	57.81	59.38	60.94
$\frac{15}{16}$	45.21	46.82	48.44	50.05	51.67	53.28	54.90	56.51	58.13	59.74	61.35	62.97
2	46.67	48.33	50.00	51.67	53.33	55.00	56.67	58.33	60.00	61.67	63.33	65.00

For Steel add 2 per cent.

Weight of Flat Rolled Iron per Lineal Foot.

For Thicknesses from 1/16 inch to 2 inches, and Widths from 10 inches to 12¾ inches.

Iron weighing 480 pounds per cubic foot.

Thickness, in inches.	Width, in inches.											
	10	10¼	10½	10¾	11	11¼	11½	11¾	12	12¼	12½	12¾
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1/16	2.08	2.14	2.19	2.24	2.29	2.34	2.40	2.45	2.50	2.55	2.60	2.66
1/8	4.17	4.27	4.38	4.48	4.58	4.69	4.79	4.90	5.00	5.10	5.21	5.31
3/16	6.25	6.41	6.56	6.72	6.88	7.03	7.19	7.34	7.50	7.66	7.81	7.97
1/4	8.33	8.54	8.75	8.96	9.17	9.38	9.58	9.79	10.00	10.21	10.42	10.63
5/16	10.42	10.68	10.94	11.20	11.46	11.72	11.98	12.24	12.50	12.76	13.02	13.28
3/8	12.50	12.81	13.13	13.44	13.75	14.06	14.38	14.69	15.00	15.31	15.63	15.94
7/16	14.58	14.95	15.31	15.68	16.04	16.41	16.77	17.14	17.50	17.86	18.23	18.59
1/2	16.67	17.08	17.50	17.92	18.33	18.75	19.17	19.58	20.00	20.42	20.83	21.25
9/16	18.75	19.22	19.69	20.16	20.63	21.09	21.56	22.03	22.50	22.97	23.44	23.91
5/8	20.83	21.35	21.88	22.40	22.92	23.44	23.96	24.48	25.00	25.52	26.04	26.56
11/16	22.92	23.49	24.06	24.64	25.21	25.78	26.35	26.93	27.50	28.07	28.65	29.22
3/4	25.00	25.62	26.25	26.88	27.50	28.13	28.75	29.38	30.00	30.63	31.25	31.88
13/16	27.08	27.76	28.44	29.11	29.79	30.47	31.15	31.82	32.50	33.18	33.85	34.53
7/8	29.17	29.90	30.63	31.35	32.08	32.81	33.54	34.27	35.00	35.73	36.46	37.19
15/16	31.25	32.03	32.81	33.59	34.38	35.16	35.94	36.72	37.50	38.28	39.06	39.84
1	33.33	34.17	35.00	35.83	36.67	37.50	38.33	39.17	40.00	40.83	41.67	42.50
1/16	35.42	36.30	37.19	38.07	38.96	39.84	40.73	41.61	42.50	43.39	44.27	45.16
1/8	37.50	38.44	39.38	40.31	41.25	42.19	43.13	44.06	45.00	45.94	46.88	47.81
3/16	39.58	40.57	41.56	42.55	43.54	44.53	45.52	46.51	47.50	48.49	49.48	50.47
1/4	41.67	42.71	43.75	44.79	45.83	46.88	47.92	48.96	50.00	51.04	52.08	53.13
5/16	43.75	44.84	45.94	47.03	48.13	49.22	50.31	51.41	52.50	53.59	54.69	55.78
3/8	45.83	46.98	48.13	49.27	50.42	51.56	52.71	53.85	55.00	56.15	57.29	58.44
7/16	47.92	49.11	50.31	51.51	52.71	53.91	55.10	56.30	57.50	58.70	59.90	61.09
1/2	50.00	51.25	52.50	53.75	55.00	56.25	57.50	58.75	60.00	61.25	62.50	63.75
9/16	52.08	53.39	54.69	55.99	57.29	58.59	59.90	61.20	62.50	63.80	65.10	66.41
5/8	54.17	55.52	56.88	58.23	59.58	60.94	62.29	63.65	65.00	66.35	67.71	69.06
11/16	56.25	57.66	59.06	60.47	61.88	63.28	64.69	66.09	67.50	68.91	70.31	71.72
3/4	58.33	59.79	61.25	62.71	64.17	65.63	67.08	68.54	70.00	71.46	72.92	74.38
13/16	60.42	61.93	63.44	64.95	66.46	67.97	69.48	70.99	72.50	74.01	75.52	77.03
7/8	62.50	64.06	65.63	67.19	68.75	70.31	71.88	73.44	75.00	76.56	78.13	79.69
15/16	64.58	66.20	67.81	69.43	71.04	72.66	74.27	75.89	77.50	79.11	80.73	82.34
2	66.67	68.33	70.00	71.67	73.33	75.00	76.67	78.33	80.00	81.67	83.33	85.00

For Steel add 2 per cent.

Weight of Square and Round Wrought-iron per Lineal Foot.

Thickness or diameter, in inches.	Weight of square bar, in pounds.	Weight of round bar, in pounds.	Thickness or diameter, in inches.	Weight of square bar, n pounds.	Weight of round bar, in pounds.	Thickness or diameter, in inches.	Weight of square bar, in pounds.	Weight of round bar, in pounds.
0			2	13.33	10.47	4	53.33	41.89
1/16	.013	.010	1/16	14.18	11.14	1/16	55.01	43.21
1/8	.052	.041	1/8	15.05	11.82	1/8	56.72	44.55
3/16	.117	.092	3/16	15.95	12.53	3/16	58.45	45.91
1/4	.208	.164	1/4	16.88	13.25	1/4	60.21	47.29
5/16	.326	.256	5/16	17.83	14.00	5/16	61.99	48.69
3/8	.469	.368	3/8	18.80	14.77	3/8	63.80	50.11
7/16	.638	.501	7/16	19.80	15.55	7/16	65.64	51.55
1/2	.833	.654	1/2	20.83	16.36	1/2	67.50	53.01
9/16	1.055	.828	9/16	21.89	17.19	9/16	69.39	54.50
5/8	1.302	1.023	5/8	22.97	18.04	5/8	71.30	56.00
11/16	1.576	1.237	11/16	24.08	18.91	11/16	73.24	57.52
3/4	1.875	1.473	3/4	25.21	19.80	3/4	75.21	59.07
13/16	2.201	1.728	13/16	26.37	20.71	13/16	77.20	60.63
7/8	2.552	2.004	7/8	27.55	21.64	7/8	79.22	62.22
15/16	2.930	2.301	15/16	28.76	22.59	15/16	81.26	63.82
1	3.333	2.618	3	30.00	23.56	5	83.33	65.45
1/16	3.763	2.955	1/16	31.26	24.55	1/16	85.43	67.10
1/8	4.219	3.313	1/8	32.55	25.57	1/8	87.55	68.76
3/16	4.701	3.692	3/16	33.87	26.60	3/16	89.70	70.45
1/4	5.208	4.091	1/4	35.21	27.65	1/4	91.88	72.16
5/16	5.742	4.510	5/16	36.58	28.73	5/16	94.08	73.89
3/8	6.302	4.950	3/8	37.97	29.82	3/8	96.30	75.64
7/16	6.888	5.410	7/16	39.39	30.94	7/16	98.55	77.40
1/2	7.500	5.890	1/2	40.83	32.07	1/2	100.8	79.19
9/16	8.138	6.392	9/16	42.30	33.23	9/16	103.1	81.00
5/8	8.802	6.913	5/8	43.80	34.40	5/8	105.5	82.83
11/16	9.492	7.455	11/16	45.33	35.60	11/16	107.8	84.69
3/4	10.21	8.018	3/4	46.88	36.82	3/4	110.2	86.56
13/16	10.95	8.601	13/16	48.45	38.05	13/16	112.6	88.45
7/8	11.72	9.204	7/8	50.05	39.31	7/8	115.1	90.36
15/16	12.51	9.828	15/16	51.68	40.59	15/16	117.5	92.29

For Steel add 2 per cent.

Weight of Square and Round Wrought-iron per Lineal Foot.

Thickness or diameter, in inches.	Weight of square bar, in pounds.	Weight of round bar, in pounds.	Thickness or diameter, in inches.	Weight of square bar, in pounds.	Weight of round bar, in pounds.	Thickness or diameter, in inches.	Weight of square bar, in pounds.	Weight of round bar, in pounds.
6	120.0	94.25	8	213.3	167.6	10	333.3	261.8
1/16	122.5	96.22	1/16	216.7	170.2	1/16	337.5	265.1
1/8	125.1	98.22	1/8	220.1	172.8	1/8	341.7	268.4
3/16	127.6	100.2	3/16	223.5	175.5	3/16	346.0	271.7
1/4	130.2	102.3	1/4	226.9	178.2	1/4	350.2	275.1
5/16	132.8	104.3	5/16	230.3	180.9	5/16	354.5	278.4
3/8	135.5	106.4	3/8	233.8	183.6	3/8	358.8	281.8
7/16	138.1	108.5	7/16	237.3	186.4	7/16	363.1	285.2
1/2	140.8	110.6	1/2	240.8	189.2	1/2	367.5	288.6
9/16	143.6	112.7	9/16	244.4	191.9	9/16	371.9	292.1
5/8	146.3	114.9	5/8	248.0	194.8	5/8	376.3	295.5
11/16	149.1	117.1	11/16	251.6	197.6	11/16	380.7	299.0
3/4	151.9	119.3	3/4	255.2	200.4	3/4	385.2	302.5
13/16	154.7	121.5	13/16	258.9	203.3	13/16	389.7	306.1
7/8	157.6	123.7	7/8	262.6	206.2	7/8	394.2	309.6
15/16	160.4	126.0	15/16	266.3	209.1	15/16	398.8	313.2
7	163.3	128.3	9	270.0	212.1	11	403.3	316.8
1/16	166.3	130.6	1/16	273.8	215.0	1/16	407.9	320.4
1/8	169.2	132.9	1/8	277.6	218.0	1/8	412.6	324.0
3/16	172.2	135.2	3/16	281.4	221.0	3/16	417.2	327.7
1/4	175.2	137.6	1/4	285.2	224.0	1/4	421.9	331.3
5/16	178.2	140.0	5/16	289.1	227.0	5/16	426.6	335.0
3/8	181.3	142.4	3/8	293.0	230.1	3/8	431.3	338.7
7/16	184.4	144.8	7/16	296.9	233.2	7/16	436.1	342.5
1/2	187.5	147.3	1/2	300.8	236.3	1/2	440.8	346.2
9/16	190.6	149.7	9/16	304.8	239.4	9/16	445.6	350.0
5/8	193.8	152.2	5/8	308.8	242.5	5/8	450.5	353.8
11/16	197.0	154.7	11/16	312.8	245.7	11/16	455.3	357.6
3/4	200.2	157.2	3/4	316.9	248.9	3/4	460.2	361.4
13/16	203.5	159.8	13/16	321.0	252.1	13/16	465.1	365.3
7/8	206.7	162.4	7/8	325.1	255.3	7/8	470.1	369.2
15/16	210.0	164.9	15/16	329.2	258.5	15/16	475.0	373.1

For Steel add 2 per cent.

Weight of Flat Rolled Iron, in Kilogrammes, per Lineal Metre.

Width.	Thickness.											
mm	2 mm	4 mm	6 mm	8 mm	10 mm	12 mm	15 mm	20 mm	25 mm	30 mm	35 mm	40 mm
5	0.078	0.155	0.233	0.310	0.388	0.465	0.581	0.775	0.969	1.163	1.356	1.550
10	0.155	0.310	0.465	0.620	0.775	0.930	1.163	1.550	1.938	2.325	2.713	3.100
15	0.233	0.465	0.698	0.930	1.163	1.395	1.744	2.325	2.906	3.488	4.069	4.650
20	0.310	0.620	0.930	1.240	1.550	1.860	2.325	3.100	3.875	4.650	5.425	6.200
25	0.388	0.775	1.163	1.550	1.938	2.325	2.906	3.875	4.844	5.813	6.781	7.750
30	0.465	0.930	1.395	1.860	2.325	2.790	3.488	4.650	5.813	6.975	8.138	9.300
35	0.543	1.085	1.628	2.170	2.713	3.255	4.069	5.425	6.781	8.138	9.494	10.850
40	0.620	1.240	1.860	2.480	3.100	3.720	4.650	6.200	7.750	9.300	10.850	12.400
45	0.698	1.395	2.093	2.790	3.488	4.185	5.231	6.975	8.719	10.463	12.206	13.950
50	0.775	1.550	2.325	3.100	3.875	4.650	5.813	7.750	9.688	11.625	13.563	15.500
55	0.853	1.705	2.558	3.410	4.263	5.115	6.394	8.525	10.656	12.788	14.919	17.050
60	0.930	1.860	2.790	3.720	4.650	5.580	6.975	9.300	11.625	13.950	16.275	18.600
65	1.008	2.015	3.023	4.030	5.038	6.045	7.556	10.075	12.594	15.113	17.631	20.150
70	1.085	2.170	3.255	4.340	5.425	6.510	8.138	10.850	13.563	16.275	18.988	21.700
75	1.163	2.325	3.488	4.650	5.813	6.975	8.719	11.625	14.531	17.438	20.344	23.250
80	1.240	2.480	3.720	4.960	6.200	7.440	9.300	12.400	15.500	18.600	21.700	24.800
85	1.318	2.635	3.953	5.270	6.588	7.905	9.862	13.175	16.469	19.763	23.056	26.350
90	1.395	2.790	4.185	5.580	6.975	8.370	10.463	13.950	17.438	20.925	24.413	27.900
95	1.473	2.945	4.418	5.890	7.363	8.835	11.044	14.725	18.406	22.088	25.769	29.450
100	1.550	3.100	4.650	6.200	7.750	9.300	11.625	15.500	19.375	23.250	27.125	31.000
110	1.705	3.410	5.115	6.820	8.525	10.230	12.789	17.050	21.314	25.575	29.838	34.100
120	1.860	3.720	5.580	7.440	9.300	11.160	13.950	18.600	23.250	27.900	32.550	37.200
130	2.015	4.030	6.045	8.060	10.075	12.090	15.113	20.150	25.188	30.225	35.263	40.300
140	2.170	4.340	6.510	8.680	10.850	13.020	16.275	21.700	27.125	32.550	37.975	43.400
150	2.325	4.650	6.975	9.300	11.625	13.950	17.438	23.250	29.062	34.875	40.688	46.500
160	2.480	4.960	7.440	9.920	12.400	14.880	18.600	24.800	31.000	37.200	43.400	49.600
170	2.635	5.270	7.905	10.540	13.175	15.810	19.763	26.350	32.938	39.525	46.113	52.700
180	2.790	5.580	8.370	11.160	13.950	16.740	20.925	27.900	34.875	41.850	48.825	55.800
190	2.945	5.890	8.835	11.780	14.725	17.670	22.088	29.450	36.813	44.175	51.538	58.900
200	3.100	6.200	9.300	12.400	15.500	18.600	23.250	31.000	38.750	46.500	54.250	62.000

For Steel add 2 per cent.

Weight of Square and Round Wrought-iron, in Kilogrammes, per Lineal Metre.

Thickness or diameter.	Square.	Round.	Thickness or diameter.	Square.	Round.
Mm.	Kg.	Kg.	Mm.	Kg.	Kg.
5	0.195	0.152	50	19.450	15.215
6	0.280	0.219	52	21.009	16.459
7	0.381	0.298	54	22.686	17.749
8	0.498	0.390	56	24.398	19.088
9	0.630	0.493	58	26.172	20.476
10	0.778	0.609	60	28.080	21.913
11	0.941	0.737	62	29.906	23 398
12	1.120	0.877	64	32.147	24.930
13	1.315	1.028	66	33.890	26.514
14	1.525	1.193	68	35.975	28.146
15	1.751	1.370	70	38.122	29.825
16	1.992	1.558	72	39.743	31.554
17	2.248	1.759	74	42.603	33.333
18	2.520	1.972	76	44.937	35.158
19	2.809	2.197	78	47.334	37.032
20	3.112	2.435	80	49.792	38.953
21	3.431	2.684	85	56.195	43.977
22	3.765	2.946	90	63.018	49.303
23	4.116	3.220	95	70.215	54.934
24	4.481	3.506	100	77.800	60.860
25	4.863	3.804	105	85.775	67.107
26	5.259	4.115	110	94.138	73.651
27	5.672	4.437	115	102.891	80.500
28	6.100	4.772	120	112.000	87.650
29	6.543	5.119	125	121.563	95.107
30	7.002	5.478	130	131.500	102.867
31	7.477	5.849	135	141.791	110.933
32	7.967	6.232	140	152.500	119.302
33	8.472	6.629	145	163.575	127.976
34	9.009	7.036	150	175.100	136.954
35	9.531	7.456	155	186.915	146.236
36	10.083	7.889	160	199.200	155.812
37	10.651	8.333	165	211.811	165.714
38	11.234	8.789	170	224.842	175.910
39	11.833	9.258	175	238.263	186.410
40	12.448	9.738	180	252.000	197.213
41	13.078	10.212	185	266.271	208.322
42	13.724	10.737	190	280.900	219.735
43	14.385	11.255	195	295.835	231.452
44	15.062	11.784	200	311.200	243.473
45	15.755	12.326	205	326.920	256.790
46	16.462	12.880	210	343.090	269.465
47	17.187	13.446	215	359.600	282.453
48	17.925	14.024	220	376.550	295.744
49	18.680	14.614	225	393.860	309.340

For Steel add 2 per cent.

Weight of Sheet-metal.

British Units.

Weight of Iron, Copper, Lead, and Zinc per Square Foot.

Thickness, in inches.	Cast-iron.	Wrought- or sheet-iron.	Sheet-copper.	Sheet-lead.	Sheet-zinc.
	Lb.	Lb.	Lb.	Lb.	Lb.
1/16	2.346	2.517	2.890	3.694	2.320
1/8	4.693	5.035	5.781	7.382	4.642
3/16	7.039	7.552	8.672	11.074	6.961
1/4	9.386	10.070	11.562	14.765	9.275
5/16	11.733	12.588	14.453	18.456	11.61
3/8	14.079	15.106	17.344	22.148	13.93
7/16	16.426	17.623	20.234	25.839	16.23
1/2	18.773	20.141	23.125	29.530	18.55
9/16	21.119	22.659	26.016	33.222	20.87
5/8	23.466	25.176	28.906	36.913	23.19
11/16	25.812	27.694	31.797	40.604	25.53
3/4	28.159	30.211	34.688	44.296	27.85
13/16	30.505	32.729	37.578	47.987	30.17
7/8	32.852	35.247	40.469	51.678	32.47
15/16	35.199	37.764	43.359	55.370	34.81
1	37.545	40.282	46.250	59.061	37.13
1/8	42.238	45.317	52.031	66.444	41.78
1/4	46.931	50.352	57.813	73.826	46.42
3/8	51.625	55.387	63.594	81.210	51.04
1/2	56.317	60.422	69.375	88.592	55.48
5/8	61.011	65.458	75.156	95.975	60.35
3/4	65.704	70.493	80.938	103.358	65.00
7/8	70.397	75.528	86.719	110.740	69.61
2	75.090	80.563	92.500	118.128	74.25

Weight of Sheet-metal.

Metric Units.

Weight, in Kilogrammes, per Square Metre.

Thickness.	Cast-iron.	Wrought-iron.	Steel.	Copper.
Mm.	Kg.	Kg.	Kg.	Kg.
0.5	3.625	3.89	3.925	4.45
1	7.25	7.78	7.85	8.90
2	14.50	15.56	15.70	17.80
3	21.75	23.34	23.55	26.70
4	29.00	31.12	31.40	35.60
5	36.25	38.90	39.25	44.50
6	43.50	46.68	47.10	53.40
7	50.75	54.46	54.95	62.30
8	58.00	62.24	62.80	71.20
9	65.25	70.02	70.65	80.10
10	72.50	77.80	78.50	89.00
11	79.75	85.58	86.35	97.90
12	87.00	93.36	94.20	106.80
13	94.25	101.14	102.05	115.70
14	101.50	108.92	109.90	124.60
15	108.75	116.70	117.75	133.50
16	116.00	124.48	125.60	142.40
17	123.25	132.26	133.45	151.30
18	130.50	140.04	141.30	160.20
19	137.75	147.82	149.15	169.10
20	145.00	155.60	157.00	178.00
21	152.25	163.38	164.85	186.90
22	159.50	171.17	172.70	195.80
23	166.75	178.94	180.55	204.70
24	174.00	186.72	188.40	213.60
25	181.25	194.50	196.25	222.50
26	188.50	202.28	204.10	231.40
27	195.75	210.06	211.95	240.30
28	203.00	217.84	219.80	249.20
29	210.25	225.62	227.65	258.10
30	217.50	233.40	235.50	267.00

Weight of Rolled Sheets of Wrought-iron and Steel.

British Units.

Specific Gravity of Iron, 7.70; of Steel, 7.85.

No. of gauge.	Birmingham Wire Gauge.			American (B. & S.) Wire Gauge.		
	Thickness, in inches.	Weight, in pounds, per square foot.		Thickness, in inches.	Weight, in pounds, per square foot.	
		Iron.	Steel.		Iron.	Steel.
0000	.454	18.16	18.52	.4600	18.40	18.76
000	.425	17.00	17.34	.4096	16.39	16.72
00	.380	15.20	15.50	.3648	14.59	14.88
0	.340	13.60	13.87	.3249	13.00	13.26
1	.300	12.00	12.24	.2893	11.57	11.80
2	.284	11.36	11.59	.2576	10.31	10.52
3	.259	10.35	10.56	.2294	9.18	9.36
4	.238	9.52	9.71	.2043	8.17	8.33
5	.220	8.80	8.98	.1819	7.27	7.42
6	.203	8.12	8.28	.1620	6.48	6.61
7	.180	7.19	7.34	.1443	5.77	5.88
8	.165	6.60	6.73	.1285	5.14	5.24
9	.148	5.92	6.04	.1144	4.57	4.66
10	.134	5.36	5.47	.1019	4.07	4.15
11	.120	4.80	4.89	.0907	3.63	3.70
12	.109	4.35	4.44	.0808	3.23	3.29
13	.095	3.80	3.87	.0720	2.88	2.93
14	.083	3.32	3.38	.0641	2.56	2.61
15	.072	2.88	2.94	.0571	2.28	2.32
16	.065	2.60	2.65	.0508	2.03	2.07
17	.058	2.32	2.37	.0453	1.81	1.84
18	.049	1.96	1.99	.0403	1.61	1.64
19	.042	1.68	1.71	.0359	1.43	1.46
20	.035	1.39	1.42	.0320	1.27	1.30
21	.032	1.27	1.30	.0285	1.13	1.16
22	.028	1.11	1.14	.0253	1.01	1.03
23	.025	.997	1.02	.0226	.903	.921
24	.022	.880	.898	.0201	.805	.821
25	.020	.800	.816	.0179	.715	.729
26	.018	.719	.734	.0159	.638	.651
27	.016	.640	.653	.0142	.570	.581
28	.014	.560	.571	.0126	.505	.515
29	.013	.520	.531	.0113	.450	.459
30	.012	.480	.489	.0100	.400	.409
31	.010	.399	.408	.0089	.357	.364
32	.009	.359	.367	.0080	.318	.324
33	.008	.320	.326	.0071	.283	.288
34	.007	.280	.286	.0063	.252	.257
35	.005	.200	.204	.0056	.224	.228
36	.004	.159	.162	.0050	.200	.204

Dimensions and Weights of Spheres.

British Units.

Sizes in inches, weights in pounds.

Diameter.	Surface.	Capacity.	Cast-iron.	Lead.	Water.
Inch.	Sq. inch.	Cub. inch.	Lb.	Lb.	Lb.
1.000	3.1416	.5236	.1365	.2147	.0188
1.125	3.9760	.7455	.1943	.3062	.0264
1.250	4.9087	1.0226	.2673	.4200	.0368
1.375	5.9395	1.3611	.3550	.5579	.0490
1.500	7.0686	1.7671	.4607	.7248	.0636
1.625	8.2957	2.2467	.5861	.9227	.0809
1.750	9.6211	2.8061	.7325	1.1528	.1050
1.875	11.044	3.4514	.9000	1.4156	.1242
2.000	12.566	4.1888	1.0920	1.7180	.1508
2.125	14.186	5.0243	1.3124	2.0631	.1809
2.250	15.904	5.9640	1.5592	2.4482	.2147
2.375	17.720	7.0143	1.8334	2.8811	.2525
2.500	19.635	8.1812	2.1328	3.3554	.2945
2.625	21.647	9.4708	2.4725	3.8892	.3410
2.750	23.758	10.889	2.8400	4.4623	.3920
2.875	25.967	12.442	3.2512	5.1056	.4479
3.000	28.274	14.137	3.6855	5.7982	.5089
3.125	30.680	15.979	4.1721	6.5568	.5752
3.250	33.183	17.974	4.6835	7.3623	.6471
3.375	35.785	20.129	5.2612	8.2521	.7246
3.500	38.484	22.449	5.8525	9.2073	.8081
3.625	41.282	24.941	6.5089	10.231	.8979
3.750	44.179	27.612	7.2135	11.323	.9941
3.875	47.173	30.466	7.9556	12.500	1.0968
4.00	50.265	33.510	8.7361	13.744	1.2064
4.25	56.745	40.194	10.510	16.482	1.4470
4.50	63.617	47.713	12.439	19.569	1.7177
4.75	70.882	56.115	14.666	23.035	2.0202
5.00	78.540	65.450	17.063	26.843	2.3562
5.25	86.590	75.766	19.810	31.089	2.7276
5.50	95.033	87.114	22.720	35.729	3.1361
5.75	103.87	99.541	26.000	40.856	3.5835
6.0	113.10	113.10	29.484	46.385	4.0716
6.5	132.73	143.79	37.453	58.976	5.1765
7.0	153.94	179.59	46.820	73.659	6.4653
7.5	176.71	220.89	57.587	90.598	7.9520
8.0	201.06	268.08	69.889	109.95	9.6509
8.5	226.98	321.55	83.839	131.38	11.576
9.0	254.47	381.70	99.51	156.55	13.741
9.5	283.53	448.92	117.03	184.12	16.161
10.0	314.16	523.60	136.50	214.75	18.850
11.0	380.13	696.91	181.76	285.83	26.289
12.0	452.39	904.78	235.87	371.09	32.572
13.0	530.92	1150.3	299.62	471.80	41.411
14.0	615.72	1436.7	374.56	589.27	51.721
15.0	706.84	1767.1	460.69	724.78	63.616
16.0	804.24	2144.6	559.11	879.61	77.206
17.0	853.96	2572.4	670.71	1055.0	92.607
18.0	1017.8	3053.6	796.08	1252.4	109.93
19.0	1134.1	3591.3	936.27	1472.9	129.29
20.0	1256.6	4188.8	1092.00	1718.0	150.80

Dimensions and Weights of Cast-iron Spheres.

Metric Units.

Sizes in centimetres, weights in kilogrammes.

For Lead, multiply by 1.575.

Diameter.	Volume.	Weight.	Diameter.	Volume.	Weight.
Cm.	Cub. cm.	Kg.	Cm.	Cub. cm.	Kg.
1.0	.524	.004	21.0	4849.05	35.16
1.5	1.767	.013	21.5	5203.72	37.73
2.0	4.189	.030	22.0	5575.28	40.42
2.5	8.181	.059	22.5	5964.12	43.24
3.0	14.137	.102	23.0	6370.63	46.19
3.5	22.449	.165	23.5	6795.20	49.27
4.0	33.510	.243	24.0	7238.23	52.48
4.5	47.713	.346	24.5	7700.11	55.83
5.0	65.45	.475	25.0	8181.23	59.31
5.5	87.11	.632	25.5	8681.98	62.94
6.0	113.10	.820	26.0	9202.77	66.72
6.5	143.79	1.043	26.5	9744.08	70.64
7.0	179.59	1.302	27.0	10305.99	74.72
7.5	220.89	1.601	27.5	10889.22	78.95
8.0	268.08	1.944	28.0	11494.04	83.33
8.5	321.56	2.331	28.5	12120.85	87.88
9.0	381.70	2.767	29.0	12770.08	92.58
9.5	448.92	3.255	29.5	13442.02	97.45
10.0	523.60	3.796	30.0	14137.17	102.49
10.5	606.13	4.394	31.0	15598.53	113.09
11.0	696.91	5.053	32.0	17157.28	124.39
11.5	796.33	5.773	33.0	18816.57	136.42
12.0	904.78	6.560	34.0	20579.53	149.20
12.5	1022.64	7.414	35.0	22449.30	162.76
13.0	1150.35	8.340	36.0	24429.02	177.11
13.5	1288.25	9.340	37.0	26521.95	192.28
14.0	1436.76	10.416	38.0	28730.91	208.30
14.5	1596.26	11.573	39.0	31059.35	225.18
15.0	1767.15	12.812	40.0	33510.32	242.95
15.5	1949.82	14.14	41.0	36086.96	261.63
16.0	2144.66	15.55	42.0	38792.39	281.24
16.5	2352.07	17.05	43.0	41629.77	301.82
17.0	2572.44	18.65	44.0	44602.24	323.37
17.5	2806.16	20.34	45.0	47712.94	345.91
18.0	3053.63	22.14	46.0	50965.01	369.50
18.5	3315.24	24.04	47.0	54361.60	394.12
19.0	3591.36	26.04	48.0	57905.58	419.82
19.5	3882.42	28.15	49.0	61600.87	446.61
20.0	4188.79	30.37	50.0	65449.85	474.51
20.5	4510.87	32.70	100.0	523598.80	3796.09

Weight of Cast-iron Pipe per Foot in Length.

British Units.

For Wrought-iron multiply by 1.067; for Lead, by 1.575; for Copper, by 1.23; for Brass, by 1.16.

Inner diam.	Thickness of pipe, in inches.												
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1¾	2
Ins.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1	3.07	....	....	....	....	....	....	....	....	....	....	....	....
¼	3.69	....	....	....	....	....	....	....	....	....	....	....	....
½	4.30	....	....	....	....	....	....	....	....	....	....	....	....
¾	4.92	....	....	....	....	....	....	....	....	....	....	....	....
2	5.53	8.76	....	....	....	....	....	....	....	....	....	....	....
¼	6.15	9.69	....	....	....	....	....	....	....	....	....	....	....
½	6.76	10.6	....	....	....	....	....	....	....	....	....	....	....
¾	7.37	11.5	....	....	....	....	....	....	....	....	....	....	....
3	7.98	12.5	17.2	22.3	....	....	....	....	....	....	....	....	....
¼	8.60	13.4	18.5	23.8	....	....	....	....	....	....	....	....	....
½	9.21	14.3	19.7	25.4	....	....	....	....	....	....	....	....	....
¾	9.83	15.2	20.9	26.9	....	....	....	....	....	....	....	....	....
4	10.3	16.1	22.2	28.5	35.1	42.0	....	....	....	....	....	....	....
¼	11.1	17.1	23.4	30.0	36.9	44.1	....	....	....	....	....	....	....
½	11.7	18.0	24.6	31.5	38.8	46.3	....	....	....	....	....	....	....
¾	12.3	18.9	25.8	33.1	40.6	48.5	....	....	....	....	....	....	....
5	12.9	19.8	27.1	34.6	42.5	50.6	59.1	67.8	....	....	....	....	....
¼	13.5	20.8	28.3	36.1	44.3	52.8	61.5	70.6	....	....	....	....	....
½	14.2	21.7	29.5	37.7	46.1	54.9	64.0	73.3	....	....	....	....	....
¾	14.8	22.6	30.8	39.2	48.0	57.1	66.4	76.1	....	....	....	....	....
6	15.4	23.5	32.0	40.8	49.8	59.2	68.9	78.9	89.2	99.8	....	....	....
½	16.6	25.4	34.5	43.8	53.5	63.5	73.8	84.4	95.3	107.0	....	....	....
7	17.8	27.2	36.9	46.9	57.2	67.8	78.7	89.4	102.0	113.0	126	151	177
½	19.1	29.1	39.4	50.0	60.9	72.1	83.7	95.5	108.0	120.0	133	159	187
8	20.3	30.9	41.8	53.1	64.6	76.4	88.6	101.0	114.0	127.0	140	168	197
½	21.5	32.8	44.3	56.1	68.3	80.7	93.5	107.0	120.0	134.0	148	177	207
9	22.8	34.6	46.8	59.2	72.0	85.1	98.4	112.0	126.0	140.0	155	185	217
½	24.0	36.4	49.2	62.3	75.7	89.3	103.0	118.0	132.0	147.0	163	194	226
10	25.1	38.3	51.7	65.3	79.4	93.6	108.0	123.0	138.0	154.0	170	202	235
½	26.4	40.1	54.1	68.4	83.0	97.9	113.2	129.0	145.0	161.0	177	211	245
11	27.6	42.0	56.6	71.5	86.7	102.0	118.0	134.0	151.0	168.0	185	220	255
½	28.8	43.8	59.1	74.6	90.4	107.0	123.0	140.0	157.0	174.0	192	228	265
12	30.0	45.7	61.5	77.7	94.1	111.0	128.0	145.0	163.0	181.0	199	237	275
13	....	....	66.4	83.8	102.0	120.0	138.0	156.0	175.0	195.0	214	254	294
14	....	....	71.4	89.4	109.0	128.0	148.0	168.0	188.0	208.0	229	271	314
15	....	....	76.3	96.1	116.0	137.0	158.0	179.0	200.0	222.0	244	289	334
16	....	....	81.2	102.0	124.0	145.0	167.0	190.0	212.0	235.0	258	306	353
17	....	....	86.1	108.0	131.0	154.0	177.0	201.0	225.0	249.0	273	323	373
18	....	....	91.0	115.0	139.0	163.0	187.0	212.0	237.0	262.0	288	340	393
19	....	....	96.0	121.0	146.0	171.0	197.0	223.0	249.0	276.0	303	357	412
20	....	....	101.0	127.0	153.0	180.0	207.0	234.0	261.0	289.0	317	375	432
21	....	....	....	133.0	161.0	188.0	217.0	245.0	274.0	303.0	332	392	452
22	....	....	....	139.0	168.0	196.0	227.0	256.0	286.0	316.0	347	409	471
23	....	....	....	145.0	175.0	206.0	236.0	267.0	298.0	330.0	362	426	491
24	....	....	....	152.0	183.0	214.0	246.0	278.0	311.0	343.0	375	444	511
25	....	....	....	....	190.0	223.0	256.0	289.0	323.0	357.0	391	461	531
26	....	....	....	....	198.0	231.0	266.0	300.0	335.0	370.0	406	478	550
27	....	....	....	....	205.0	240.0	276.0	311.0	348.0	384.0	421	495	570
28	....	....	....	....	212.0	249.0	286.0	323.0	360.0	397.0	436	512	590
30	....	....	....	....	227.0	266.0	305.0	345.0	384.0	424.0	465	547	629
32	....	....	....	....	242.0	283.0	325.0	367.0	409.0	451.0	495	581	668
34	....	....	....	....	257.0	300.0	345.0	389.0	434.0	479.0	524	616	708
36	....	....	....	....	271.0	318.0	364.0	411.0	458.0	506.0	554	650	746
42	....	....	....	....	315.0	370.0	423.0	478.0	532.0	588.0	644	753	864
48	....	....	....	....	359.0	422.0	482.0	544.0	605.0	669.0	733	856	982

The weight of a spigot and faucet joint may be taken as equal to 8 inches of straight pipe, and the weight of two flanges as equal to 12 inches of straight pipe.

Weight of Cast-iron Pipe.

Metric Units.

Weight, in Kilogrammes, per Metre of Length.

For Wrought-iron multiply by 1.067; for Lead, by 1.575; for Copper, by 1.23; for Brass, by 1.16.

Inside diameter.	Thickness, in millimetres.					
	10	15	20	25	30	40
Mm.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.
25	8.0	13.7	20.5	28.5	....	....
30	9.1	15.4	22.8	31.3	41.0	....
35	10.2	17.1	25.1	34.4	44.4	68.3
40	11.4	18.8	27.3	37.0	47.8	72.9
45	12.5	20.5	29.6	39.9	51.2	77.4
50	13.7	22.2	31.8	42.8	54.7	82.0
60	15.9	25.6	36.5	48.4	61.5	91.1
70	18.2	29.0	41.0	54.1	68.3	100.2
80	20.5	32.5	45.6	59.8	75.2	109.3
90	22.8	36.0	50.1	65.5	82.0	118.4
100	25.1	39.3	54.7	71.1	88.8	129.8
110	27.3	42.8	59.2	76.9	95.7	136.7
120	29.6	46.1	63.8	82.5	102.5	144.9
130	31.8	49.5	68.3	88.3	109.3	154.9
140	34.4	52.8	72.9	94.0	116.2	164.0
150	36.5	56.5	77.4	99.6	123.0	173.1
160	38.7	59.8	82.0	105.4	129.8	182.2
170	41.8	63.2	86.5	110.7	136.7	191.3
180	43.3	66.6	91.5	116.6	143.5	200.4
190	45.6	70.1	97.7	122.3	150.3	209.5
200	47.8	73.5	100.2	128.1	157.2	218.7
210	50.1	76.9	104.8	133.8	164.0	227.8
220	52.4	80.2	109.3	139.5	170.8	236.9
230	54.7	83.9	113.8	145.2	177.7	246.0
240	56.9	86.8	118.4	150.9	184.5	255.1
250	59.2	90.5	123.0	156.6	191.3	264.2
260	61.5	94.0	127.6	162.3	198.2	273.3
270	63.8	97.3	132.1	168.0	205.0	282.4
280	66.1	100.8	136.7	173.7	211.8	291.5
290	68.3	104.9	141.2	179.4	218.7	300.7
300	70.6	107.6	144.9	185.1	225.5	309.7
325	....	116.2	157.2	199.3	242.6	332.5
350	....	124.7	168.5	213.5	259.7	355.3
375	....	133.2	179.9	227.8	276.7	378.1
400	....	141.8	191.3	241.9	293.8	400.9
450	....	158.9	214.1	270.5	328.9	446.4
500	....	175.9	236.9	298.9	362.1	492.0

The weight of spigot and faucet joint = 0.2 metre.
The weight of two flanges = 0.3 metre.

Weight of Bridge Rivets per Hundred.

This table also applies to Button-headed Bolts.

Length of rivet under head.	Diameter of rivet, in inches.							
	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
Inch.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1 1/4	5.7	12.8	22.0	29.3	43.9	66.6	93.3	127.1
3/8	6.1	13.5	23.1	30.9	46.1	69.4	96.9	131.5
1/2	6.5	14.2	24.1	32.4	48.2	72.1	100.4	135.8
5/8	6.9	14.8	25.2	34.0	50.3	74.9	103.9	140.2
3/4	7.3	15.5	26.3	35.5	52.5	77.7	107.4	144.5
7/8	7.7	16.2	27.4	37.1	54.6	80.5	110.9	148.9
2	8.0	16.9	28.5	38.7	56.7	83.3	114.5	153.2
1/8	8.4	17.6	29.6	40.2	58.8	86.0	118.0	157.5
1/4	8.8	18.3	30.7	41.8	61.0	88.8	121.5	161.9
3/8	9.2	19.0	31.7	43.3	63.1	91.6	125.0	166.2
1/2	9.6	19.7	32.8	44.9	65.2	94.4	128.5	170.6
5/8	10.0	20.4	33.9	46.5	67.4	97.2	132.1	174.9
3/4	10.4	21.1	35.0	48.0	69.5	99.9	135.6	179.3
7/8	10.8	21.8	36.1	49.6	71.6	102.7	139.1	183.6
3	11.2	22.5	37.2	51.1	73.7	105.5	142.6	188.0
1/8	11.6	23.2	38.3	52.7	75.9	108.3	146.1	192.3
1/4	11.9	23.9	39.3	54.3	78.0	111.1	149.7	196.7
3/8	12.3	24.6	40.4	55.8	80.1	113.8	153.1	201.0
1/2	12.7	25.3	41.5	57.4	82.3	116.6	156.7	205.4
5/8	13.1	26.0	42.6	58.9	84.4	119.4	160.2	209.7
3/4	13.5	26.7	43.7	60.5	86.5	122.2	163.7	214.1
7/8	13.9	27.4	44.8	62.1	88.6	125.0	167.3	218.4
4	14.3	28.1	45.9	63.6	90.8	127.8	170.8	222.8
1/8	14.7	28.7	46.9	65.2	92.9	130.5	174.3	227.1
1/4	15.1	29.4	48.0	66.7	95.0	133.3	177.8	231.4
3/8	15.5	30.1	49.1	68.3	97.2	136.1	181.3	235.8
1/2	15.8	30.8	50.2	69.9	99.3	138.9	184.9	240.1
5/8	16.2	31.5	51.3	71.4	101.4	141.7	188.4	244.5
3/4	16.6	32.2	52.4	73.0	103.5	144.4	191.9	248.8
7/8	17.0	32.9	53.5	74.5	105.7	147.2	195.4	253.2
5	17.4	33.6	54.5	76.1	107.8	150.0	198.9	257.5
1/4	18.2	35.0	56.7	79.2	112.1	155.6	206.0	266.2
1/2	19.0	36.4	58.9	82.3	116.3	161.1	213.1	274.9
3/4	19.7	37.8	61.1	85.5	120.6	166.7	220.1	283.6
6	20.5	39.2	63.2	88.6	124.8	172.2	227.1	292.3
7	23.6	44.7	71.9	101.1	142.0	194.5	255.3	327.1
8	26.8	50.3	80.6	113.7	158.9	216.7	283.4	361.9
9	29.9	55.9	89.3	126.2	175.9	239.0	311.6	396.6
10	33.0	61.4	98.0	138.7	193.0	261.2	339.7	431.4
12	39.3	72.5	115.4	163.7	227.0	305.7	367.9	501.0
Weight of Two (2) Rivet Heads, in Pounds.								
Before driving	.037	.116	.222	.273	.453	.780	1.16	1.67
After driving	.032	.082	.147	.246	.369	.545	.746	1.02
Weight of Body per Inch of Length, in Pounds.								
Before driving	.031	.056	.087	.125	.170	.223	.282	.348

Weight of Bolts per Hundred.

Square Heads and Nuts.

Dimensions in inches.

Length.	Diameter.										
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1½	3.9	9.7	20.4	37.0	58.0	......	....	....	....	....	....
¾	4.2	10.5	21.3	37.9	60.5	......	....	....	....	....	....
2	4.6	11.3	22.4	39.9	63.2	97.7	145	....	....	....	....
¼	5.0	12.1	23.6	42.0	66.0	101.6	149	....	....	....	....
½	5.4	12.9	25.0	44.4	69.0	105.6	153	....	....	....	....
¾	5.8	13.7	26.4	46.2	72.1	109.7	158	....	....	....	....
3	6.2	14.5	27.8	48.3	75.2	113.8	163	200	289	350	480
½	6.9	16.1	30.6	52.5	81.4	122.0	174	213	305	370	500
4	7.6	17.7	33.4	56.7	87.6	130.2	185	226	322	390	520
½	8.3	19.2	36.2	60.9	93.8	138.4	196	240	339	410	545
5	9.0	20.7	39.0	65.1	100.0	146.6	207	255	356	430	570
½	9.7	22.2	41.8	69.2	106.1	154.9	218	270	373	450	595
6	10.4	23.7	44.6	73.4	112.2	163.2	229	285	390	470	620
½	11.1	25.2	47.4	77.6	118.3	171.5	240	300	407	490	645
7	11.8	26.7	50.2	81.8	124.4	179.8	251	315	434	510	670
½	12.5	28.2	53.1	86.0	130.5	187.1	262	330	451	530	695
8	13.2	29.7	56.0	90.0	136.6	195.4	273	345	468	550	725
9	......	33.1	61.5	98.0	148.8	212.0	295	375	505	590	775
10	......	36.5	67.0	106.3	161.0	229.0	317	405	540	630	825
11	......	40.0	72.5	114.6	173.2	246.0	339	435	575	670	875
12	......	43.5	78.0	122.9	184.4	263.0	361	465	610	710	925
13	......	......	83.5	131.2	196.6	280.0	383	495	645	751	975
14	......	......	89.0	139.5	208.8	297.0	405	525	680	793	1025
15	......	......	94.5	148.0	221.0	314.0	427	555	715	835	1075
16	......	......	100.0	156.5	233.2	331.0	449	585	750	877	1125
17	......	......	105.5	165.0	245.4	348.0	471	615	785	919	1175
18	......	......	111.0	173.5	257.6	365.0	493	645	820	961	1225

Weight of Bolts per Hundred.

Hexagon Heads and Nuts.

Dimensions in inches.

Length.	Diameter.										
	¼	⅜	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½
	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1½	3.4	8.5	17.7	32.5	49.0	......	....	....	....	....	....
¾	3.7	9.3	18.6	33.4	51.5	......	....	....	....	....	....
2	4.1	10.1	19.7	35.4	54.2	86.6	128	....	....	....	....
¼	4.5	10.9	20.9	37.5	57.0	90.6	132	....	....	....	....
½	4.9	11.7	22.3	39.9	60.0	94.6	136	....	....	....	....
¾	5.3	12.5	23.7	41.7	63.1	98.7	141	....	....	....	....
3	5.7	13.3	25.1	43.8	66.2	102.8	144	174	255	310	430
½	6.1	14.1	26.6	45.7	69.4	107.0	151	187	271	330	450
4	6.8	15.7	29.4	49.9	75.6	115.2	162	200	288	350	470
½	7.5	17.0	32.2	56.1	81.8	123.4	173	214	305	370	495
5	8.2	18.7	35.0	58.3	88.0	131.6	184	229	322	390	520
½	8.9	20.2	37.8	62.4	94.1	139.9	195	244	339	410	545
6	9.6	21.7	40.6	66.6	100.2	148.2	206	259	356	430	570
½	10.3	23.2	43.4	70.8	106.3	156.5	217	274	373	450	595
7	11.0	24.7	46.2	75.0	112.4	164.8	228	289	400	470	620
½	11.7	26.2	49.1	79.2	118.5	172.1	239	304	417	490	645
8	12.4	27.7	52.0	83.2	124.6	180.3	250	319	424	510	670
9	......	29.7	54.8	87.0	130.8	189.0	262	336	465	530	700
10	......	33.1	60.3	95.0	143.0	206.0	284	366	500	570	750
11	......	36.6	65.8	103.6	155.2	223.0	306	396	535	610	800
12	......	40.1	71.3	111.9	166.4	240.0	328	426	570	650	850
13	......	......	76.8	120.2	178.6	257.0	350	456	605	691	900
14	......	......	82.3	128.5	190.8	274.0	372	486	640	733	950
15	......	......	87.8	137.0	203.0	291.0	384	516	675	775	1000
16	......	......	93.3	145.5	215.2	308.0	416	546	710	817	1050
17	......	......	98.8	154.0	227.4	325.0	438	576	745	859	1100
18	......	......	104.3	162.5	239.6	342.0	460	606	780	901	1150

United States Standard Screw Threads.

Diameter.	Threads per inch.	Diameter of root of thread.	Width of flat.	Area of bolt body.	Area at root of thread.	Short diameter, rough.	Short diameter, finish.	Long diameter, rough.	Long diameter, rough.	Thickness, rough.	Thickness, finish.
In.		Inch.	Inch.	Sq. in.	Sq. in.	Inch.	Inch.	Inch.	Inch.	Inch.	In.
1/4	20	.185	.0062	.049	.027	1/2	7/16	37/64	7/10	1/4	3/16
5/16	18	.240	.0074	.077	.045	19/32	17/32	11/16	10/12	5/16	1/4
3/8	16	.294	.0078	.110	.068	11/16	5/8	51/64	63/64	3/8	5/16
7/16	14	.344	.0089	.150	.093	25/32	23/32	9/10	1 7/64	7/16	3/8
1/2	13	.400	.0096	.196	.126	7/8	13/16	1	1 15/64	1/2	7/16
9/16	12	.454	.0104	.249	.162	31/32	29/32	1 1/8	1 23/64	9/16	1/2
5/8	11	.507	.0113	.307	.202	1 1/16	1	1 7/32	1 1/2	5/8	9/16
3/4	10	.620	.0125	.442	.302	1 1/4	1 3/16	1 7/16	1 49/64	3/4	11/16
7/8	9	.731	.0138	.601	.420	1 7/16	1 3/8	1 21/32	2 1/32	7/8	13/16
1	8	.837	.0156	.785	.550	1 5/8	1 9/16	1 7/8	2 19/64	1	15/16
1/8	7	.940	.0178	.994	.694	1 13/16	1 3/4	2 3/32	2 9/16	1 1/8	1 1/16
1/4	7	1.065	.0178	1.227	.893	2	1 15/16	2 5/16	2 53/64	1 1/4	1 3/16
3/8	6	1.160	.0208	1.485	1.057	2 3/16	2 1/8	2 17/32	3 3/32	1 3/8	1 5/16
1/2	6	1.284	.0208	1.767	1.295	2 3/8	2 5/16	2 3/4	3 23/64	1 1/2	1 7/16
5/8	5 1/2	1.389	.0227	2.074	1.515	2 9/16	2 1/2	2 31/32	3 5/8	1 5/8	1 9/16
3/4	5	1.491	.0250	2.405	1.746	2 3/4	2 11/16	3 3/16	3 57/64	1 3/4	1 11/16
7/8	5	1.616	.0250	2.761	2.051	2 15/16	2 7/8	3 13/32	4 5/32	1 7/8	1 13/16
2	4 1/2	1.712	.0277	3.142	2.302	3 1/8	3 1/16	3 5/8	4 27/64	2	1 15/16
1/4	4 1/2	1.962	.0277	3.976	3.023	3 1/2	3 7/16	4 1/16	4 61/64	2 1/4	2 3/16
1/2	4	2.176	.0312	4.909	3.719	3 7/8	3 13/16	4 1/2	5 31/64	2 1/2	2 7/16
3/4	4	2.426	.0312	5.940	4.620	4 1/4	4 3/16	4 29/32	6	2 3/4	2 11/16
3	3 1/2	2.629	.0357	7.069	5.428	4 5/8	4 9/16	5 3/8	6 17/32	3	2 15/16
1/4	3 1/2	2.879	.0357	8.296	6.510	5	4 15/16	5 13/16	7 1/16	3 1/4	3 3/16
1/2	3 1/4	3.100	.0384	9.621	7.548	5 3/8	5 5/16	6 7/64	7 39/64	3 1/2	3 7/16
3/4	3	3.317	.0413	11.045	8.641	5 3/4	5 11/16	6 21/32	8 1/8	3 3/4	3 11/16
4	3	3.567	.0413	12.566	9.963	6 1/8	6 1/16	7 3/32	8 41/64	4	3 15/16
1/4	2 7/8	3.798	.0435	14.186	11.329	6 1/2	6 7/16	7 9/16	9 3/16	4 1/4	4 3/16
1/2	2 3/4	4.028	.0454	15.904	12.753	6 7/8	6 13/16	7 31/32	9 3/4	4 1/2	4 7/16
3/4	2 5/8	4.256	.0476	17.721	14.226	7 1/4	7 3/16	8 13/32	10 1/4	4 3/4	4 11/16
5	2 1/2	4.480	.0500	19.635	15.763	7 5/8	7 9/16	8 27/32	10 49/64	5	4 15/16
1/4	2 1/2	4.730	.0500	21.648	17.572	8	7 15/16	9 9/32	11 23/64	5 1/4	5 3/16
1/2	2 3/8	4.953	.0526	23.758	19.267	8 3/8	8 5/16	9 23/32	11 7/8	5 1/2	5 7/16
3/4	2 3/8	5.203	.0526	25.967	21.262	8 3/4	8 11/16	10 5/32	12 3/8	5 3/4	5 11/16
6	2 1/4	5.423	.0555	28.274	23.098	9 1/8	9 1/16	10 19/32	12 15/16	6	5 15/16

Whitworth Screw Bolts and Nuts.

Size of bolt and thickness of nut.	Number of threads per inch.	Diameter at bottom of thread.	Area at bottom of thread.	Thickness of bolt head.	Nut across plate.	Nut across corners.
Inch.		Inch.	Inch.	Inch.	Inch.	Inch.
1/8	40	.093	.0067	.109	.338	.390
3/16	24	.134	.0141	.164	.448	.517
1/4	20	.186	.0271	.219	.525	.606
5/16	18	.241	.0458	.273	.601	.694
3/8	16	.295	.0683	.328	.709	.819
7/16	14	.346	.0940	.383	.820	.947
1/2	12	.393	.1215	.437	.919	1.06
9/16	12	.456	.1633	.492	1.011	1.16
5/8	11	.508	.2032	.547	1.101	1.27
11/16	11	.571	.2560	.601	1.201	1.38
3/4	10	.622	.3038	.656	1.301	1.50
13/16	10	.684	.3674	.711	1.390	1.60
7/8	9	.733	.422	.766	1.479	1.70
15/16	9	.795	.496	.820	1.574	1.82
1	8	.840	.554	.875	1.670	1.95
1/8	7	.942	.697	.984	1.860	2.15
1/4	7	1.067	.894	1.094	2.048	2.36
3/8	6	1.161	1.059	1.203	2.215	2.55
1/2	6	1.286	1.30	1.312	2.413	2.78
5/8	5	1.369	1.47	1.422	2.576	2.97
3/4	5	1.494	1.75	1.531	2.758	3.18
7/8	4½	1 590	1.99	1.641	3.018	3.48
2	4½	1.715	2.31	1.750	3.149	3.63
1/8	4½	1.840	2.66	1.859	3.337	3.85
1/4	4	1.930	2.92	1.969	3.546	4.09
3/8	4	2.055	3.31	2.078	3.750	4.33
1/2	4	2.180	3.73	2.187	3.894	4.49
5/8	4	2.305	4.17	2.297	4.049	4.67
3/4	3½	2.384	4.46	2.406	4.181	4.82
7/8	3½	2.509	4.92	2.516	4.346	5.02
3	3½	2.634	5.45	2.625	4.530	5.23

Whitworth threads are inclined at an angle of 55 degrees, and have one-sixth of the total depth of thread rounded off at the top and also at the bottom.

The Whitworth system is the standard for Great Britain, and is also used extensively on the Continent pending the adoption of a satisfactory international metric screw thread system. In many of the leading machine shops of France, Switzerland, Belgium, and Germany the bolts are made in English units with Whitworth threads, all other parts of the machines being in metric units.

Wrought-iron Steam Pipe.

United States Standard.

Inner diameter. Nominal.	Inner diameter. Actual.	Thickness.	Weight per foot.	Threads per inch of screw.	Inner diameter. Nominal.	Inner diameter. Actual.	Thickness.	Weight per foot.	Threads per inch of screw.
Inch.	Inch.	Inch.	Lb.		Inch.	Inch.	Inch.	Lb.	
1/8	.270	.068	.24	27	4 1/2	4.508	.246	12.49	8
1/4	.364	.088	.42	18	5	5.045	.259	14.50	8
3/8	.494	.091	.56	18	6	6.065	.280	18.76	8
1/2	.623	.109	.84	14	7	7.023	.301	23.27	8
3/4	.824	.113	1.12	14	8	7.982	.322	28.18	8
1	1.048	.134	1.67	11 1/2	9	9.001	.344	33.70	8
1/4	1.380	.140	2.24	11 1/2	10	10.019	.366	40.00	8
1/2	1.611	.145	2.68	11 1/2	11	11.00	.375	45.00	8
2	2.067	.154	3.61	11 1/2	12	12.00	.375	49.00	8
1/2	2.468	.204	5.74	8	13	13.25	.375	54.00	8
3	3.067	.217	7.54	8	14	14.25	.375	58.00	8
1/2	3.548	.226	9.00	8	15	15.25	.375	62.00	8
4	4.026	.237	10.66	8					

Whitworth or British Standard.

Pipes having an internal diameter of 1/4 inch or 3/8 inch have 19 threads to the inch. Those of 1/2 inch, 5/8 inch, 3/4 inch, and 7/8 inch have 14 threads to the inch, and all other sizes of pipes have 11 threads to the inch.

Nominal size.	Diameter. Internal.	Diameter. External.	Nominal size.	Diameter. Internal.	Diameter. External.	Nominal size.	Diameter. Internal.	Diameter. External.
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1/8	.27	.40	2 1/2	2.47	2.87	9	8.94	9.62
1/4	.36	.54	3	3.07	3.50	10	10.02	10.75
3/8	.49	.67	1/2	3.55	4.00	11	11.00	11.75
1/2	.62	.84	4	4.03	4.50	12	12.00	12.75
3/4	.82	1.05	1/2	4.51	5.00	13	13.25	14.00
1	1.05	1.31	5	5.04	5.56	14	14.25	15.00
1/4	1.38	1.66	6	6.06	6.62	15	15.43	16.00
1/2	1.61	1.90	7	7.02	7.62	16	16.40	17.00
2	2.07	2.37	8	7.98	8.62	17	17.32	18.00

Standard Cast-iron Pipe.

Metric System.

Inside diameter.	Thickness.	Outside diameter.	Weight per metre.	Inside diameter.	Thickness.	Outside diameter.	Weight per metre.
Mm.	Mm.	Mm.	Kilo.	Mm.	Mm.	Mm.	Kilo.
40	8.0	56	8.75	375	14.0	403	124.04
50	8.0	66	10.57	400	14.5	429	136.89
60	8.5	77	13.26	425	14.5	454	145.15
70	8.5	87	15.20	450	15.0	480	158.87
80	9.0	98	18.24	475	15.5	506	173.17
90	9.0	108	20.29	500	16.0	532	188.04
100	9.0	118	22.34	550	16.5	583	212.90
125	9.5	144	29.10	600	17.0	634	238.90
150	10.0	170	36.44	650	18.0	686	273.86
175	10.5	196	44.36	700	19.0	738	311.15
200	11.0	222	52.86	750	20.0	790	350.76
225	11.5	248	61.95	800	21.0	842	392.69
250	12.0	274	71.61	900	22.5	945	472.76
275	12.5	300	81.85	1000	24.0	1048	559.76
300	13.0	326	92.68	1100	26.0	1152	666.81
325	13.5	352	104.08	1200	28.0	1256	783.15
350	14.0	378	116.07				

Cast-iron Pipe.

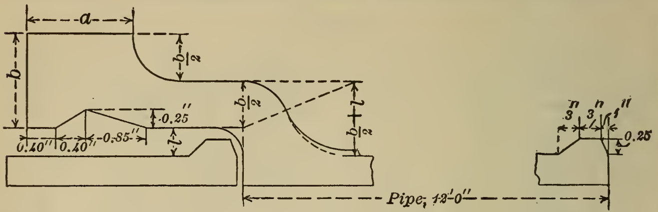
The following tables of dimensions and weights of standard cast-iron pipe for water mains are those adopted by the New England Water Works Association at its meeting in September, 1902. Ten classes of pipe are given, designated by the letters of the alphabet, the difference between the various classes being in the matter of thickness,—the inside diameter remaining constant for any size pipe, and the variation in thickness affecting the outside diameter. Table No. 1, herewith, gives the dimensions of the various sizes, and Table No. 2 gives the weights for standard 12-foot lengths of the various diameters and thicknesses.

The various classes are required to stand hydrostatic tests, as follows:

	Pounds per square inch for diameters of 20 inches and larger.	Less than 20 inches.
Class A	150	300
Class B	200	300
Class C	250	300
Class D	300	300
Class E	350	350
Class F	350	350

TABLE No. 1.

General Dimensions of Pipes and Special Castings.



Nominal diameter.	Classes.	Actual outside diameter.	Diam. of sockets.		Depth of sockets.		"a"	"b"
			Pipe.	Special castings.	Pipe.	Special castings.		
Inch.		Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch
4	A, C, E	4.80	5.60	5.70	3.0	4.0	1.50	1.3
4	G, I, K	5.00	5.80	5.70	3.0	4.0	1.50	1.3
6	A, C, E	6.90	7.77	7.80	3.0	4.0	1.50	1.4
6	G, I	7.10	7.90	7.80	3.0	4.0	1.50	1.4
8	A, C, E	9.05	9.85	10.00	3.5	4.0	1.50	1.5
8	G, I	9.30	10.10	10.00	3.5	4.0	1.50	1.5
10	A, B, C, D	11.10	11.90	12.10	3.5	4.5	1.50	1.5
10	E, F, G, H	11.40	12.20	12.10	3.5	4.5	1.50	1.5
12	A, B, C, D	13.20	14.00	14.20	3.5	4.5	1.50	1.6
12	E, F, G, H	13.50	14.30	14.20	3.5	4.5	1.50	1.6
14	A, B, C, D	15.30	16.10	16.35	3.5	4.5	1.50	1.7
14	E, F, G, H	15.65	16.45	16.35	3.5	4.5	1.50	1.7
16	A, B, C, D	17.40	18.40	18.60	4.0	5.0	1.75	1.8
16	E, F, G, H	17.80	18.80	18.60	4.0	5.0	1.75	1.8
18	A, B	19.25	20.25	20.40	4.0	5.0	1.75	1.9
18	C, D	19.50	20.50	20.40	4.0	5.0	1.75	1.9
18	E, F	19.70	20.70	20.70	4.0	5.0	1.75	1.9
20	A, B	21.30	22.30	22.50	4.0	5.0	1.75	2.0
20	C, D	21.60	22.60	22.50	4.0	5.0	1.75	2.0
20	E, F	21.90	22.90	23.00	4.0	5.0	1.75	2.0
24	A, B	25.40	26.40	26.60	4.0	5.0	2.00	2.1
24	C, D	25.80	26.80	26.60	4.0	5.0	2.00	2.1
24	E, F	26.10	27.10	27.10	4.0	5.0	2.00	2.1
30	A, B	31.60	32.60	32.60	4.5	5.0	2.00	2.3
30	C, D	32.00	33.00	33.00	4.5	5.0	2.00	2.3
30	E, F	32.40	33.40	33.40	4.5	5.0	2.00	2.3
36	A, B	37.80	38.80	38.80	4.5	5.0	2.00	2.5
36	C, D	38.30	39.30	39.30	4.5	5.0	2.00	2.5
36	E, F	38.70	39.70	39.70	4.5	5.0	2.00	2.5
42	A, B	44.00	45.00	45.00	5.0	5.0	2.00	2.8
42	C, D	44.50	45.50	45.50	5.0	5.0	2.00	2.8
42	E, F	45.10	46.10	46.10	5.0	5.0	2.00	2.8
48	A, B	50.20	51.20	51.20	5.0	5.0	2.00	3.0
48	C, D	50.80	51.80	51.80	5.0	5.0	2.00	3.0
48	E, F	51.40	52.40	52.40	5.0	5.0	2.00	3.0
54	A, B	56.40	57.40	57.40	5.5	5.5	2.25	3.2
54	C, D	57.10	58.10	58.10	5.5	5.5	2.25	3.2
54	E, F	57.80	58.80	58.80	5.5	5.5	2.25	3.8
60	A, B	62.60	63.60	63.60	5.5	5.5	2.25	3.4
60	C, D	63.40	64.40	64.40	5.5	5.5	2.25	3.4
60	E, F	64.20	65.20	65.20	5.5	5.5	2.25	4.0

TABLE NO. 2.

Standard Thicknesses and Weights of Cast-iron Pipes

12 Feet in Length, exclusive of Socket.

Nominal diameter of pipe.	Class A.		Class B.		Class C.		Class D.		Class E.		Class F.		Class G.		Class H.		Class I.		Class K.	
	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.	Thickness of shell.	Weight per length.
Inch.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.	Inch.	Lb.
4	.34	200	....	.....	.36	215	....	......	.39	230	....	......	.42	245	....	......	.45	260	.48	275
6	.38	330	....	......	.42	355	....	......	.46	385	....	......	.50	415	....	......	.54	440	....	......
8	.42	475	....	......	.48	525	....	......	.53	575	....	......	.58	630	....	......	.63	685	....	......
10	.47	650	.50	685	.53	725	.56	765	.60	805	.63	845	.67	885	.70	925	....	......	....	......
12	.49	810	.53	865	.57	920	.61	980	.65	1035	.69	1090	.73	1150	.77	1210	....	......	....	......
14	.53	1010	.57	1085	.61	1155	.66	1230	.70	1300	.75	1380	.79	1450	.83	1530	....	......	....	......
16	.55	1215	.60	1310	.65	1410	.70	1500	.75	1600	.80	1700	.85	1800	.90	1900	....	......	....	......
18	.57	1410	.63	1540	.69	1660	.75	1790	.80	1910	.86	2040	....	......	....	......	....	......	....	......
20	.60	1610	.66	1760	.72	1920	.79	2080	.85	2250	.92	2410	....	......	....	......	....	......	....	......
24	.64	2050	.72	2290	.80	2530	.88	2770	.95	3000	1.03	3240	....	......	....	......	....	......	....	......
30	.71	2860	.81	3230	.91	3600	1.01	3950	1.10	4320	1.20	4680	....	......	....	......	....	......	....	......
36	.79	3800	.90	4270	1.02	4830	1.13	5300	1.25	5900	1.37	6360	....	......	....	......	....	......	....	......
42	.87	4900	1.00	5560	1.13	6270	1.27	6970	1.40	7700	1.53	8350	....	......	....	......	....	......	....	......
48	.95	6130	1.10	6970	1.25	7900	1.40	8780	1.55	9740	1.70	10600	....	......	....	......	....	......	....	......
54	1.03	7510	1.20	8600	1.37	9800	1.54	10900	1.72	12350	1.90	13500	....	......	....	......	....	......	....	......
60	1.10	8900	1.30	10300	1.50	11900	1.70	13300	1.90	15100	2.10	16500	....	......	....	......	....	......	....	......

Sizes and Weights of Cast-iron Pipe Connections.

Crosses.

Inch.	Lb.
2	40
3	104
3 × 2	90
4	150
4 × 3	114
4 × 2	110
6	200
6 × 4	150
6 × 3	150
8	325
8 × 6	265
8 × 4	265
8 × 3	225
10	510
10 × 8	415
10 × 6	388
10 × 4	338
10 × 3	350
12	700
12 × 10	650
12 × 8	615
12 × 6	540
12 × 4	525
12 × 3	495
14 × 10	750
14 × 8	635
14 × 6	570
16	1025
16 × 14	1070
16 × 12	1025
16 × 10	1010
16 × 8	825
16 × 6	700
16 × 4	650
20	1790
20 × 12	1370
20 × 10	1225
20 × 8	1000
20 × 6	1000
20 × 4	1000
24	2190
24 × 20	2020
24 × 6	1340
30 × 20	2635
30 × 12	2250
30 × 8	1995

Tees.

Inch.	Lb.
2	28
3	76
3 × 2	76
4	100
4 × 3	90
4 × 2	87
6	150
6 × 4	130
6 × 3	125
6 × 2	120
8	266
8 × 6	252
8 × 4	222
8 × 3	220
10	390
10 × 8	330
10 × 6	312
10 × 4	292
10 × 3	290
12	565
12 × 10	510
12 × 8	492
12 × 6	484
12 × 4	460
14 × 12	650
14 × 10	650
14 × 8	575
14 × 6	545
14 × 4	525
14 × 3	490
16	790
16 × 14	850
16 × 12	825
16 × 10	890
16 × 8	755
16 × 6	630
16 × 4	655
20	1375
20 × 16	1115
20 × 12	1025
20 × 10	1090
20 × 8	900
20 × 6	875
20 × 4	845
21 × 10	1465
24	1875
24 × 12	1425
24 × 8	1375
24 × 6	1375
30	3025
30 × 24	2640
30 × 20	2200
30 × 12	2035
30 × 10	2050
30 × 6	1825
36	5140
36 × 30	4200
36 × 12	4050

45° Branch Pipes.

Inch.	Lb.
3	90
6 × 6 × 4	145
8	300
8 × 6	290
24	2765
24 × 24 × 20	2145
30	4170
36	10300

Sleeves.

Inch.	Lb.
2	10
3	20
4	44
6	65
8	86
10	140
12	176
14	208
16	340
20	500
24	710
30	965
36	1500

90° Elbows.

Inch.	Lb.
2	14
3	34
4	48
6	110
8	145
10	225
12	370
14	450
16	525
20	900
24	1400

Plugs.

Inch.	Lb.
2	2
3	5
4	8
6	12
8	26
10	46
12	66
14	70
16	100
20	150
24	185
30	370

1/8 or 45° Bends.

Inch.	Lb.
3	30
4	65
6	85
8	160
10	190
12	290
16	510
20	740
24	1425
30	2000

1/16 or 22½° Bends.

Inch.	Lb.
6	150
8	155
10	165
12	260
16	500
24	1280
30	1735

Reducers.

Inch.	Lb.
3 × 2	35
4 × 3	42
4 × 2	40
6 × 4	95
6 × 3	80
8 × 6	126
8 × 4	116
8 × 3	116
10 × 8	212
10 × 6	150
10 × 4	128
12 × 10	278
12 × 8	254
12 × 6	250
12 × 4	250
14 × 12	475
14 × 10	430
14 × 8	340
14 × 6	285
16 × 12	475
16 × 10	435
20 × 16	690
20 × 14	575
20 × 12	540
20 × 8	300
24 × 20	745
30 × 24	1305
30 × 18	1385
36 × 30	1730

Caps.

Inch.	Lb.
3	15
4	25
6	60
8	75
10	100
12	120

Drip-boxes.

Inch.	Lb.
4	235
8	355
10	760
20	1420

Lap-welded American Charcoal Iron Boiler Tubes.

Tables of Standard Sizes.

Morris, Tasker & Co.

External diameter.	Internal diameter.	Thickness.	External circumference.	Internal circumference.	Length of pipe per square foot, inside surface.	Length of pipe per square foot, outside surface.	Internal area.	External area.	Weight per foot.
Inch.	Inch.	Inch.	Inch.	Inch.	Feet.	Feet.	Inch.	Inch.	Lb.
1	.856	.072	3.142	2.689	4.460	3.819	.575	.785	.708
¼	1.106	.072	3.927	3.474	3.455	3.056	.960	1.227	.900
½	1.334	.083	4.712	4.191	2.863	2.547	1.396	1.767	1.250
¾	1.560	.095	5.498	4.901	2.448	2.183	1.911	2.405	1.665
2	1.804	.098	6.283	5.667	2.118	1.909	2.556	3.142	1.981
¼	2.054	.098	7.069	6.484	1.850	1.698	3.314	3.976	2.238
½	2.283	.109	7.854	7.172	1.673	1.528	4.094	4.909	2.755
¾	2.533	.109	8.639	7.957	1.508	1.390	5.039	5.940	3.045
3	2.783	.109	9.425	8.743	1.373	1.273	6.083	7.069	3.333
¼	3.012	.119	10.210	9.462	1.268	1.175	7.125	8.296	3.958
½	3.262	.119	10.995	10.248	1.171	1.091	8.357	9.621	4.272
¾	3.512	.119	11.781	11.033	1.088	1.018	9.687	11.045	4.590
4	3.741	.130	12.566	11.753	1.023	.955	10.992	12.566	5.320
½	4.241	.130	14.137	13.323	.901	.849	14.126	15.904	6.010
5	4.720	.140	15.708	14.818	.809	.764	17.497	19.635	7.226
6	5.699	.151	18.849	17.904	.670	.637	25.509	28.274	9.346
7	6.657	.172	21.991	20.914	.574	.545	34.805	38.484	12.435
8	7.636	.182	25.132	23.989	.500	.478	45.795	50.265	15.109
9	8.615	.193	28.274	27.055	.444	.424	58.291	63.617	18.002
10	9.573	.214	31.416	30.074	.399	.382	71.975	78.540	22.190

Wrought-iron Welded Tubes.

Extra strong.

Nominal diameter.	Actual outside diameter.	Thickness. Extra strong.	Thickness. Double extra strong.	Actual inside diameter. Extra strong.	Actual inside diameter. Double extra strong.
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
⅛	.405	.100	....	.205	....
¼	.540	.123	....	.294	....
⅜	.675	.127	....	.421	....
½	.840	.149	.298	.542	.244
¾	1.050	.157	.314	.736	.422
1	1.315	.182	.364	.951	.587
¼	1.660	.194	.388	1.272	.884
½	1.900	.203	.406	1.494	1.088
2	2.375	.221	.442	1.933	1.491
½	2.875	.280	.560	2.315	1.755
3	3.5	.304	.608	2.892	2.284
½	4.0	.321	.642	3.358	2.716
4	4.5	.341	.682	3.818	3.136

Different Standards for Wire Gauge in Use in the United States.

Dimensions in decimal parts of an inch.

Number of wire gauge.	American, or Brown & Sharpe.	Birmingham, or Stubs'.	Washburn & Moen Mfg. Co., Worcester, Mass.	Trenton Iron Co., Trenton, N. J.	United States Standard.	Old English, from Brass Mfrs. List.
000000	....	....	.46	....	.46875	....
00000	....	....	.43	.45	.43750	....
0000	.460 000	.454	.393	.40	.40625	....
000	.409 640	.425	.362	.36	.37500	....
00	.364 800	.380	.331	.33	.34375	....
0	.324 950	.340	.307	.305	.31250	....
1	.289 300	.300	.283	.285	.28125	....
2	.257 630	.284	.263	.265	.26563	....
3	.229 420	.259	.244	.245	.25000	....
4	.204 310	.238	.225	.225	.23438	....
5	.181 940	.220	.207	.205	.21875	....
6	.162 020	.203	.192	.190	.20313	....
7	.144 280	.180	.177	.175	.18750	....
8	.128 490	.165	.162	.160	.17188	....
9	.114 430	.148	.148	.145	.15625	....
10	.101 890	.134	.135	.130	.14063	....
11	.090 742	.120	.120	.1175	.12500	....
12	.080 808	.109	.105	.1050	.10938	....
13	.071 961	.095	.092	.0925	.09375	....
14	.064 084	.083	.080	.0800	.07813	.083
15	.057 068	.072	.072	.0700	.07031	.072
16	.050 820	.065	.063	.0610	.06250	.065
17	.045 257	.058	.054	.0525	.05625	.058
18	.040 303	.049	.047	.0450	.05000	.049
19	.035 390	.042	.041	.0390	.04375	.040
20	.031 961	.035	.035	.0340	.03750	.035
21	.028 462	.032	.032	.0300	.03438	.0315
22	.025 347	.028	.028	.0270	.03125	.0295
23	.022 571	.025	.025	.0240	.02813	.0270
24	.020 100	.022	.023	.0215	.02500	.0250
25	.017 900	.020	.020	.0190	.02188	.0230
26	.015 940	.018	.018	.0180	.01875	.0205
27	.014 195	.016	.017	.0170	.01719	.01875
28	.012 641	.014	.016	.0160	.01563	.01650
29	.011 257	.013	.015	.0150	.01406	.01550
30	.010 025	.012	.014	.0140	.01250	.01375
31	.008 928	.010	.0135	.0130	.01094	.01225
32	.007 950	.009	.0130	.0120	.01016	.01125
33	.007 080	.008	.0110	.0110	.00938	.01025
34	.006 304	.007	.0100	.0100	.00859	.00950
35	.005 614	.005	.0095	.0090	.00781	.00900

Wire.—Iron, Steel, Copper, Brass.

Weight, in Pounds, of 100 Feet.

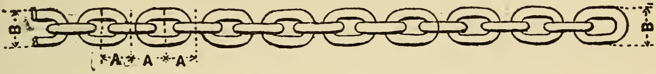
Birmingham Wire Gauge.

Number of gauge.	Per 100 lineal feet.			
	Iron.	Steel.	Copper.	Brass.
	Lb.	Lb.	Lb.	Lb.
0000	54.62	55.13	62.39	58.93
000	47.86	48.32	54.67	51.64
00	38.27	38.63	43.71	41.28
0	30.63	30.92	34.99	33.05
1	23.85	24.07	27.24	25.73
2	21.37	21.57	24.41	23.06
3	17.78	17.94	20.30	19.18
4	15.01	15.15	17.15	16.19
5	12.82	12.95	14.65	13.84
6	10.92	11.02	12.47	11.78
7	8.586	8.667	9.807	9.263
8	7.214	7.283	8.241	7.783
9	5.805	5.859	6.630	6.262
10	4.758	4.803	5.435	5.133
11	3.816	3.852	4.359	4.117
12	3.148	3.178	3.596	3.397
13	2.392	2.414	2.732	2.580
14	1.826	1.843	2.085	1.969
15	1.374	1.387	1.569	1.482
16	1.119	1.130	1.279	1.208
17	.8915	.900	1.018	.9618
18	.6363	.6423	.7268	.6864
19	.4675	.4720	.5340	.5043
20	.3246	.3277	.3709	.3502
21	.2714	.2740	.3100	.2929
22	.2079	.2098	.2373	.2241
23	.1656	.1672	.1892	.1788
24	.1283	.1295	.1465	.1384
25	.1060	.1070	.1211	.1144
26	.0859	.0867	.0981	.0926
27	.0678	.0685	.0775	.0732
28	.0519	.0524	.0593	.0560
29	.0448	.0452	.0511	.0483
30	.0382	.0385	.0436	.0412
31	.0265	.0267	.0303	.0286
32	.0215	.0217	.0245	.0231
33	.0170	.0171	.0194	.0183
34	.0130	.0131	.0148	.0140
35	.0066	.0067	.0076	.0071
36	.0042	.0043	.0048	.0046

United States Standard Gauge for Sheet- and Plate-iron and Steel, 1893.

Number of gauge.	Approximate thickness, in fractions of an inch.	Approximate thickness, in decimal parts of an inch.	Approximate thickness, in millimetres.	Weight per square foot, in ounces avoirdupois.	Weight per square foot, in pounds avoirdupois.	Weight per square foot, in kilogrammes.	Weight per square metre, in kilogrammes.	Weight per square metre, in pounds avoirdupois.
0000000	$\frac{1}{2}$	.500 000	12.700 000	320	20.000	9.072	97.65	215.28
000000	$\frac{15}{32}$	.468 750	11.906 250	300	18.750	8.505	91.55	201.82
00000	$\frac{7}{16}$	.437 500	11.112 500	280	17.500	7.938	85.44	188.37
0000	$\frac{13}{32}$	.406 250	10.318 750	260	16.250	7.371	79.33	174.91
000	$\frac{3}{8}$	.375 000	9.525 000	240	15.000	6.804	73.24	161.46
00	$\frac{11}{32}$	.343 750	8.731 250	220	13.750	6.237	67.13	148.00
0	$\frac{5}{16}$	.312 500	7.937 500	200	12.500	5.670	61.03	134.55
1	$\frac{9}{32}$	.281 250	7.143 750	180	11.250	5.103	54.93	121.09
2	$\frac{17}{64}$	.265 625	6.746 875	170	10.625	4.819	51.88	114.37
3	$\frac{1}{4}$	.250 000	6.350 000	160	10.000	4.536	48.82	107.64
4	$\frac{15}{64}$	.234 375	5.953 125	150	9.375	4.252	45.77	100.91
5	$\frac{7}{32}$	.218 750	5.556 250	140	8.750	3.969	42.72	94.18
6	$\frac{13}{64}$	.203 125	5.159 375	130	8.125	3.685	39.67	87.45
7	$\frac{3}{16}$	.187 500	4.762 500	120	7.500	3.402	36.62	80.72
8	$\frac{11}{64}$	.171 875	4.365 625	110	6.875	3.118	33.57	74.00
9	$\frac{5}{32}$	.156 250	3.968 750	100	6.250	2.835	30.52	67.27
10	$\frac{9}{64}$	.140 625	3.571 875	90	5.625	2.552	27.46	60.55
11	$\frac{1}{8}$	.125 000	3.175 000	80	5.000	2.268	24.41	53.82
12	$\frac{7}{64}$	.109 375	2.778 125	70	4.375	1.984	21.36	47.09
13	$\frac{3}{32}$	.093 750	2.381 250	60	3.750	1.701	18.31	40.36
14	$\frac{5}{64}$	.078 125	1.984 375	50	3.125	1.417	15.26	33.64
15	$\frac{9}{128}$	.070 312 500	1.785 937 500	45	2.812 500	1.276	13.73	30.27
16	$\frac{1}{16}$	.062 500 000	1.587 500 000	40	2.500 000	1.134	12.21	26.91
17	$\frac{9}{160}$	.056 250 000	1.428 750 000	36	2.250 000	1.021	10.99	24.22
18	$\frac{1}{20}$	.050 000 000	1.270 000 000	32	2.000 000	.9072	9.765	21.53
19	$\frac{7}{160}$	.043 750 000	1.111 250 000	28	1.750 000	.7938	8.544	18.84
20	$\frac{3}{80}$	.037 500 000	.952 500 000	24	1.500 000	.6804	7.324	16.15
21	$\frac{11}{320}$	.034 375 000	.873 125 000	22	1.375 000	.6237	6.713	14.80
22	$\frac{1}{32}$	.031 250 000	.793 750 000	20	1.250 000	.5670	6.103	13.46
23	$\frac{9}{320}$	.028 125 000	.714 375 000	18	1.125 000	.5103	5.493	12.11
24	$\frac{1}{40}$	.025 000 000	.635 000 000	16	1.000 000	.4536	4.882	10.76
25	$\frac{7}{320}$	.021 875 000	.555 625 000	14	.875 000	.3969	4.272	9.42
26	$\frac{3}{160}$	.018 750 000	.476 250 000	12	.750 000	.3402	3.662	8.07
27	$\frac{11}{640}$	.017 187 500	.436 562 500	11	.687 500	.3119	3.357	7.40
28	$\frac{1}{64}$	.015 625 000	.396 875 000	10	.625 000	.2835	3.052	6.73
29	$\frac{9}{640}$	.014 062 500	.357 187 500	9	.562 500	.2551	2.746	6.05
30	$\frac{1}{80}$	.012 500 000	.317 500 000	8	.500 000	.2268	2.441	5.38
31	$\frac{7}{640}$	.010 937 500	.277 812 500	7	.437 500	.1984	2.136	4.71
32	$\frac{13}{1280}$	.010 156 250	.257 968 750	6½	.406 250	.1843	1.983	4.37
33	$\frac{3}{320}$	.009 375 000	.238 125 000	6	.375 000	.1701	1.831	4.04
34	$\frac{11}{1280}$	.008 593 750	.218 281 250	5½	.343 750	.1559	1.678	3.70
35	$\frac{5}{640}$	.007 812 500	.198 437 500	5	.312 500	.1417	1.526	3.36
36	$\frac{9}{1280}$	.007 031 250	.178 593 750	4½	.281 250	.1276	1.373	3.03
37	$\frac{17}{2560}$	.006 640 625	.168 671 875	4¼	.265 625	.1205	1.297	2.87
38	$\frac{1}{160}$	.006 250 000	.158 750 000	4	.250 000	.1134	1.221	2.69

Crane Chains.



"D. B. G." Special Crane.							Crane.		
Size of chain.	Pitch A. Approximately.	Weight per foot. Approximately.	Outside width. B.	Proof test.	Average breakage strain.	Ordinary safe load. General use.	Proof test.	Average breakage strain.	Ordinary safe load. General use.
Inch.	Inch.	Lb.	Inch.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
$\frac{1}{4}$	$\frac{25}{32}$	$\frac{7}{8}$	$\frac{7}{8}$	1 932	3 864	1 288	1 680	3 360	1 120
$\frac{5}{16}$	$\frac{27}{32}$	1	$1\frac{1}{16}$	2 898	5 796	1 932	2 520	5 040	1 680
$\frac{3}{8}$	$\frac{31}{32}$	$1\frac{7}{10}$	$1\frac{1}{4}$	4 186	8 372	2 790	3 640	7 280	2 427
$\frac{7}{16}$	$1\frac{5}{32}$	2	$1\frac{3}{8}$	5 796	11 592	3 864	5 040	10 080	3 360
$\frac{1}{2}$	$1\frac{11}{32}$	$2\frac{1}{2}$	$1\frac{11}{16}$	7 728	15 456	5 182	6 720	13 440	4 480
$\frac{9}{16}$	$1\frac{15}{32}$	$3\frac{2}{10}$	$1\frac{7}{8}$	9 660	19 320	6 440	8 400	16 800	5 600
$\frac{5}{8}$	$1\frac{23}{32}$	$4\frac{1}{8}$	$2\frac{1}{16}$	11 914	22 828	7 942	10 360	20 720	6 907
$\frac{11}{16}$	$1\frac{27}{32}$	5	$2\frac{1}{4}$	14 490	28 980	9 660	12 600	25 200	8 400
$\frac{3}{4}$	$1\frac{31}{32}$	$5\frac{7}{8}$	$2\frac{1}{2}$	17 388	34 776	11 592	15 120	30 240	10 080
$\frac{13}{16}$	$2\frac{3}{32}$	$6\frac{7}{10}$	$2\frac{11}{16}$	20 286	40 572	13 524	17 640	35 280	11 760
$\frac{7}{8}$	$2\frac{7}{32}$	8	$2\frac{7}{8}$	22 484	44 968	14 989	20 440	40 880	13 627
$\frac{15}{16}$	$2\frac{15}{32}$	9	$3\frac{1}{16}$	25 872	51 744	17 248	23 520	47 040	15 680
1	$2\frac{19}{32}$	$10\frac{7}{10}$	$3\frac{1}{4}$	29 568	59 136	19 712	26 880	53 760	17 920
$\frac{1}{16}$	$2\frac{23}{32}$	$11\frac{2}{10}$	$3\frac{5}{16}$	33 264	66 538	22 176	30 240	60 480	20 160
$\frac{1}{8}$	$2\frac{27}{32}$	$12\frac{1}{2}$	$3\frac{3}{4}$	37 576	75 152	25 050	34 160	68 320	22 773
$\frac{3}{16}$	$3\frac{5}{32}$	$13\frac{7}{10}$	$3\frac{7}{8}$	41 888	83 776	27 925	38 080	76 160	25 387
$\frac{1}{4}$	$3\frac{7}{32}$	16	$4\frac{1}{8}$	46 200	92 400	30 800	42 000	84 000	28 000
$\frac{5}{16}$	$3\frac{15}{32}$	$16\frac{1}{2}$	$4\frac{3}{8}$	50 512	101 024	33 674	45 920	91 840	30 613
$\frac{3}{8}$	$3\frac{5}{8}$	$18\frac{4}{10}$	$4\frac{9}{16}$	55 748	111 496	37 165	50 680	101 360	33 787
$\frac{7}{16}$	$3\frac{25}{32}$	$19\frac{7}{10}$	$4\frac{3}{4}$	60 368	120 736	40 245	54 880	109 760	36 587
$\frac{1}{2}$	$3\frac{31}{32}$	$21\frac{7}{10}$	5	66 528	133 056	44 352	60 480	120 960	40 320

The distance from centre of one link to centre of next is equal to the inside length of link, but in practice $\frac{1}{32}$ inch is allowed for weld. This is approximate, and, where exactness is required, chain should be made so.

For **Chain Sheaves.**—The diameter, if possible, should be not less than twenty times the diameter of chain used.

Example. For 1-inch chain use 20-inch sheaves.

Window Glass.

Number of Lights per Box of 50 Feet.

Inch.	No.	Inch.	No.	Inch.	No.	Inch.	No.
6 × 8	150	12 × 18	33	16 × 44	10	26 × 32	9
7 × 9	115	12 × 20	30	18 × 20	20	26 × 34	8
8 × 10	90	12 × 22	27	18 × 22	18	26 × 36	8
8 × 11	82	12 × 24	25	18 × 24	17	26 × 40	7
8 × 12	75	12 × 26	23	18 × 26	15	26 × 42	7
8 × 13	70	12 × 28	21	18 × 28	14	26 × 44	6
8 × 14	64	12 × 30	20	18 × 30	13	26 × 48	6
8 × 15	60	12 × 32	18	18 × 32	13	26 × 50	6
8 × 16	55	12 × 34	17	18 × 34	12	26 × 54	5
9 × 11	72	13 × 14	40	18 × 36	11	26 × 58	5
9 × 12	67	13 × 16	35	18 × 38	11	28 × 30	9
9 × 13	62	13 × 18	31	18 × 40	10	28 × 32	8
9 × 14	57	13 × 20	28	18 × 44	9	28 × 34	8
9 × 15	53	13 × 22	25	20 × 22	16	28 × 36	7
9 × 16	50	13 × 24	23	20 × 24	15	28 × 38	7
9 × 17	47	13 × 26	21	20 × 26	14	28 × 40	6
9 × 18	44	13 × 28	19	20 × 28	13	28 × 44	6
9 × 20	40	13 × 30	18	20 × 30	12	28 × 46	6
10 × 12	60	14 × 16	32	20 × 32	11	28 × 50	5
10 × 13	55	14 × 18	29	20 × 34	11	28 × 52	5
10 × 14	52	14 × 20	26	20 × 36	10	28 × 56	4
10 × 15	48	14 × 22	23	20 × 38	9	30 × 36	7
10 × 16	45	14 × 24	22	20 × 40	9	30 × 40	6
10 × 17	42	14 × 26	20	20 × 44	8	30 × 42	6
10 × 18	40	14 × 28	18	20 × 46	8	30 × 44	5
10 × 20	36	14 × 30	17	20 × 48	8	30 × 46	5
10 × 22	33	14 × 32	16	20 × 50	7	30 × 48	5
10 × 24	30	14 × 34	15	20 × 60	6	30 × 50	5
10 × 26	28	14 × 36	14	22 × 24	14	30 × 54	4
10 × 28	26	14 × 40	13	22 × 26	13	30 × 56	4
10 × 30	24	14 × 44	11	22 × 28	12	30 × 60	4
10 × 32	22	15 × 18	27	22 × 30	11	32 × 42	5
10 × 34	21	15 × 20	24	22 × 32	10	32 × 44	5
11 × 13	50	15 × 22	22	22 × 34	10	32 × 46	5
11 × 14	47	15 × 24	20	22 × 36	9	32 × 48	5
11 × 15	44	15 × 26	18	22 × 38	9	32 × 50	4
11 × 16	41	15 × 28	17	22 × 40	8	32 × 54	4
11 × 17	39	15 × 30	16	22 × 44	8	32 × 56	4
11 × 18	36	15 × 32	15	22 × 46	7	32 × 60	4
11 × 20	33	16 × 18	25	22 × 50	7	34 × 40	5
11 × 22	30	16 × 20	23	24 × 28	11	34 × 44	5
11 × 24	27	16 × 22	20	24 × 30	10	34 × 46	5
11 × 26	25	16 × 24	19	24 × 32	9	34 × 50	4
11 × 28	23	16 × 26	17	24 × 36	8	34 × 52	4
11 × 30	21	16 × 28	16	24 × 40	8	34 × 56	4
11 × 32	20	16 × 30	15	24 × 44	7	36 × 44	5
11 × 34	19	16 × 32	14	24 × 46	7	36 × 50	4
12 × 14	43	16 × 34	13	24 × 48	6	36 × 56	4
12 × 15	40	16 × 36	12	24 × 50	6	36 × 60	3
12 × 16	38	16 × 38	12	24 × 54	5	36 × 64	3
12 × 17	35	16 × 40	11	24 × 56	5	40 × 60	3

Roofing Slate.

A square of slating is 100 square feet of finished roofing. Slating is usually so laid that the third slate laps the first slate by three inches. To compute the number of slates of a given size required to cover a square of roof, subtract 3 inches from the length of the slate, multiply the remainder by the width of the slate, and divide by 2; the result is the number of square inches of roof covered per slate. Divide 14,400 (the number of square inches in a square) by the number thus found, and the result will be the number of slates required for a square.

Slate.

Dimensions and Number per Square.

Dimensions, in inches.	Number per square.	Dimensions, in inches.	Number per square.	Dimensions, in inches.	Number per square.
6 × 12	533	9 × 16	246	16 × 20	137
7 × 12	457	10 × 16	221	12 × 22	126
8 × 12	400	9 × 18	213	14 × 22	108
9 × 12	355	10 × 18	192	12 × 24	114
7 × 14	374	12 × 18	160	14 × 24	98
8 × 14	327	10 × 20	169	16 × 24	86
9 × 14	291	11 × 20	154	14 × 26	89
10 × 14	261	12 × 20	141	16 × 26	78
8 × 16	277	14 × 20	121		

Thickness, $\frac{1}{8}$ inch, $\frac{3}{16}$ inch, $\frac{1}{4}$ inch, increasing by eighths to 1 inch.

The weight of slate is about 174 pounds per cubic foot, or, per square foot of various thicknesses, as follows:

Thickness, in inches..	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
Weight, in pounds.....	1.81	2.71	3.62	5.43	7.25	9.06	10.88	12.69	14.50

Tin Plates (Tinned Sheet-steel).

Brand	....	IC	IX	IXX	....	....	....	......
Thickness, B. W. gauge	....	29	27	26	....	....	....	......
Number of sheets per box	....	225	225	225	....	....	....	......
	Inch.	Lb.	Lb.	Lb.				
Net weight per box	10 × 14	108	135	160	....	....	....	......
	12 × 12	110	138	165	....	....	....	......
	13 × 13	132	162	192	....	....	....	......
	14 × 14	155	193	230	....	....	....	......
	15 × 15	178	218	260	....	....	....	......
	16 × 16	200	248	290	....	....	....	......
	17 × 17	230	289	340	....	....	....	......
	10 × 20	160	195	222	....	....	....	......
	11 × 22	190	235	275	....	....	....	......

Brand	....	IC	IX	IXX	IXXX	IXXXX	IX	IXX
Thickness, B. W. gauge	....	29	27	26	25	24½	27	26
Number of sheets per box	....	112	112	112	112	112	56	56
	Inch.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
Net weight per box	14 × 20	108	135	160	180	200	....	......
	20 × 28	216	270	320	....	....	180	......
	18 × 18	138	158	178	....	....	....	......
	20 × 20	160	195	222	....	....	....	......
	22 × 22	190	235	275	....	....	....	......
	24 × 24	220	276	330	....	....	....	......
	12 × 24	110	138	165	....	....	....	......
	13 × 26	132	162	192	....	....	....	......
	14 × 22	120	148	174	....	....	....	......
	14 × 24	130	161	190	....	....	....	......
	14 × 28	155	193	230	....	....	....	......
	14 × 56	....	....	......	....	....	185	220
	14 × 31	178	210	240	....	....	....	......
	14 × 60	....	....	......	....	....	200	240
	15 × 21	120	152	176	....	....	....	......
	16 × 19	120	147	170	....	....	....	......
	16 × 20	127	154	180	....	....	....	......
	16 × 22	138	170	200	....	....	....	......

Brand	....	DC	DX	DXX	DXXX	DXXXX
Thickness, B. W. gauge	....	28	25	24	23	22
	Inch.	Lb.	Lb.	Lb.	Lb.	Lb.
Net weight per box of 100 sheets	12½ × 17	94	122	143	164	185
	15 × 21	130	180	213	244	275
Net weight per box of 50 sheets	17 × 25	94	122	143	164	185

Weight and Thickness of Lead Pipe.

Caliber.	Mark.	Weight per foot.		Thickness.	Mean bursting pressure.	Safe working pressure.	Caliber.	Mark.	Weight per foot.		Thickness.	Mean bursting pressure.	Safe working pressure.
In.		Lb.	Oz.	In.	Lb.	Lb.	In.		Lb.	Oz.	In.	Lb.	Lb.
⅜	AAA	1	12	.180	1968	492	1	A	4	0	.210	857	214
⅜	AA	1	5	.150	1627	406	1	B	3	4	.170	745	186
⅜	A	1	2	.130	1381	347	1	C	2	8	.140	562	140
⅜	B	1	0	.125	1342	335	1	D	2	4	.125	518	129
⅜	C	0	14	.110	1187	296	1	E	2	0	.100	475	118
⅜	......	0	10	.087	1085	271	1	......	1	8	.090	325	81
7/16	......	0	9½	.080	775	193	1¼	AAA	6	12	.275	962	240
½	AAA	3	0	.250	1787	446	1¼	AA	5	12	.250	823	205
½	......	2	8	.225	1655	413	1¼	A	4	11	.210	685	171
½	AA	2	0	.180	1393	343	1¼	B	3	11	.170	546	136
½	A	1	10	.160	1285	321	1¼	C	3	0	.135	420	105
½	B	1	3	.125	980	245	1¼	D	2	8	.125	350	87
½	C	1	0	.100	782	195	1¼	......	2	0	.095	322	80
½	D	0	9	.065	468	117	1½	AAA	8	0	.290	742	185
½	......	0	10	.070	556	139	1½	AA	7	0	.250	700	175
½	......	0	12	.090	625	156	1½	A	6	4	.220	628	157
⅝	AAA	3	8	.230	1548	387	1½	B	5	0	.180	506	126
⅝	AA	2	12	.210	1380	345	1½	C	4	4	.150	430	107
⅝	A	2	8	.180	1152	288	1½	D	3	8	.140	315	78
⅝	B	2	0	.160	987	246	1½	......	3	0	.120	245	61
⅝	C	1	7	.117	795	198	1¾	B	5	0	....	....	116
⅝	D	1	4	.100	708	177	1¾	C	4	0	....	....	93
¾	AAA	4	14	.290	1462	365	1¾	D	3	10	.125	318	79
¾	AA	3	8	.225	1225	306	2	AAA	10	11	.300	611	152
¾	A	3	0	.190	1072	268	2	AA	8	14	.250	511	127
¾	B	2	3	.150	865	216	2	A	7	0	.210	405	101
¾	C	1	12	.125	782	195	2	B	6	0	.190	360	90
¾	D	1	3	.090	505	126	2	C	5	0	.160	260	65
1	AAA	6	0	.300	1230	307	2	D	4	0	.090	200	50
1	AA	4	8	.230	910	227							

Corrugated Iron.

The following table is calculated for sheets 30½ inches wide before corrugating.

Number by Birmingham gauge.	Thickness.	Weight per square foot, flat.	Weight per square foot, corrugated.	Weight per square of 100 square feet, when laid, allowing 6 inches lap in length and 2½ inches, or one corrugation, in width of sheet, for sheet lengths of						Weight per square foot, flat, galvanized.
				5′	6′	7′	8′	9′	10′	
	Inch.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
16	.065	2.61	3.28	365	358	353	350	348	346	2.95
18	.049	1.97	2.48	275	270	267	264	262	261	2.31
20	.035	1.40	1.76	196	192	190	188	186	185	1.74
22	.028	1.12	1.41	156	154	152	150	149	148	1.46
24	.022	.88	1.11	123	121	119	118	117	117	1.22
26	.018	.72	.91	101	99	97	97	96	95	1.06

Skylight and Floor Glass.

Weight per Cubic Foot, 156 Pounds.

Weight per Square Foot.

Thickness, in inches. ..	⅛	$\frac{3}{16}$	¼	⅜	½	⅝	¾	1
Weight, in pounds......	1.62	2.43	3.25	4.88	6.50	8.13	9.75	13

Flagging.

Weight per Cubic Foot, 168 Pounds.

Weight per Square Foot.

Thickness, in inches ...	1	2	3	4	5	6	7	8
Weight, in pounds......	14	28	42	56	70	84	98	112

Number and Weight of Shingles (Pine) per Square of 100 Feet.

Number of inches exposed to weather.	Number of shingles per square.	Weight per square, in pounds.
4	900	216
4½	800	192
5	720	173
5½	655	157
6	600	144

Shipping Weights of Corrugated Iron.

United States Standard Gauge.

Black. No.	Lb. per square foot.	*Galvanized.* No.	Lb. per square foot.
16	2.75	16	2.91
18	2.20	18	2.36
20	1.65	20	1.82
22	1.38	22	1.54
24	1.11	24	1.27
26	0.84	26	0.99

Add to net surface 23 to 26 per cent. for roofing with 6-inch end laps.
Add to net surface 20 to 22 per cent. for siding with 4-inch end laps.
All side laps = 1 corrugation = 2½ inches.
Example. 1600 square feet roof = 2000 square feet sheeting = 2000 × 1.65 pounds = 3300 pounds black corrugated iron.

Weight of Roofing.

Table for Computing Loads upon Roofs.

Pounds per Square of 100 Square Feet.

Yellow pine, Northern, sheathing, 1 inch thick	300
Yellow pine, Southern, sheathing, 1 inch thick	400
Spruce, sheathing, 1 inch thick	200
Chestnut or maple, sheathing, 1 inch thick	400
Ash or oak, sheathing, 1 inch thick	500
Shingles, pine	200
Slates, ¼ inch thick	900
Sheet-iron, $\frac{1}{16}$ inch thick	300
Sheet-iron, $\frac{1}{16}$ inch thick, and laths	500
Iron, corrugated	100 to 375
Iron, galvanized, flat	100 to 350
Tin	70 to 125
Felt and asphalt	100
Felt and gravel	800 to 1000
Skylights, glass, $\frac{3}{16}$ inch to ½ inch thick	250 to 700
Sheet-lead	500 to 800
Copper	80 to 125
Zinc	100 to 200
Tiles, flat	1500 to 2000
Tiles, flat, with mortar	2000 to 3000
Tiles, pan	1000

Timber Measurement.

Two methods are in use for the measurement of timber: the method of board measure, and the use of the cubic foot.

Board Measure, abbreviated **B.M.,** employs as a unit one square foot of surface by one inch in thickness. For boards one inch thick the board measure, therefore, is equal to the number of square feet. For any thickness the board measure is obtained by multiplying the width in inches by the thickness in inches and by the length in feet, and dividing by 12. The table on page 322 gives the board measure for various widths and thicknesses for a length of one foot, and hence the tabular figures must be multiplied by the length of the board in feet.

Table of Board Measure.

Width, in inches.	Thickness, in inches. 1	1½	2	2½	3	3½	4	4½	5	5½	6	7	8	9	10	11	12	14	16
1	.0833	.1250	.1667	.2083	.2500	.2917	.3333	.3750	.4167	.4583	.50	.5833	.6667	.750	.8333	.9167	1.00	1.167	1.333
1½	.1250	.1875	.2500	.3125	.3750	.4063	.5000	.5625	.6250	.6875	.75	.875	1.000	1.125	1.250	1.375	1.50	1.750	2.000
2	.1667	.2500	.3333	.4688	.5625	.5833	.6667	.7500	.8333	.9167	1.00	1.167	1.333	1.500	1.667	1.833	2.00	2.333	2.667
2½	.2083	.3125	.4167	.5208	.625	.7292	.8333	.9375	1.042	1.146	1.25	1.458	1.667	1.875	2.083	2.292	2.50	2.917	3.333
3	.2500	.3750	.5000	.6250	.750	.875	1.000	1.125	1.250	1.375	1.50	1.750	2.000	2.250	2.500	2.750	3.00	3.500	4.000
3½	.2917	.4375	.5833	.7292	.875	1.021	1.167	1.313	1.458	1.604	1.75	2.042	2.333	2.625	2.917	3.208	3.50	4.083	4.667
4	.3333	.5000	.6667	.8333	1.000	1.167	1.333	1.500	1.667	1.833	2.00	2.333	2.667	3.000	3.333	3.667	4.00	4.667	5.333
4½	.3750	.5625	.7500	.9375	1.125	1.313	1.500	1.688	1.875	2.063	2.25	2.625	3.000	3.375	3.750	4.125	4.50	5.250	6.000
5	.4167	.6250	.8333	1.042	1.250	1.457	1.666	1.875	2.083	2.292	2.50	2.917	3.333	3.750	4.167	4.583	5.00	5.833	6.667
5½	.4583	.6875	.9167	1.146	1.375	1.603	1.833	2.063	2.292	2.521	2.75	3.208	3.667	4.125	4.583	5.042	5.50	6.417	7.333
6	.5000	.750	1.000	1.250	1.50	1.750	2.000	2.250	2.500	2.750	3.00	3.500	4.000	4.50	5.000	5.500	6.00	7.000	8.000
7	.5833	.875	1.167	1.458	1.75	2.042	2.333	2.625	2.917	3.208	3.50	4.083	4.667	5.25	5.833	6.417	7.00	8.167	9.333
8	.6667	1.000	1.333	1.667	2.00	2.333	2.667	3.000	3.333	3.667	4.00	4.667	5.333	6.00	6.667	7.333	8.00	9.333	10.67
9	.7500	1.125	1.500	1.875	2.25	2.625	3.000	3.375	3.750	4.125	4.50	5.249	6.000	6.75	7.500	8.250	9.00	10.50	12.00
10	.8333	1.250	1.667	2.083	2.50	2.917	3.333	3.750	4.167	4.583	5.00	5.833	6.667	7.50	8.333	9.167	10.00	11.67	13.33
11	.9167	1.375	1.833	2.292	2.75	3.208	3.666	4.125	4.583	5.042	5.50	6.417	7.333	8.25	9.167	10.08	11.00	12.83	14.67
12	1.000	1.500	2.000	2.500	3.00	3.500	4.000	4.500	5.000	5.500	6.00	7.000	8.000	9.00	10.00	11.00	12.00	14.00	16.00
13	1.083	1.625	2.167	2.708	3.25	3.792	4.333	4.875	5.417	5.958	6.50	7.583	8.666	9.75	10.83	11.92	13.00	15.17	17.33
14	1.167	1.750	2.333	2.917	3.50	4.083	4.667	5.250	5.833	6.417	7.00	8.167	9.333	10.50	11.67	12.83	14.00	16.33	18.67
15	1.250	1.875	2.500	3.125	3.75	4.375	5.000	5.625	6.250	6.875	7.50	8.750	10.00	11.25	12.50	13.75	15.00	17.50	20.00
16	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00	6.667	7.333	8.00	9.333	10.67	12.00	13.33	14.67	16.00	18.67	21.33
18	1.500	2.25	3.000	3.750	4.50	5.250	6.000	6.75	7.500	8.250	9.00	10.50	12.00	13.50	15.00	16.50	18.00	21.00	24.00
20	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50	8.333	9.167	10.00	11.67	13.33	15.00	16.67	18.33	20.00	23.33	26.67

Multiply the tabular value for the given width and thickness by the length in feet to obtain the board measure.
Round and square timber is measured in cubic feet.

For round timber, taking all dimensions in feet, multiply the length by one-fourth the product of the mean girth and diameter, and the result will be in cubic feet.

The volume of square timber is obtained by the ordinary rules of mensuration.

Wrought Spikes.

Size and Number in Keg of 150 Pounds.

Length, in inches.	Diameter, in inches.					Length, in inches.	Diameter, in inches.				
	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$		$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$
3	2250	......	......	......	......	7	1161	662	482	445	306
3½	1890	1208	......	......	......	8	......	635	455	384	256
4	1650	1135	......	......	......	9	......	573	424	300	240
4½	1464	1064	......	......	......	10	......	......	391	270	222
5	1380	930	742	......	......	11	......	......	......	249	203
6	1292	868	570	......	......	12	......	......	......	236	180

Wire Spikes.

Size and Number to the Pound.

Title.	Number of wire.	Length, in inches.	Number per pound.	Title.	Number of wire.	Length, in inches.	Number per pound.
10d.	7	3	50	60d.	1	6	10
16d.	6	3½	35	6½ in.	1	6½	9
20d.	5	4	26	7 in.	0	7	7
30d.	4	4½	20	8 in.	00	8	5
40d.	3	5	15	9 in.	00	9	4½
50d.	2	5½	12				

Wire Nails.

Length and Number to the Pound.

Title.	Length, in inches.	Common nails and brads.	Barbed, common.	Clinch.	Fence.	Smooth and barbed.	Fine.	Casing and smooth and barbed finishing.	Flooring-brads.	Slating.	Barbed roofing.	Shingle.
......	¾	......	......	....	....	......	....	....	......	....	714	....
......	⅞	......	......	....	....	......	....	....	......	....	469	....
2d.	1	1200	876	710	....	1558	1550	1350	......	411	411	....
3d.	1⅛	......	......	....	....	......	1140	....	......	....	......	....
3d.	1¼	720	568	429	....	980	....	913	......	251	251	....
4d.	1½	432	357	274	....	760	760	584	......	209	165	....
5d.	1¾	300	235	235	142	575	....	410	......	142	142	270
6d.	2	252	204	157	124	350	....	310	157	....	103	204
7d.	2¼	186	139	139	92	275	....	238	139	182	......	....
8d.	2½	132	99	99	82	190	....	170	99	125	......	....
9d.	2¾	105	90	90	62	173	....	150	90	114	......	....
10d.	3	87	69	83	50	137	....	121	67	83	......	....
12d.	3¼	66	53	64	38	98	....	97	53	....	......	....
16d.	3½	51	43	59	30	81	....	72	43	....	......	....
20d.	4	35	31	43	23	71	....	54	......	....	......	....
30d.	4½	27	24	....	....	......	....	46	......	....	......	....
40d.	5	21	18	....	....	......	....	36	......	....	......	....
50d.	5½	15	......	....	....	......	....	....	......	....	......	....
60d.	6	12	......	....	....	......	....	....	......	....	......	....

Size and Weight of Lag Screws.

Length, in inches.	Diameter, in inches.				
	⅜	7/16	½	⅝	¾
	Lb. per 100.	Lb. per 100.	Lb. per 100.	Lb. per 100.	Lb. per 100.
1½	6.88	....	....	....	....
1¾	7.50	11.75	16.88	....	....
2	8.25	12.62	17.18	....	....
2¼	9.25	12.88	18.07	....	....
2½	9.62	13.28	19.18	....	....
3	10.82	16.62	22.00	34.07	....
3½	11.50	18.18	24.00	35.88	....
4	13.31	18.88	26.82	39.25	64.00
4½	14.82	19.50	28.25	42.62	67.88
5	16.50	21.25	30.37	47.75	71.37
5½	17.37	23.56	33.88	51.62	79.37
6	18.82	25.31	35.37	55.12	86.62
7	....	....	38.94	61.88	92.75
8	....	....	44.37	68.75	97.50
9	....	....	....	77.00	108.75
10	....	....	....	90.00	124.75

Dimensions of Wood Screws.

Number.	Threads per inch.	Diameter of body.	Diameter of flat head.	Diameter of round head.	Diameter of filister head.	Lengths.
		Inch.	Inch.	Inch.	Inch.	Inch.
2	56	.0842	.1631	.1544	.1332	3/16 to ½
3	48	.0973	.1894	.1786	.1545	3/16 to ⅝
4	32, 36, 40	.1105	.2158	.2028	.1747	3/16 to ¾
5	32, 36, 40	.1236	.2421	.2270	.1985	3/16 to ⅞
6	30, 32	.1368	.2684	.2512	.2175	3/16 to 1
7	30, 32	.1500	.2947	.2754	.2392	¼ to 1⅛
8	30, 32	.1631	.3210	.2936	.2610	¼ to 1¼
9	24, 30, 32	.1763	.3474	.3238	.2805	¼ to 1⅜
10	24, 30, 32	.1894	.3737	.3480	.3035	¼ to 1½
12	20, 24	.2158	.4263	.3922	.3445	⅜ to 1¾
14	20, 24	.2421	.4790	.4364	.3885	⅜ to 2
16	16, 18, 20	.2684	.5316	.4866	.4300	⅜ to 2¼
18	16, 18	.2947	.5842	.5248	.4710	½ to 2½
20	16, 18	.3210	.6368	.5690	.5200	½ to 2¾
22	16, 18	.3474	.6894	.6106	.5557	½ to 3
24	14, 16	.3737	.7420	.6522	.6005	½ to 3
26	14, 16	.4000	.7420	.6938	.6525	¾ to 3
28	14, 16	.4263	.7946	.7354	.6920	⅞ to 3
30	14, 16	.4520	.8473	.7770	.7240	1 to 3

Lengths vary by 16ths from 3/16 to ½; by 8ths, from ½ to 1½; by 4ths, from 1½ to 3.

Wrought-iron Welded Extra Strong Pipe.

Diameter.			Thickness.	Nominal weight per foot.
Nominal internal.	Actual external.	Actual internal.		
Inch.	Inch.	Inch.	Inch.	Lb.
⅛	.405	.205	.100	.29
¼	.540	.294	.123	.54
⅜	.675	.421	.127	.74
½	.840	.542	.149	1.09
¾	1.050	.736	.157	1.39
1	1.315	.951	.182	2.17
1¼	1.660	1.272	.194	3.00
1½	1.900	1.494	.203	3.63
2	2.375	1.933	.221	5.02
2½	2.875	2.315	.280	7.67
3	3.500	2.892	.304	10.25
3½	4.000	3.358	.321	12.47
4	4.500	3.818	.341	14.97
5	5.563	4.813	.375	20.54
6	6.625	5.750	.437	28.58

Wrought-iron Welded Double Extra Strong Pipe.

Diameter.			Thickness.	Nominal weight per foot.
Nominal internal.	Actual external.	Actual internal.		
Inch.	Inch.	Inch.	Inch.	Lb.
½	.840	.244	.298	1.70
¾	1.050	.422	.314	2.44
1	1.315	.587	.364	3.65
1¼	1.660	.885	.388	5.20
1½	1.900	1.088	.406	6.40
2	2.375	1.491	.442	9.02
2½	2.875	1.755	.560	13.68
3	3.500	2.284	.608	18.56
3½	4.000	2.716	.642	22.75
4	4.500	3.136	.682	27.48
5	5.563	4.063	.750	38.12
6	6.625	4.875	.875	53.11

Lap-welded Charcoal Iron Boiler Tubes.

Diameter.		Thickness.	Length of tube per square foot of		Nominal weight per foot.
External.	Internal.		External surface.	Internal surface.	
Inch.	Inch.	Inch.	Feet.	Feet.	Lb.
3	2.782	.109	1.273	1.373	3.33
3¼	3.010	.120	1.175	1.260	3.96
3½	3.260	.120	1.091	1.172	4.28
3¾	3.510	.120	1.018	1.088	4.60
4	3.732	.134	.955	1.024	5.47
4¼	3.982	.134	.899	.959	5.82
4½	4.232	.134	.849	.902	6.17
4¾	4.482	.134	.804	.852	6.53
5	4.704	.148	.764	.812	7.58
5¼	4.954	.148	.728	.771	7.97
5½	5.204	.148	.694	.734	8.36
6	5.670	.165	.637	.673	10.16

Double Galvanized Spiral Riveted Pressure Pipe..

For Compressed Air.

Inside diameter, in inches.	Thickness.		Approximate weight per foot, in pounds.	Approximate bursting pressure, in pounds, per square inch.	Safe working pressure, in pounds, per square inch.
	B. W. G.	Inches.			
3	20	.035	2¼	900	300
4	20	.035	3	700	220
5	20	.035	4	550	175
6	18	.049	5	700	220
7	18	.049	6	600	185
8	18	.049	7	500	150
9	18	.049	8	450	135
10	16	.065	11	500	150
11	16	.065	12	450	135
12	16	.065	14	400	120
13	16	.065	15	380	115
14	14	.083	20	470	140
15	14	.083	22	450	135
16	14	.083	24	400	120
18	14	.083	29	370	110
20	14	.083	34	325	100
22	12	.109	40	365	110
24	12	.109	50	335	100

A variety of joints can be used to connect lengths, but the surest are bolted joints where the pipe is to carry an excessive pressure. Flanged, leaded, and cement joints may also be conveniently used according to pressure and permanency of pipe line.

Riveted Hydraulic Pipe.

Pelton Water-wheel Company.

Diameter of pipe, in inches.	Thickness of material, U. S. standard gauge.	Equivalent thickness, in inches.	Head, in feet, pipe will safely stand.	Weight per lineal foot, in pounds.	Diameter of pipe, in inches.	Thickness of material, U. S. standard gauge.	Equivalent thickness, in inches.	Head, in feet, pipe will safely stand.	Weight per lineal foot, in pounds.
3	18	.05	810	2.25	18	12	.109	295	25.25
4	18	.05	607	3.00	18	11	.125	337	29.00
4	16	.062	760	3.75	18	10	.140	378	32.50
5	18	.050	485	3.75	18	8	.171	460	40.00
5	16	.062	605	4.50	20	16	.062	151	16.00
5	14	.078	757	5.75	20	14	.078	189	19.75
6	18	.050	405	4.25	20	12	.109	265	27.50
6	16	.062	505	5.25	20	11	.125	304	31.50
6	14	.078	630	6.50	20	10	.140	340	35.00
7	18	.050	346	4.75	20	8	.171	415	45.50
7	16	.062	433	6.00	22	16	.062	138	17.75
7	14	.078	540	7.50	22	14	.078	172	22.00
8	16	.062	378	7.00	22	12	.109	240	30.50
8	14	.078	472	8.75	22	11	.125	276	34.50
8	12	.109	660	12.00	22	10	.140	309	39.00
9	16	.062	336	7.50	22	8	.171	376	50.00
9	14	.078	420	9.25	24	14	.078	158	23.75
9	12	.109	587	12.75	24	12	.109	220	32.00
10	16	.062	307	8.25	24	11	.125	253	37.50
10	14	.078	378	10.25	24	10	.140	283	42.00
10	12	.109	530	14.25	24	8	.171	346	50.00
10	11	.125	607	16.25	24	6	.200	405	59.00
10	10	.140	680	18.25	26	14	.078	145	25.50
11	16	.062	275	9.00	26	12	.109	203	35.50
11	14	.078	344	11.00	26	11	.125	233	39.50
11	12	.109	480	15.25	26	10	.140	261	44.25
11	11	.125	553	17.50	26	8	.171	319	54.00
11	10	.140	617	19.50	26	6	.200	373	64.00
12	16	.062	252	10.00	28	14	.078	135	27.25
12	14	.078	316	12.25	28	12	.109	188	38.00
12	12	.109	442	17.00	28	11	.125	216	42.25
12	11	.125	506	19.50	28	10	.140	242	47.50
12	10	.140	567	21.75	28	8	.171	295	58.00
13	16	.062	233	10.50	28	6	.200	346	69.00
13	14	.078	291	13.00	30	12	.109	176	39.50
13	12	.109	407	18.00	30	11	.125	202	45.00
13	11	.125	467	20.50	30	10	.140	226	50.50
13	10	.140	522	23.00	30	8	.171	276	61.75
14	16	.062	216	11.25	30	6	.200	323	73.00
14	14	.078	271	14.00	30	¼	.250	404	90.00
14	12	.109	378	19.50	36	11	.125	168	54.00
14	11	.125	433	22.25	36	10	.140	189	60.50
14	10	.140	485	25.00	36	$\frac{3}{16}$	.187	252	81.00
15	16	.062	202	11.75	36	¼	.250	337	109.00
15	14	.078	252	14.75	36	$\frac{5}{16}$	.312	420	135.00
15	12	.109	352	20.50	40	10	.140	170	67.50
15	11	.125	405	23.25	40	$\frac{3}{16}$	.187	226	90.00
15	10	.140	453	26.00	40	¼	.250	303	120.00
16	16	.062	190	13.00	40	$\frac{5}{16}$	.312	378	150.00
16	14	.078	237	16.00	40	⅜	.375	455	180.00
16	12	.109	332	22.25	42	10	.140	162	71.00
16	11	.125	379	24.50	42	$\frac{3}{16}$	.187	216	94.50
16	10	.140	425	28.50	42	¼	.250	289	126.00
18	16	.062	168	14.75	42	$\frac{5}{16}$	.312	360	158.00
18	14	.078	210	18.50	42	⅜	.375	435	190.00

Weight of Wrought-iron Pipe.

Metric System.

Weight, in Kilogrammes, per Metre.

Internal diameter.	Thickness, in millimetres.							
	2	3	4	5	6	7	8	10
Mm.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.
10	.58	.95	1.36	1.83	2.34	2.90	2.51	4.87
13	.73	1.17	1.65	2.20	2.78	3.41	4.09	5.60
15	.83	1.31	1.85	2.43	3.07	3.75	4.51	6.09
20	1.07	1.68	2.34	3.04	3.80	4.60	5.46	7.30
25	1.31	2.05	2.83	3.65	4.54	5.46	6.43	8.50
30	1.56	2.41	3.42	4.26	5.26	6.31	7.40	9.74
35	1.80	2.78	3.80	4.87	5.99	7.16	8.38	10.96
40	2.05	3.14	4.26	5.48	6.72	8.00	9.31	12.18
45	2.29	3.51	4.77	6.09	7.45	8.86	10.32	13.39
50	2.53	3.87	5.26	6.69	8.18	9.72	11.30	14.61
55	2.78	4.24	5.75	7.30	8.91	10.57	12.27	15.83
60	3.02	4.60	6.23	7.92	9.64	11.42	13.25	17.04
70	3.51	5.33	7.21	9.13	11.10	13.01	15.20	19.48
80	3.99	6.06	8.18	10.35	12.56	14.83	17.14	21.91

Weight of Copper Pipe.

Metric System.

Weight, in Kilogrammes, per Metre.

Internal diameter.	Thickness, in millimetres.							
	2	3	4	5	6	7	8	10
Mm.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.	Kg.
10	.68	1.10	1.58	2.12	2.71	3.37	4.07	5.66
13	.85	1.36	1.92	2.55	3.22	3.96	4.75	6.50
15	.96	1.53	2.15	2.83	3.56	4.35	5.24	7.07
20	1.24	1.95	2.71	3.53	4.41	5.34	6.33	8.48
25	1.52	2.38	3.28	4.24	5.26	6.33	7.46	9.90
30	1.81	2.80	3.85	4.93	6.11	7.32	8.60	11.31
35	2.09	3.22	4.41	5.66	6.96	8.31	9.73	12.72
40	2.38	3.65	4.98	6.36	7.80	9.30	10.86	14.14
45	2.66	4.07	5.54	7.07	8.65	10.29	11.99	15.55
50	2.94	4.50	6.11	7.78	9.50	11.28	13.12	16.96
55	3.22	4.92	6.67	8.48	10.35	12.27	14.25	18.38
60	3.51	5.34	7.24	9.19	11.20	13.26	15.38	19.79
70	4.07	6.19	8.37	10.60	12.89	15.24	17.64	22.62

Standard Flanges.

The following standard dimensions for pipe flanges were prepared by a committee of the American Society of Mechanical Engineers in 1892.

Pipe size, in inches.	Pipe thickness, $\frac{P+100}{.4S}d + .333\left(1-\frac{d}{100}\right)$.	Thickness, nearest fraction, in inches.	Stress on pipe per square inch at 200 pounds.	Radius of fillet, in inches.	Flange diameters, in inches.		Flange thickness, in inches.		Width of flange face, in inches.		Bolt circle diameter, in inches.		Number of bolts.	Bolt size diameters, in inches.		Bolt length, in inches.	Stress on each bolt per square inch, at bottom of thread, at 200 pounds.
2	.409	7/16	460	1/8		6		5/8		2		4 3/4	4	1/2	5/8	2	825
2 1/2	.429	7/16	550	1/8		6 3/4		11/16		2 1/8		5 1/4	4	1/2	5/8	2 1/4	1050
3	.448	7/16	690	1/8		7 1/2		3/4		2 1/4		6	4	1/2	5/8	2 1/2	1330
3 1/2	.466	1/2	700	1/8		8		13/16		2 1/4		6 1/2	4	1/2	5/8	2 1/2	2530
4	.486	1/2	800	1/8		8 3/4		15/16		2 3/8		7 1/4	4	5/8	3/4	2 3/4	2100
4 1/2	.498	1/2	900	1/8		9 1/4		15/16		2 3/8		7 3/4	8	5/8	3/4	3	1430
5	.525	1/2	1000	1/8		10		15/16		2 1/2		8 1/2	8	5/8	3/4	3	1630
6	.563	9/16	1060	1/8		11 1/4		1		2 5/8		9 5/8	8	5/8	3/4	3	2360
7	.600	5/8	1120	1/8		12 1/2		1 1/16		2 5/8		10 3/4	8	5/8	3/4	3 1/4	3200
8	.639	5/8	1280	1/8		13 1/2		1 1/8		2 3/4		11 3/4	8	5/8	3/4	3 1/2	4190
9	.678	11/16	1310	3/16		14 3/4		1 1/8		2 7/8		13	12	5/8	3/4	3 1/2	3610
10	.713	3/4	1330	3/16		16		1 3/16		3		14 1/4	12	3/4	7/8	3 5/8	2970
12	.790	13/16	1470	3/16		18 1/2		1 1/4		3 1/4		16 1/2	12	3/4	7/8	3 3/4	4280
14	.864	7/8	1600	3/16		21		1 3/8		3 1/2		18 3/4	12	7/8	1	4 1/4	4280
15	.904	15/16	1600	3/16		22 1/4		1 3/8		3 5/8		20	16	7/8	1	4 1/4	3660
16	.946	1	1600	3/16		23 1/2		1 7/16		3 3/4		21 1/4	16	7/8	1	4 1/4	4210
18	1.02	1 1/16	1690	3/16		25		1 9/16		3 1/2		22 3/4	16	1	1 1/8	4 3/4	4540
20	1.09	1 1/8	1780	3/16		27 1/2		1 11/16		3 5/8		25	20	1	1 1/8	5	4490
22	1.18	1 3/16	1850	1/4		29 1/2		1 13/16		3 5/8		27 1/4	20	1	1 1/4	5 1/2	4320
24	1.25	1 1/4	1920	1/4	31 1/2	32	1 1/4	1 7/8	3 3/4	4	29 1/4	29 1/2	20	1	1 1/4	5 1/2	5130
26	1.30	1 5/16	1980	1/4	33 3/4	34 1/4	1 7/8	2	3 7/8	4 1/8	31 1/4	31 3/4	24	1	1 1/4	5 3/4	5030
28	1.38	1 3/8	2040	1/4	36	36 1/2	1 15/16	2 1/16	4	4 1/4	33 1/2	34	28	1	1 1/4	6	5000
30	1.48	1 1/2	2000	1/4	38	38 3/4	1 1/2	2 1/8	4	4 3/8	35 1/2	36	28	1 1/8	1 3/8	6 1/4	4590
36	1.71	1 3/4	1920	1/4	44 1/2	45 3/4	1 3/4	2 3/8	4 1/4	4 7/8	42	42 3/4	32	1 1/8	1 3/8	6 1/2	5790
42	1.87	2	2100	1/4	51	52 3/4	1 7/8	2 5/8	4 1/2	5 3/8	48 1/2	49 1/2	36	1 1/4	1 1/2	7 1/4	5700
48	2.17	2 1/4	2130	1/4	57 1/2	59 1/2	2	2 3/4	4 3/4	5 3/4	54 3/4	56	44	1 3/8	1 1/2	7 3/4	6090

Sizes up to 24 inches are designed for 200 pounds or less.

Sizes from 24 to 48 inches are divided into two scales, one for 200 pounds, the other for less.

The two sizes of bolts given are for medium and high pressures.

The sudden increase in diameters at 16 inches is due to the possible insertion of wrought-iron pipe, making with a nearly constant width of gasket, a greater diameter desirable.

When wrought-iron pipe is used, if thinner flanges than those given are sufficient, it is proposed that bosses be used to bring the nuts up to the standard lengths. This avoids the use of a reënforcement around the pipe.

Figures in the third, fourth, fifth, and last columns refer only to pipe for 200 pounds pressure.

The above standards, while not officially adopted, are used by many manufacturers.

Extra Heavy Flanges.

Standard dimensions for extra heavy flanges for pipe fittings and valves, adopted by leading manufacturers in the United States, January 1, 1902.

Size of pipe.	Diameter of flange.	Thickness of flange.	Diameter of bolt circle.	Number of bolts.	Size of bolts.	Size of pipe.	Diameter of flange.	Thickness of flange.	Diameter of bolt circle.	Number of bolts.	Size of bolts.
In.	Inch.	Inch.	Inch.		Inch.	In.	Inch.	Inch.	Inch.		Inch.
2	6½	⅞	5	4	⅝	9	16	1¾	14	12	⅞
2½	7½	1	5⅞	4	¾	10	17½	1⅞	15¼	16	⅞
3	8¼	1⅛	6⅝	8	⅝	12	20	2	17¾	16	⅞
3½	9	1 3/16	7¼	8	⅝	14	22½	2⅛	20	20	⅞
4	10	1¼	7⅞	8	¾	15	23½	2 3/16	21	20	1
4½	10½	1 5/16	8½	8	¾	16	25	2¼	22½	20	1
5	11	1⅜	9¼	8	¾	18	27	2⅜	24½	24	1
6	12½	1 7/16	10⅝	12	¾	20	29½	2½	26¾	24	1⅛
7	14	1½	11⅞	12	⅞	22	31½	2⅝	28¾	28	1⅛
8	15	1⅝	13	12	⅞	24	34	2¾	31¼	28	1⅛

The foregoing table includes the following features: bolt holes are in multiples of four, in order to enable the positions of connections to be varied by right angles; bolt holes to be drilled to straddle vertical axis; the distance between bolt centres not to exceed 3⅝ inches, which is accomplished on all but the 2½-inch size; distance from centre of bolt to edge of the flange should always equal or exceed the diameter of bolt plus ⅛ inch for 9-inch valves and under, and diameter of bolt plus not less than ¼ inch for sizes larger.

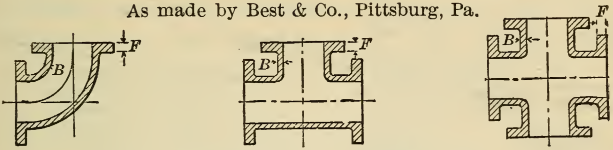
The bolt circle diameters, as above stated, will allow the use of calking recess on pipe flanges, provided such device is specified.

The above standard sizes have been adopted by the following firms, and will be furnished by other firms to order:

The Eaton, Cole & Burnham Company............Bridgeport, Conn.
Chapman Valve Manufacturing Company..........Indian Orchard, Mass.
Walworth Manufacturing Company...............Boston, Mass.
Crane Company................................Chicago, Ill.
The Pratt & Cady Company.....................Hartford, Conn.
Jenkins Bros.................................New York City.
General Fire Extinguisher Company............Providence, R. I.
Builders' Iron Foundry.......................Providence, R. I.
Jarecki Manufacturing Company................Erie, Penna.
Crosby Steam Gauge and Valve Company.........Boston, Mass.
The Kennedy Valve Manufacturing Company......New York City.
The Ludlow Valve Manufacturing Company.......Troy, N. Y.
The Lunkheimer Company.......................Cincinnati, Ohio.
The Michigan Brass and Iron Works............Detroit, Mich.
The Kelly & Jones Company....................New York City.
Eastwood Wire Manufacturing Company..........Belleville, N. J.
National Tube Company........................Pittsburg, Penna.
Coffin Valve Company.........................Boston, Mass.
Rensselaer Manufacturing Company.............Troy, N. Y.
The Mason Regulator Company..................Boston, Mass.
McNab & Harlin Manufacturing Company.........New York City.
The John Davis Company.......................Chicago, Ill.
Watson & McDaniel Company....................Philadelphia, Penna.
Ross Valve Company...........................Troy, N. Y.
Edward P. Bates..............................Syracuse, N. Y.

Dimensions of Cast-iron Pipe Fittings.

As made by Best & Co., Pittsburg, Pa.



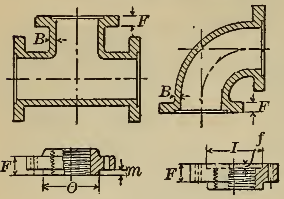
For Pressures from 50 to 1000 Pounds.

B = thickness of body. F = thickness of flanges.

Size.	50 lb.		100 lb.		150 lb.		200 lb.		300 lb.		500 lb.		1000 lb.	
	B	F	B	F	B	F	B	F	B	F	B	F	B	F
Inch.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1	⅜	¾	⅜	13/16	⅜	13/16	7/16	13/16	7/16	13/16	½	⅞	½	⅞
1¼	7/16	⅞	7/16	15/16	7/16	15/16	½	1	½	1	17/32	1	17/32	1
1½	7/16	⅞	7/16	15/16	7/16	15/16	½	1	½	1	⅝	1¼	⅝	1¼
2	7/16	⅞	7/16	15/16	7/16	15/16	9/16	1	9/16	1	⅝	1¼	⅝	1¼
2½	½	1	½	1	½	1	9/16	1	9/16	1	11/16	1⅜	25/32	1½
3	½	1	½	1	½	1	⅝	1⅛	⅝	1⅛	¾	1½	27/32	1¾
3½	9/16	1⅛	9/16	1⅛	9/16	1⅛	⅝	1⅛	⅝	1⅛	¾	1½	1	1¾
4	9/16	1⅛	9/16	1⅛	9/16	1⅛	11/16	1¼	11/16	1¼	⅞	1¾	1⅛	2
4½	9/16	1⅛	9/16	1⅛	9/16	1⅛	11/16	1¼	11/16	1¼	⅞	1¾	1¼	2
5	9/16	1⅛	9/16	1⅛	⅝	1¼	¾	1¼	¾	1¼	1	1⅞	1⅜	2⅛
6	9/16	1⅛	9/16	1⅛	11/16	1⅜	13/16	1⅜	13/16	1⅜	1⅛	2	1½	2¼
7	9/16	1⅛	⅝	1¼	11/16	1 7/16	13/16	1½	15/16	1½	1¼	2⅛	1⅝	2⅜
8	9/16	1⅛	⅝	1¼	¾	1½	⅞	1½	1	1½	1⅜	2¼	1¾	2½
9	9/16	1⅛	⅝	1¼	¾	1½	15/16	1⅝	1 1/16	1⅝	1½	2⅜	1⅞	2⅝
10	9/16	1⅛	11/16	1⅜	13/16	1 9/16	1	1¾	1⅛	1¾	1¾	2½	2	2¾
11	⅝	1 5/16	11/16	1½	⅞	1 11/16	1 1/16	1⅞	1 3/16	1⅞	1¾	2⅝	2¼	2⅞
12	⅝	1 5/16	¾	1 9/16	⅞	1¾	1⅛	2	1 5/16	2	1⅞	2¾	2½	3
*O. D.														
14	11/16	1 7/16	13/16	1⅝	15/16	1⅞	1 3/16	2⅛	1 7/16	2⅛	2	3	2¾	3½
15	11/16	1 7/16	13/16	1 11/16	15/16	1 15/16	1 3/16	2⅛	1½	2¼	....	....	....	....
16	¾	1 9/16	⅞	1¾	1	2	1¼	2¼	1 9/16	2⅜	....	....	....	....
18	¾	1⅝	⅞	1⅞	1 1/16	2 3/16	1⅜	2⅜	1 11/16	2½	....	....	....	....
20	13/16	1¾	15/16	1 15/16	1⅛	2¼	1½	2½	1⅞	2¾	....	....	....	....
22	13/16	1¾	1	2 1/16	1 3/16	2⅜	1⅝	2⅝	2	3	....	....	....	....
24	⅞	1⅞	1 1/16	2⅛	1 5/16	2⅝	1 11/16	2¾	2⅛	3¼	....	....	....	....
26	15/16	2	1⅛	2¼	1 5/16	2 11/16	1 13/16	2⅞	2¼	3⅜	....	....	....	....
28	15/16	2	1 3/16	2⅜	1⅜	2¾	1 15/16	3	2⅜	3½	....	....	....	....
30	1	2	1¼	2½	1 7/16	2 13/16	2	3¼	2½	3¾	....	....	....	....
32	1	2	1¼	2½	1½	2 15/16	2⅛	3⅜	2⅝	4	....	....	....	....
34	1 1/16	2⅛	1 5/16	2⅝	1 9/16	3 1/16	2 3/16	3½	2¾	4⅛	....	....	....	....
36	1 1/16	2⅛	1⅜	2¾	1⅝	3 3/16	2 5/16	3⅝	2⅞	4⅜	....	....	....	....

*O. D. 14 inches and larger is for lap-weld steel pipe whose outside diameter is of sizes given.

Dimensions of Steel Fittings and Flanges.



As made by Best & Co., Pittsburg, Pa.

For Pressures from 150 to 1000 Pounds.

B = thickness of body.
F = thickness of flange.
O = diameter of male.
I = diameter of female
m = height of male.
f = depth of female.

Size.	Steel Fittings. 150 to 300 lb.		500 lb.		1000 lb.		Cast and Rolled Steel Flanges. 150 and 200 lb.		300 lb.		500 lb.		1000 lb.		For all pressures.	
	B	F	B	F	B	F	O	I	O	I	O	I	O	I	m	f
Inch.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
1	3/8	1	7/16	1	7/16	1	2 5/16	2 3/8	2 5/16	2 3/8	2 1/4	2 5/16	2 1/4	2 5/16	1/4	3/16
1 1/4	7/16	1	1/2	1	1/2	1	2 9/16	2 5/8	2 9/16	2 5/8	2 1/2	2 9/16	2 1/2	2 9/16	1/4	3/16
1 1/2	7/16	1	1/2	1	1/2	1	2 13/16	2 7/8	2 13/16	2 7/8	3	3 1/16	3	3 1/16	1/4	3/16
2	7/16	1 1/16	9/16	1 3/16	9/16	1 3/16	3 5/16	3 3/8	3 5/16	3 3/8	3 1/2	3 9/16	3 1/2	3 9/16	1/4	3/16
2 1/2	1/2	1 1/8	9/16	1 3/16	9/16	1 3/16	3 15/16	4	3 15/16	4	4	4 1/16	4	4 1/16	1/4	3/16
3	1/2	1 1/8	5/8	1 5/16	5/8	1 5/16	4 15/16	5	4 15/16	5	4 3/4	4 13/16	4 3/4	4 13/16	1/4	3/16
3 1/2	9/16	1 3/16	5/8	1 5/16	5/8	1 5/16	5 7/16	5 1/2	5 7/16	5 1/2	6	6 1/16	6	6 1/16	1/4	3/16
4	9/16	1 3/16	11/16	1 7/16	11/16	1 7/16	6 1/16	6 1/8	6 1/16	6 1/8	6	6 1/16	6	6 1/16	1/4	3/16
4 1/2	9/16	1 3/16	11/16	1 7/16	11/16	1 7/16	6 9/16	6 5/8	6 9/16	6 5/8	7 1/4	7 5/16	7 1/4	7 5/16	1/4	3/16
5	9/16	1 3/16	3/4	1 7/16	3/4	1 7/16	7 9/16	7 5/8	7 9/16	7 5/8	7 1/4	7 5/16	7 1/4	7 5/16	1/4	3/16
6	9/16	1 3/16	13/16	1 9/16	13/16	1 9/16	8 9/16	8 5/8	8 9/16	8 5/8	8 1/2	8 9/16	8 1/2	8 9/16	1/4	3/16
7	5/8	1 5/16	13/16	1 9/16	15/16	1 9/16	9 9/16	9 5/8	9 9/16	9 5/8	9 5/8	9 11/16	9 5/8	9 11/16	1/4	3/16
8	5/8	1 5/16	7/8	1 11/16	1	1 11/16	11 9/16	11 5/8	11 15/16	12	10 5/8	10 11/16	10 5/8	10 11/16	1/4	3/16
9	5/8	1 5/16	15/16	1 13/16	1 1/16	1 13/16	12 9/16	12 5/8	12 15/16	13	11 5/8	10 11/16	11 3/4	11 13/16	1/4	3/16
10	11/16	1 7/16	1	2	1 1/3	2	13 13/16	13 7/8	14 11/16	14 3/4	12 3/4	12 13/16	13	13 1/16	1/4	3/16
11	11/16	1 1/2	1 1/16	2 1/16	1 3/16	2 1/16	14 13/16	14 7/8	15 11/16	15 3/4	14	14 1/16	15	15 1/16	1/4	3/16
12	3/4	1 5/8	1 1/8	2 3/16	1 5/16	2 3/16	16 3/16	16 1/4	16 11/16	16 3/4	15	15 1/16	15	15 1/16	1/4	3/16
*O.D.																
14	13/16	1 3/4	1 3/16	2 3/8	1 7/16	2 3/8	17 5/16	17 3/8	17 15/16	18	17	17 1/16	17	17 1/16	5/16	1/4
15	13/16	1 3/4	....	....	....	....	18 5/16	18 3/8	19 5/16	19 3/8	....	....	....	....	5/16	1/4
16	7/8	1 7/8	....	....	....	....	19 5/16	19 3/8	20 5/16	20 3/8	....	....	....	....	5/16	1/4
18	7/8	2	....	....	....	....	21 11/16	21 3/4	22 7/16	22 1/2	....	....	....	....	5/16	1/4
20	15/16	2 1/16	....	....	....	....	23 11/16	23 3/4	24 3/16	24 1/4	....	....	....	....	5/16	1/4
22	1	2 1/8	....	....	....	....	26 5/16	26 3/8	27 1/16	27 1/8	....	....	....	....	5/16	1/4
24	1 1/16	2 3/16	....	....	....	....	28 5/16	28 3/8	29 1/16	29 1/8	....	....	....	....	5/16	1/4
26	1 1/8	2 1/4					30 7/16	30 1/2								
28	1 3/16	2 3/8					32 7/16	32 1/2								
30	1 1/4	2 1/2					34 7/16	34 1/2								
32	1 1/4	2 1/2					36 9/16	36 5/8								
34	1 5/16	2 5/8					38 9/16	38 5/8								
36	1 3/8	2 3/4					40 13/16	40 7/8								

Elbows, Tees, and Crosses, 200 pounds and above. Female on all ends.

Flanges and Fittings 150 pounds, and below plain face. For pressures 200 pounds and above. Fittings, Female; Valves, Male; according to dimensions on this table.

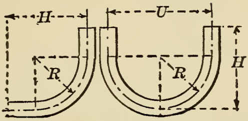
*O. D. 14 inches and larger is for lap-weld steel pipe whose outside diameter is of sizes given.

Dimensions of Pipe Fittings.

As made by Best & Co., Pittsburg, Pa.

Pipe Bends.

Made of Standard and Extra Heavy Pipe.



R = radius. H = centre to face. U = centre to centre.

Size.	Standard radius.						Minimum radius.					
	R		H		U		R		H		U	
Inch.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.	Ft.	In.
⅛	..	3	..	3¾	..	6	..	1	..	1¾	..	2
¼	..	3½	..	4½	..	7	..	1¼	..	2¼	..	2½
⅜	..	4	..	5¼	..	8	..	1½	..	2¾	..	3
½	..	5	..	6½	..	10	..	1¾	..	3¼	..	3½
¾	..	6	..	7¾	..	12	..	2	..	3¾	..	4
1	..	7	..	9¼	..	14	..	2¼	..	4½	..	4½
1¼	..	8	..	10½	..	16	..	2¾	..	5¼	..	5½
1½	..	10	..	12¾	..	20	..	3¼	..	6	..	6½
2	..	12	..	15	2	4	..	4½	..	7½	..	9
2½	..	14	..	17½	2	4	..	6	..	9½	..	12
3	..	18	..	22	3	..	..	7	..	11	..	14
3½	..	20	2	½	3	4	..	9	..	13½	..	18
4	..	24	2	5	4	..	..	12	..	17	..	24
4½	2	2	2	7½	4	4	..	15	..	20½	2	6
5	2	6	3	..	5	..	..	18	..	24	3	..
6	3	..	3	6½	6	..	..	24	2	6½	4	..
7	4	..	4	7	8	..	2	6	3	1	5	..
8	4	6	5	2	9	..	3	2	3	10	6	4
10	5	6	6	6	11	..	4	4	5	4	8	8
12	7	6	8	8	15	..	6	6	7	8	13	..
* O. D.												
14	8	..	9	6	16	..	7	6	9	..	15	..
16	10	..	11	10	20	..	8	6	10	4	17	..
18	12	..	14	..	24	..	9	6	11	6	19	..
20	14	..	16	..	28	..	10	6	12	6	21	..
22	16	..	18	..	32	..	11	6	13	6	23	..
24	18	..	20	..	36	..	12	6	14	6	25	..

* O. D. 14 inches and larger is for lap-weld steel pipe whose outside diameter is of sizes given.

Bends below heavy lines made in two pieces.

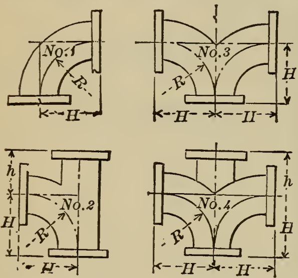
Dimensions of Pipe Fittings.

As made by Best & Co., Pittsburg, Pa.

Pipe Bends.

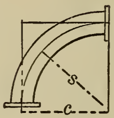
Made of Standard and Extra Heavy Pipe.

Long Radius, for Pressures from 50 to 300 Pounds.



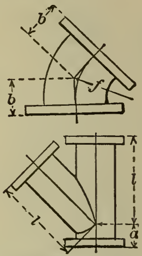
R = radius.
H = centre to face.
h = centre to face, on short end.

Extra Long Radius, for Pressures from 50 to 300 Pounds.



S = radius.
C = centre to face.

45° Elbows and Y's, for Pressures from 50 to 150 Pounds.



f = radius.
b = centre to face.
l = centre to face.
a = centre to face, on short end.

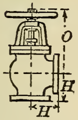
Size.	R	H	h	S	C	f	b	l	a
Inch.	Ft. In.	Ft. In.	Inch.	Ft. In.	Ft. In.	Inch.	Inch.	Ft. In.	In.
2	.. $5\frac{13}{16}$	.. 7	4¾	.. $9\frac{5}{16}$	.. 10½	4⅛	2⅞	.. 9	2½
2½	.. 6½	.. 7¾	5	.. 10¼	.. 11½	5	3⅜	.. 10	3
3	.. 7¼	.. 8½	5½	.. 11½	.. 12¾	$5\frac{11}{16}$	3⅝	.. 11½	3
3½	.. 7⅞	.. 9¼	6	.. 12⅝	.. 14	$5\frac{15}{16}$	3⅞	.. 12¼	3¼
4	.. $8\frac{9}{16}$	.. 10	6¾	.. $13\frac{9}{16}$	.. 15	6⅝	4⅛	.. 13½	3½
4½	.. $9\frac{1}{16}$	.. 10½	7¼	.. $14\frac{5}{16}$	.. 15¾	7⅜	4½	.. 14¼	3¾
5	.. $9\frac{7}{16}$	.. 11	7¾	.. $14\frac{15}{16}$	.. 16½	$7\frac{15}{16}$	4⅞	.. 16¼	3¾
6	.. 10¼	.. 12	8½	.. 16¼	.. 18	$8\frac{5}{16}$	5⅛	.. 17¼	4¼
7	.. $11\frac{3}{16}$	.. 13	9	.. $17\frac{11}{16}$	.. 19½	9⅛	5⅝	.. 18½	4½
8	.. 12⅛	.. 14	9¾	.. 19⅛	.. 21	9⅞	6	.. 21	4½
10	.. $14\frac{9}{16}$	.. 16½	11¼	.. $22\frac{13}{16}$	2 ¾	11	6½	.. 24	5
12	.. 16⅞	.. 19	12½	2 2⅜	2 4½	12	7¼	2 3½	6
*O. D.									
14	.. $18\frac{11}{16}$	.. 21	14	2 $5\frac{3}{16}$	2 7½	13¼	7¾	2 5	6½
16	.. $21\frac{1}{16}$	.. 23½	16	2 $8\frac{13}{16}$	2 11¼	14⅛	8⅜	2 7¾	7¼
18	.. 23⅜	2 2	17	3 ⅜	3 3	15½	9⅛	2 10¾	7¾
20	2 $1\frac{3}{16}$	2 4	18	3 $3\frac{5}{16}$	3 6	16⅜	9½	3 1¾	8¼
22	2 3⅛	2 6	19	3 6⅛	3 9	18¾	10¾	3 5¾	8¾
24	2 4⅞	2 8	21	3 8⅞	4 ..	20	11⅜	3 8¾	9¼

* O. D. 14 inches and larger is for lap-weld steel pipe whose outside diameter is of sizes given.

Dimensions of Angle, Globe, and Check Valves.

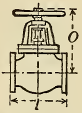
As made by Best & Co., Pittsburg, Pa.

Angle.



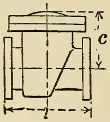
H = centre to face.
o = centre to top of wheel, when open.

Globe.



l = face to face.
o = centre to top of wheel, when open.

Check.

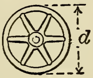


l = face to face.
c = centre to top of cover.

Size.	Angle. Hy. and Ex. Hy.		Globe. Hy. and Ex. Hy.		Check. Hy. and Ex. Hy.	
	H	o	l	o	l	c
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
2½	4⅝	12¼	9¼	12¼	8	5¼
3	6¼	16¼	12½	16¼	11	6½
3½	6¾	17¼	13½	17¼	12	7½
4	7	18¼	14	18¼	13½	8½
5	7⅞	19¾	15¾	19¾	15½	10
6	8¾	21½	17½	21½	17	11
7	9¾	23⅜	19½	23⅜	18	11¾
8	10½	25¼	21	25¼	19½	12¾
10	12¼	29½	24½	29½	22½	14½
12	14	35½	28	35½	25	16½
14	15	39	30	39	28	18½
16	15½	42½	30½	42½	32	20¾
18	....	....	....	....	34	22
20	....	....	....	....	36	24
22	....	....	....	....	38	26¾
24	....	....	....	....	42	28¾

Hy. for 150 pounds pressure.
X Hy. for 200 pounds pressure.

Diameter of Wheels for Valves.



Dimensions of hydraulic valves to 5000 pounds on application.

Angle Check Valves to order.

Size of valves, in inches.	1	1¼	1½	2	2½	3	3½	4	4½	5	6	7	8
Angle and Globe Valves.....	..	..	..	8	9	10	12	12	14	14	16	16	18
Lt., Std., and Hy. Gate Valves	4½	6	6	7	7	10	10	10	10	12	13	13	14
X Hy., XX Hy. Gate Valves.	4½	6	6	7	8½	10	10	12	12	13	14	14	15
Hyd. Gate Valves...........	4½	6	6	7	8½	12	12	14	14	14	16	16	18
X Hyd. Gate Valves.........	6	6	7	8½	10	12	12	14	16	16	18	20	24

Size of valves, in inches.	9	10	12	14	15	16	18	20	22	24	26	28	30
Angle and Globe Valves.....	18	20	24	30	....	..	..	..	..	..	..	..	..
Lt., Std., and Hy. Gate Valves	14	15	15	18	20	24	24	30	32	32	32	36	36
X Hy., XX Hy. Gate Valves.	16	18	20	22	24	24	30	30	32	36	..	..	..
Hyd. Gate Valves...........	18	20	20	24	....	..	..	..	..	..	..	..	..
X Hyd. Gate Valves.........	24	24	30	30	....	..	..	..	..	..	..	..	..

Butterfly Valves.

Light, Standard, and X Hy.—Dimensions in inches.

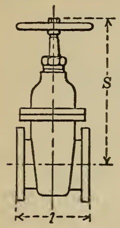
Size of valve.........	4	5	6	7	8	10	12	14	15	16	18	20	22	24
Length, face to face..	4½	5¼	6	7	8	10	12	12¼	13	13¾	15	16½	18½	19¼

Double Butterfly Valves to order.

Dimensions of Gate Valves.

As made by Best & Co., Pittsburg, Pa.

Inside Screw.

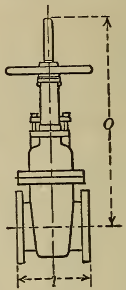


Pressures Indicated.

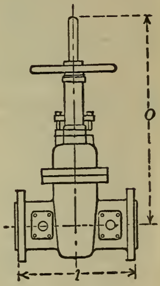
Light,	50 lb.	
Standard,	100 lb.	
Hy.,	150 lb.	
X Hy.,	200 lb.	Male.
XX Hy.,	300 lb.	Male.
Hydc.,	500 lb.	Male.
X Hydc.,	1000 lb.	Male.

Outside Screw and Yoke.

No By-pass.



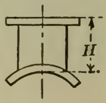
With By-pass.



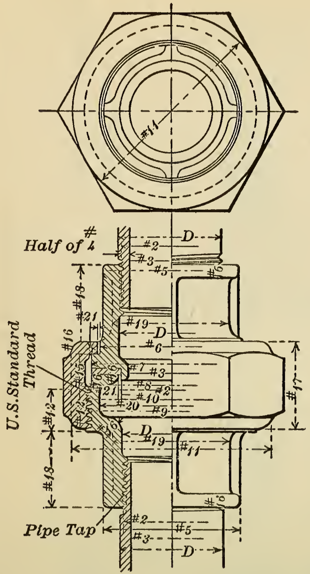
l = face to face.
L = face to face by-pass valves.
S = centre to top of wheel.
O = centre to top of stem, when open.

Size.	Light and Standard.						Hy., X Hy., XX Hy.						Hydc., X Hydc.	
	l	*S*		*O*		*L*	*l*	*S*		*O*		*L*	*l*	*S*
Inch.	In.	Ft.	In.	Ft.	In.	In.	In.	Ft.	In.	Ft.	In.	In.	Inch.	Inch.
1	....	..	....	..	....	....	....	..	....	..	....	....	6¾	8½
1¼	....	..	....	..	....	....	....	..	....	..	....	....	7½	10½
1½	....	..	....	..	....	....	....	..	....	..	....	....	11½	12¾
2	8	..	11⅜	..	14	....	....	..	....	..	....	....	12	14⅛
2½	8¾	..	12½	..	15⅝	....	9½	..	12⅜	..	15	....	14	17
3	8¾	..	14	..	19⅜	....	11⅛	..	16⅛	..	19⅜	....	14½	18
3½	....	..	....	..	....	....	11⅞	..	18⅛	..	21¾	....	14½	18
4	9⅜	..	16¾	..	23	....	12	..	19	..	23⅜	....	20½	22
4½	10¼	..	19	..	24	....	13¼	..	21¾	..	26½	....	21½	23
5	9⅝	..	19½	..	26	....	15	..	22⅝	..	27⅞	18½	22½	24
6	10⅞	..	22¼	..	31¾	....	15⅞	..	25½	..	31⅞	19	24	25½
7	11½	..	24	..	36¼	....	16¼	..	29⅛	..	36½	20	25	28
8	11⅞	..	25¼	3	4	18½	16½	..	32¾	3	5¼	20⅝	26	29½
9	12⅞	..	26¾	3	7½	19½	17	..	34¾	3	8⅛	21⅞	26	32¼
10	13⅝	..	30⅜	3	10¼	21	18	3	1⅝	4	⅛	22¾	27	35
12	14⅝	..	33⅜	4	6¼	22	19¾	3	8¼	4	8¾	23¾	30½	39¼
14	15⅞	3	1½	5	2½	24	21½	4	2	5	4½	25⅞	32½	43¼
15	....	..	....	..	....	....	21½	4	5¾	5	9¼	26⅛		
16	18¾	3	7	6	½	26½	....	4	8	6	½	32⅞		
18	20	3	10¼	7	7¾	26¾	....	5	2¾	6	9½	33¼		
20	21	4	2	7	4¾	27	....	5	8½	7	5¼	35¼		
22	22½	4	6⅜	8	....	32	....	6	2¾	8	1½	35½		
24	24	4	10	8	8	32⅞	....	6	8	8	8⅞	35⅞		
26	26	5	2½	9	4¾	32⅞								
28	28	5	5¾	10	1	34½								
30	30	5	9	10	7	36								

Necks.



Cast-iron, cast-steel, forged.
Size 4 to 6, 8 in. *H*. Size 7 to 10, 8½ in. *H*.
Size 12 to 16, 9 in. *H*.



Dimensions of the various parts, according to the corresponding numbers in the accompanying table.

Standard Pipe Unions.

According to the Report of the Committee of the American Society of Mechanical Engineers, 1901.

Dimensions for Standard Pipe Unions.

Numbers indicate corresponding dimensions in the illustration.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
In.																					
⅛	.375	.270	.105	.59	.63	.78	.80	.85	.89	1.05	.26	¼	27	.2225	.08	.5625	½	.590	.615	.006	.05
¼	.496	.364	.132	.76	.80	.96	.98	1.05	1.09	1.29	.33	$\frac{5}{16}$	18	.2625	.10	.6925	$\frac{9}{16}$	.760	.760	.006	.06
⅜	.630	.494	.136	.90	.95	1.11	1.13	1.20	1.24	1.45	.34	⅜	18	.2825	.11	.7325	⅝	.900	.905	.006	.07
½	.783	.623	.160	1.16	1.21	1.38	1.40	1.49	1.54	1.78	.40	$\frac{7}{16}$	14	.3025	.12	.8225	$\frac{11}{16}$	1.030	1.20	.006	.08
¾	.992	.824	.168	1.38	1.43	1.61	1.63	1.72	1.77	2.02	.42	½	14	.3225	.13	.8725	¾	1.240	1.43	.007	.09
1	1.246	1.048	.198	1.74	1.79	1.98	2.01	2.13	2.19	2.49	.49	$\frac{9}{16}$	11	.3625	.15	1.0025	$\frac{13}{16}$	1.565	1.76	.007	.10
1¼	1.592	1.380	.212	2.12	2.18	2.37	2.40	2.52	2.58	2.90	.53	.6	11	.3825	.16	1.0725	.9	1.91	2.15	.007	.11
1½	1.831	1.610	.221	2.40	2.46	2.66	2.69	2.81	2.87	3.20	.55	.7	11	.4025	.17	1.1225	1.0	2.18	2.40	.007	.13
2	2.306	2.067	.239	2.89	2.95	3.16	3.19	3.31	3.38	3.74	.60	.8	11	.4225	.18	1.2025	1.1	2.66	2.90	.008	.14
2½	2.775	2.468	.307	3.39	3.45	3.67	3.70	3.86	3.93	4.39	.77	9	8	.5225	.23	1.5225	1.2	3.16	3.41	.008	.16
3	3.401	3.067	.334	4.07	4.13	4.36	4.40	4.56	4.63	5.13	.84	1.0	8	.5625	.25	1.6525	1.3	3.81	4.08	.008	.18
3½	3.901	3.548	.353	4.61	4.68	4.91	4.95	5.11	5.19	5.72	.88	1.1	8	.6025	.27	1.7525	1.4	4.31	4.63	.008	.20
4	4.4	4.026	.374	5.15	5.22	5.47	5.51	5.67	5.75	6.31	.94	1.2	8	.6225	.28	1.8425	1.5	4.81	5.19	.008	.22

Machine Screws.

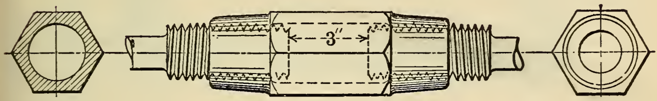
Screw gauge size.	Threads per inch.	Outside diameter, in inches.	Approximate diameter, in inches.	Tap drill, B. & S. drill gauge.
2	56	.08420	5/64	No. 49
3	48	.09730	3/32	No. 45
4	36	.11050	7/64	No. 42
5	36	.12360	1/8	No. 38
6	32	.13680	9/64	No. 35
7	32	.15000	5/32	No. 30
8	32	.16310	5/32	No. 29
9	30	.17630	11/64	No. 27
10	24	.18940	3/16	No. 25
11	24	.20206	13/64	No. 21
12	24	.2158	7/32	No. 17
13	22	.2289	15/64	No. 15
14	20	.2421	15/64	No. 13
15	20	.2552	1/4	No. 8
16	18	.2684	17/64	No. 6
17	18	.2816	9/32	No. 2
18	18	.2947	19/64	No. 1
19	18	.3079	5/16	1/4″
20	16	.3210	21/64	1/4″
22	16	.3474	11/32	9/32″
24	16	.3737	3/8	19/64″
26	16	.4000	13/32	21/64″
28	14	.4263	27/64	11/32″
30	14	.4526	7/16	23/64″

Set Screws.

Outside diameter, in inches.	Short diameter of square head, in inches.	Threads per inch.	Size of tap drill.	Lengths under head, in inches.
1/4	1/4	20	No. 5	3/4 to 3
5/16	5/16	18	17/64″	3/4 to 3 1/4
3/8	3/8	16	21/64″	3/4 to 3 1/2
7/16	7/16	14	3/8″	3/4 to 3 3/4
1/2	1/2	12	27/64″	3/4 to 4
9/16	9/16	12	31/64″	3/4 to 4 1/4
5/8	5/8	11	17/32″	3/4 to 4 1/2
3/4	3/4	10	21/32″	1 to 4 3/4
7/8	7/8	9	49/64″	1 1/4 to 5
1	1	8	7/8″	1 1/2 to 5

Standard Sleeve Nuts and Upsets.

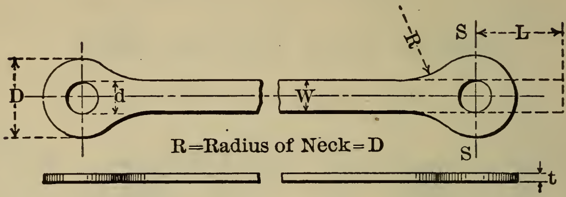
Passaic Rolling Mill Company.



Diameter of ● rods.	Side of ■ rods.	Diameter of upset.	Length of upset.	Short diameter of hexagon.	Long diameter of hexagon.	Number of threads per inch.	Length of sleeve nut.	Weight of sleeve nut.	Additional length of rod required for one upset. ●	Additional length of rod required for one upset. ■
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.		Inch.	Lb.	Inch.	Inch.
¾	⅝	1	4	2¼	2⅝	8	8¼	4	3¾	4¾
⅞	¾	1⅛	4	2¼	2⅝	7	8½	5	3¼	3¾
1	⅞	1⅜	4½	2⅜	2¾	6	9¼	7	4¾	5
1⅛	1	1½	4½	2⅞	3 5/16	6	9¼	8	4¼	4¼
1¼	1⅛	1⅝	4½	2⅞	3 5/16	5½	9½	9	3¾	3½
1⅜	1¼	1⅞	5	3¼	3¾	5	10¼	13	5¼	4½
1½	1⅜	2	5	3¼	3¾	4½	10¼	13	4¾	4
1⅝	1½	2⅛	5	3⅝	4 3/16	4½	10½	16	4¼	3½
1¾	....	2¼	5½	3¾	4 5/16	4½	11	18	4¼	......
1⅞	1⅝	2⅜	5½	4	4⅝	4	11¼	21	4	4½
2	1¾	2½	5½	4	4⅝	4	11¼	22	3¾	4
2⅛	1⅞	2⅝	6	4⅝	5⅜	4	12	29	3¾	4
2¼	2	2⅞	6	4¾	5½	3½	12¼	33	4½	4½
2½	2¼	3¼	6	5⅛	5 15/16	3½	12½	40	5	4¾
2¾	2½	3½	6	5½	6⅜	3¼	12¾	47	4½	4
3	....	3¾	6	5⅞	6¾	3	13	58	4	......

Standard Steel Eye Bars.

Passaic Rolling Mill Company.



W	*t*	*D*	*d*	*S—S*	*L*
Width of bar.	Minimum thickness of bar.	Diameter of head.	Diameter of largest pin-hole.	Sectional area of head on lines *S—S* in excess of that in body of bar.	Additional length of bar beyond centre of pin-hole to form one head.
Inch.	Inch.	Inch.	Inch.	Per cent.	Inch.
3	¾	7	$2\frac{11}{16}$	42	14½
3	¾	8	$3\frac{11}{16}$	42	18½
4	¾	9½	$3\frac{15}{16}$	37½	18½
4	¾	10½	4⅞	39	23½
5	¾	11½	4⅜	41	21
5	¾	12½	5⅜	41	25½
6	⅞	13½	4⅞	42	22
6	⅞	14½	5⅞	42	26½
7	1	16	5⅞	43	28
8	1⅛	18	7	37½	32½
10	1¼	23	9	40	40

Notes on Passaic Steel Eye Bars.

Passaic standard steel eye bars are forged without the addition of extraneous metal and without welds of any kind, and are guaranteed under the conditions given in the above table to develop the full strength of the bar when tested to destruction.

The maximum sizes of pin-holes, given in the above table, allow an excess in the net section of the head over that of the body of the bar of 40 per cent. when the thickness of the head is the same as the thickness of the body of the bar. The thickness of the head is usually $\frac{1}{16}$ of an inch thicker than the body of the bar; and where a number of eye bars are to be placed closely together, as at a joint, the thicknesses of the heads should be considered ⅛ of an inch greater than the bodies of the bars, in order to allow for the increased thickness of the heads and for the usual roughness of forged work.

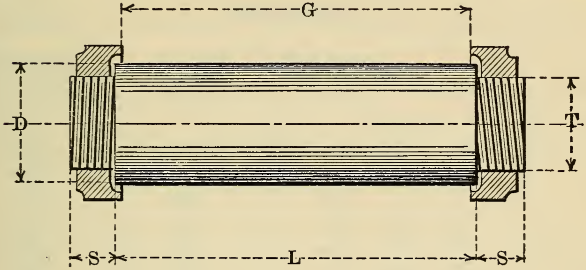
Unless otherwise specified, the steel manufactured by the Passaic Rolling Mill Company for the use of eye bars is open-hearth medium steel, conforming with the standard specifications of the Association of American Steel Manufacturers.

All eye bars are finished to length, and the eyes bored at the specified distances, centre to centre, according to United States standard measurements.

Eye bars having larger or smaller heads than the above standards can be furnished by special arrangement.

Standard Pins and Nuts.

Passaic Rolling Mill Company.



$G =$ grip. $L = G + \frac{3}{8}$ inch.

D Diameter of pin.	*T* Diameter of thread.	*S* Length of thread.	Short diameter of nut.	Long diameter of nut.	Weight of one nut.
Inch.	Inch.	Inch.	Inch.	Inch.	Lb.
$1\frac{3}{16}$	1	$1\frac{1}{2}$	$1\frac{3}{4}$	2	....
$1\frac{7}{16}$	1	$1\frac{1}{2}$	$1\frac{3}{4}$	2	....
$1\frac{11}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{3}{4}$	1.5
$1\frac{15}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{3}{4}$	1.5
$2\frac{3}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{3}{4}$	1.5
$2\frac{7}{16}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$3\frac{1}{4}$	$3\frac{3}{4}$	1.5
$2\frac{11}{16}$	2	$1\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{1}{4}$	2.5
$2\frac{15}{16}$	$2\frac{1}{4}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{4}$	3.0
$3\frac{3}{16}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{4}$	2.8
$3\frac{7}{16}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{4}$	2.8
$3\frac{11}{16}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$4\frac{3}{4}$	$5\frac{1}{2}$	3.0
$3\frac{15}{16}$	3	$1\frac{1}{2}$	$4\frac{3}{4}$	$5\frac{1}{2}$	3.0
$4\frac{3}{8}$	$3\frac{1}{2}$	$1\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{4}$	3.8
$4\frac{5}{8}$	$3\frac{1}{2}$	$1\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{4}$	3.8
$4\frac{7}{8}$	4	$1\frac{1}{2}$	6	7	6.7
$5\frac{3}{8}$	4	2	6	7	6.7
$5\frac{7}{8}$	4	2	7	8	9.1
7	5	$2\frac{1}{4}$	8	$9\frac{1}{4}$	12.0
8	6	$2\frac{1}{4}$	$10\frac{1}{2}$	12	18.8
9	7	$2\frac{1}{4}$	$10\frac{1}{2}$	12	22.8

Standard Wire Hoisting Rope.

John A. Roebling's Sons Company.

Composed of 6 Strands and a Hemp Centre, 19 Wires to the Strand.

Swedish Iron.

Trade number.	Diameter, in inches.	Approximate circumference, in inches.	Weight per foot, in pounds.	Approximate breaking strain, in tons of 2000 pounds.
....	2¾	8⅝	11.95	114.0
....	2½	7⅞	9.85	95.0
1	2¼	7⅛	8.00	78.0
2	2	6¼	6.30	62.0
3	1¾	5½	4.85	48.0
4	1⅝	5	4.15	42.0
5	1½	4¾	3.55	36.0
5½	1⅜	4¼	3.00	31.0
6	1¼	4	2.45	25.0
7	1⅛	3½	2.00	21.0
8	1	3	1.58	17.0
9	⅞	2¾	1.20	13.0
10	¾	2¼	.89	9.7
10¼	⅝	2	.62	6.8
10½	$\frac{9}{16}$	1¾	.50	5.5
10¾	½	1½	.39	4.4
10*a*	$\frac{7}{16}$	1¼	.30	3.4
10*b*	⅜	1⅛	.22	2.5
10*c*	$\frac{5}{16}$	1	.15	1.7
10*d*	¼	¾	.10	1.2
		Cast-steel.		
1	2¼	7⅛	8.00	156.0
2	2	6¼	6.30	124.0
3	1¾	5½	4.85	96.0
4	1⅝	5	4.15	84.0
5	1½	4¾	3.55	72.0
5½	1⅜	4¼	3.00	62.0
6	1¼	4	2.45	50.0
7	1⅛	3½	2.00	42.0
8	1	3	1.58	34.0
9	⅞	2¾	1.20	26.0
10	¾	2¼	.89	19.4
10¼	⅝	2	.62	13.6
10½	$\frac{9}{16}$	1¾	.50	11.0
10¾	½	1½	.39	8.8
10*a*	$\frac{7}{16}$	1¼	.30	6.8
10*b*	⅜	1⅛	.22	5.0
10*c*	$\frac{5}{16}$	1	.15	3.4
10*d*	¼	¾	.10	2.4

Transmission or Haulage Rope.

John A. Roebling's Sons Company.

Composed of 6 Strands and a Hemp Centre, 7 Wires to the Strand.

Swedish Iron.

Trade number.	Diameter, in inches.	Approximate circumference, in inches.	Weight per foot, in pounds.	Approximate breaking strain, in tons of 2000 pounds.
11	1½	4¾	3.55	34.0
12	1⅜	4¼	3.00	29.0
13	1¼	4	2.45	24.0
14	1⅛	3½	2.00	20.0
15	1	3	1.58	16.0
16	⅞	2¾	1.20	12.0
17	¾	2¼	.89	9.3
18	11/16	2⅛	.75	7.9
19	⅝	2	.62	6.6
20	9/16	1¾	.50	5.3
21	½	1½	.39	4.2
22	7/16	1¼	.30	3.3
23	⅜	1⅛	.22	2.4
24	5/16	1	.15	1.7
25	9/32	⅞	.125	1.4
		Cast-steel.		
11	1½	4¾	3.55	68.0
12	1⅜	4¼	3.00	58.0
13	1¼	4	2.45	48.0
14	1⅛	3½	2.00	40.0
15	1	3	1.58	32.0
16	⅞	2¾	1.20	24.0
17	¾	2¼	.89	18.6
18	11/16	2⅛	.75	15.8
19	⅝	2	.62	13.2
20	9/16	1¾	.50	10.6
21	½	1½	.39	8.4
22	7/16	1¼	.30	6.6
23	⅜	1⅛	.22	4.8
24	5/16	1	.15	3.4
25	9/32	⅞	.125	2.8

Extra Strong Crucible Cast-steel Rope.

John A. Roebling's Sons Company.

Composed of 6 Strands and a Hemp Centre, 19 Wires to the Strand.

Trade number.	Diameter, in inches.	Approximate circumference, in inches.	Weight per foot, in pounds.	Approximate breaking strain, in tons of 2000 pounds.
....	2¾	8⅝	11.95	266.0
....	2½	7⅞	9.85	222.0
1	2¼	7⅛	8.00	182.0
2	2	6¼	6.30	144.0
3	1¾	5½	4.85	112.0
4	1⅝	5	4.15	97.0
5	1½	4¾	3.55	84.0
5½	1⅜	4¼	3.00	72.0
6	1¼	4	2.45	58.0
7	1⅛	3½	2.00	49.0
8	1	3	1.58	39.0
9	⅞	2¾	1.20	30.0
10	¾	2¼	.89	22.0
10¼	⅝	2	.62	15.8
10½	$\frac{9}{16}$	1¾	.50	12.7
10¾	½	1½	.39	10.1
10*a*	$\frac{7}{16}$	1¼	.30	7.8
10*b*	⅜	1⅛	.22	5.78
10*c*	$\frac{5}{16}$	1	.15	4.05
10*d*	¼	¾	.10	2.70
		7 Wires to the Strand.		
11	1½	4¾	3.55	79.0
12	1⅜	4¼	3.00	68.0
13	1¼	4	2.45	56.0
14	1⅛	3½	2.00	46.0
15	1	3	1.58	37.0
16	⅞	2¾	1.20	28.0
17	¾	2¼	.89	21.0
18	$\frac{11}{16}$	2⅛	.75	18.4
19	⅝	2	.62	15.1
20	$\frac{9}{16}$	1¾	.50	12.3
21	½	1½	.39	9.70
22	$\frac{7}{16}$	1¼	.30	7.50
23	⅜	1⅛	.22	5.58
24	$\frac{5}{16}$	1	.15	3.88
25	$\frac{9}{32}$	⅞	.125	3.22

Weight, Length, and Strength of Steel Wire.

John A. Roebling's Sons Company.

Number, Roebling gauge.	Diameter, in inches.	Area, in square inches.	Breaking load at rate of 100,000 pounds per square inch.	Weight, in pounds.		Number of feet in 2000 pounds.
				Per 1000 feet.	Per mile.	
000000	.460	.166 191	16619.0	558.4	2948.0	3 582
00000	.430	.145 221	14522.0	487.9	2576.0	4 099
0000	.393	.121 304	12130.0	407.6	2152.0	4 907
000	.362	.102 922	10292.0	345.8	1826.0	5 783
00	.331	.086 049	8605.0	289.1	1527.0	6 917
0	.307	.074 023	7402.0	248.7	1313.0	8 041
1	.283	.062 902	6290.0	211.4	1116.0	9 463
2	.263	.054 325	5433.0	182.5	964.0	10 957
3	.244	.046 760	4676.0	157.1	830.0	12 730
4	.225	.039 761	3976.0	133.6	705.0	14 970
5	.207	.033 654	3365.0	113.1	597.0	17 687
6	.192	.028 953	2895.0	97.3	514.0	20 559
7	.177	.024 606	2461.0	82.7	437.0	24 191
8	.162	.020 612	2061.0	69.3	366.0	28 878
9	.148	.017 203	1720.0	57.8	305.0	34 600
10	.135	.014 314	1431.0	48.1	254.0	41 584
11	.120	.011 310	1131.0	38.0	201.0	52 631
12	.105	.008 659	866.0	29.1	154.0	68 752
13	.092	.006 648	665.0	22.3	118.0	89 525
14	.080	.005 027	503.0	16.9	89.2	118 413
15	.072	.004 071	407.0	13.7	72.2	146 198
16	.063	.003 117	312.0	10.5	55.3	191 022
17	.054	.002 290	229.0	7.70	40.6	259 909
18	.047	.001 735	174.0	5.83	30.8	343 112
19	.041	.001 320	132.0	4.44	23.4	450 856
20	.035	.000 962	96.0	3.23	17.1	618 620
21	.032	.000 804	80.0	2.70	14.3	740 193
22	.028	.000 616	62.0	2.07	10.9	966 651
23	.025	.000 491	49.0	1.65	8.71	....
24	.023	.000 415	42.0	1.40	7.37	....
25	.020	.000 314	31.0	1.06	5.58	....
26	.018	.000 254	25.0	.855	4.51	....
27	.017	.000 227	23.0	.763	4.03	....
28	.016	.000 201	20.0	.676	3.57	....
29	.015	.000 177	18.0	.594	3.14	....
30	.014	.000 154	15.0	.517	2.73	....
31	.0135	.000 143	14.0	.481	2.54	....
32	.0130	.000 133	13.0	.446	2.36	....
33	.0110	.000 095	9.5	.319	1.69	....
34	.0100	.000 079	7.9	.264	1.39	....
35	.0095	.000 071	7.1	.238	1.26	....
36	.0090	.000 064	6.4	.214	1.13	....

This table was calculated on a basis of 483.84 pounds per cubic foot for steel wire.

The breaking loads were calculated for 100,000 pounds per square inch throughout, simply for convenience, so that the breaking loads for wires of any strength per square inch may be quickly determined by multiplying the values given in the table by the ratio between the strength per square inch and 100,000. Thus, a No. 15 wire, with a strength per square inch of 150,000 pounds, will break with a load of $407 \times \frac{150,000}{100,000} = 610.5$ pounds.

It must not be thought from this table that steel wire invariably has a strength of 100,000 pounds per square inch. As a matter of fact, it ranges from 45,000 pounds per square inch for soft annealed wire to over 400,000 pounds per square inch for hard wire.

STRENGTH OF MATERIALS.

When a body is subjected to the action of external forces certain deformations are produced. These deformations are called *strains*, and the forces by which they are produced are called *stresses.*

The application of the external stresses is opposed by the production of internal stresses. The extent to which these internal stresses are capable of resisting the external stresses constitutes the *strength* of the material.

Thus, when a man lifts a weight so that it is suspended from his arm, stresses of sufficient magnitude to sustain the weight against the action of gravity are produced in the muscles. In like manner, a weight suspended from an iron rod produces internal stresses upon the fibres of the metal; and since equilibrium exists,—the weight being sustained,—the internal forces must balance the external ones.

Since the internal forces are brought into play by the production of deformation, or *strain*, it follows that every force, however slight, when acting upon a resistant body must produce some deformation, in order that the internal fibre stress by which the external force is to be opposed shall appear. No body, therefore, is absolutely rigid; since, if a body could be entirely rigid, no internal stresses could be produced and no external stresses could be resisted.

In the use of materials of engineering it is necessary to know the extent to which they may safely be subjected to certain external forces. It is also important to know the extent to which they become deformed under determinate loads, as well as the manner in which the stresses and strains are distributed. These various properties constitute the *resistance* of the materials, and it is upon a knowledge of the resistance of materials that the ability to make a correct distribution of them in any given structure depends.

The manner in which the fibres of a material act in resisting deformation is not entirely understood. Apparently, the first and smaller deformations act only to separate the particles to distances within their range of attraction for each other, so that, when the external force is removed, the original relation of the particles is resumed. When, however, the deformation becomes sufficiently great for the range of attraction of the particles to be exceeded, the original relations are not resumed upon the removal of the external stresses, but a portion of the deformation remains, the structure of the material being more or less broken down. This is very clearly shown by the change in the appearance of the polished surface of a metal under stress, the bright surface suddenly becoming dulled when the stress exceeds a magnitude which affects the permanent structure. This was first observed by Professor J. B. Johnson, as long ago as 1892, and has recently been further investigated in France by M. Frémont.

Further increase of external stress after the structure of the material has broken down is followed by rapidly-increasing deformation and rupture.

It is obvious that no material should ever be subjected to such stresses in practice as will result in the breaking down of its molecular structure, since no further effective resistance can then be expected of it. It is, therefore, of little importance to know the force which produces rupture in a material. The important thing to know is the magnitude of the load at which the break-down begins, so that the structure under consideration may be so proportioned that this load is approached only within a certain known limit. In other words, it is not the breaking load which is required, but *the permissible fibre stress* to which the material may be subjected.

The external forces which act to cause strains in a material may produce

Tension,
Compression or **Crushing,**
Bending,
Shearing,
Torsion;

and, in most instances, several of these actions are produced at the same time.

Up to the point at which the molecular structure of the material breaks down under stress, the deformation produced exists only during the appli-

cation of the stress, and the material returns to its original dimensions and form upon the removal of the stress. This property of returning to its original form and dimensions is called the *elasticity* of the material. Since a body does not return to its original dimensions and form when loaded to the point of structural break-down, its elasticity is then said to have been overcome, or its *elastic limit* reached.

Up to the elastic limit the deformation of a body is directly proportional to the load,—that is, the strain is proportional to the stress. If a certain elongation is observed with a load of 100 pounds, double that elongation will be produced by 200 pounds, and so on until the elastic limit is reached. This is known as Hooke's Law.

If the material is subjected to a continually-increasing load in a testing machine provided with an autographic recorder, the line of the record will be a straight one, making a constant angle to the axes; while as soon as the elastic limit is closely approached it becomes a curve, the curvature rapidly changing until rupture occurs.

The determination of the true elastic limit has been a matter of much discussion. Theoretically, it is the point at which "set" first occurs; practically, it is often assumed to correspond to the load at which the weigh-beam of the testing machine drops, showing the sudden yield of the material. An examination of autographic test diagrams shows that the departure from Hooke's law begins gradually, not suddenly, the deviation from a straight line being at first slight, but rapidly increasing.

The true elastic limit is determined as the point at which strain ceases to be proportional to stress; but this point is not readily determined in practice, except with precise and accurate testing machines, and hence the yield point, or point at which the drop of the beam of the ordinary testing machine occurs, is usually substituted for it. While admitting that this is not absolutely correct, it is quite within the working limits of accuracy under existing shop conditions.

In practice, the loads or stresses upon a body are expressed in units of weight per unit of area, as pounds per square inch or kilogrammes per square centimetre. Elongations are expressed either as the extension of a unit of length or in percentages of the length of test-piece.

The Modulus of Elasticity of a material is the result obtained by dividing the stress per unit of area by the strain per unit of length. If we call the stress per unit of area $= S$, and the corresponding elongation per unit of length $= e$, we have

$$\text{Modulus of elasticity} = E = \frac{S}{e}.$$

Thus, if an elongation of 0.01 is produced in a bar of 10 inches in length and 1 square inch cross-section by a load of 30,000 pounds, the modulus of elasticity will be

$$E = \frac{30{,}000}{0.001} = 30{,}000{,}000.$$

Since this is constant up to the yield point, it may be used for the determination of the elongation produced by any other load. Thus,

$$e = \frac{S}{E} = \frac{S}{30{,}000{,}000},$$

and any value of e can be obtained for any given value of S.

In the use of any material in the construction of a framework, a machine, or any kind of mechanism, it is most important to use judgment and common sense in a careful examination of the case under consideration before attempting to apply any of the rules or tables. The actual magnitude and direction of the forces acting should be determined as closely as possible, for we may be well assured that they will be in action whether taken into account or not. It has been well said that "theory takes into account all the conditions which can be ascertained, but practice has to take into account all the conditions there *are*."

In the design of structural work, such as bridges, roofs, buildings, etc., the size and direction of action of loads can generally be determined with a fair degree of accuracy, the principal uncertainty being as to the action of wind pressure. In machine design, however, the stresses are much

more difficult of determination, the number and complex action of forces often rendering determinate analysis impossible. Under such circumstances recourse must often be had to empirical rules, based upon the experience gained in practice. In such cases, also, careful exercise of judgment is demanded, in order that one may be assured that the case under consideration is similar to those from which the experience has been derived; and too frequently errors have been made by blindly following the precedent set by some excellent authority, but wholly inapplicable to the case in hand.

Before applying any rules, tables, or formulas, the end to be obtained should be intelligently considered. In some instances it is the actual strength of the material which must be taken into account, but in machine design this is not often the case. More generally, it is the *stiffness* which must be considered. It is always necessary that a machine should retain the relative position of its parts to such an extent that the movements may continue within determinate limits of accuracy, and that no undue binding or friction be created in the running parts.

Steam machinery must be so rigid that valve seats, etc., will remain tight and true, lathe beds must not spring under heavy cuts, planer uprights must stand firmly to their work; and all these and many other parts must be made far heavier than would be necessary for mere strength, in order that ample rigidity may be obtained.

In many instances the principal value to be obtained from a study of the distribution of stresses in a machine is to ascertain the *relative* disposition of the material, and not the absolute strength to be used. Experience has shown how heavy certain portions must be, in order that deflection or spring may be kept within working limits, while a graphical analysis of the distribution of the stresses will then show where metal may safely be spared and where it must be lavishly disposed.

It must be remembered, also, that break-downs usually occur by reason of unusual or abnormal stresses. Machines rarely break down under regular working loads. It is when some sudden shock occurs that the rupture takes place. While it is not to be expected that provision can be made for all accidents, yet the possible accidents should be considered in the original design; and often a little forethought as to whence the unusual stress may be expected will materially modify the disposition of the material.

Bearing the preceding considerations in mind, the following rules, formulas, and tables may be used, as representing both theory and practice.

Tension.

By far the greater number of tests of materials are made by pulling a test-piece, and observing or recording successive stages in the strains produced by the increasing stresses. The points usually observed are

Elastic Limit,
Ultimate Strength,
Ductility,
Stiffness,
Resilience.

As already stated, the elastic limit is the point at which the strain ceases to be proportional to the stress. In testing machines which are not provided with a recording attachment the nearest approach which can usually be had to this value is the stress observed at the moment of the drop of the beam. When a diagram of the test is automatically produced the point at which the line distinctly deviates from a straight one, at a definite angle with the axes, shows the elastic limit.

The ultimate strength is found when the material yields so rapidly that no further increase in load can be made. Both the elastic limit and the ultimate strength are always referred to the original area of the test specimen. In general, the elastic limit is reached at a stress about equal to six-tenths of the ultimate, but this varies for different materials and conditions.

The ductility of a material when subjected to tension is measured by the elongation in a given length or by the reduction of fractured area.

The stiffness is measured by the angle which the test-line makes with the coördinate axes, the portion within the elastic limit alone being considered.

Resilience is the amount of work performed in the production of strain by stress. It is, therefore, expressed in terms of force by length, usually in inch-pounds. When a piece is strained to the elastic limit, the work required is called the elastic resilience. When the load is applied gradually, the work done is equal to one-half the product of the stress at the elastic limit by the extension. When the load is applied instantaneously, the elastic deformation is double that produced by the same load applied slowly. When the force is applied by a drop, producing percussion, the product of the weight by the fall will give the work.

An examination of the following table, from data of the Pencoyd Iron Works, will serve to show the relations which exist in open-hearth basic steel, such as is used in structural work.

Open-hearth Basic Structural Steel.

Pencoyd Iron Works.

Percentage of carbon.	Tensile strength, in pounds, per square inch.		Ductility.	
	Ultimate strength.	Elastic limit.	Stretch in 8 inches.	Reduction of fractured area.
			Per cent.	Per cent.
.08	54000	32500	32	60
.09	54800	33000	31	58
.10	55700	33500	31	57
.11	56500	34000	30	56
.12	57400	34500	30	55
.13	58200	35000	29	54
.14	59100	35500	29	53
.15	60000	36000	28	52
.16	60800	36500	28	51
.17	61600	37000	27	50
.18	62500	37500	27	49
.19	63300	38000	26	48
.20	64200	38500	26	47
.21	65000	39000	25	46
.22	65800	39500	25	45
.23	66600	40000	24	44
.24	67400	40500	24	43
.25	68200	41000	23	42

The predominant elements other than carbon in the above steels average as follows: manganese, 0.40 per cent.; phosphorus, 0.04 per cent.; sulphur, 0.05 per cent. Any increase of these constituents is attended by an increase of tensile strength and a diminished ductility. The tensile strength of steel is also affected to some extent by the heat treatment to which it has been subjected. Bessemer or open-hearth acid steel will generally show a higher tensile strength than basic steel, owing to the higher proportion of phosphorus, sulphur, and manganese present.

For convenient distinguishing terms it is customary to classify steel in three grades: "mild or soft," "medium," and "hard," and although the different grades blend into each other, so that no line of distinction exists, in a general sense the grades below 0.15 carbon may be considered as "soft" steel, from 0.15 to 0.30 carbon as "medium," and above that "hard" steel. Each grade has its own advantages for the particular purpose to which it is adapted. The soft steel is well adapted for boiler plate and similar uses, where its high ductility is advantageous. The medium grades are used for general structural purposes, while harder steel is especially adapted for axles and shafts and any service where good wearing surfaces are desired. Mild steel has superior welding property as compared to hard steel, and will endure higher heat without injury. Steel below 0.10 carbon should be capable of doubling flat without fracture, after being chilled from a red heat in cold water. Steel of 0.15 carbon will occasionally submit to the same treatment, but will usually bend around a curve whose radius is equal to the thickness of the specimen; about 90 per cent. of specimens stand the latter bending test without fracture. As the steel becomes harder, its ability to endure this bending test becomes more exceptional, and when the carbon ratio becomes 0.20, little over 25 per cent. of specimens will stand the last-described bending test. Steel having about 0.40 per cent. carbon will usually harden sufficiently to cut soft iron and maintain an edge.

Compression.

When a material is subjected to a compressive load a *crushing* action is produced. This is frequently misunderstood, many assuming that the material is really compressed into a smaller volume than before. As a matter of fact, the only reduction in volume which can be produced is that permitted by the presence of voids in the material, the matter being pressed into the spaces existing in it. Liquids, in which no voids exist, are practically incompressible, while most metals may be materially increased in density under the hammer or the forging press; but it must be understood in all such cases that the increased density is due to the reduction in voids, and not the crowding of the actual particles of the metal closer together.

Crushing, however, is the usual effect of a heavy compressive stress, the material spreading in some other dimension as the yielding occurs along the line of compression. For ductile materials no definite point of rupture can be determined, since the change of shape becomes too great before any sign of rupture appears. Brittle materials, such as cast-iron, stone, bricks, cement, etc., have crushing points which may be more clearly determined. Many materials show a fairly distinct elastic limit under compression, the upsetting being proportional to stress within such limit.

The manner of rupture under crushing is a matter of less importance than the determination of a safe working stress, and this is generally taken as the upsetting or yield point. For brittle materials, in which no such yield point can be determined, the actual crushing load must be used, the safe working load being made a certain proportion of the crushing load.

Shearing.

By shearing is understood the resistance which a material opposes to displacement in a plane. This action rarely, if ever, takes place alone. When a cutting edge begins to shear a bar, for example, true shearing takes place only for a very short distance, the material then bending and flowing down with further pressure, so that with a thick bar the fibres are torn apart before the shearing edge has passed entirely through, and the divided piece falls off, the fracture clearly indicating the combing actions to which it has been subjected. These actions are still more clearly shown by polishing the surface of the metal and etching it to bring out the distortion of the fibres. The relation of the shearing to the tensile strength cannot be expressed as any definite ratio, varying with the materials and their disposition.

Bending.

When a body, such as a beam, is subjected to the action of a force producing deflection, there are reactions at the supports, and if no motion is produced these external forces must be equal to each other, or in equilibrium. In like manner, these external forces are opposed by internal forces acting upon the fibres of the material. In the case of a horizontal beam, the fibres in the upper portion are subjected to compression and those in the lower portion to tension, there being a portion between these where the reversal of stress takes place and where the fibre stress is zero.

In such materials as steel and wrought-iron the resistance to compression and tension may be taken as equal, and this *neutral axis*, as it is called, then coincides with the centre of gravity of the section of the beam. When the beam is of symmetrical section the neutral axis naturally coincides with the centre of figure. If the beam is to resist the external forces, the internal stresses upon its fibres at any point must be equal to the bending moment of the external forces at the same point. The sum of the moments of the internal forces about the neutral axis is called the *moment of resistance.*

This moment of resistance is determined as follows:

Let S be the stress per unit of area in the extreme outer fibre of the cross-section; a, the cross-section of a fibre; y, the distance of any other fibre from the neutral axis. Then the moment of any fibre stress at a distance, y, from the neutral axis will be

$$\frac{S}{v}ay^2;$$

and the sum of all the fibre-stress moments of the cross-section, taken with reference to the neutral axis, is

$$\frac{S}{v}\Sigma\, ay^2.$$

The quantity Σay^2, or the sum of all the elements of the area multiplied by the squares of their respective distances from the neutral axis, is called the *moment of inertia* of the section, and is always symbolized as I, so that we have for the moment of resistance of any section

$$M = \frac{S}{v}I.$$

The value of the moment of inertia depends upon the form of the cross-section; the value of v is also dependent upon the shape of the section, while the value of S, the maximum permissible fibre stress, is governed by the material. These formulas are true only when the material is subjected to strains within the elastic limit, and the value of S should always be chosen within that limit. As a general rule, the maximum fibre stress should not exceed one-half the elastic limit of the material.

Since both I and v depend upon the shape of the section, we may consider them by themselves, and write the moment of resistance

$$M = S\frac{I}{v}.$$

The factor $\frac{I}{v}$, or the moment of inertia divided by the distance of the extreme fibre from the neutral axis, is called by Reuleaux and by Unwin the Section Modulus. It may be called

$$Z = \frac{I}{v}.$$

The radius of gyration of any section may be obtained by taking the square root of the quotient obtained by dividing the moment of inertia by

the area of the section. Thus, if R be the radius of gyration, I the moment of inertia, and A the area of the section, we have

$$R = \sqrt{\frac{I}{A}}.$$

This will be seen to be of use in connection with struts and pillars.

We thus see that an expression for the internal forces in a body subjected to bending stresses—such as a beam—has been obtained, and that it contains but two elements, the fibre stress on the material and the shape of the cross-section of the beam. It is only necessary, therefore, to place this expression for the moment of resistance, $S\frac{I}{v}$, equal to the moment of the external forces, to have their relation fully expressed. Thus, for a cantilever or projecting beam of a length, l, carrying a load, W, at its extremity, we have

$$Wl = S\frac{I}{v}, \quad \text{or} \quad W = \frac{S}{l} \cdot \frac{I}{v}.$$

For a cantilever carrying a load, W, uniformly distributed, the lever arm, l, is one-half as long, and we have

$$W = 2\frac{S}{l} \cdot \frac{I}{v}.$$

For a beam carrying a load, W, in the middle, we have

$$W = 4\frac{S}{l} \cdot \frac{I}{c};$$

and for a beam carrying a load, W, uniformly distributed, we have

$$W = 8\frac{S}{l} \cdot \frac{I}{c}.$$

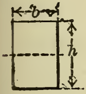
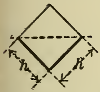
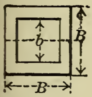
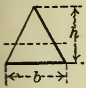
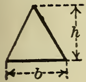
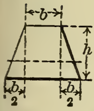
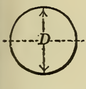
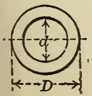
In the preceding formulas W is the load, in pounds, which will produce a fibre stress, S, in pounds; l being the length of the beam, in inches; I, the moment of inertia of the section; and v, the distance of the most remote fibre from the neutral axis. By taking S as about one-half the elastic limit of the material, the proper working load, W, can readily be determined. Good practice takes S at 14,000 pounds for wrought-iron and 16,000 pounds for structural steel. Other values will be tabulated hereafter.

The determination of the value of the moment of inertia for the section used is evidently the principal feature in the problem. Most of the important sections have been reduced to formulas, as in the following tables:

Elements of Usual Sections.

Pencoyd Iron Works.

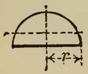
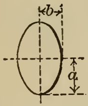
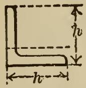
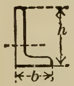
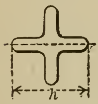
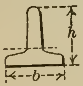
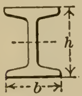
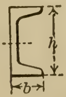
Moments refer to horizontal axis, as shown. This table is intended for convenient application where extreme accuracy is not important. Some of the terms are only approximate; those marked * are correct. Values for radius of gyration in flanged beams apply to standard minimum sections only. A = area of section.

Shape of section.	Moment of inertia.	Section modulus.	Distance of base from centre of gravity.	Least radius of gyration.
	$\frac{bh^3}{12}$ *	$\frac{bh^2}{6}$ *	$\frac{h}{2}$	$\frac{\text{Least side}}{3.46}$ *
	$\frac{h^4}{12}$ *	$0.1178h^3$ *	....	$\frac{h}{3.46}$ *
	$\frac{B^4 - b^4}{12}$ *	$\frac{1}{6}\frac{B^4 - b^4}{B}$ *	$\frac{B}{2}$	$\sqrt{\frac{B^2 + b^2}{12}}$ *
	$\frac{bh^3}{36}$ *	$\frac{bh^2}{24}$ *	$\frac{1}{3}h$	The least of the two: $\frac{h}{4.24}$ or $\frac{b}{4.9}$ *
	$\frac{bh^3}{12}$ *	....	....	....
	$\frac{6b^2 + 6bb_1 + b_1^2}{36(2b + b_1)}h^3$*	....	$\frac{1}{3}\frac{3b + b_1}{2b + b_1}h$	....
	$\frac{AD^2}{16}$ *	$\frac{AD}{8}$ *	$\frac{D}{2}$	$\frac{D}{4}$ *
	$0.0491(D^4 - d^4)$*	$0.0982\frac{D^4 - d^4}{D}$*	$\frac{D}{2}$	$\frac{1}{4}\sqrt{(D^2 + d^2)}$ *

Elements of Usual Sections.

Pencoyd Iron Works.

Moments refer to horizontal axis, as shown. This table is intended for convenient application where extreme accuracy is not important. Some of the terms are only approximate; those marked * are correct. Values for radius of gyration in flanged beams apply to standard minimum sections only. A = area of section.

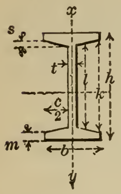
Shape of section.	Moment of inertia.	Section modulus.	Distance of base from centre of gravity.	Least radius of gyration.
	$0.1098r^4$ *	$W_1 = 0.1908r^3$ * $W_2 = 0.2587r^3$	$0.4244r$	$0.0699r^2$ *
	$0.7854ba^3$ *	$0.7854ba^2$ *	....	....
	$\frac{Ah^2}{10.4}$	$\frac{Ah}{7.4}$	$\frac{h}{3.5}$	$\frac{h}{5}$
	$\frac{Ah^2}{9.9}$	$\frac{Ah}{6.7}$	$\frac{h}{3.1}$	$\frac{hb}{2.6(h+b)}$
	$\frac{Ah^2}{19}$	$\frac{Ah}{9.5}$	$\frac{h}{2}$	$\frac{h}{4.74}$
	$\frac{Ah^2}{10.9}$	$\frac{Ah}{7.6}$	$\frac{h}{3.3}$	$\frac{b}{4.66}$
	$\frac{Ah^2}{6.1}$	$\frac{Ah}{3.0}$	$\frac{h}{2}$	$\frac{b}{5.2}$
	$\frac{Ah^2}{6.73}$	$\frac{Ah}{3.3}$	$\frac{h}{2}$	$\frac{b}{3.56}$

Moment of Inertia of Rectangles.

Depth, in inches.	Width of rectangle, in inches.						
	¼	⅜	½	⅝	¾	⅞	1
6	4.50	6.75	9.00	11.25	13.50	15.75	18.00
7	7.15	10.72	14.29	17.86	21.44	25.01	28.58
8	10.67	16.00	21.33	26.67	32.00	37.33	42.67
9	15.19	22.78	30.38	37.97	45.56	53.16	60.75
10	20.83	31.25	41.67	52.08	62.50	72.92	83.33
11	27.73	41.59	55.46	69.32	83.18	97.06	110.92
12	36.00	54.00	72.00	90.00	108.00	126.00	144.00
13	45.77	68.66	91.54	114.43	137.31	160.20	183.08
14	57.17	85.75	114.33	142.92	171.50	200.08	228.67
15	70.31	105.47	140.63	175.78	210.94	246.09	281.25
16	85.33	128.00	170.67	213.33	256.00	298.67	341.33
17	102.35	153.53	204.71	255.89	307.06	358.24	409.42
18	121.50	182.25	243.00	303.75	364.50	425.25	486.00
19	142.90	214.34	285.79	357.24	428.68	500.14	571.58
20	166.67	250.00	333.33	416.67	500.00	583.33	666.67
21	192.94	289.41	385.88	482.34	578.81	675.28	771.75
22	221.83	332.75	443.67	554.58	665.50	776.42	887.33
23	253.48	380.22	506.96	633.70	760.44	887.18	1013.92
24	288.00	432.00	576.00	720.00	864.00	1008.00	1152.00
25	325.52	488.28	651.04	813.80	976.56	1139.32	1302.08
26	366.17	549.25	732.33	915.42	1098.50	1281.58	1464.67
27	410.06	615.09	820.13	1025.16	1230.19	1435.22	1640.25
28	457.33	686.00	914.67	1143.33	1372.00	1600.67	1829.33
29	508.10	762.16	1016.21	1270.26	1524.31	1778.36	2032.42
30	562.50	843.75	1125.00	1406.25	1687.50	1968.75	2250.00
31	620.65	930.97	1241.30	1551.62	1861.94	2172.26	2482.60
32	682.67	1024.00	1365.33	1706.67	2048.00	2389.33	2730.67
33	748.69	1123.03	1497.38	1871.72	2246.06	2620.40	2994.76
34	818.83	1228.25	1637.67	2047.08	2456.50	2865.92	3275.33
35	893.23	1339.84	1786.46	2233.07	2679.68	3126.30	3572.92
36	972.00	1458.00	1944.00	2430.00	2916.00	3402.00	3888.00
37	1055.27	1582.90	2110.54	2638.17	3165.80	3693.44	4221.08
38	1143.17	1714.75	2286.33	2857.92	3429.50	4001.08	4572.67
39	1235.81	1853.72	2471.62	3089.53	3707.44	4325.34	4943.24
40	1333.33	2000.00	2666.67	3333.33	4000.00	4666.67	5333.33

Moments of Inertia of Standard Sections.

Pencoyd Iron Works.



When not otherwise specified, the inertia is the greatest around centre of gravity, or for horizontal axis in figures.

A = total area of section.

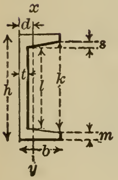
I Beam Section.

s = taper of flange. $\qquad l = k - \frac{2s}{3}$.

$$I = \frac{bh^3 - ck^3}{12} + \frac{cs^3}{18} + \frac{csl^2}{4}.$$

$$I, \text{ axis } xy = \frac{mb^3}{6} + \frac{kt^3}{12} + \frac{s\left(\frac{b-t}{2}\right)^3}{9} + 2s\left(\frac{b-t}{2}\right)\left(\frac{b}{6} + \frac{t}{3}\right)^2.$$

Channel Section.



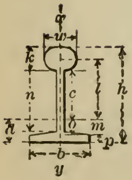
s = taper of flange. $\qquad r = \frac{s}{b-t}$.

$$I = \frac{bh^3 - \frac{1}{8r}(k^4 - l^4)}{12}.$$

$$I, \text{ axis } xy = \frac{2mb^3 + lt^3 + \frac{r}{2}(b^4 - t^4)}{3} - Ad^2.$$

$$d = \frac{mb^2 + \frac{kt^2}{2} + \frac{s}{3}(b-t)(b+2t)}{A}.$$

Deck Beam Section.



s = taper of flange. $\qquad a$ = area of bulb.

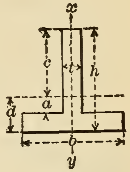
$$o = m - \frac{s}{3}.$$

$$I = \frac{aw^2}{15} + al^2 + \frac{tc^3}{3} + \frac{bd^3}{3} - \frac{m^3(b-t)}{3} + \frac{(b-t)s^3}{36} + \frac{s(b-t)o^2}{2}.$$

$$I, \text{ axis } xy = \frac{ak^2}{12.4} + \frac{nt^3}{12} + \frac{\left(p + \frac{s}{4}\right)b^3}{12}.$$

$$d = \frac{a(2h-k) + t(h-k)^2 + (b-t)p^2 + s(b-t)\left(p + \frac{s}{3}\right)}{2A}.$$

Tee Section.

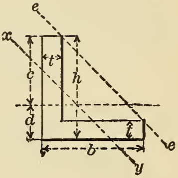


$$I = \frac{tc^3 + bd^3 - (b - t)a^3}{3}.$$

$$I, \text{ axis } xy = \frac{fb^3 + (h - f)t^3}{12}.$$

$$d = \frac{bf^2 + t(h^2 - f^2)}{2A}.$$

Angle Section.



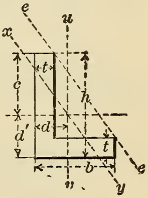
$$I = \frac{tc^3 + bd^3 - (b - t)(d - t)^3}{3}, \text{ for even or uneven angles.}$$

$$I, \text{ axis } uv = \frac{t(b - d_1)^3 + hd_1^3 - (h - t)(d_1 - t)^3}{3}, \text{ for uneven angles.}$$

xy passes through centre of gravity parallel to ee.

$$I, \text{ axis } xy = \frac{2d^4 - 2(d - t)^4 + t\left[b - \left(2d - \frac{t}{2}\right)\right]^3}{3}, \text{ for even angles.}$$

A close approximation for the latter is the following:



$$I, \text{ axis } xy = \frac{Ab^2}{25}, \text{ for even angles.}$$

$$I, \text{ axis } xy = \frac{Ah^2b^2}{13(h^2 + b^2)}, \text{ for uneven angles.}$$

$$d = \frac{bt^2 + t(h^2 - t^2)}{2A}, \text{ for even and uneven angles.}$$

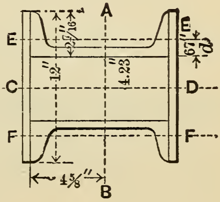
$$d' = \frac{ht^2 + t(b^2 - t^2)}{2A}, \text{ for uneven angles.}$$

In uneven angles the distance from centre of gravity in direction of the long leg exceeds that in the direction of the short leg by half the difference in the length of the two legs.

In angles and tees of equal legs and thickness

$$d = \tfrac{1}{4}\left(b + \frac{3}{2}t\right), \text{ nearly.}$$

Inertia of Compound Shapes.



"The moment of inertia of any section about any axis is equal to the I about a parallel axis passing through its centre of gravity + the area of the section multiplied by the square of the distance between the axes."

By use of this rule, the moments of inertia or radii of gyration of any single sections being known, corresponding values can readily be obtained for any combination of these sections.

Example 1. A combination of two 9-inch channels of 3.89 square inches section and two 12″ × ¼″ plates, as shown.

Axis AB of Section.

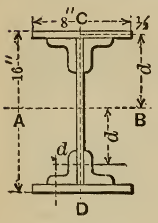
I for two channels, column VI., page 362,		$= 95.78$
I for two plates $= \dfrac{12 \times .25^3}{12} \times 2 =$	$.03125$	
6 (area of plates) $\times 4\frac{5}{8}^2$	$= 128.34375$	
		128.375
I for combined section		$= 224.155$

which, divided by area (13.78), gives $16.27 = R^2$ or 4.03 radius of combined section.

Axis CD.

Find distance, $d = (.60)$, from column XII., page 363, then obtaining the distance (4.17) between axes CD and EF.

I for two channels around axis EF from column VI. $=$	3.54
Area of channels $\times$ square of distance $= 7.78 \times 4.17^2 =$	135.286
I for two plates $= \dfrac{.5 \times 12^3}{12}$	$= 72.000$
I for combined section	$= 210.826$



Radius of gyration $= \sqrt{\dfrac{210.826}{13.78}} = 3.91.$

By similar methods inertia or radius of gyration for any combination of shapes can readily be obtained.

Example 2. A "built-up beam" composed of

4 angles $3'' \times 3'' \times \frac{1}{4}''$.
2 plates $8'' \times \frac{1}{2}''$.
1 plate $15'' \times \frac{3}{8}''$.

Axis AB.

I of two $8'' \times \frac{1}{2}''$ plates $= \dfrac{8 \times \frac{1}{2}^3}{12} \times 2$	$=$	$.167$	
$+ 8$ (area) $\times 7\frac{3}{4}^2$ (square of distance, d)	$=$	480.500	
			480.667
I of one $15'' \times \frac{3}{8}''$ plate $= \dfrac{15^3 \times \frac{3}{8}}{12}$	$=$		105.469
I of four $3'' \times 3'' \times \frac{1}{4}''$ angles $= 4 \times 1.25$	$=$	5.000	
$+ 5.76$ (area) $\times 6.66^2$ (square of distance, d^1)	$=$	255.488	
			260.488
Inertia of combined section around AB	$=$		846.624

Radius of gyration $= \sqrt{\dfrac{846.624}{19.385}} = 6.61.$

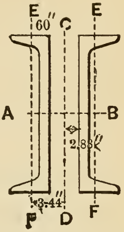
Radius of Gyration of Compound Shapes.

In the case of a pair of any shape without a web the value of R can always be readily found without considering the moment of inertia.

The radius of gyration for any section around an axis parallel to another axis passing through its centre of gravity is found as follows:

Let r = radius of gyration around axis through centre of gravity; R = radius of gyration around another axis parallel to above; d = distance between axis.

$$R = \sqrt{d^2 + r^2}.$$



When r is small, R may be taken as equal to d without material error. Thus, in the case of a pair of channels latticed together, or a similar construction.

Example 1. Two 9″ channels of 3.89 square inches section placed 5.68″ apart; required the radius of gyration around axis CD for combined section.

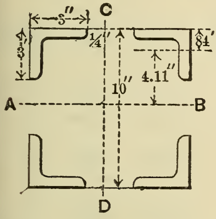
Find in column X., page 362, $r = .67$ and $r^2 = 0.45$.

Find distance from base of channel to neutral axis, same page, $= .60$, this added to one-half the distance between the two bars, $2.84'' = 3.44''\ d$, and $d^2 = 11.8336$.

Radius of gyration of the pair as placed =

$$\sqrt{11.8336 + 0.45} = 3.505.$$

The value of R for the whole section in relation to the axis AB is the same as for the single channel, to be found in the tables.



Example 2. Four $3'' \times 3'' \times \frac{1}{4}''$ angles, placed as shown, form a column of 10 inches square; required the radius of gyration.

Find in column VIII., page 381, $r = .93$ and $r^2 = .8649$.

Find distance from side of angle to neutral axis, same page, $= .84$. Subtract this from half the width of column $= 5 - .84 = 4.16 = d$, or distance between two axes. $d^2 = 17.3056$.

Radius of gyration of four angles as placed =

$$\sqrt{17.3056 + .8649} = 4.26.$$

When the angles are large, as compared with the outer dimensions of the combined section, the radius of gyration can be taken without serious error from the table of radii of gyration for square columns, on page 353.

Elements of Pencoyd Structural Shapes.

In the following tables various fundamental properties of rolled sections are given, whereby the strength or stiffness of each can be readily determined.

The calculations are made for the least and greatest thickness of the various shapes; intermediate thicknesses of these can be approximated by interpolation.

Moments of Inertia for the sections are obtained as hereafter described.

Radius of Gyration, equal to $\sqrt{\frac{\text{Inertia}}{\text{area}}}$, is used for determining the resistance of struts or columns.

Section Modulus, equal to $\frac{\text{Inertia}}{\text{distance from axis to extreme fibres}}$, is used for determining transverse strength in beams, etc.

Coefficient for Safe Load is the calculated load, in net tons, on a beam one foot between supports, that produces fibre strains of 16,000 pounds per square inch. A corresponding load for any beam is found by dividing this coefficient by the length of span in feet.

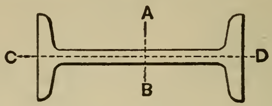
Coefficients for Deflection are based on a modulus of elasticity of 28,000,000 pounds. They apply to beams one foot long, bearing one ton (2000 pounds). The deflection of any beam, in inches, is found by multiplying its coefficient by the load in net tons and by the cube of the length in feet.

Maximum Load, in Net Tons, indicates the greatest load that a beam, however short, should carry, unless its web is reinforced to prevent crippling. This load is obtained by the formula:

$$W = \frac{2xdt}{1 + \frac{l^2}{3000t^2}}$$

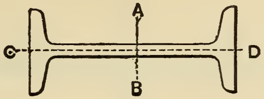
$x = 8$ tons.
$d =$ depth of beam.
$t =$ thickness of web.
$l = d \times$ secant 45° ($l^2 = 2d^2$).

Elements of Pencoyd Beams.



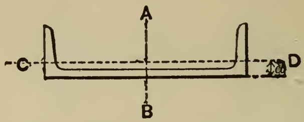
I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Size, in inches.	Section number.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.		Square of radius of gyration.	
				Axis *AB*.	Axis *CD*.	Axis *AB*.	Axis *CD*.
24	240B	23.53	80.0	2111.40	42.84	89.73	1.82
24	244B	29.42	100.0	2497.30	57.53	84.88	1.96
20	200B	19.10	65.0	1179.71	27.72	61.76	1.45
20	207B	29.42	100.0	1649.55	55.57	56.07	1.89
18	180B	16.13	55.0	809.05	21.17	50.16	1.31
18	187B	26.46	90.0	1187.99	46.03	44.90	1.74
15	150B	12.35	42.0	443.71	14.43	35.93	1.17
15	158B	23.54	80.0	773.84	40.69	32.87	1.73
12	120B	9.27	31.5	218.71	9.45	23.59	1.02
12	127B	19.12	65.0	403.48	28.93	21.10	1.51
10	100B	7.34	25.0	123.07	6.81	16.77	0.93
10	103B	11.75	40.0	175.48	12.36	14.93	1.05
9	90B	6.17	21.0	84.94	5.06	13.77	0.82
9	93B	10.30	35.0	112.76	7.25	10.95	0.70
8	80B	5.29	18.0	57.36	3.72	10.84	0.70
8	83B	7.50	25.5	69.14	4.70	9.22	0.63
7	70B	4.42	15.0	36.61	2.64	8.28	0.60
7	72B	5.88	20.0	42.55	3.20	7.24	0.54
6	60B	3.60	12.25	22.09	1.83	6.14	0.51
6	68B	7.03	23.90	41.98	7.89	5.97	1.12
		to	to				
6	68B	8.15	27.70	45.36	8.99	5.57	1.10
5	50B	2.87	9.75	12.12	1.21	4.22	0.42
5	52B	4.34	14.75	15.18	1.67	3.50	0.39
4	40B	2.20	7.50	5.90	0.76	2.68	0.34
4	43B	3.08	10.50	7.07	1.00	2.30	0.32
3	30B	1.62	5.50	2.43	0.45	1.50	0.28
3	32B	2.20	7.50	2.87	0.59	1.30	0.27

Elements of Pencoyd Beams.



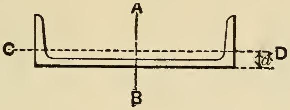
IX.	X.	XI.	XII.	XIII.	XIV.	XV.	IV.	I.
Radius of gyration.		Section modulus.	Coefficient for greatest safe load, in net tons.	Coefficient for deflection.		Maximum load, in net tons.	Weight per foot, in pounds.	Size, in inches.
Axis *AB*.	Axis *CD*.	Axis *AB*.		Distributed load.	Centre load.			
9.47	1.35	176.0	938.4	.000 00076	.000 00122	75.8	80.0	24
9.21	1.40	208.1	1109.9	.000 00064	.000 00103	143.4	100.0	24
7.86	1.20	118.0	629.2	.000 00137	.000 00217	74.2	65.0	20
7.49	1.37	165.0	889.8	.000 00097	.000 00155	184.8	100.0	20
7.08	1.14	89.9	479.4	.000 00198	.000 00317	65.6	55.0	18
6.70	1.32	132.0	712.9	.000 00135	.000 00216	178.2	90.0	18
5.99	1.08	59.2	315.5	.000 00357	.000 00578	47.6	42.0	15
5.73	1.32	103.2	550.3	.000 00207	.000 00331	162.6	80.0	15
4.86	1.01	36.5	194.4	.000 00727	.000 01172	35.6	31.5	12
4.59	1.23	67.3	358.7	.000 00397	.000 00635	134.4	65.0	12
4.10	0.96	24.6	131.3	.000 0129	.000 0208	27.0	25.0	10
3.86	1.03	35.1	187.2	.000 0091	.000 0146	78.8	40.0	10
3.71	0.91	18.9	100.7	.000 0185	.000 0302	21.2	21.0	9
3.31	0.84	25.1	133.6	.000 0142	.000 0227	93.8	35.0	9
3.29	0.84	14.3	76.5	.000 0275	.000 0447	19.4	18.0	8
3.04	0.79	17.3	92.2	.000 0231	.000 0371	58.8	25.5	8
2.88	0.78	10.5	55.8	.000 0433	.000 0700	17.2	15.0	7
2.69	0.74	12.2	64.8	.000 0376	.000 0603	43.2	20.0	7
2.48	0.71	7.4	39.3	.000 0717	.000 1161	13.8	12.25	6
2.44	1.06	14.0	74.6	.000 0370	.000 0591	30.8	23.90	6
2.36	1.05	15.1	80.6	.000 0342	.000 0547	50.2	27.70	6
2.05	0.65	4.9	25.9	.000 1305	.000 2115	11.0	9.75	5
1.87	0.62	6.1	32.4	.000 1054	.000 1689	36.8	14.75	5
1.64	0.58	3.0	15.7	.000 2671	.000 4346	8.2	7.50	4
1.52	0.57	3.5	18.9	.000 2263	.000 3627	23.4	10.50	4
1.23	0.53	1.6	8.6	.000 6452	.001 0552	5.4	5.50	3
1.14	0.52	1.9	10.2	.000 5575	.000 8934	15.6	7.50	3

Elements of Pencoyd Channels.



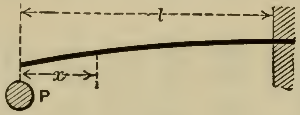
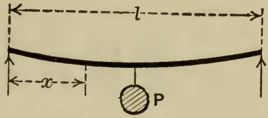
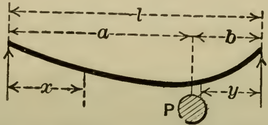
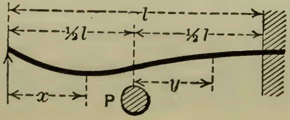
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Size, in inches.	Section number.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.		Square of radius of gyration.		Radius of gyration.	
				Axis *AB.*	Axis *CD.*	Axis *AB.*	Axis *CD.*	Axis *AB.*	Axis *CD.*
15	150C	9.69	33.0	311.21	3.10	32.12	0.84	5.67	0.91
15	155C	16.17	55.0	469.85	17.20	29.06	1.06	5.39	1.03
13	130C	9.39	31.9	238.26	11.48	25.38	1.22	5.04	1.11
		to	to						
13	130C	14.27	48.5	306.25	16.22	21.47	1.14	4.63	1.07
12	120C	6.02	20.5	129.27	3.90	21.47	0.65	4.63	0.81
12	128C	6.01	20.5	123.98	3.10	20.63	0.52	4.54	0.72
		to	to						
12	128C	9.40	32.0	164.30	4.42	17.48	0.47	4.18	0.69
10	100C	4.41	15.0	67.11	2.28	15.22	0.52	3.90	0.72
10	104C	10.29	35.0	124.61	5.99	12.11	0.58	3.48	0.76
9	90C	3.89	13.25	47.89	1.77	12.31	0.45	3.51	0.67
9	95C	5.98	20.30	70.21	3.99	11.65	0.66	3.41	0.81
		to	to						
9	95C	8.23	28.00	85.40	5.17	10.32	0.63	3.21	0.79
8	80C	3.31	11.25	32.51	1.32	9.82	0.40	3.13	0.63
8	84C	6.25	21.25	51.85	2.97	8.30	0.48	2.88	0.69
7	70C	2.86	9.75	21.37	0.98	7.47	0.34	2.73	0.59
7	74C	5.81	19.75	35.85	2.49	6.17	0.43	2.48	0.66
6	60C	2.35	8.00	13.07	0.69	5.56	0.29	2.36	0.54
6	65C	4.46	15.10	25.15	5.20	5.64	1.17	2.38	1.08
5	50C	1.91	6.50	7.37	0.47	3.86	0.25	1.96	0.50
5	52C	3.38	11.50	10.43	0.82	3.09	0.24	1.76	0.49
4	40C	1.54	5.25	3.74	0.32	2.43	0.21	1.56	0.45
4	42C	2.13	7.25	4.52	0.44	2.12	0.21	1.46	0.46
3	30C	1.18	4.00	1.61	0.20	1.36	0.17	1.17	0.41
3	32C	1.76	6.00	2.05	0.31	1.16	0.18	1.07	0.42
2¼	22C	1.12	3.80	0.80	0.19	0.71	0.17	0.85	0.42
2	20C	0.87	2.90	0.48	0.08	0.55	0.10	0.74	0.31
2	20C	1.06	3.60	0.54	0.11	0.51	0.10	0.71	0.32
1¾	17C	0.33	1.13	0.15	0.01	0.46	0.03	0.67	0.16

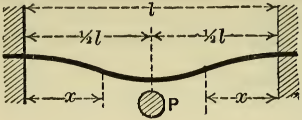
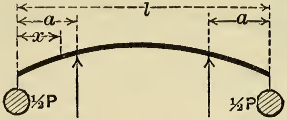
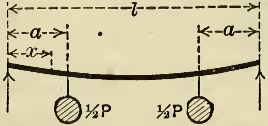
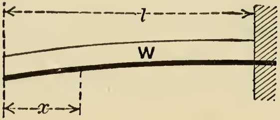
Elements of Pencoyd Channels.



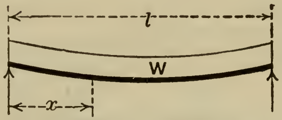
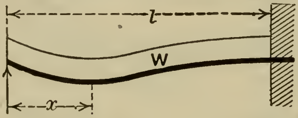
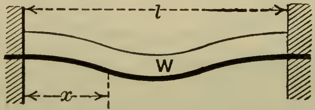
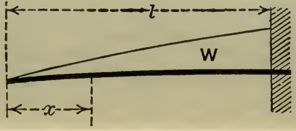
XI.	XII.	XIII.	XIV.	XV.	XVI.	I.
Distance, *d*, from base to neutral axis.	Section modulus.	Coefficient for greatest safe load, in net tons.	Coefficient for deflection.		Maximum load, in net tons.	Size, in inches.
	Axis *AB*.		Distributed load.	Centre load.		
0.79	41.5	221.3	.000 00514	.000 00826	45.0	15
0.95	62.7	334.1	.000 00340	.000 00546	135.4	15
1.01	36.7	195.5	.000 00651	.000 01042	43.4	13
0.97	47.1	251.3	.000 00507	.000 00811	130.0	13
0.70	21.6	114.9	.000 01237	.000 01986	23.4	12
0.62	20.7	110.2	.000 01290	.000 02072	24.2	12
0.62	27.4	146.0	.000 00974	.000 01564	82.4	12
0.64	13.4	71.6	.000 02384	.000 03838	16.4	10
0.76	24.9	132.9	.000 01284	.000 02067	106.8	10
0.60	10.6	56.8	.000 03341	.000 05379	15.6	9
0.74	15.6	83.2	.000 02210	.000 03536	39.0	9
0.75	19.0	101.2	.000 01817	.000 02907	79.0	9
0.57	8.1	43.4	.000 04921	.000 07923	13.6	8
0.66	13.0	69.1	.000 03086	.000 04968	57.0	8
0.54	6.1	32.6	.000 07487	.000 12054	13.2	7
0.65	10.2	54.6	.000 04463	.000 07185	57.8	7
0.51	4.4	23.2	.000 12242	.000 19709	10.8	6
1.07	8.4	44.7	.000 06170	.000 09872	28.2	6
0.49	3.0	15.7	.000 21710	.000 34953	9.2	5
0.50	4.2	22.3	.000 15340	.000 24697	21.4	5
0.46	1.9	10.0	.000 42781	.000 68877	8.2	4
0.46	2.3	12.1	.000 35398	.000 56991	18.4	4
0.43	1.1	5.7	.000 99377	.001 59997	6.0	3
0.45	1.4	7.3	.000 78050	.001 25660	16.0	3
0.47	0.7	3.8	.002 00000	.003 22000	8.6	2¼
0.36	0.5	2.6	.003 33333	.005 36666	6.6	2
0.37	0.5	2.9	.002 96980	.004 78138	9.6	2
0.18	0.2	0.9	.010 66672	.017 17342	2.0	1¾

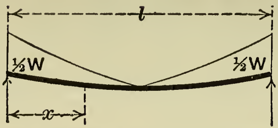
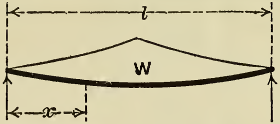
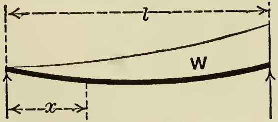
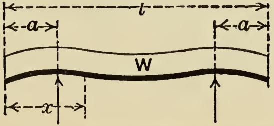
Bending Moments, etc., for Beams of Uniform Section.

Mode of loading. Lengths, in inches; loads, in pounds.		Bending moment, in inch-pounds.	Maximum load, in pounds.	Deflection, in inches.	Remarks.
One end firmly fixed, other end loaded.		Px Maximum when $x = l$	$\frac{SQ}{l}$	$\frac{Pl^3}{3EI}$	Weakest section at right support.
Supported at both ends, loaded at centre.		$\frac{Px}{2}$ Maximum $= \frac{Pl}{4}$	$\frac{4SQ}{l}$	$\frac{Pl^3}{48EI}$	Weakest section at centre of beam.
Supported at both ends, loaded any place.		For the left side, $\frac{Pbx}{l}$ For the right side, $\frac{Pay}{l}$	$\frac{lSQ}{ab}$	$\frac{Pab(2l-a)\sqrt{3a(2l-a)}}{27lEI}$	Weakest section at point of application of load.
One end fixed, other end supported, loaded at centre.		For the left side, $\frac{5Px}{16}$ For the right side, $Pl\left(\frac{5}{32} - \frac{11y}{16l}\right)$	$\frac{16SQ}{3l}$	$\frac{3Pl^3}{322EI}$	Weakest section at right support.

Both ends fixed, load at centre.		$\frac{Pl}{2}\left(\frac{x}{l}-\frac{1}{4}\right)$ Maximum $=\frac{Pl}{8}$	$\frac{8SQ}{l}$	$\frac{Pl^3}{192EI}$	Weakest sections at either support and at centre.
Loaded at each end, two supports between ends.		$\frac{Px}{2}$ Maximum $=\frac{Pa}{2}$	$\frac{2SQ}{a}$	For overhang, $\frac{Pa}{12EI}(3al-4a^2)$ For part between supports, $\frac{Pa}{16EI}(l-2a)^2$	Weakest sections at either support and at all points between supports.
Both ends supported, two symmetrical loads.		$\frac{Px}{2}$ Maximum $=\frac{Pa}{2}$	$\frac{2SQ}{a}$	$\frac{Pa}{48EI}(3l^2-4a^2)$	Weakest sections at points of application of loads and at all points between loads.
One end fixed, load uniformly distributed.		$\frac{Wx^2}{2l}$ Maximum $=\frac{Wl}{2}$	$\frac{2SQ}{l}$	$\frac{Wl^3}{8EI}$	Weakest section at right support.

Bending Moments, etc., for Beams of Uniform Section.

Mode of loading. Lengths, in inches; loads, in pounds.		Bending moment, in inch-pounds.	Maximum load, in pounds.	Deflection, in inches.	Remarks.
Both ends supported, load uniformly distributed.		$\frac{Wx}{2}\left(1-\frac{x}{l}\right)$ Maximum $=\frac{Wl}{8}$	$\frac{8SQ}{l}$	$\frac{5Wl^3}{384EI}$	Weakest section at centre.
One end supported, other end fixed, load uniformly distributed.		$\frac{Wx}{2}\left(\frac{3}{4}-\frac{x}{l}\right)$ Maximum $=\frac{Wl}{8}$	$\frac{8SQ}{l}$	$\frac{5Wl^3}{926EI}$	Weakest section at right support.
Both ends fixed, load uniformly distributed.		$\frac{Wl}{2}\left(\frac{x}{l}-\frac{x^2}{l^2}-\frac{1}{6}\right)$ Maximum $=\frac{Wl}{12}$	$\frac{12SQ}{l}$	$\frac{Wl^3}{384EI}$	Weakest section at either support.
One end fixed, load distributed, increasing uniformly towards the fixed end.		$\frac{Wx^3}{3l^2}$ Maximum $=\frac{Wl}{3}$	$\frac{3SQ}{l}$	$\frac{Wl^3}{15EI}$	Weakest section at right support.

Both ends supported, load distributed, decreasing uniformly towards the centre.		$Wx\left(\frac{1}{2}-\frac{x}{l}+\frac{2x^2}{3l^2}\right)$ Maximum $=\frac{Wl}{12}$	$\frac{12SQ}{l}$	$\frac{3Wl^3}{320EI}$	Weakest section at centre of span.
Both ends supported, load distributed, increasing uniformly towards the centre.		$Wx\left(\frac{1}{2}-\frac{2x^2}{3l^2}\right)$ Maximum $=\frac{Wl}{6}$	$\frac{6SQ}{l}$	$\frac{Wl^3}{60EI}$	Weakest section at centre of span.
Both ends supported, load distributed, increasing uniformly towards one end.		$\frac{Wx}{3}\left(1-\frac{x^2}{l^2}\right)$ Maximum $=\frac{104Wl}{810}$	$\frac{810SQ}{104l}$	$\frac{47Wl^3}{3600EI}$	Weakest section at $x=0.52l$.
Two symmetrical supports, load uniformly distributed.		At either support, $\frac{Wa^2}{2l}$ At centre of span, $\frac{W}{2}\left(a-\frac{l}{4}\right)$	The supporting power varies with the relation of a to l, and becomes a maximum when $a = 0.207l$, in which case Maximum bending moment $=\frac{3Wl}{140}$; Maximum load $=\frac{140SQ}{3l}$.		

Thrust.

Bodies subjected to thrust, such as columns, struts, etc., generally fail by bending sideways,—this showing the practical impossibility of maintaining the thrust in the exact axial line. As in the case of beams, the shape of the cross-section of the column is an important element in the supporting power, but with this must be considered the length and the manner in which the ends are held.

The cross-section is best represented by the least radius of gyration, usually indicated by r. The length of the column or strut being taken as l, in inches, we have the ratio, $\frac{l}{r}$, as representing the proportions of the column. The manner of supporting the ends are classified according to the extent to which the column is secured and the degree to which it is maintained in the line of the thrust.

Owing to the complex nature of the stresses in columns it is difficult to determine the maximum fibre stresses, and the various formulas which have been devised are the consequence of attempts to embody the results of experimental investigations. These have been conducted to determine the crippling loads required for the various conditions, the safe load then being taken as a certain portion of the crippling load, the latter being divided by a so-called factor of safety.

The following discussion of the subject, prepared by Mr. James Christie to accompany the tabulated results of his experiments for the Pencoyd Iron Company, represent standard current practice.

Struts are generally classified in four divisions, with respect to the manner in which the ends are secured,—viz., "fixed-ended," "flat-ended," "hinged-ended," and "round-ended."

In the class of "fixed ends" the struts are supposed to be so rigidly attached at both ends to the contiguous parts of the structure that the attachment would not be severed if the member was subjected to the ultimate load. "Flat-ended" struts are supposed to have their ends flat and normal to the axis of length, but not rigidly attached to the adjoining parts. "Hinged ends" embrace the class which have both ends properly fitted with pins or ball and socket joints of substantial dimensions, as compared with the section of the strut, the centres of these end joints being practically coincident with an axis passing through the centre of gravity of the section of the strut. "Round-ended" struts are those which have only central points of contact, such as balls or pins resting on flat plates, but still the centres of the balls or pins coincident with the proper axis of the strut.

If in hinged-ended struts the balls or pins are of comparatively insignificant diameter, it will be safest in such cases to consider the struts as round-ended.

If there should be any serious deviation of the centres of round or hinged ends from the proper axis of the strut there will be a reduction of resistance that cannot be estimated without knowing the exact conditions.

When the pins of hinged-end struts are of substantial diameter, well fitted and exactly centred, experiment shows that the hinged-ended will be equally as strong as flat-ended struts. But a very slight inaccuracy of the centring rapidly reduces the resistance to lateral bending, and, as it is almost impossible in practice to uniformly maintain the rigid accuracy required, it is considered best to allow for such inaccuracies to the extent given in the tables, which are the average of many experiments.

It is considered good practice to increase the factors of safety as the length of the strut is increased, owing to the greater inability of the long struts to resist cross strains, etc. For similar reasons it is considered advisable to increase the factor of safety for hinged and round ends in a greater ratio than for fixed or flat ends.

Presuming that one-third of the ultimate load would constitute the greatest safe load for the shortest struts, the following progressive factors of safety are adopted for the increasing lengths:

$$3 + .01\,\frac{l}{r} \text{ for flat and fixed ends.}$$

$$3 + .015 \frac{l}{r} \text{ for hinged and round ends.}$$

l = length of strut. r = least radius of gyration.

From the above we derive the following factors of safety:

$\frac{l}{r}$	Fixed and flat ends.	Hinged and round ends.	$\frac{l}{r}$	Fixed and flat ends.	Hinged and round ends.	$\frac{l}{r}$	Fixed and flat ends.	Hinged and round ends.
20	3.2	3.30	110	4.1	4.65	200	5.0	6.00
30	3.3	3.45	120	4.2	4.80	210	5.1	6.15
40	3.4	3.60	130	4.3	4.95	220	5.2	6.30
50	3.5	3.75	140	4.4	5.10	230	5.3	6.45
60	3.6	3.90	150	4.5	5.25	240	5.4	6.60
70	3.7	4.05	160	4.6	5.40	250	5.5	6.75
80	3.8	4.20	170	4.7	5.55	260	5.6	6.90
90	3.9	4.35	180	4.8	5.70	270	5.7	7.05
100	4.0	4.50	190	4.9	5.85	280	5.8	7.20

Cast-iron Columns.

Cast-iron columns are sometimes used in buildings of moderate height, but their use is not to be recommended for buildings where the iron framework must be rigid and afford sufficient lateral stability. The manner in which cast-iron columns are connected together and the mode of attaching beams and girders to them does not permit of obtaining sufficient rigidity for such buildings. Cast-iron columns have more or less internal strains, due to the unequal cooling of the metal in the moulds, which makes it necessary to employ a large factor of safety. No cast-iron column should be used in a building with a factor of safety less than 8. Particular attention should be paid to the designing of the cast-iron brackets for supporting the beams and girders, in order that they may not be subjected to large internal strains, making them liable to break off under a sudden shock. The tables on pages 391 and 392 furnish an easy method of determining the safe loads on round and square cast-iron columns. Where the loads are eccentrically applied, producing bending strains in the columns, cast-iron columns are inadmissible, because of their inability to resist such strains.

TABLE No. 1.

Struts of Wrought-iron or Extreme Soft Steel.

Destructive pressure, in pounds, per square inch.

Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	46000	46000	46000	44000
30	43000	43000	43000	40250
40	40000	40000	40000	36500
50	38000	38000	38000	33500
60	36000	36000	36000	30500
70	34000	34000	33750	27750
80	32000	32000	31500	25000
90	31000	30900	29750	22750
100	30000	29800	28000	20500
110	29000	28050	26150	18500
120	28000	26300	24300	16500
130	26750	24900	22650	14650
140	25500	23500	21000	12800
150	24250	21750	18750	11150
160	23000	20000	16500	9500
170	21500	18400	14650	8500
180	20000	16800	12800	7500
190	18750	15650	11800	6750
200	17500	14500	10800	6000
210	16250	13600	9800	5500
220	15000	12700	8800	5000
230	14000	11950	8150	4650
240	13000	11200	7500	4300
250	12000	10500	7000	4050
260	11000	9800	6500	3800
270	10500	9150	6100	3500
280	10000	8500	5700	3200
290	9500	7850	5350	3000
300	9000	7200	5000	2800
310	8500	6600	4750	2650
320	8000	6000	4500	2500
330	7500	5550	4250	2300
340	7000	5100	4000	2100
350	6750	4700	3750	2000
360	6500	4300	3500	1900
370	6150	3900	3250	1800
380	5800	3500	3000	1700
390	5500	3250	2750	1600
400	5200	3000	2500	1500

TABLE No. 2.

Struts of Wrought-iron or Extreme Soft Steel.

Greatest safe load, in pounds per square inch of cross-section, for vertical struts. Both ends are supposed to be secured as indicated at the head of each column.

If both ends are not secured alike, take a mean proportional between the values given for the classes to which each end belongs.

If the strut is hinged by any uncertain method, so that the centres of pins and axis of strut may not coincide, or the pins may be relatively small and loosely fitted, it is best in such cases to consider the strut as "round-ended."

Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	14380	14380	13940	13330
30	13030	13030	12460	11670
40	11760	11760	11110	10140
50	10860	10860	10130	8930
60	10000	10000	9230	7820
70	9190	9190	8330	6850
80	8420	8420	7500	5950
90	7950	7920	6840	5230
100	7500	7450	6220	4560
110	7070	6840	5620	3980
120	6670	6260	5060	3440
130	6220	5790	4580	2960
140	5800	5340	4120	2510
150	5390	4830	3570	2120
160	5000	4350	3060	1760
170	4570	3920	2640	1530
180	4170	3500	2250	1310
190	3830	3190	2020	1150
200	3500	2900	1800	1000
210	3190	2670	1590	890
220	2880	2440	1400	790
230	2640	2250	1260	720
240	2410	2070	1140	650
250	2180	1910	1040	600
260	1960	1750	940	550
270	1840	1610	870	500
280	1720	1460	790	440
290	1610	1330	730	410
300	1500	1200	670	370
310	1390	1080	620	350
320	1290	970	580	320
330	1190	880	540	290
340	1090	800	490	260
350	1040	720	450	240
360	980	650	420	230
370	920	580	380	210
380	850	510	340	200
390	800	470	310	80
400	740	430	280	70

TABLE No. 3.

Struts of Medium Steel.

Destructive pressure, in pounds per square inch, for steel of medium grade, tensile strength about 70,000 pounds per square inch.
For extreme soft steel, use Table No. 1.

Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	70000	70000	70000	66900
30	51000	51000	51000	47700
40	46000	46000	46000	41900
50	44000	44000	44000	38800
60	42000	42000	42000	35600
70	40000	40000	39700	32600
80	38000	38000	37400	29700
90	36100	36000	34700	26500
100	34200	34000	31900	23400
110	33100	32000	29800	21100
120	31900	30000	27700	18800
130	30100	28000	25500	16500
140	28200	26000	23200	14200
150	26800	24000	20700	12300
160	25300	22000	18100	10400
170	23400	20000	15900	9240
180	21400	18000	13700	8030
190	19400	16200	12200	6990
200	17900	14800	11000	6120
210	16200	13600	9800	5500
220	15000	12700	8800	5000
230	14000	11950	8100	4650
240	13000	11200	7500	4300
250	12000	10500	7000	4050
260	11000	9800	6500	3800
270	10500	9150	6100	3500
280	10000	8500	5700	3200
290	9500	7850	5330	3000
300	9000	7200	5000	2800

TABLE NO. 4.

Struts of Medium Steel.

Greatest safe load for steel of medium grade, tensile strength about 70,000 pounds.

For extreme soft steel, use Table No. 2.

The figures are the working loads, in pounds per square inch, for vertical struts.

Both ends are supposed to be secured as indicated at the head of each column.

If both ends are not secured alike, take a mean proportional between the values given for the classes to which each end belongs.

If the strut is hinged by any uncertain method, so that the centres of pins and axis of strut may not coincide, or the pins may be relatively small and loosely fitted, it is best in such cases to consider the strut as "round-ended."

Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	21900	21900	21200	20300
30	15400	15400	14800	13800
40	13500	13500	12800	11600
50	12600	12600	11700	10300
60	11700	11700	10800	9130
70	10800	10800	9800	8050
80	10000	10000	8900	7070
90	9260	9230	7980	6090
100	8550	8500	7090	5200
110	8070	7800	6410	4540
120	7590	7140	5770	3920
130	7000	6510	5150	3330
140	6410	5910	4550	2780
150	5950	5330	3940	2340
160	5500	4780	3350	1920
170	4980	4250	2860	1660
180	4460	3750	2400	1410
190	3960	3310	2080	1190
200	3580	2960	1830	1020
210	3180	2670	1590	890
220	2880	2440	1400	790
230	2640	2250	1250	720
240	2410	2070	1140	650
250	2180	1910	1040	600
260	1960	1750	940	550
270	1840	1610	860	500
280	1720	1460	790	440
290	1610	1330	720	410
300	1500	1200	670	370

TABLE No. 5.

Struts of Hard Steel.

Destructive pressure, in pounds per square inch, for hard steel, tensile strength about 100,000 pounds.

For softer steel, see Table No. 3.

Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	100000	100000	100000	95600
30	74000	74000	74000	69300
40	62000	62000	62000	56600
50	60000	60000	60000	52900
60	58000	58000	58000	49100
70	55500	55500	55100	45300
80	53000	53000	52200	41400
90	49900	49700	47800	36600
100	46800	46500	43700	32000
110	44700	43200	40400	28500
120	42600	40000	36900	25100
130	39400	36700	33500	21600
140	36300	33500	29900	18200
150	34200	30700	26500	15700
160	32200	28000	23100	13300
170	29800	25500	20300	11800
180	27400	23000	17500	10300
190	25100	21000	15800	9060
200	22900	19000	14100	7860
210	20300	17200	12400	6950
220	18300	15500	10700	6100
230	16900	14400	9820	5600
240	15500	13400	8960	5140
250	14200	12400	8270	4780
260	12900	11500	7630	4460
270	12200	10600	7060	4050
280	11400	9700	6500	3650
290	10900	9000	6130	3440
300	10600	8500	5890	3300

TABLE No. 6.

Struts of Hard Steel.

Greatest safe load for hard steel, tensile strength about 100,000 pounds.
For soft steel, see Table No. 4.

The figures are the working loads, in pounds per square inch, for vertical struts.

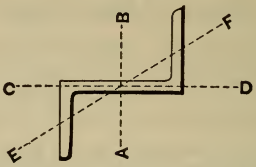
Both ends are supposed to be secured as indicated at the head of each column.

If both ends are not secured alike, take a mean proportional between the values given for the classes to which each end belongs.

If the strut is hinged by any uncertain method, so that the centres of pins and axis of strut may not coincide, or the pins may be relatively small and loosely fitted, it is best in such cases to consider the strut as "round-ended."

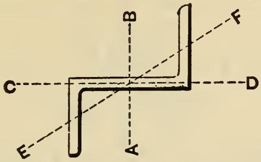
Length. / Least radius of gyration.	Fixed ends.	Flat ends.	Hinged ends.	Round ends.
20	31200	31200	30300	29000
30	22400	22400	21400	20100
40	18200	18200	17200	15700
50	17100	17100	16000	14100
60	16100	16100	14900	12600
70	15000	15000	13600	11200
80	13900	13900	12400	9860
90	12800	12700	11000	8410
100	11700	11600	9710	7110
110	10900	10500	8670	6130
120	10100	9520	7690	5230
130	9160	8530	6770	4360
140	8250	7610	5860	3570
150	7600	6820	5050	2990
160	7000	6090	4280	2460
170	6340	5420	3660	2130
180	5710	4790	3070	1810
190	5120	4280	2700	1550
200	4580	3800	2350	1310
210	3980	3370	2020	1130
220	3520	2980	1700	970
230	3190	2720	1500	870
240	2870	2480	1360	780
250	2580	2250	1220	710
260	2300	2050	1100	650
270	2240	1860	1000	570
280	1960	1670	900	510
290	1850	1520	830	470
300	1800	1420	780	440

Elements of Pencoyd Z-Bars.



Section number.	Size, in inches.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.			Resistance.	
				Axis *AB.*	Axis *CD.*	Axis *EF.*	Axis *AB.*	Axis *CD.*
30Z	$2\frac{5}{8} \times 3 \times 2\frac{5}{8} \times \frac{1}{4}$	1.94	6.60	2.81	2.61	0.59	1.9	1.0
31Z	$2\frac{11}{16} \times 3\frac{1}{16} \times 2\frac{11}{16} \times \frac{5}{16}$	2.44	8.29	3.52	3.38	0.74	2.3	1.3
32Z	$2\frac{3}{4} \times 3\frac{1}{8} \times 2\frac{3}{4} \times \frac{3}{8}$	2.94	10.00	4.34	4.22	0.92	2.8	1.7
33Z	$2\frac{21}{32} \times 3 \times 2\frac{21}{32} \times \frac{7}{16}$	3.25	11.15	4.20	4.24	0.95	2.8	1.7
34Z	$2\frac{11}{16} \times 3\frac{1}{32} \times 2\frac{11}{16} \times \frac{15}{32}$	3.51	11.93	4.54	4.64	1.01	3.0	1.9
35Z	$2\frac{23}{32} \times 3\frac{1}{16} \times 2\frac{23}{32} \times \frac{1}{2}$	3.75	12.75	4.88	5.04	1.11	3.2	2.0
40Z	$2\frac{7}{8} \times 4 \times 2\frac{7}{8} \times \frac{1}{4}$	2.32	7.88	5.95	3.47	0.95	3.0	1.3
41Z	$2\frac{15}{16} \times 4\frac{1}{16} \times 2\frac{15}{16} \times \frac{5}{16}$	2.91	9.89	7.52	4.49	1.23	3.7	1.6
42Z	$3 \times 4\frac{1}{8} \times 3 \times \frac{3}{8}$	3.52	11.90	9.14	5.58	1.53	4.4	2.0
43Z	$2\frac{31}{32} \times 4 \times 2\frac{31}{32} \times \frac{7}{16}$	3.96	13.46	9.40	6.09	1.63	4.7	2.2
44Z	$3\frac{1}{32} \times 4\frac{1}{16} \times 3\frac{1}{32} \times \frac{1}{2}$	4.56	15.50	10.92	7.21	1.94	5.4	2.6
45Z	$3\frac{3}{32} \times 4\frac{1}{8} \times 3\frac{3}{32} \times \frac{9}{16}$	5.16	17.54	12.40	8.40	2.27	6.0	3.0
46Z	$3\frac{1}{16} \times 4 \times 3\frac{1}{16} \times \frac{5}{8}$	5.55	18.80	12.11	8.73	2.32	6.1	3.2
47Z	$3\frac{1}{8} \times 4\frac{1}{16} \times 3\frac{1}{8} \times \frac{11}{16}$	6.14	20.87	13.52	9.95	2.67	6.7	3.6
48Z	$3\frac{3}{16} \times 4\frac{1}{8} \times 3\frac{3}{16} \times \frac{3}{4}$	6.75	22.95	14.97	11.24	3.03	7.3	4.0
50Z	$3\frac{3}{16} \times 5 \times 3\frac{3}{16} \times \frac{5}{16}$	3.36	11.42	13.14	5.81	1.86	5.3	1.9
51Z	$3\frac{1}{4} \times 5\frac{1}{16} \times 3\frac{1}{4} \times \frac{3}{8}$	4.05	13.77	15.93	7.20	2.28	6.3	2.4
52Z	$3\frac{5}{16} \times 5\frac{1}{8} \times 3\frac{5}{16} \times \frac{7}{16}$	4.75	16.15	18.76	8.67	2.75	7.3	2.8
53Z	$3\frac{7}{32} \times 5 \times 3\frac{7}{32} \times \frac{1}{2}$	5.23	17.78	19.03	8.77	2.76	7.6	3.0
54Z	$3\frac{9}{32} \times 5\frac{1}{16} \times 3\frac{9}{32} \times \frac{9}{16}$	5.91	20.09	21.65	10.19	3.20	8.6	3.4
55Z	$3\frac{11}{32} \times 5\frac{1}{8} \times 3\frac{11}{32} \times \frac{5}{8}$	6.60	22.44	24.33	11.70	3.73	9.5	3.9
56Z	$3\frac{1}{4} \times 5 \times 3\frac{1}{4} \times \frac{11}{16}$	6.96	23.66	23.68	11.37	3.59	9.5	3.9
57Z	$3\frac{5}{16} \times 5\frac{1}{16} \times 3\frac{5}{16} \times \frac{3}{4}$	7.64	25.97	26.16	12.83	4.12	10.3	4.4
60Z	$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{3}{8}$	4.59	15.61	25.32	9.11	3.11	8.4	2.8
61Z	$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{7}{16}$	5.39	18.32	29.80	10.95	3.74	9.8	3.3
62Z	$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{1}{2}$	6.19	21.05	34.36	12.87	4.37	11.2	3.8
63Z	$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{9}{16}$	6.68	22.71	34.64	12.59	4.37	11.6	3.9
64Z	$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{5}{8}$	7.46	25.36	38.86	14.42	4.92	12.8	4.4
65Z	$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{11}{16}$	8.25	28.05	43.18	16.34	5.66	14.1	5.0
66Z	$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{3}{4}$	8.64	29.37	42.12	15.44	5.61	14.0	4.9
67Z	$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{13}{16}$	9.38	31.89	46.13	17.27	6.16	15.2	5.5
68Z	$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{7}{8}$	10.16	34.54	50.22	19.18	6.85	16.4	6.0

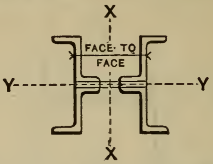
Elements of Pencoyd Z-Bars.



Radius of gyration.			Coefficient, in net tons, for greatest safe load distributed.		Coefficient for deflection about axis *AB*.		Maximum load, in net tons.	Section number.
Axis *AB*.	Axis *CD*.	Least. Axis *EF*.	Fibre stress, 16,000 pounds.	Fibre stress, 12,000 pounds.	Distributed.	Centre.		
1.20	1.16	0.55	10 0	7.5	.000 5694	.000 9167	11.0	30Z
1.20	1.18	0.55	12.3	9.2	.000 4545	.000 7317	14.4	31Z
1.21	1.20	0.56	14.8	11.1	.000 3687	.000 5937	18.0	32Z
1.13	1.14	0.54	14.9	11.2	.000 3809	.000 6132	20.4	33Z
1.14	1.15	0.54	16.0	12.0	000 3524	.000 5674	22.2	34Z
1.14	1.16	0.55	17.0	12.8	.000 3279	.000 5279	24.0	35Z
1.60	1.22	0.64	15.9	11.9	.000 2689	.000 4329	13.6	40Z
1.61	1.24	0.65	19.7	14.8	000 2128	.000 3426	18.2	41Z
1.62	1.26	0.66	23.6	17.7	.000 1750	.000 2817	23.0	42Z
1.54	1.24	0.64	25.1	18.8	.000 1702	.000 2740	26.6	43Z
1.55	1.27	0.65	28.7	21.5	.000 1465	.000 2359	31.2	44Z
1.55	1.28	0.66	32.1	24.1	.000 1290	.000 2077	35.8	45Z
1.48	1.26	0.65	32.3	24.2	.000 1321	.000 2127	39.0	46Z
1.48	1.27	0.66	35.5	26.6	.000 1183	.000 1905	43.6	47Z
1.49	1.29	0.67	38.7	29.0	.000 1069	.000 1721	48.6	48Z
1.98	1.32	0.74	28.0	21.0	.000 1218	.000 1961	21.4	50Z
1.98	1.33	0.75	33.6	25.2	.000 1005	000 1618	27.0	51Z
1.99	1.35	0.76	39.1	29.3	.000 0853	.000 1373	32.8	52Z
1.91	1.30	0.73	40.6	30.5	.000 0841	.000 1354	37.6	53Z
1.91	1.31	0.74	45.6	34.2	.000 0739	.000 1190	43.2	54Z
1.92	1.33	0.75	50.6	38.0	.000 0658	.000 1059	49.0	55Z
1.84	1.28	0.72	50.5	37.9	.000 0676	.000 1088	53.2	56Z
1.85	1.30	0.73	55.1	41.3	.000 0612	.000 0984	59.0	57Z
2.35	1.41	0.82	45.0	33.8	.000 0632	.000 1017	30.8	60Z
2.35	1.43	0.83	52.4	39.3	.000 0537	.000 0864	37.6	61Z
2.36	1.44	0.84	59.8	44.9	.000 0466	.000 0750	44.6	62Z
2.28	1.37	0.81	61.6	46.2	.000 0462	.000 0744	50.2	63Z
2.28	1.39	0.81	68.4	51.3	.000 0412	.000 0663	57.0	64Z
2.29	1.41	0.83	75.2	56.4	.000 0370	.000 0596	64.0	65Z
2.21	1.34	0.81	74.9	56.2	.000 0380	.000 0612	69.0	66Z
2.22	1.36	0.81	81.2	60.9	.000 0347	.000 0559	76.0	67Z
2.22	1.37	0.82	87.5	65.6	.000 0319	.000 0513	83.0	68Z

Elements of Z-Bar Columns.

I = moment of inertia.



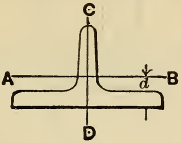
R = radius of gyration.

The thicknesses of Web Plate and Z-Bars are the same.

Size of Z-bar, in inches.	7″ web plate. $7\frac{1}{4}$″ face to face.					8″ web plate. $8\frac{1}{4}$″ face to face.				
	Area of 4 Z-bars and 1 plate.	Axis XX.		Axis YY.		Area of 4 Z-bars and 1 plate.	Axis XX.		Axis YY.	
		I.	R.	I.	R.		I.	R.	I.	R.
$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{3}{8}$	20.99	264.11	3.55	287.85	3.70	21.36	337.09	3.97	287.86	3.67
$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{7}{16}$	24.62	306.46	3.53	346.98	3.75	25.06	391.45	3.95	346.99	3.72
$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{1}{2}$	28.26	347.80	3.51	409.27	3.80	28.76	444.60	3.93	409.29	3.77
$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{9}{16}$	30.66	365.19	3.45	426.34	3.73	31.22	469.13	3.88	426.36	3.69
$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{5}{8}$	34.22	402.96	3.43	489.21	3.78	34.84	518.08	3.86	489.23	3.75
$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{11}{16}$	37.81	440.31	3.41	555.79	3.83	38.50	566.52	3.83	555.82	3.80
$3\frac{1}{2} \times 6 \times 3\frac{1}{2} \times \frac{3}{4}$	39.81	448.24	3.36	562.39	3.76	40.56	579.76	3.78	562.43	3.72
$3\frac{9}{16} \times 6\frac{1}{16} \times 3\frac{9}{16} \times \frac{13}{16}$	43.21	481.03	3.34	628.18	3.81	44.02	622.55	3.76	628.23	3.78
$3\frac{5}{8} \times 6\frac{1}{8} \times 3\frac{5}{8} \times \frac{7}{8}$	46.77	514.64	3.32	699.11	3.87	47.64	666.66	3.74	699.17	3.83
	7″ web plate. $7\frac{3}{4}$″ face to face.					8″ web plate. $8\frac{1}{4}$″ face to face.				
$3\frac{3}{16} \times 5 \times 3\frac{3}{16} \times \frac{5}{16}$	15.63	193.88	3.52	147.41	3.07	15.94	248.26	3.95	147.41	3.04
$3\frac{1}{4} \times 5\frac{1}{16} \times 3\frac{1}{4} \times \frac{3}{8}$	18.83	231.00	3.50	183.49	3.12	19.20	295.96	3.92	183.50	3.09
$3\frac{5}{16} \times 5\frac{1}{8} \times 3\frac{5}{16} \times \frac{7}{16}$	22.06	267.64	3.48	222.06	3.17	22.50	343.27	3.91	222.07	3.14
$3\frac{7}{32} \times 5 \times 3\frac{7}{32} \times \frac{1}{2}$	24.42	287.66	3.43	234.48	3.10	24.92	370.54	3.86	234.50	3.07
$3\frac{9}{32} \times 5\frac{1}{16} \times 3\frac{9}{32} \times \frac{9}{16}$	27.58	321.15	3.41	273.70	3.15	28.14	414.03	3.83	273.72	3.12
$3\frac{11}{32} \times 5\frac{1}{8} \times 3\frac{11}{32} \times \frac{5}{8}$	30.78	354.33	3.39	315.67	3.20	31.40	457.20	3.81	315.69	3.17
$3\frac{1}{4} \times 5 \times 3\frac{1}{4} \times \frac{11}{16}$	32.65	364.87	3.34	320.05	3.13	33.34	472.86	3.77	320.08	3.10
$3\frac{5}{16} \times 5\frac{1}{16} \times 3\frac{5}{16} \times \frac{3}{4}$	35.81	395.55	3.32	363.02	3.18	36.56	513.07	3.74	363.05	3.15
	6″ web plate. $6\frac{1}{4}$″ face to face.					7″ web plate. $7\frac{1}{4}$″ face to face.				
$2\frac{7}{8} \times 4 \times 2\frac{7}{8} \times \frac{1}{4}$	10.78	101.90	3.07	65.71	2.47	11.03	134.71	3.49	65.79	2.44
$2\frac{15}{16} \times 4\frac{1}{16} \times 2\frac{15}{16} \times \frac{5}{16}$	13.52	126.14	3.05	85.80	2.52	13.83	166.94	3.47	85.80	2.49
$3 \times 4\frac{1}{8} \times 3 \times \frac{3}{8}$	16.33	150.56	3.04	107.87	2.57	16.71	199.42	3.45	107.87	2.54
$2\frac{31}{32} \times 4 \times 2\frac{31}{32} \times \frac{7}{16}$	18.47	166.03	3.00	115.62	2.50	18.90	220.65	3.42	115.63	2.47
$3\frac{1}{32} \times 4\frac{1}{16} \times 3\frac{1}{32} \times \frac{1}{2}$	21.24	188.60	2.98	138.66	2.55	21.74	250.89	3.40	138.67	2.52
$3\frac{3}{32} \times 4\frac{1}{8} \times 3\frac{3}{32} \times \frac{9}{16}$	24.02	210.64	2.96	163.07	2.60	24.58	280.45	3.38	163.08	2.58
$3\frac{1}{16} \times 4 \times 3\frac{1}{16} \times \frac{5}{8}$	25.95	221.78	2.92	167.28	2.54	26.58	296.36	3.34	167.30	2.51
$3\frac{1}{8} \times 4\frac{1}{16} \times 3\frac{1}{8} \times \frac{11}{16}$	28.69	242.16	2.91	192.77	2.59	29.37	323.88	3.32	192.80	2.56
$3\frac{3}{16} \times 4\frac{1}{8} \times 3\frac{3}{16} \times \frac{3}{4}$	31.50	262.65	2.89	220.51	2.64	32.25	351.59	3.30	220.55	2.61
	6″ web plate. $6\frac{1}{4}$″ face to face.					7″ web plate. $7\frac{1}{4}$″ face to face.				
$2\frac{5}{8} \times 3 \times 2\frac{5}{8} \times \frac{1}{4}$	9.26	84.78	3.03	31.74	1.85	9.51	112.65	3.44	31.74	1.83
$2\frac{11}{16} \times 3\frac{1}{16} \times 2\frac{11}{16} \times \frac{5}{16}$	11.64	105.17	3.01	41.89	1.90	11.95	139.88	3.42	41.89	1.87
$2\frac{3}{4} \times 3\frac{1}{8} \times 2\frac{3}{4} \times \frac{3}{8}$	14.01	125.10	2.99	53.41	1.95	14.39	166.56	3.40	53.42	1.93
$2\frac{21}{32} \times 3 \times 2\frac{21}{32} \times \frac{7}{16}$	15.63	134.64	2.93	55.24	1.88	16.06	180.30	3.35	55.25	1.85
$2\frac{23}{32} \times 3\frac{1}{16} \times 2\frac{23}{32} \times \frac{1}{2}$	18.00	153.14	2.92	67.17	1.93	18.50	205.32	3.33	67.18	1.90

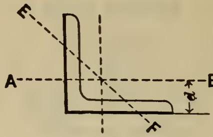
Elements of Pencoyd Tees.

Uneven Legs.



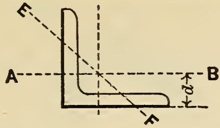
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Section number.	Size, in inches.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.		Resistance.		Radius of gyration.		Dist., *d*, from base to neutral axis.
				Axis *AB.*	Axis *CD.*	Axis *AB.*	Axis *CD.*	Axis *AB.*	Axis *CD.*	
66T	6 × 4½	8.21	28.2	14.74	13.81	4.71	4.60	1.33	1.29	1.37
64T	6 × 4	4.61	15.6	5.82	8.19	1.92	2.73	1.12	1.33	0.97
65T	6 × 5¼	11.58	39.0	28.68	18.75	8.19	6.25	1.57	1.27	1.75
53T	5 × 3½	4.95	17.0	5.29	5.47	2.17	2.19	1.03	1.05	1.06
54T	5 × 4	4.54	15.3	6.16	5.41	2.11	2.16	1.17	1.09	1.08
42T	4 × 2	1.93	6.5	0.53	1.75	0.34	0.87	0.52	0.95	0.46
43T	4 × 3	2.67	9.0	1.99	2.10	0.90	1.05	0.87	0.89	0.78
44T	4 × 3	3.05	10.2	2.24	2.44	1.02	1.22	0.85	0.89	0.81
45T	4 × 4½	4.29	14.6	7.87	2.80	2.50	1.40	1.37	0.81	1.37
46T	4½ × 3½	4.65	15.8	4.93	3.67	2.05	1.63	1.03	0.89	1.11
47T	4 × 4½	3.38	11.4	6.31	2.11	1.96	1.06	1.37	0.79	1.28
38T	3½ × 3	2.11	7.0	1.65	1.18	0.75	0.67	0.88	0.75	0.80
39T	3½ × 3	2.46	8.5	1.91	1.41	0.88	0.81	0.88	0.75	0.83
30T	3 × 1½	1.20	4.0	0.18	0.60	0.16	0.40	0.39	0.71	0.36
31T	3 × 2½	1.46	5.0	0.78	0.60	0.42	0.40	0.73	0.64	0.66
32T	3 × 2½	1.76	6.0	0.93	0.74	0.51	0.49	0.73	0.65	0.68
33T	3 × 2½	2.06	7.0	1.08	0.89	0.60	0.59	0.72	0.66	0.71
34T	3 × 2½	2.38	8.0	1.32	0.91	0.78	0.61	0.74	0.62	0.80
35T	3 × 3½	2.46	8.3	2.82	0.89	1.17	0.59	1.07	0.60	1.08
36T	3 × 3½	2.81	9.5	3.19	1.04	1.33	0.69	1.07	0.61	1.10
28T	2¾ × 1¾	1.96	6.6	0.56	0.60	0.50	0.44	0.54	0.56	0.64
29T	2¾ × 2	2.14	7.2	0.82	0.61	0.66	0.44	0.62	0.54	0.75
25T	2½ × 1¼	0.97	3.3	0.10	0.33	0.11	0.26	0.32	0.58	0.31
26T	2½ × 2¾	1.68	5.7	1.16	0.43	0.60	0.34	0.83	0.51	0.83
27T	2½ × 3	1.76	6.0	1.48	0.44	0.71	0.35	0.92	0.50	0.93
24T	2¼ × $\frac{9}{16}$	0.66	2.2	0.01	0.24	0.03	0.21	0.14	0.60	0.17
20T	2 × $\frac{9}{16}$	0.60	2.0	0.01	0.17	0.03	0.17	0.14	0.53	0.17
22T	2 × 1$\frac{1}{16}$	0.62	2.0	0.04	0.16	0.05	0.16	0.24	0.51	0.23
21T	2 × 1	0.72	2.5	0.05	0.17	0.07	0.17	0.26	0.49	0.27
23T	2 × 1½	0.91	3.0	0.16	0.17	0.15	0.17	0.42	0.44	0.45
17T	1¾ × 1$\frac{1}{16}$	0.56	1.9	0.05	0.11	0.06	0.13	0.30	0.45	0.24
18T	1¾ × 1¼	1.04	3.5	0.12	0.21	0.14	0.24	0.35	0.45	0.40
15T	1½ × $\frac{15}{16}$	0.41	1.4	0.02	0.07	0.03	0.09	0.22	0.41	0.21
12T	1¼ × $\frac{15}{16}$	0.35	1.2	0.02	0.03	0.03	0.05	0.24	0.30	0.22

Elements of Pencoyd Angles.



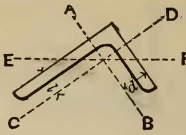
I.	II.	III.	IV.	V.	VI.	VII.
Section number.	Size, in inches.	Thickness.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.	
					Axis *AB*.	Axis *EF*.
880A	8 × 8	½	7.75	26.4	48.47	19.60
888A	8¼ × 8¼	1	15.29	52.8	94.14	39.01
660A	6 × 6	⅜	4.36	14.8	15.37	6.20
669A	6¼ × 6¼	15/16	10.65	35.9	36.69	15.48
550A	5 × 5	⅜	3.61	12.3	8.73	3.54
559A	5¼ × 5¼	15/16	8.77	29.4	20.72	9.09
440A	4 × 4	5/16	2.40	8.2	3.69	1.50
447A	4¼ × 4¼	¾	5.69	18.6	8.71	3.82
350A	3½ × 3½	5/16	2.09	7.1	2.45	0.99
355A	3⅝ × 3⅝	⅝	4.06	13.7	4.60	1.97
330A	3 × 3	¼	1.44	4.9	1.25	0.50
336A	3 3/16 × 3 3/16	⅝	3.51	11.5	3.01	1.32
275A	2¾ × 2¾	¼	1.31	4.5	0.95	0.39
279A	3 × 3	½	2.70	8.6	2.11	0.90
250A	2½ × 2½	3/16	0.90	3.1	0.54	0.22
255A	2⅝ × 2⅝	½	2.33	7.8	1.33	0.59
225A	2¼ × 2¼	3/16	0.81	2.7	0.39	0.16
228A	2 7/16 × 2 7/16	⅜	1.66	5.4	0.85	0.37
220A	2 × 2	3/16	0.71	2.5	0.27	0.11
223A	2 3/16 × 2 3/16	⅜	1.47	4.8	0.61	0.26
175A	1¾ × 1¾	3/16	0.62	2.1	0.18	0.08
178A	1 15/16 × 1 15/16	⅜	1.28	4.1	0.39	0.18
150A	1½ × 1½	⅛	0.36	1.2	0.08	0.03
154A	1¾ × 1¾	⅜	1.14	3.5	0.29	0.13
125A	1¼ × 1¼	⅛	0.30	1.0	0.05	0.02
127A	1⅜ × 1⅜	¼	0.62	2.0	0.10	0.04
110A	1 × 1	⅛	0.23	0.8	0.02	0.01
112A	1⅛ × 1⅛	¼	0.49	1.5	0.05	0.02

Elements of Pencoyd Angles.



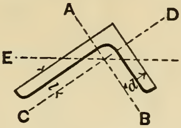
VIII.	IX.	X.	XI.	I.
Radius of gyration.		Resistance.	Distance from base to neutral axis.	Section number.
Axis *AB*.	Axis *EF*.	Axis *AB*.	*d*.	
2.50	1.59	8.34	2.19	880A
2.48	1.60	16.18	2.43	888A
1.88	1.19	3.53	1.64	660A
1.86	1.21	8.43	1.90	669A
1.56	0.99	2.42	1.39	550A
1.54	1.02	5.76	1.65	559A
1.24	0.79	1.28	1.12	440A
1.24	0.82	3.10	1.34	447A
1.08	0.69	0.98	0.99	350A
1.06	0.70	1.84	1.13	355A
0.93	0.59	0.58	0.84	330A
0.93	0.61	1.39	1.02	336A
0.85	0.55	0.48	0.78	275A
0.88	0.58	1.02	0.93	279A
0.77	0.49	0.30	0.70	250A
0.76	0.50	0.75	0.84	255A
0.69	0.44	0.24	0.63	225A
0.72	0.47	0.50	0.75	228A
0.62	0.39	0.19	0.58	220A
0.64	0.42	0.40	0.68	223A
0.54	0.36	0.15	0.51	175A
0.55	0.38	0.30	0.63	178A
0.47	0.28	0.07	0.42	150A
0.50	0.34	0.25	0.57	154A
0.41	0.26	0.06	0.35	125A
0.40	0.25	0.11	0.43	127A
0.29	0.21	0.03	0.30	110A
0.32	0.20	0.07	0.37	112A

Elements of Pencoyd Angles.



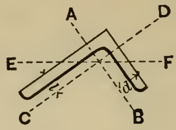
I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Section number.	Size, in inches.	Thickness.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.		
					Axis *AB.*	Axis *CD.*	Axis *EF.*
860A	8 × 6	½	6.75	23.0	44.38	21.73	12.04
868A	8¼ × 6¼	1	13.29	45.6	85.34	41.67	24.76
730A	7 × 3½	½	5.00	17.0	25.29	4.37	3.64
738A	7¼ × 3¾	1	9.79	32.5	48.59	8.47	7.47
650A	6½ × 4	⅜	3.80	12.9	16.83	5.03	3.29
659A	6⅞ × 4⅜	$\frac{15}{16}$	9.48	31.9	42.40	12.91	9.28
640A	6 × 4	⅜	3.61	12.2	13.48	4.91	3.04
649A	6⅜ × 4⅜	$\frac{15}{16}$	9.01	29.4	33.95	12.47	8.57
630A	6 × 3½	⅜	3.42	11.6	12.82	3.32	2.39
639A	6⅜ × 3⅞	$\frac{15}{16}$	8.54	28.6	32.56	7.74	6.50
500A	5½ × 3½	⅜	3.23	11.0	10.15	3.28	2.14
504A	5¾ × 3¾	⅝	5.47	17.9	17.62	5.85	3.82
540A	5 × 4	⅜	3.23	11.0	8.13	4.65	2.50
546A	5$\frac{3}{16}$ × 4$\frac{3}{16}$	¾	6.35	21.3	15.65	8.74	4.95
510A	5 × 3½	$\frac{5}{16}$	2.56	8.7	6.58	2.71	1.65
517A	5¼ × 3¾	¾	6.07	20.0	15.51	6.41	4.17
530A	5 × 3	$\frac{5}{16}$	2.40	8.2	6.27	1.75	1.20
537A	5¼ × 3¼	¾	5.69	18.7	14.75	4.18	3.05
450A	4½ × 3	$\frac{5}{16}$	2.25	7.7	4.72	1.72	1.10
457A	4¾ × 3¼	¾	5.32	17.4	11.04	4.07	2.96
410A	4 × 3½	$\frac{5}{16}$	2.25	7.7	3.57	2.56	1.18
417A	4¼ × 3¾	¾	5.32	17.4	8.42	6.06	3.08

Elements of Pencoyd Angles.



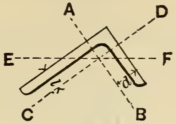
IX.	X.	XI.	XII.	XIII.	XIV.	XV.	I.
Radius of gyration.			Resistance.		Distance from base to neutral axis.		Section number.
Axis *AB.*	Axis *CD.*	Axis *EF.*	Axis *AB.*	Axis *CD.*	*d.*	*l.*	
2.56	1.79	1.34	8.03	4.80	2.47	1.47	860A
2.53	1.77	1.37	15.43	9.20	2.72	1.72	868A
2.25	0.93	0.85	5.66	1.61	2.53	0.78	730A
2.23	0.93	0.87	10.85	3.10	2.77	1.02	738A
2.10	1.15	0.93	3.87	1.62	2.15	0.90	650A
2.12	1.17	0.99	9.58	4.07	2.45	1.20	659A
1.93	1.17	0.92	3.32	1.60	1.94	0.94	640A
1.94	1.18	0.98	8.21	3.98	2.24	1.24	649A
1.94	0.99	0.84	3.24	1.23	2.04	0.79	630A
1.95	0.95	0.87	8.05	2.77	2.33	1.08	639A
1.77	1.01	0.81	2.76	1.22	1.82	0.82	500A
1.79	1.03	0.84	4.66	2.10	1.97	0.97	504A
1.59	1.20	0.88	2.34	1.57	1.53	1.03	540A
1.57	1.17	0.88	4.50	2.93	1.71	1.21	546A
1.60	1.03	0.80	1.93	1.02	1.59	0.84	510A
1.60	1.03	0.83	4.51	2.38	1.81	1.06	517A
1.62	0.85	0.71	1.89	0.75	1.68	0.68	530A
1.61	0.86	0.73	4.40	1.78	1.90	0.90	537A
1.45	0.87	0.70	1.55	0.75	1.46	0.71	450A
1.44	0.87	0.75	3.61	1.76	1.69	0.94	457A
1.26	1.07	0.72	1.27	1.00	1.18	0.93	410A
1.26	1.07	0.76	2.95	2.33	1.40	1.15	417A

Elements of Pencoyd Angles.



I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Section number.	Size, in inches.	Thick-ness.	Area, in square inches.	Weight per foot, in pounds.	Moments of inertia.		
					Axis *AB.*	Axis *CD.*	Axis *EF.*
430A	4 × 3	$\frac{5}{16}$	2.09	7.1	3.38	1.64	0.93
435A	$4\frac{1}{8} \times 3\frac{1}{8}$	$\frac{5}{8}$	4.06	13.8	6.36	2.59	1.80
300A	$3\frac{1}{2} \times 3$	$\frac{5}{16}$	1.93	6.6	2.33	1.59	0.80
305A	$3\frac{13}{16} \times 3\frac{5}{16}$	$\frac{5}{8}$	3.98	12.9	5.12	3.54	1.88
310A	$3\frac{1}{2} \times 2\frac{1}{2}$	$\frac{1}{4}$	1.44	4.9	1.81	0.78	0.45
314A	$3\frac{3}{4} \times 2\frac{3}{4}$	$\frac{1}{2}$	2.95	9.4	3.93	1.76	1.01
316A	$3\frac{1}{2} \times 2$	$\frac{1}{4}$	1.31	4.5	1.66	0.41	0.30
318A	$3\frac{5}{8} \times 2\frac{1}{8}$	$\frac{3}{8}$	1.99	6.6	2.55	0.65	0.45
325A	$3 \times 2\frac{1}{2}$	$\frac{1}{4}$	1.31	4.5	1.15	0.73	0.41
329A	$3\frac{1}{4} \times 2\frac{3}{4}$	$\frac{1}{2}$	2.70	8.7	2.64	1.71	0.76
320A	3×2	$\frac{1}{4}$	1.19	4.1	1.09	0.40	0.24
324A	$3\frac{1}{4} \times 2\frac{1}{4}$	$\frac{1}{2}$	2.45	7.9	2.41	0.92	0.57
200A	$2\frac{1}{2} \times 2$	$\frac{3}{16}$	0.81	2.7	0.51	0.29	0.13
205A	$2\frac{13}{16} \times 2\frac{5}{16}$	$\frac{1}{2}$	2.26	7.0	1.64	0.97	0.44
206A	$2\frac{1}{4} \times 1\frac{1}{2}$	$\frac{3}{16}$	0.67	2.3	0.35	0.12	0.08
209A	$2\frac{7}{16} \times 1\frac{11}{16}$	$\frac{3}{8}$	1.38	4.4	0.73	0.29	0.18
215A	$2 \times 1\frac{1}{2}$	$\frac{3}{16}$	0.62	2.1	0.25	0.12	0.07
218A	$2\frac{3}{16} \times 1\frac{11}{16}$	$\frac{3}{8}$	1.28	4.3	0.52	0.29	0.15
210A	$2 \times 1\frac{1}{4}$	$\frac{3}{16}$	0.57	1.9	0.23	0.07	0.05
213A	$2\frac{3}{16} \times 1\frac{7}{16}$	$\frac{3}{8}$	1.19	3.9	0.50	0.17	0.12

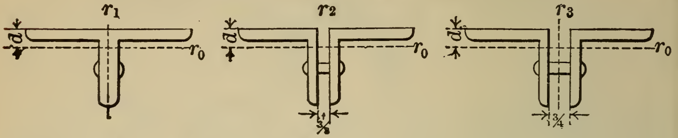
Elements of Pencoyd Angles.



IX.	X.	XI.	XII.	XIII.	XIV.	XV.	I.
Radius of gyration.			Resistance.		Distance from base to neutral axis.		Section number.
Axis *AB.*	Axis *CD.*	Axis *EF.*	Axis *AB.*	Axis *CD.*	*d.*	*l.*	
1.27	0.89	0.67	1.23	0.73	1.26	0.76	430A
1.25	0.80	0.67	2.33	1.16	1.40	0.90	435A
1.10	0.91	0.64	0.95	0.73	1.06	0.81	300A
1.13	0.95	0.69	2.00	1.53	1.25	1.00	305A
1.12	0.74	0.56	0.76	0.41	1.11	0.61	310A
1.15	0.77	0.59	1.58	0.88	1.26	0.76	314A
1.13	0.56	0.48	0.72	0.27	1.21	0.46	316A
1.13	0.57	0.48	1.09	0.41	1.28	0.53	318A
0.94	0.75	0.56	0.55	0.40	0.92	0.67	325A
0.99	0.80	0.53	1.20	0.88	1.05	0.80	329A
0.96	0.58	0.45	0.54	0.26	0.99	0.49	320A
0.99	0.61	0.48	1.14	0.57	1.14	0.64	324A
0.79	0.60	0.40	0.29	0.19	0.76	0.51	200A
0.85	0.66	0.44	0.88	0.60	0.94	0.69	205A
0.72	0.42	0.35	0.23	0.11	0.74	0.37	206A
0.73	0.46	0.36	0.46	0.24	0.86	0.48	209A
0.63	0.44	0.34	0.18	0.11	0.64	0.39	215A
0.64	0.48	0.34	0.36	0.24	0.76	0.50	218A
0.64	0.35	0.30	0.18	0.07	0.69	0.31	210A
0.65	0.38	0.32	0.36	0.17	0.80	0.42	213A

Radii of Gyration for Two Angles, with Sides Parallel.

The radii of gyration correspond to axes shown.

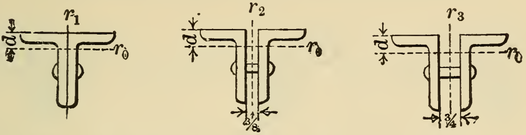


Size, in inches.	Thick-ness.	Weight per foot, in pounds.	d.	Radius of gyration. r_0.	r_1.	r_2.	r_3.
8 × 8	½	26.4	2.19	2.50	3.32	3.45	3.58
8¼ × 8¼	1	52.8	2.43	2.48	3.47	3.61	3.74
6 × 6	⅜	14.8	1.64	1.88	2.49	2.62	2.76
6¼ × 6¼	$\frac{15}{16}$	35.9	1.90	1.86	2.66	2.80	2.94
5 × 5	⅜	12.3	1.39	1.56	2.09	2.22	2.35
5¼ × 5¼	$\frac{15}{16}$	29.4	1.65	1.54	2.26	2.40	2.54
4 × 4	$\frac{5}{16}$	8.2	1.12	1.24	1.67	1.80	1.94
4¼ × 4¼	¾	18.6	1.34	1.24	1.82	1.97	2.12
3½ × 3½	$\frac{5}{16}$	7.1	0.99	1.08	1.46	1.60	1.74
3⅝ × 3⅝	⅝	13.7	1.13	1.06	1.55	1.69	1.84
3 × 3	¼	4.9	0.84	0.93	1.25	1.39	1.53
$3\frac{3}{16}$ × $3\frac{3}{16}$	⅝	11.5	1.02	0.93	1.38	1.52	1.68
2¾ × 2¾	¼	4.5	0.78	0.85	1.15	1.29	1.43
3 × 3	½	8.6	0.93	0.88	1.28	1.42	1.57
2½ × 2½	$\frac{3}{16}$	3.1	0.70	0.77	1.04	1.17	1.32
2⅝ × 2⅝	½	7.8	0.84	0.76	1.13	1.28	1.43
2¼ × 2¼	$\frac{3}{16}$	2.7	0.63	0.69	0.93	1.07	1.21
$2\frac{7}{16}$ × $2\frac{7}{16}$	⅜	5.4	0.75	0.72	1.04	1.18	1.34
2 × 2	$\frac{3}{16}$	2.5	0.58	0.62	0.85	0.99	1.14
$2\frac{3}{16}$ × $2\frac{3}{16}$	⅜	4.8	0.68	0.64	0.93	1.08	1.23
1¾ × 1¾	$\frac{3}{16}$	2.1	0.51	0.54	0.74	0.88	1.04
$1\frac{15}{16}$ × $1\frac{15}{16}$	⅜	4.1	0.63	0.55	0.84	0.98	1.14
1½ × 1½	⅛	1.2	0.42	0.47	0.63	0.77	0.92
1¾ × 1¾	⅜	3.5	0.57	0.50	0.76	0.91	1.07
1¼ × 1¼	⅛	1.0	0.35	0.41	0.54	0.68	0.83
1⅜ × 1⅜	¼	2.0	0.43	0.40	0.59	0.73	0.90
1 × 1	⅛	0.8	0.30	0.29	0.42	0.57	0.73
1⅛ × 1⅛	¼	1.5	0.37	0.32	0.49	0.64	0.81

r_1, r_2, and r_3 will also be radii of gyration for star columns.

Radii of Gyration for Two Angles, with Sides Parallel.

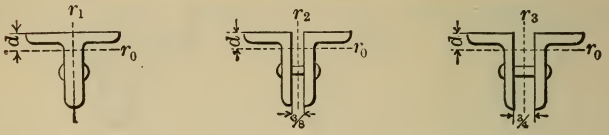
The radii of gyration correspond to axes shown.



Size, in inches.	Thick-ness.	Weight per foot, in pounds.	d.	Radius of gyration.			
				r_0.	r_1.	r_2.	r_3.
8 × 6	½	23.0	2.47	2.56	2.32	2.44	2.57
8¼ × 6¼	1	45.6	2.72	2.53	2.47	2.60	2.74
7 × 3½	½	17.0	2.53	2.25	1.21	1.34	1.48
7¼ × 3¾	1	32.5	2.77	2.23	1.38	1.51	1.68
6½ × 4	⅜	12.9	2.15	2.10	1.46	1.58	1.72
6⅞ × 4⅜	15/16	31.9	2.45	2.12	1.68	1.81	1.96
6 × 4	⅜	12.2	1.94	1.93	1.50	1.62	1.76
6⅜ × 4⅜	15/16	29.4	2.24	1.94	1.71	1.85	2.00
6 × 3½	⅜	11.6	2.04	1.94	1.27	1.39	1.53
6⅜ × 3⅞	15/16	28.6	2.33	1.95	1.44	1.58	1.74
5½ × 3½	⅜	11.0	1.82	1.77	1.30	1.43	1.56
5¾ × 3¾	⅝	17.9	1.97	1.79	1.41	1.55	1.69
5 × 4	⅜	11.0	1.53	1.59	1.58	1.71	1.85
5 3/16 × 4 3/16	¾	21.3	1.71	1.57	1.68	1.82	1.97
5 × 3½	5/16	8.7	1.59	1.60	1.33	1.45	1.59
5¼ × 3¾	¾	20.0	1.81	1.60	1.48	1.62	1.77
5 × 3	5/16	8.2	1.68	1.62	1.09	1.21	1.35
5¼ × 3¼	¾	18.7	1.90	1.61	1.24	1.39	1.54
4½ × 3	5/16	7.7	1.46	1.45	1.12	1.25	1.39
4¾ × 3¼	¾	17.4	1.69	1.44	1.28	1.42	1.58

Radii of Gyration for Two Angles, with Sides Parallel.

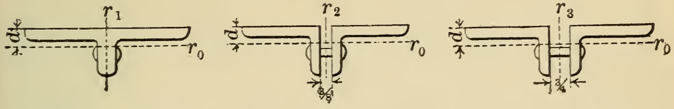
The radii of gyration correspond to axes shown.



Size, in inches.	Thick-ness.	Weight per foot, in pounds.	d.	Radius of gyration.			
				r_0.	r_1.	r_2.	r_3.
4 × 3½	$\frac{5}{16}$	7.7	1.18	1.26	1.42	1.55	1.69
4¼ × 3¾	¾	17.4	1.40	1.26	1.57	1.71	1.86
4 × 3	$\frac{5}{16}$	7.1	1.26	1.27	1.17	1.30	1.44
4⅛ × 3⅛	⅝	13.8	1.40	1.25	1.20	1.35	1.50
3½ × 3	$\frac{5}{16}$	6.6	1.06	1.10	1.22	1.35	1.49
$3\frac{13}{16}$ × $3\frac{5}{16}$	⅝	12.9	1.25	1.13	1.38	1.52	1.67
3½ × 2½	¼	4.9	1.11	1.12	0.97	1.09	1.23
3¾ × 2¾	½	9.4	1.26	1.15	1.08	1.22	1.37
3½ × 2	¼	4.5	1.21	1.13	0.72	0.86	1.00
3⅝ × 2⅛	⅜	6.6	1.28	1.13	0.78	0.92	1.07
3 × 2½	¼	4.5	0.92	0.94	1.00	1.13	1.29
3¼ × 2¾	½	8.7	1.05	0.99	1.13	1.27	1.42
3 × 2	¼	4.1	0.99	0.96	0.76	0.89	1.04
3¼ × 2¼	½	7.9	1.14	0.99	0.88	1.03	1.18
2½ × 2	$\frac{3}{16}$	2.7	0.76	0.79	0.79	0.92	1.07
$2\frac{13}{16}$ × $2\frac{5}{16}$	½	7.0	0.94	0.85	0.95	1.09	1.24
2¼ × 1½	$\frac{3}{16}$	2.3	0.74	0.72	0.56	0.70	0.85
$2\frac{7}{16}$ × $1\frac{11}{16}$	⅜	4.4	0.86	0.73	0.66	0.81	0.97
2 × 1½	$\frac{3}{16}$	2.1	0.64	0.63	0.59	0.73	0.88
$2\frac{3}{16}$ × $1\frac{11}{16}$	⅜	4.3	0.76	0.64	0.69	0.84	1.00
2 × 1¼	$\frac{3}{16}$	1.9	0.69	0.64	0.47	0.61	0.77
$2\frac{3}{16}$ × $1\frac{7}{16}$	⅜	3.9	0.80	0.65	0.57	0.72	0.88

Radii of Gyration for Two Angles, with Sides Parallel.

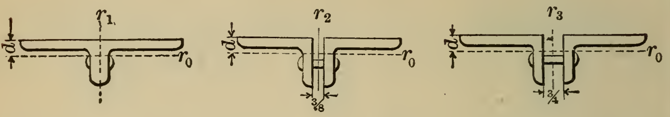
The radii of gyration correspond to axes shown.



Size, in inches.	Thickness.	Weight per foot, in pounds.	d.	Radius of gyration.			
				r_0.	r_1.	r_2.	r_3.
8 × 6	½	23.0	1.47	1.79	3.56	3.69	3.83
8¼ × 6¼	1	45.6	1.72	1.77	3.71	3.85	4.00
7 × 3½	½	17.0	0.78	0.93	3.38	3.53	3.67
7¼ × 3¾	1	32.5	1.02	0.93	3.56	3.70	3.85
6½ × 4	⅜	12.9	0.90	1.15	3.00	3.14	3.28
6⅞ × 4⅜	15/16	31.9	1.20	1.17	3.24	3.38	3.53
6 × 4	⅜	12.2	0.94	1.17	2.74	2.87	3.01
6⅜ × 4⅜	15/16	29.4	1.24	1.18	2.96	3.11	3.26
6 × 3½	⅜	11.6	0.79	0.99	2.81	2.95	3.10
6⅜ × 3⅞	15/16	28.6	1.08	0.95	3.04	3.18	3.33
5½ × 3½	⅜	11.0	0.82	1.01	2.54	2.68	2.82
5¾ × 3¾	⅝	17.9	0.97	1.03	2.66	2.80	2.95
5 × 4	⅜	11.0	1.03	1.20	2.21	2.34	2.48
5 3/16 × 4 3/16	¾	21.3	1.21	1.17	2.32	2.46	2.61
5 × 3½	5/16	8.7	0.84	1.03	2.25	2.39	2.53
5¼ × 3¾	¾	20.0	1.06	1.03	2.41	2.56	2.71
5 × 3	5/16	8.2	0.68	0.85	2.33	2.47	2.62
5¼ × 3¼	¾	18.7	0.90	0.86	2.49	2.64	2.79
4½ × 3	5/16	7.7	0.71	0.87	2.06	2.19	2.34
4¾ × 3¼	¾	17.4	0.94	0.87	2.22	2.37	2.52

Radii of Gyration for Two Angles, with Sides Parallel.

The radii of gyration correspond to axes shown.



Size, in inches.	Thickness.	Weight per foot, in pounds.	d.	Radius of gyration. r_0.	r_1.	r_2.	r_3.
4 × 3½	$\frac{5}{16}$	7.7	0.93	1.07	1.73	1.86	2.00
4¼ × 3¾	¾	17.4	1.15	1.07	1.88	2.03	2.18
4 × 3	$\frac{5}{16}$	7.1	0.76	0.89	1.79	1.92	2.07
4⅛ × 3⅛	⅝	13.8	0.90	0.80	1.88	2.02	2.17
3½ × 3	$\frac{5}{16}$	6.6	0.81	0.91	1.53	1.66	1.81
3$\frac{13}{16}$ × 3$\frac{5}{16}$	⅝	12.9	1.00	0.95	1.68	1.82	1.98
3½ × 2½	¼	4.9	0.61	0.74	1.58	1.71	1.86
3¾ × 2¾	½	9.4	0.76	0.77	1.71	1.85	2.00
3½ × 2	¼	4.5	0.46	0.56	1.65	1.80	1.95
3⅝ × 2⅛	⅜	6.6	0.53	0.57	1.71	1.85	2.00
3 × 2½	¼	4.5	0.67	0.75	1.31	1.45	1.60
3¼ × 2¾	½	8.7	0.80	0.80	1.44	1.58	1.73
3 × 2	¼	4.1	0.49	0.58	1.38	1.52	1.67
3¼ × 2¼	½	7.9	0.64	0.61	1.51	1.66	1.81
2½ × 2	$\frac{3}{16}$	2.7	0.51	0.60	1.10	1.23	1.38
2$\frac{13}{16}$ × 2$\frac{5}{16}$	½	7.0	0.69	0.66	1.27	1.41	1.56
2¼ × 1½	$\frac{3}{16}$	2.3	0.37	0.42	1.03	1.17	1.33
2$\frac{7}{16}$ × 1$\frac{11}{16}$	⅜	4.4	0.48	0.46	1.13	1.28	1.43
2 × 1½	$\frac{3}{16}$	2.1	0.39	0.44	0.90	1.04	1.19
2$\frac{3}{16}$ × 1$\frac{11}{16}$	⅜	4.3	0.50	0.48	0.99	1.14	1.30
2 × 1¼	$\frac{3}{16}$	1.9	0.31	0.35	0.94	1.09	1.24
3$\frac{3}{16}$ × 1$\frac{7}{16}$	⅜	3.9	0.42	0.38	1.03	1.18	1.34

Safe Loads, in Tons of 2000 Pounds, for Hollow Cylindrical Cast-iron Columns.

Passaic Rolling Mill Company.

Square ends. Factor of safety of 8.

Outside diameter, in inches.	Thickness of metal, in inches.	Length of column, in feet.									Area of section, in square inches.	Weight per foot of columns, in pounds.
		8	10	12	14	16	18	20	22	24		
6	¾	47	41	36	31	27	24	21	...	...	12.4	39
6	1	60	52	46	40	35	30	26	...	...	15.7	49
7	¾	60	54	48	43	38	34	30	27	24	14.7	46
7	1	76	69	62	55	49	43	38	34	30	18.9	60
8	¾	72	67	61	55	50	45	40	36	33	17.1	53
8	1	93	86	78	71	64	58	52	47	42	22.0	69
8	1¼	112	104	94	86	77	69	62	56	51	26.5	83
9	¾	85	80	74	68	62	57	52	47	43	19.4	61
9	1	110	103	95	88	80	73	67	61	55	25.1	78
9	1¼	133	125	115	106	97	89	81	73	67	30.4	95
9	1½	155	145	134	123	113	103	94	85	78	35.3	110
10	1	127	120	112	105	97	89	82	76	69	28.3	88
10	1¼	154	146	136	127	118	109	100	92	84	34.4	107
10	1½	180	170	159	148	137	127	117	107	98	40.1	125
10	1¾	203	192	180	168	155	143	132	121	111	45.4	142
11	1	144	137	129	122	114	106	100	91	85	31.4	98
11	1¼	175	167	158	148	139	129	122	112	103	38.3	119
11	1½	204	195	184	173	161	151	143	130	121	44.8	140
11	1¾	232	221	209	197	184	172	162	148	137	50.9	159
11	2	258	246	233	219	205	191	181	164	152	56.6	176
12	1	160	154	147	139	131	123	115	108	101	34.6	108
12	1¼	196	188	180	170	160	150	141	132	123	42.2	131
12	1½	229	220	210	199	187	176	165	154	144	49.5	154
12	1¾	261	251	239	226	213	201	188	176	164	56.4	176
12	2	291	279	266	252	238	224	210	196	183	62.8	196
13	1	177	170	163	156	148	140	132	124	117	37.7	118
13	1¼	216	209	200	191	181	172	162	152	143	46.1	144
13	1½	254	245	235	224	213	201	190	179	168	54.2	169
13	1¾	289	280	268	256	243	229	217	204	192	61.9	193
13	2	324	312	300	286	272	257	242	228	214	69.1	216
14	1	193	187	180	173	165	157	149	141	134	40.8	128
14	1¼	237	229	221	212	203	193	183	173	164	50.1	156
14	1½	278	270	260	250	239	227	215	204	193	58.9	184
14	1¾	318	308	297	285	273	260	246	233	220	67.4	210
14	2	356	345	333	320	305	291	276	261	247	75.4	235
15	1	209	204	197	190	183	175	167	159	151	44.0	137
15	1¼	257	250	242	233	224	214	205	195	185	54.0	168
15	1½	303	295	285	275	264	253	241	229	218	63.6	199
15	1¾	347	337	327	315	302	289	276	263	249	72.9	227
15	2	389	378	366	353	339	324	309	294	280	81.7	255
16	1¼	277	270	262	254	245	235	225	216	206	57.8	180
16	1½	327	319	311	300	290	278	267	255	244	68.4	214
16	1¾	375	366	356	344	332	319	306	292	279	78.4	245
16	2	421	411	400	387	373	358	343	328	313	88.0	275
16	2¼	465	454	441	427	412	396	379	363	346	97.2	304

Safe Loads, in Tons of 2000 Pounds, for Hollow Square Cast-iron Columns.

Passaic Rolling Mill Company.

Square ends. Factor of safety of 8.

Outside diameter, in inches.	Thickness of metal, in inches.	Length of column, in feet.									Area of section, in square inches.	Weight per foot of columns, in pounds.
		8	10	12	14	16	18	20	22	24		
6	¾	64	57	51	45	40	36	32	...	...	15.8	49
6	1	81	73	65	58	51	45	40	...	...	20.0	63
7	¾	80	73	67	61	55	50	45	...	...	18.8	59
7	1	102	94	86	78	70	63	57	...	...	24.0	75
8	¾	96	90	83	77	71	65	59	54	49	21.8	68
8	1	123	116	107	99	91	83	76	69	63	28.0	88
8	1¼	149	139	129	119	110	100	92	84	76	33.8	106
9	¾	112	106	100	93	87	80	74	69	63	24.8	77
9	1	144	137	129	121	112	104	96	89	82	32.0	100
9	1¼	175	166	156	146	136	126	116	107	99	38.8	121
9	1½	203	193	182	170	158	146	135	125	115	45.0	141
10	1	166	159	151	142	134	125	117	109	101	36.0	113
10	1¼	201	193	183	173	163	152	142	132	123	43.8	137
10	1½	235	225	214	202	189	177	166	154	143	51.0	159
10	1¾	266	254	242	228	215	201	188	175	162	57.8	181
11	1	187	180	172	164	156	147	138	130	122	40.0	125
11	1¼	227	219	210	200	190	179	169	158	148	48.8	152
11	1½	266	256	246	234	222	209	197	185	174	57.0	178
11	1¾	302	291	279	266	252	238	224	210	197	64.8	202
11	2	336	324	310	295	280	264	249	234	219	72.0	225
12	1	208	201	194	186	177	169	160	151	143	44.0	138
12	1¼	254	246	237	227	217	206	196	185	174	53.8	168
12	1½	297	288	278	266	254	242	229	217	205	63.0	197
12	1¾	338	328	316	303	289	275	261	247	233	71.8	224
12	2	377	366	352	338	323	307	291	275	260	80.0	250
13	1	228	222	215	208	199	191	182	173	164	48.0	150
13	1¼	279	272	263	254	244	233	223	212	201	58.8	184
13	1½	328	319	309	298	286	274	261	249	236	69.0	216
13	1¾	375	365	353	341	327	313	298	284	270	78.8	246
13	2	419	407	394	380	365	350	334	317	301	88.0	275
14	1	249	243	236	229	221	213	204	195	186	52.0	163
14	1¼	305	298	290	281	271	261	250	239	228	63.8	199
14	1½	359	351	341	330	319	307	294	281	268	75.0	234
14	1¾	411	401	390	378	365	351	336	322	307	85.8	268
14	2	460	449	437	423	408	393	376	360	344	96.0	300
15	1	270	264	258	250	243	235	226	217	208	56.0	175
15	1¼	331	324	316	308	298	288	277	266	255	68.8	215
15	1½	390	382	373	362	351	339	327	314	301	81.0	253
15	1¾	446	437	427	415	402	388	374	359	345	92.8	289
15	2	501	490	479	465	451	436	420	403	386	104.0	325
16	1¼	357	350	343	334	325	315	305	294	286	73.8	231
16	1½	421	413	404	394	383	372	359	347	334	87.0	272
16	1¾	482	474	463	452	440	426	412	397	383	99.8	312
16	2	541	532	520	507	493	478	463	446	429	112.0	350
16	2¼	598	588	575	561	545	529	511	493	475	123.8	387

Torsion.

When a prismatic body is subjected to the action of a force tending to rotate it about its geometric axis, it opposes to such a force its resistance to torsion. This resistance consists of the moments of the fibre stresses in the cross-section of the prism, and, until the elastic limit is reached, there exists an equilibrium between the external rotating forces on the one hand, and the stress moments of the various elements of the section on the other hand; both being taken with regard to the polar axis through the centre of gravity of the section, and at right angles to it.

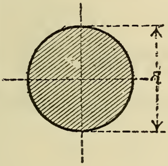
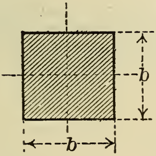
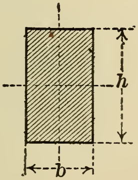
In computing these relations it is necessary to use the *polar* moment of inertia of the section, which may be indicated by I_p, and determined from the two moments of inertia of the section, taken at right angles to each other through the centre of gravity of the section.

If we have I_1 and I_2 to be the moments of inertia of the section of the prism under consideration, we have

$$\text{Polar Moment of Inertia} = I_p = I_1 + I_2.$$

The most usual sections for bodies under torsion are the circle, as in shafting, and the square and rectangle, which occur in various examples of machine framing. The polar moments of inertia for these are given in the annexed table.

Torsion Sections.

Number.	Section.	Polar moment of inertia, I_p.	Polar section modulus, $Z_p = \frac{I_p}{a}$.
I.		$\frac{\pi}{32}d^4$	$\frac{\pi}{16}d^3$
II.		$\frac{b^4}{6}$	$\frac{b^3}{3\sqrt{2}}$
III.		$\frac{1}{3}\frac{b^3h^3}{b^2+h^2}$	$\frac{b^2h^2}{3\sqrt{b^2+h^2}}$ Approximately, $\frac{b^2h^2}{3(0.4b+0.96h)}$

We then have the following relation:

Let M be the statical moment of the external forces at any section of the prism; I_p, the polar moment of inertia for that section; v, the distance

of the furthest element of the section from the centre of gravity of the section; S, the shearing fibre stress of the material at the distance, a, being taken at $\frac{4}{5}$ the permissible fibre stress for direct tension.

Then we have

$$M = S\frac{I_p}{v}.$$

The relative rotation which two sections of a prism at a given distance apart make with each other is called the angle of torsion. This may be represented by ϑ.

If we call the distance between two sections x, we have

$$\frac{d\vartheta}{dx} = \frac{M}{I_p G},$$

in which G is the modulus of torsion for the material used, and is equal to $\frac{2}{5}$ of the modulus of elasticity, E.

An example will make the application of the formulas clear.

Suppose a round shaft of wrought-iron, 4 inches in diameter and 48 inches long, is held at one end. A twisting force of 1000 pounds is applied at the other end, with a lever arm of 24 inches.

From the equation, $M = S\frac{I_p}{v}$, we have

$$S = \frac{v}{I_p}M.$$

For a circular section, $I_p = \frac{\pi}{32}d^4$, and we have $M = 24{,}000$, $d = 4$, and $v = \frac{d}{2}$, $\pi = 3.1416$.

Hence, $$S = \frac{16d}{\pi d^4} \,.\, 24{,}000 = 1909 \text{ pounds}.$$

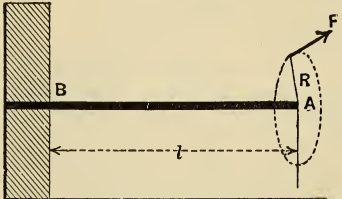
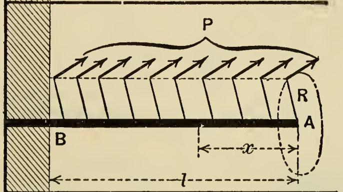
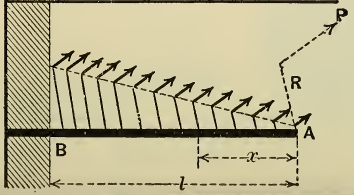
To get the angle of torsion we have, from the torsion table,

$$\vartheta = \frac{S}{G} \cdot \frac{l}{v} = \frac{1909}{11{,}200{,}000} \cdot \frac{48}{2} = 0.004,$$

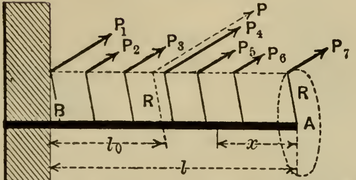
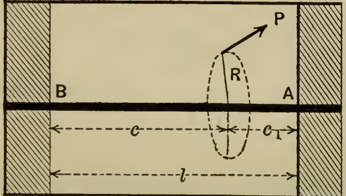
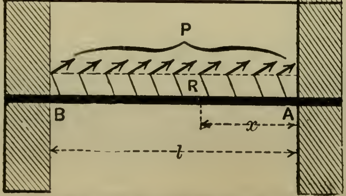
which is the length of arc of torsion for a radius 1, practically equal to the tangent of the angle; the corresponding angle being 0° 14′.

The following tables give the essential elements for all the conditions of torsion which are of probable occurrence in practice.

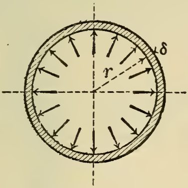
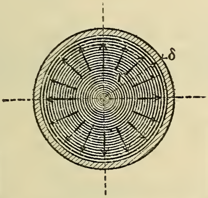
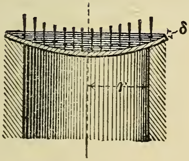
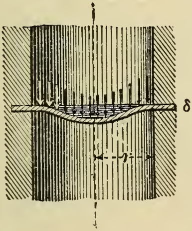
Torsion Table.

Number.	Application.	Moment, M.	Twisting force, P.	Angle of torsion, ϑ.	Remarks.
I.		$M = PR$ for all points between A and B.	$P = \frac{SI_p}{vR}$	$\vartheta = \frac{PRl}{I_pG}$ $= \frac{S}{G} \cdot \frac{l}{v}$	All sections between A and B are equally strong.
II.		$M = PR\frac{x}{l}$	$P = \frac{SI_p}{vR}$	$\vartheta = \frac{1}{2}\frac{PRl}{I_pG}$ $= \frac{1}{2}\frac{S}{G} \cdot \frac{l}{v}$	Weakest section at B.
III.		$M = PR\frac{x^2}{l^2}$ PR = the collected moment of all the twisting forces.	$P = \frac{SI_p}{vR}$	$\vartheta = \frac{1}{3}\frac{PRl}{I_pG}$ $= \frac{1}{3}\frac{S}{G} \cdot \frac{l}{v}$	Twisting forces decrease uniformly from B to A. Weakest section at B.

Torsion Table.

Number.	Application.	Moment, M.	Twisting force, P.	Angle of torsion, ϑ.	Remarks.
IV.		$M =$ the sum of the moments within x.	$P = \frac{SI_p}{vR}$	$\vartheta = \frac{PRl_o}{I_pG}$ $= \frac{S}{G} \cdot \frac{l_o}{v}$	General form of Cases I., II., and III. Weakest section at B. The value of ϑ in III. will be reached in IV., when $l_o = \frac{l}{3}$.
V.		In the portion c, $M = PR\frac{c_1}{l}$ In the portion c_1, $M = PR\frac{c}{l}$	When $c_1 < c$ $P = \frac{SI_p}{vR} \cdot \frac{l}{c}$	$\vartheta = \frac{PR}{I_pG} \cdot \frac{cc_1}{l}$ $= \frac{S}{G} \cdot \frac{c_1}{l}$	The shorter portion, c_1, is the weaker.
VI.		$M = PR\left(\frac{1}{2} - \frac{x}{l}\right)$	$P = 2\frac{SI_p}{vR}$	$\vartheta = \frac{1}{8}\frac{PRl}{I_pG}$ $= \frac{1}{8}\frac{S}{G} \cdot \frac{l}{v}$	Weakest points at A_1 and B.

Resistance to Internal Pressure.

Number.	Application.	Pressure, p.	Thickness, δ.
I. Cylinder.		$p = S\left(\sqrt{1+\frac{2\delta}{r}} - 1\right)$	$\frac{\delta}{r} = \frac{p}{S}\left(1+\frac{p}{S}\right)$
II. Sphere.		$p = 2S\frac{\delta}{r}$	$\frac{\delta}{r} = \frac{p}{2S}$
III. Round plate.		$p = S\left(\frac{\delta}{r}\right)^2$	$\frac{\delta}{r} = \sqrt{\frac{p}{S}}$
IV. Round plate.		$p = \frac{3}{2}S\left(\frac{\delta}{r}\right)^2$	$\frac{\delta}{r} = \sqrt{\frac{2}{3}}\sqrt{\frac{p}{S}}$

p = internal pressure, in pounds per square inch.
S = fibre stress upon material.
E = modulus of elasticity.
δ = thickness of plate, in inches.

For the deflection, f, we have for Case III.,

$$\frac{f}{\delta} = \frac{5}{6}\left(\frac{r}{\delta}\right)^4\frac{p}{E},$$

and for Case IV.,

$$\frac{f}{\delta} = \frac{1}{6}\left(\frac{r}{\delta}\right)^4\frac{p}{E}.$$

Thick Cylinders.

When the walls of a cylinder are very thick, as in the case of a hydraulic press, the material is not all strained uniformly for a given internal stress, the greater strain taking place upon the inner portion. The resistance under such conditions may be found by the formulas of Lamé:

$$p = S\frac{(r + \delta)^2 - r^2}{(r + \delta)^2 + r^2} \quad \text{and} \quad \frac{\delta}{r} = \sqrt{\frac{S + p}{S - p}} - 1,$$

in which r is the internal radius, δ is the thickness, and S is the fibre stress.

When p reaches the elastic limit of the material the inner fibres will begin to yield, regardless of the thickness of the walls.

For a discussion of the strengthening of cylinders by hooping, see Reuleaux's "Constructor."

Springs.

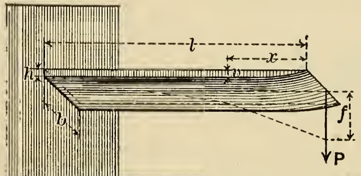
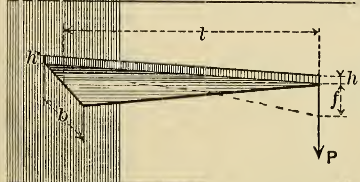
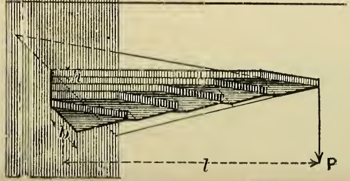
The deflection and supporting power of the various forms of springs exhibit very fully the laws of bending, torsion, and elasticity; and the data for various forms are given in the following tables, as prepared by Reuleaux.

The quantities in the tables are

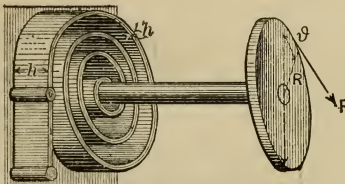
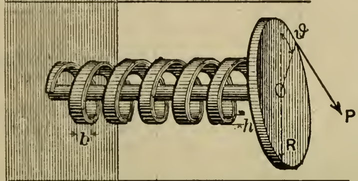
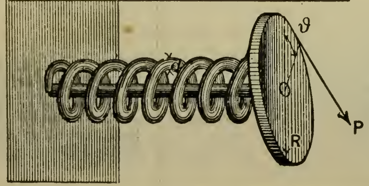
E = modulus of elasticity = 30,000,000 for steel;
G = modulus of torsion = $\frac{2}{5}E$;
S = fibre stress;
ϑ = angle of torsion = length of arc for radius 1.

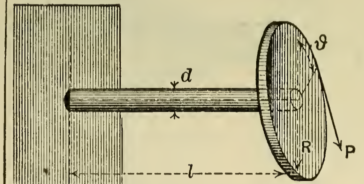
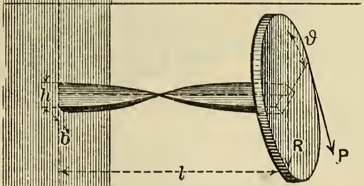
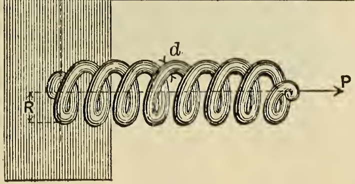
The other data used in the formulas are indicated in the illustrations.

Springs.

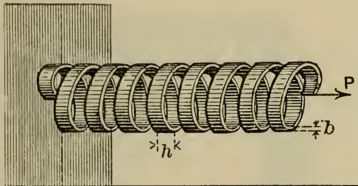
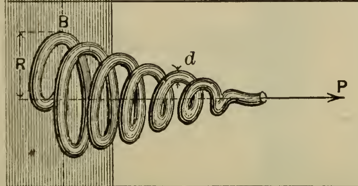
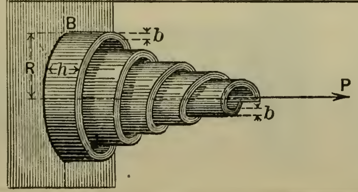
No.	Form.	Name.	Supporting power.	Deflection.	Elasticity.	Remarks.
I.		Rectangular spring.	$P = \frac{S}{6} \cdot \frac{bh^2}{l}$	$f = 6\frac{Pl^3}{Ebh^3}$	$\frac{f}{l} = \frac{S}{E} \cdot \frac{l}{h}$	An approximation to $\frac{y}{h} = \sqrt{\frac{x}{l}}$ will be secured by making the end $= \frac{2}{3}h$.
II.		Simple triangular spring.	$P = \frac{S}{6} \cdot \frac{bh^2}{l}$	$f = 6\frac{Pl^3}{Ebh^3}$	$\frac{f}{l} = \frac{S}{E} \cdot \frac{l}{h}$	Body of uniform resistance to bending. In practice the end is made somewhat thicker.
III.		Compound triangular spring.	$P = \frac{S}{6} \cdot \frac{ibh^2}{l}$ i = number of plates.	$f = 6\frac{Pl^3}{Eibh^3}$	$\frac{f}{l} = \frac{S}{E} \cdot \frac{l}{h}$	This is equivalent to a simple triangular spring, with a base $= ib$, as shown by the dotted lines.

Springs.

No.	Form.	Name.	Supporting power.	Deflection.	Elasticity.	Remarks.
IV.		Flat spiral spring.	$P = \frac{S}{6} \cdot \frac{bh^2}{R}$	$f = R\vartheta = 12\frac{PlR^2}{Ebh^3}$	$\frac{f}{R} = 2\frac{S}{E} \cdot \frac{l}{h}$	
V.		Flat helical spring.	$P = \frac{S}{6} \cdot \frac{bh^2}{R}$	$f = R\vartheta = 12\frac{PlR^2}{Ebh^3}$	$\frac{f}{R} = 2\frac{S}{E} \cdot \frac{l}{h}$	l = the developed length of the spiral. All three forms of uniform resistance. The value $\frac{f}{R}$ is the angle of rotation, ϑ, produced by the load, P.
VI.		Round helical spring.	$P = \frac{S\pi}{32} \cdot \frac{d^3}{R}$	$f = R\vartheta = \frac{64}{\pi} \cdot \frac{Pl}{E} \cdot \frac{R^2}{d^4}$	$\frac{f}{R} = 2\frac{S}{E} \cdot \frac{l}{d}$	

VII.		Simple round torsion spring.	$P = S\frac{\pi}{16} \cdot \frac{d^3}{R}$	$f = R\vartheta = \frac{32}{\pi} \cdot \frac{P}{G} \cdot \frac{R^2 l}{d^4}$	$\frac{f}{R} = 2\frac{S}{G} \cdot \frac{l}{d}$	Cases VII. to X. are bodies of uniform resistance to torsion.
VIII.		Simple flat torsion spring.	$P = \frac{S}{3R} \cdot \frac{b^2h^2}{\sqrt{b^2 + h^2}}$ Approximately, when $h > b$, $P = \frac{S}{R} \cdot \frac{b^2h^2}{3(0.4b + 0.96h)}$	$f = R\vartheta = 3\frac{PR^2l}{G} \cdot \frac{b^2 + h^2}{b^3h^3}$	$\frac{f}{R} = \frac{S}{G} \cdot \frac{l\sqrt{b^2 + h^2}}{bh}$	Springs of the form of VII. and VIII. may also be combined into compound forms.
IX.		Helical spring of round wire.	$P = S\frac{\pi}{16} \cdot \frac{d^3}{R}$	$f = \frac{32}{\pi} \cdot \frac{PR^2l}{Gd^4}$	$\frac{f}{R} = 2\frac{S}{G} \cdot \frac{l}{d}$	In cases IX. to XII. l is always the developed length of the spring.

Springs.

No.	Form.	Name.	Supporting power.	Deflection.	Elasticity.	Remarks.
X.		Helical spring of flat wire.	$P = \frac{S}{3R} \cdot \frac{b^2h^2}{\sqrt{b^2 + h^2}}$ Approximately, when $h > b$, $P = \frac{S}{R} \cdot \frac{b^2h^2}{3(0.4b + 0.96h)}$	$f = 3\frac{PR^2l}{G} \cdot \frac{b^2 + h^2}{b^3h^3}$	$\frac{f}{R} = \frac{S}{G} \cdot \frac{l\sqrt{b^2 + h^2}}{bh}$	It is immaterial whether the breadth of the plate is parallel, normal, or oblique to the axis.
XI.		Conical spring of round wire.	$P = S\frac{\pi}{16} \cdot \frac{d^3}{R}$	Approximately, $f = \frac{16}{\pi} \cdot \frac{PR^2l}{Gd^4}$	$\frac{f}{R} = \frac{S}{G} \cdot \frac{l}{d}$	Here, as in case XII., also, the spring is measured to the apex of the cone. The weakest point is at *B*.
XII.		Flat volute spring.	$P = \frac{S}{3R} \cdot \frac{b^2h^2}{\sqrt{b^2 + h^2}}$ Approximately, when $h > b$, $P = \frac{S}{R} \cdot \frac{b^2h^2}{3(0.4b + 0.96h)}$	Approximately, $f = \frac{3}{2}\frac{PR^2l}{G} \cdot \frac{b^2 + h^2}{b^3h^3}$	$\frac{f}{R} = \frac{1}{2}\frac{S}{G} \cdot \frac{l\sqrt{b^2 + h^2}}{bh}$	By making a gradual reduction in the value of *h*, from *B* to the end, this may be made a form of uniform resistance.

Specifications for Structural Steel.

Condensed from the Standard Specifications of the Association of American Steel Manufacturers.

Process of Manufacture.

1. Steel shall be made by either the open hearth or Bessemer process.

Test Pieces.

2. All tests and inspections shall be made at place of manufacture prior to shipment.

3. The tensile strength, limit of elasticity, and ductility shall be determined from a standard test piece, planed or turned parallel throughout its entire length, cut from the finished material. The elongation shall be measured on an original length of 8 inches, except when the thickness of the finished material is $\frac{5}{16}$ of an inch or less, in which case the elongation shall be measured in a length equal to sixteen times the thickness; and except in rounds of $\frac{5}{8}$ of an inch or less in diameter, in which case the elongation shall be measured in a length equal to eight times the diameter of section tested. Two test pieces shall be taken from each heat of finished material, one for tension and one for bending.

4. Every finished piece of steel shall be stamped with the heat number. Steel for pins shall have the heat numbers stamped on the ends. Rivet and lacing steel, and small pieces for tie plates and stiffeners, may be shipped in bundles securely wired together, with the heat number on a metal tag attached.

Finish.

5. Finished bars must be free from injurious seams, flaws, or cracks, and have a workmanlike finish.

Chemical Properties.

6. Steel for buildings, train sheds, highway bridges, and similar structures shall not contain more than 0.10 per cent. of phosphorus.

7. Steel for railway bridges shall not contain more than 0.08 per cent. of phosphorus.

Physical Properties.

8. Structural steel shall be of three grades: rivet steel, soft steel, and medium steel.

Rivet Steel.

9. Rivet steel shall have an ultimate strength of 48,000 to 58,000 pounds per square inch, an elastic limit of not less than one-half the ultimate strength, and an elongation of 26 per cent., and shall bend 180 degrees, flat on itself, without fracture on the outside of the bent portion.

Soft Steel.

10. Soft steel shall have an ultimate strength of 52,000 to 62,000 pounds per square inch, an elastic limit of not less than one-half the ultimate strength, and an elongation of 25 per cent., and shall bend 180 degrees, flat on itself, without fracture on the outside of the bent portion.

Medium Steel.

11. Medium steel shall have an ultimate strength of 60,000 to 70,000 pounds per square inch, an elastic limit of not less than one-half the ultimate strength, and an elongation of 22 per cent., and shall bend 180 degrees, around a curve having a diameter equal to the thickness of the piece tested, without fracture on the outside of the bent portion.

Pin Steel.

12. Pins made from either of the above-mentioned grades of steel shall, on specimen test pieces cut at a depth of 1 inch from the surface of finished material, fill the physical requirements of the grade of steel from which they are rolled for ultimate strength, elastic limit, and bending, but the required percentage of elongation shall be decreased 5 per cent.

Eye Bar Steel.

13. Eye bar material 1½ inches and less in thickness, made of either of the above-mentioned grades of steel, shall, on test pieces cut from finished material, fill the requirements of the grade of steel from which it is rolled. For thicknesses greater than 1½ inches there will be allowed a reduction in percentage of elongation of 1 per cent. for each ⅛ of an inch increase in thickness, to a minimum of 20 per cent. for medium steel and 22 per cent. for soft steel.

Full Size Test of Steel Eye Bars.

14. Full size tests of steel eye bars shall be required to show not less than 10 per cent. elongation in the body of the bar, and a tensile strength not more than 5000 pounds below the minimum tensile strength required in specimen tests of the grade of steel from which the bars are rolled. The bars will be required to break in the body; should a bar break in the head, but develop 10 per cent. elongation and the ultimate strength specified, it shall not be cause for rejection, provided not more than one-third of the total number of bars tested break in the head.

Variation in Weight.

15. A variation in cross-section or weight of more than 2½ per cent. from that specified will be sufficient cause for rejection, except in the case of sheared plates.

When sheared plates are ordered by weight, the permissible variation shall not be more than 2½ per cent. from that specified, except for plates ¼″ to $\frac{5}{16}$″ thick (10.2 to 12.75 pounds per square foot), which, when ordered to weight, shall not average a variation greater than 5 per cent. above or below the theoretical weight for plates over 75 inches wide.

When sheared plates are ordered to gauge, the overweight shall not exceed the percentages given in the following table:

Percentages of Allowable Overweights for Sheared Plates when ordered to Gauge.

Thickness of plate.	Width of plate.		
	Up to 75 inches.	75 to 100 inches.	Over 100 inches.
¼ inch	10	14	18
$\frac{5}{16}$ inch	8	12	16
⅜ inch	7	10	13
$\frac{7}{16}$ inch	6	8	10
½ inch	5.	7	9
$\frac{9}{16}$ inch	4½	6½	8½
⅝ inch	4	6	8
Over ⅝ inch	3½	5	6½

Timber.

The following data for the strength of wooden posts and beams are based upon tests made at the government arsenal at Watertown and by the Forestry Division of the United States Department of Agriculture.

Thus, tests made on pillars of white and yellow pine at Watertown gave the following results for the breaking loads in pounds per square inch:

	Ratio of length to thickness.										
	10	15	20	25	30	35	40	45	50	55	60
Yellow pine....	4400	4275	4100	3875	3600	3275	2900	2475	2130	1760	1480
White pine	2450	2390	2300	2190	2000	1890	1700	1490	1320	1090	910
Hemlock	2200	2150	2050	1950	1850	1700	1530	1340	1190	980	820

The following general facts concerning the physical properties of timber are deduced from the experiments of the Forestry Division:

1. That bleeding (the experiments were made on long leaf yellow pine) has no material effect on the strength of timber; the flexibility is slightly increased, but the bled timber will probably endure exposure to the weather as well as the other.

2. That moisture reduces the strength of timber, whether that moisture be the sap or water absorbed after seasoning. In general, seasoned timber, or with not more than 12 per cent. moisture, is from 75 per cent. to 100 per cent. stronger than green timber.

3. When artificially dried, timber contains a uniform percentage of moisture throughout, a condition requiring months or even years to attain in air-dried, heavy timber.

When kiln-dried at usual temperatures, wood shows no loss of strength compared with air-dried timber of the same percentage of moisture. The effect of very high temperatures and pressures (as used in vulcanizing) is lower strengths than when air-dried.

4. Large timbers are equal in strength per square inch of section, tested every way, to small timbers, provided they are equally sound and contain the same percentage of moisture.

5. The tests seem to indicate that the strength of woods of uniform structure increases with the specific gravity, irrespective of species,—*i.e.*, in general, the heaviest wood is the strongest. Oak seems not to belong to the list of woods to which this general remark applies.

The data on properties of timbers must be used with considerable judgment and caution. Seasoned wood will gain weight to the extent of 5 to 15 per cent. if exposed to the weather, and this excess will be reduced if the wood is kept a week in a warm, dry place.

Some of the individual tests made by the United States Forestry Division varied considerably from the mean values given in the table. In the case of tension tests, which varied most from the average, a few were as low as 25 per cent., while others reached 190 per cent. of the mean.

The elastic limit given in connection with the data from the United States Forestry Division is the relative elastic limit suggested by Professor Johnson, as there is no definite "elastic limit" in timber similar to that in some metals. This relative elastic limit is taken where the rate of deflection is 50 per cent. more than it is under initial loads.

Modulus of ultimate bending is extreme fibre stress on beam at rupture. The modulus of elastic bending is the fibre stress when the rate of deflection is increased 50 per cent. The modulus of elasticity is derived from transverse tests.

Physical Properties of Wood.

Seasoned timber, moisture 12 per cent. and under.
Stresses given in pounds per square inch.

Name of material.	Ultimate resistance to tension.	Ultimate resistance to compression, length.	Ultimate resistance to compression, cross.	Ultimate resistance to shearing, length.	Ultimate resistance to shearing, cross.
Ash (American)	17000	7200	1900	1100	6280
Birch	15000	8000	....	....	5600
Box	20000	10300	....	....	....
Cedar (white)	....	5200	700	400	1370
Cedar (American red)	10800	6000	....	....	....
Chestnut	11500	5300	....	....	1530
Cottonwood (see poplar)	....	....	....	....	....
Douglas spruce (Oregon pine)	13000	5700	800	500	....
Fir	13000	....	....	1300	....
Gum	....	7100	1400	800	5890
Hemlock	8700	5700	....	400	2750
Hickory (American average)	19600	9500	2700	1100	6000
Lignum vitæ	11800	9900	....	....	....
Mahogany (Spanish)	14900	8200	....	....	....
Maple	11150	7150	1800	500	6350
Oregon pine (see Douglas spruce)	....	....	....	....	....
Oak (red)	10250	7200	2300	1100	....
Oak (black or yellow)	10000	7300	1800	1100	....
Oak (white)	13600	8500	2200	1000	4400
Oak (live)	....	10400	....	....	8480
Pine (Southern yellow, long-leafed)	13000	8000	1260	835	5600
Pine (Cuban)	13000	8700	1200	770	....
Pine (loblolly)	13000	7400	1150	800	....
Pine (white)	10000	5400	700	400	2500
Poplar	7000	5000	....	....	....
Spruce (Northern)	11000	6000	....	400	3250
Spruce pine (pinus glabra of Southern States)	12000	7300	1200	800	....
Walnut (black)	10500	7500	2500	....	4700

Physical Properties of Wood.

Seasoned timber, moisture 12 per cent. and under.
Stresses given in pounds per square inch.

Elastic limit.	Modulus of elasticity.	Modulus of ultimate bending.	Modulus of elastic bending.	Ordinary working stress.			Weight, in pounds, per cubic foot.
				Tension.	Compression.	Transverse.	
7900	1 640 000	10800	7900	2000	1000	1200	39
....	1 645 000	11700	....	2000	1000	1200	33
....	....	....	....	2500	1200	1500	....
5800	910 000	6300	5800	1200	600	800	23
....	....	7200	....	1400	700	900	....
....	1 140 000	8100	....	1400	600	900	41
....	....	....	....	....	....	....	....
6400	1 680 000	7900	6400	1400	700	1000	32
....	1 530 000	....	....	....	....	....	....
7800	1 700 000	9500	7800	1200	900	900	37
....	....	7100	....	....	....	750	25
11200	2 390 000	16000	11000	2000	1200	1800	50
....	....	11700	....	1500	1200	1500	83
....	1 255 000	9550	....	1500	1200	1500	53
....	....	10000	....	....	....	....	49
....	....	....	....	....	....	....	....
9200	1 970 000	11400	9200	1400	900	1200	45
8100	1 740 000	10800	8100	1400	900	1200	45
9600	2 090 000	13100	9600	1700	1000	1500	50
9040	1 851 500	11300	....	....	....	....	....
10000	2 070 000	12600	9500	1600	1000	1500	38
11100	2 370 000	13600	10640	....	....	....	....
9200	2 050 000	11300	9400	1600	900	1200	33
6400	1 390 000	7900	6400	1200	700	900	24
....	....	6500	....	900	600	750	....
....	1 400 000	8000	....	1200	700	900	26
8400	1 640 000	10000	8400	1200	700	900	30
5700	1 306 000	8000	....	1000	1000	900	38

Greatest Safe Load, Uniformly Distributed, for Rectangular Wooden Beams One Inch Thick.

Depth, in inches.	Kind of timber.	Length of span, in feet.									
		4	6	8	10	12	14	16	18	20	22
		Safe loads, in pounds, per inch of thickness.									
5	Hemlock	520	350	260	210	170	150	130	120	100	90
	Spruce or pine	620	420	310	250	210	180	160	140	120	110
	Oak	830	550	420	330	280	240	210	190	170	150
	Yellow pine	*800	700	520	420	350	300	260	230	210	190
6	Hemlock	*640	500	380	300	250	210	190	170	150	140
	Spruce or pine	900	600	450	360	300	260	220	200	180	160
	Oak	1200	800	600	480	400	340	300	270	240	220
	Yellow pine	*960	*960	750	600	500	430	370	330	300	270
7	Hemlock	*750	680	510	410	340	290	260	230	200	190
	Spruce or pine	*1120	820	610	490	410	350	310	270	240	220
	Oak	1630	1090	820	650	540	470	410	360	330	300
	Yellow pine	*1120	*1120	1020	820	680	580	510	450	410	370
8	Hemlock	*850	*850	670	530	440	380	330	300	270	240
	Spruce or pine	*1280	1060	800	640	530	460	400	360	320	290
	Oak	*2130	1420	1070	850	710	610	530	470	430	390
	Yellow pine	*1280	*1280	*1280	1070	890	760	670	590	530	480
9	Hemlock	*960	*960	840	670	560	480	420	370	340	310
	Spruce or pine	*1440	1350	1010	810	670	580	510	450	400	370
	Oak	*2400	1800	1350	1080	900	770	670	600	540	490
	Yellow pine	*1440	*1440	*1440	1350	1120	960	840	750	670	610
10	Hemlock	*1070	*1070	1040	830	690	590	520	460	420	380
	Spruce or pine	*1600	*1600	1250	1000	830	710	620	560	500	450
	Oak	*2670	2220	1670	1330	1110	950	830	740	670	610
	Yellow pine	*1600	*1600	*1600	*1600	1390	1190	1040	930	830	760
12	Hemlock	*1280	*1280	*1280	1200	1000	860	750	670	600	540
	Spruce or pine	*1920	*1920	1800	1440	1200	1030	900	800	720	650
	Oak	*3200	*3200	2400	1920	1600	1370	1200	1070	960	870
	Yellow pine	*1920	*1920	*1920	*1920	*1920	1710	1500	1330	1200	1090
14	Hemlock	*1490	*1490	*1490	*1490	1360	1170	1020	900	820	740
	Spruce or pine	*2240	*2240	*2240	1960	1630	1400	1220	1090	980	890
	Oak	*3730	*3730	3270	2610	2180	1870	1630	1450	1310	1190
	Yellow pine	*2240	*2240	*2240	*2240	*2240	*2240	2040	1810	1630	1480
16	Hemlock	*1710	*1710	*1710	*1710	*1710	1520	1330	1180	1070	970
	Spruce or pine	*2560	*2560	*2560	2550	2130	1830	1600	1420	1280	1160
	Oak	*4270	*4270	*4270	3410	2840	2440	2130	1900	1710	1550
	Yellow pine	*2560	*2560	*2560	*2560	*2560	*2560	*2560	2370	2130	1940
18	Hemlock	*1920	*1920	*1920	*1920	*1920	*1920	1690	1500	1350	1230
	Spruce or pine	*2880	*2880	*2880	*2880	2700	2310	2030	1800	1620	1470
	Oak	*4800	*4800	*4800	4320	3600	3090	2700	2400	2160	1960
	Yellow pine	*2880	*2880	*2880	*2880	*2880	*2880	*2880	*2880	2700	2450

The short lengths, marked with a star, are computed to resist longitudinal shearing.

Greatest Safe Central Loads for Rectangular Wooden Beams One Inch Thick.

Depth, in inches.	Kind of timber.	Length of span, in feet.									
		4	6	8	10	12	14	16	18	20	22
		Safe loads, in pounds, per inch of thickness.									
5	Hemlock	260	170	130	100	90	75	65	60	50	45
	Spruce or pine.	310	210	160	120	100	90	80	70	60	55
	Oak	420	280	210	170	140	120	100	95	85	75
	Yellow pine ...	520	350	260	210	170	150	130	120	100	95
6	Hemlock	380	250	190	150	120	110	95	85	75	70
	Spruce or pine.	450	300	220	180	150	130	110	100	90	80
	Oak	600	400	300	240	200	170	150	130	120	110
	Yellow pine ...	750	500	370	300	250	210	190	170	150	140
7	Hemlock	510	340	260	200	170	150	130	110	100	95
	Spruce or pine.	610	410	310	240	200	170	150	140	120	110
	Oak	820	540	410	330	270	230	200	180	160	150
	Yellow pine ...	1020	680	510	410	340	290	260	230	200	190
8	Hemlock	670	440	330	270	220	190	170	150	130	120
	Spruce or pine.	800	530	400	320	270	230	200	180	160	150
	Oak	1070	710	530	430	360	300	270	240	210	190
	Yellow pine ...	1330	890	670	530	440	380	330	300	270	240
9	Hemlock	840	560	420	340	280	240	210	190	170	150
	Spruce or pine.	1010	670	510	400	340	290	250	220	200	190
	Oak	1350	900	670	540	450	390	340	300	270	250
	Yellow pine ...	*1440	1120	840	670	560	480	420	370	340	310
10	Hemlock	1040	690	520	420	350	300	260	230	210	190
	Spruce or pine.	1250	830	620	500	410	360	310	280	250	230
	Oak	1670	1110	830	670	550	480	420	370	330	300
	Yellow pine ...	*1600	1390	1040	830	690	590	520	460	410	380
12	Hemlock	*1280	1000	750	600	500	430	370	330	300	270
	Spruce or pine.	1800	1200	900	720	600	510	450	400	360	330
	Oak	2400	1600	1200	960	800	690	600	530	480	440
	Yellow pine ...	*1920	*1920	1500	1200	1000	860	750	670	600	540
14	Hemlock	*1490	1360	1020	820	680	580	510	450	410	370
	Spruce or pine.	*2240	1630	1220	980	810	700	610	540	490	440
	Oak	3270	2180	1630	1310	1090	930	820	730	650	590
	Yellow pine ...	*2240	*2240	2040	1630	1360	1170	1020	910	810	740
16	Hemlock	*1710	*1710	1330	1070	890	760	660	590	530	480
	Spruce or pine.	*2560	2130	1600	1280	1060	910	800	710	640	580
	Oak	*4270	2840	2130	1710	1420	1220	1060	950	850	780
	Yellow pine ...	*2560	*2560	*2560	2130	1780	1520	1330	1180	1060	970
18	Hemlock	*1920	*1920	1690	1350	1120	960	840	750	670	610
	Spruce or pine.	*2880	2700	2030	1620	1350	1160	1010	900	810	740
	Oak	*4800	3600	2700	2160	1800	1540	1350	1200	1080	980
	Yellow pine ...	*2880	*2880	*2880	2700	2250	1930	1690	1500	1350	1230

The short lengths, marked with a star, are computed to resist longitudinal shearing.

Total Safe Load, in Net Tons, for Square Pillars.

For Hemlock Pillars take $\frac{9}{10}$ of Load for White Pine.

Height, in feet.	Kind of timber.	Side of square pillar, in inches. 4	5	6	7	8	9	10	12	14	16
		Total safe load, in net tons.									
6	Yellow pine...	7.3	11.8	17.3	23.8	31.3	39.8	49.3	71.3	97.3	127.3
	White pine....	4.5	7.2	10.5	14.4	18.9	24.0	29.7	42.9	58.5	76.5
7	Yellow pine...	7.1	11.6	17.1	23.6	31.1	39.6	49.1	71.1	97.1	127.1
	White pine....	4.4	7.1	10.3	14.3	18.7	23.6	29.5	42.8	58.4	76.4
8	Yellow pine...	6.8	11.3	16.8	23.3	30.8	39.3	48.8	70.8	96.8	126.8
	White pine....	4.2	6.9	10.2	14.1	18.6	23.7	29.4	42.6	58.2	76.2
9	Yellow pine...	6.5	11.0	16.5	23.0	30.5	39.0	48.5	70.5	96.5	126.5
	White pine....	4.1	6.8	10.1	14.0	18.5	23.6	29.3	42.5	58.1	76.1
10	Yellow pine...	6.2	10.7	16.2	22.7	30.2	38.7	48.2	70.2	96.2	126.2
	White pine....	3.9	6.6	9.9	13.8	18.3	23.4	29.1	42.3	57.9	75.9
11	Yellow pine...	5.8	10.3	15.8	22.3	29.8	38.3	47.8	69.8	95.8	125.8
	White pine....	3.7	6.4	9.7	13.6	18.1	23.2	28.9	42.1	57.7	75.7
12	Yellow pine...	5.4	9.9	15.4	21.9	29.4	37.9	47.4	69.4	95.4	125.4
	White pine....	3.5	6.2	9.5	13.4	17.9	23.0	28.7	41.9	57.5	75.5
13	Yellow pine...	5.0	9.5	15.0	21.5	29.0	37.5	47.0	69.0	95.0	125.0
	White pine....	3.3	6.0	9.3	13.2	17.7	22.8	28.5	41.7	57.3	75.3
14	Yellow pine...	4.5	9.0	14.5	21.0	28.5	37.0	46.5	68.5	94.5	124.5
	White pine....	3.0	5.7	9.0	12.9	17.4	22.5	28.2	41.4	57.0	75.0
15	Yellow pine...	3.9	8.4	13.9	20.5	27.9	36.4	45.9	67.9	93.9	123.9
	White pine....	2.8	5.5	8.8	12.7	17.2	22.3	28.0	41.2	56.8	74.8
16	Yellow pine...	3.4	7.9	13.4	19.9	27.4	35.9	45.4	67.4	93.4	123.4
	White pine....	2.5	5.2	8.5	12.4	16.9	22.0	27.7	40.9	56.5	74.5
17	Yellow pine...	2.5	7.3	12.8	19.3	26.8	35.3	44.8	66.8	92.8	122.8
	White pine....	1.7	4.9	8.2	12.1	16.6	21.7	27.4	40.6	56.2	74.2
18	Yellow pine...	2.3	6.7	12.2	18.7	26.2	34.7	44.2	66.2	92.2	122.2
	White pine....	1.5	4.6	7.9	11.8	16.3	21.4	27.1	40.3	55.9	73.9
19	Yellow pine...	2.1	6.0	11.5	18.0	25.5	34.0	43.5	65.5	91.5	121.5
	White pine....	1.4	4.2	7.6	11.4	15.9	21.0	26.7	39.9	55.6	73.5
20	Yellow pine...	1.9	5.3	10.8	17.3	24.8	33.3	42.8	64.8	90.8	120.8
	White pine....	1.2	3.9	7.2	11.1	15.6	20.7	26.4	39.6	55.2	73.2
21	Yellow pine...	1.7	2.6	10.1	16.6	24.1	32.6	42.1	64.0	90.1	120.1
	White pine....	1.1	1.7	6.8	10.7	15.2	20.3	26.0	39.2	54.8	72.8
22	Yellow pine...	1.5	2.4	9.3	15.8	23.3	31.8	41.3	63.3	89.3	119.3
	White pine....	1.0	1.6	6.4	10.3	14.8	19.9	25.6	38.8	54.4	72.4

Average Strengths of Materials.

In the foregoing discussion of the strength of materials it has been assumed that the elastic limit, ultimate strength, modulus of elasticity, and similar data concerning the materials to be used are known for the especial case under consideration, and attention has mainly been given to the distribution of stresses and strains. In all important works the material should be tested and its properties ascertained, and during the conduct of the work frequent tests should be made by competent persons, using reliable testing machines; the test pieces being selected with care to represent the actual material employed.

In the absence of specific data concerning the actual materials to be used, the values in the following tables may be taken as representing fairly average results.

The tables which have been given of the strength of standard rolled sections represent experimental results made by the makers, and may be accepted, also, as closely corresponding to the similar sections of other mills.

Data concerning the strength and proportions of various machine parts will be discussed in connection with the subject of machine design.

In the use of materials of construction judgment should be used in connection with the results of tests, since it is manifestly absurd to take the resistance of the material to the pound when the load may be known only to the nearest ton. Care should also be taken to use records of strength of materials in the same general sense in which the original experiments were made, so far as can be ascertained, otherwise there can be no certainty that the conditions under which the resistance was ascertained are reproduced in the case in point. No records of experimental work can take the place of sound judgment on the part of the engineer, and he should always be liberal in his allowances for unforeseen stresses and shocks.

In many cases it must be remembered that strength is not the only element to be taken into account, but that stability and massiveness may sometimes demand far more material than the mere stresses would indicate. The effects of impact may require masses of metal for their reception, while in other cases the section may depend upon the amount of heat to be carried away. When it is realized that the actual strength is but one of several elements involved in engineering design, it will be understood that the main thing is to be on the right side, and that an extreme apparent precision may be far from representing true accuracy.

Average Ultimate Strengths of Materials.

Pounds per Square Inch.

Material.	Tension.		Compression.			Transverse.		Shearing.	
			With grain.						
	With grain.	Across grain.	End bearing.	Columns under 15 diams. long.	Across grain.	Extreme fibre stress.	Modulus of elasticity.	With grain.	Across grain.
Timbers.									
White oak	10000	2000	7000	4500	2000	6000	1 100 000	800	4000
White pine	7000	500	5500	3500	800	4000	1 000 000	400	2000
Southern long-leaf, or Georgia yellow pine	12000	600	8000	5000	1400	7000	1 700 000	600	5000
Douglas, Oregon, and Washington fir or pine { yellow fir	12000	....	8000	6000	1200	6500	1 400 000	600	....
Douglas, Oregon, and Washington fir or pine { red fir	10000	....	....	....	....	5000	....	....	....
Northern, or short-leaf yellow pine	9000	500	6000	4000	1000	6000	1 200 000	400	4000
Red pine	9000	500	6000	4000	800	5000	1 200 000	....	....
Norway pine	8000	....	6000	4000	800	4000	1 200 000	....	....
Canadian (Ottawa) white pine	10000	....	....	5000	....	....	....	350	....
Canadian (Ontario) red pine	10000	....	....	5000	....	5000	1 400 000	400	....
Spruce and Eastern fir	8000	500	6000	4000	700	4000	1 200 000	400	3000
Hemlock	6000	....	....	4000	600	3500	900 000	350	2500
Cypress	6000	....	6000	4000	700	5000	900 000	....	....
Cedar	8000	....	6000	4000	700	5000	700 000	....	1500
Chestnut	9000	....	....	5000	900	5000	1 000 000	600	1500
California redwood	7000	....	....	4000	800	4500	700 000	400	....
California spruce	....	....	....	4000	....	5000	1 200 000	....	....

For quiescent loads, as in buildings, divide above values by the following factors: tension, 10; compression, 5; transverse, 6; shearing, 5.

Average Ultimate Strengths of Materials.

Pounds per Square Inch.

Material.	Compression.	Tension.	Elastic limit.	Shearing.	Modulus of rupture.	Modulus of elasticity.
Metals.						
Aluminum, commercial	12 000	15000	6500	12000	....	11 000 000
Aluminum, nickel	....	40000	22000	....	....	....
Brass, cast	(30 000)	24000	6000	36000	20000	9 000 000
Brass wire, annealed	....	50000	....	....	....	....
Brass wire, unannealed	....	80000	16000	....	....	14 000 000
Bronze, aluminum	120 000	75000	....	....	....	....
Bronze, gun-metal	(20 000)	32000	10000	....	53000	10 000 000
Bronze, manganese	120 000	60000	30000	....	....	....
Bronze, phosphor	....	50000	24000	....	....	14 000 000
Bronze, Tobin	....	66000	40000	....	....	4 500 000
Copper bolts	30 000	30000	....	....	....	....
Copper, cast	(40 000)	24000	6000	30000	22000	10 000 000
Copper wire, annealed	....	36000	....	....	....	15 000 000
Copper wire, unannealed	....	60000	10000	....	....	18 000 000
Gold, cast	....	20000	4000	....	....	8 000 000
Iron, cast	80 000	15000	6000	18000	30000	12 000 000
Iron chains	....	35000	....	....	....	....
Iron, corrugated	....	....	....	....	40000	....
Iron wire, annealed	....	60000	....	....	....	15 000 000
Iron wire, unannealed	....	80000	27000	....	....	25 000 000
Iron, wrought, shapes	46 000	48000	26000	40000	44000	27 000 000
Iron, wrought, rerolled bars	48 000	50000	27000	40000	48000	26 000 000

Compression values enclosed in parentheses indicate loads producing 10 per cent. reduction in original lengths.

Average Ultimate Strengths of Materials.

Pounds per Square Inch.

Material.	Compression.	Tension.	Elastic limit.	Shearing.	Modulus of rupture.	Modulus of elasticity.
Metals.—*Continued.*						
Lead, cast	....	2 000	1000	....	....	1 000 000
Lead-pipe	....	1 600	....	....	....	....
Silver, cast	....	40 000	4000	....	....	10 000 000
Steel castings	70000	70 000	40000	60000	70000	30 000 000
Steel, structural, 0.10 per cent. carbon	56000	56 000	30000	48000	54000	29 000 000
Steel, structural, 0.15 per cent. carbon	64000	64 000	33000	50000	60000	29 000 000
Steel wire, annealed	....	80 000	40000	....	....	29 000 000
Steel wire, unannealed	....	120 000	60000	....	....	30 000 000
Steel wire, crucible	....	180 000	80000	....	....	30 000 000
Steel wire for suspension bridges	....	200 000	90000	....	....	30 000 000
Steel wire, special tempered	....	300 000	....	....	....	....
Tin, cast	(6000)	3 500	1800	....	4000	4 000 000
Zinc, cast	(20000)	5 000	4000	....	7000	13 000 000
Miscellaneous.						
Flax yarn	....	25 000	....	....	....	....
Glass, common green	20000	3 000	3000	....	4000	8 000 000
Glass flooring	10000	3 000	....	....	3000	....
Glass wire, for skylights	....	....	5000	....	5000	....
Leather, ox	....	4 000	....	....	....	240 000
Rope, hemp	....	8 000	....	....	....	....
Rope, manila	....	9 000	....	....	....	....
Silk fibre	....	5 000	....	....	....	1 300 000

Compression values enclosed in parentheses indicate loads producing 10 per cent. reduction in original lengths.

Average Ultimate Strengths of Materials.

Pounds per Square Inch.

Material.	Compression.	Tension.	Modulus of rupture.
Building Stones.			
Bluestone	13500	1400	2700
Granite, average	15000	600	1800
Granite, Connecticut	12000	....	....
Granite, New Hampshire	15000	....	1500
Granite, Massachusetts	16000	....	1800
Granite, New York	15000	....	....
Limestone, average	7000	1000	1500
Limestone, Hudson River, New York	17000	....	....
Limestone, Ohio	12000	....	1500
Marble, average	8000	700	....
Marble, Vermont	8000	700	1200
Sandstone, average	5000	150	1200
Sandstone, New Jersey	12000	....	650
Sandstone, New York	10000	....	1700
Sandstone, Ohio	9000	100	700
Slate	10000	10000	5000
Stonework	($\frac{4}{10}$ strength of stone)		
Bricks.			
Bricks, light red	1000	40	....
Bricks, good common	10000	200	600
Bricks, best hard	12000	400	800
Bricks, Philadelphia pressed	6000	200	600
Brickwork, common (lime mortar)	1000	50	....
Brickwork, good (cement and lime mortar)	1500	100	....
Brickwork, best (cement mortar)	2000	300	....
Terra-cotta	5000	....	....
Terra-cotta work	2000	....	....
Cements, etc.			
Cement, Rosendale, one month old	1200	200	200
Cement, Portland, one month old	2000	400	400
Cement, Rosendale, one year old	2000	300	400
Cement, Portland, one year old	3000	500	800
Mortar, lime, one year old	400	50	100
Mortar, lime and Rosendale, one year old	600	75	200
Mortar, Rosendale cement, one year old	1000	125	300
Mortar, Portland cement, one year old	2000	250	600
Concrete, Portland, one month old	1000	200	100
Concrete, Rosendale, one month old	500	100	50
Concrete, Portland, one year old	2000	400	150
Concrete, Rosendale, one year old	1000	200	75

Safe strengths of stone, brick, and cement, $\frac{1}{10}$ to $\frac{1}{30}$ of ultimate.

MACHINE DESIGN.

"A machine," according to the definition of Professor Reuleaux, is "a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions."

In designing a machine, therefore, it is essential to consider the resistance or strength of the bodies of which it is composed, also the work which it is to perform and the determinate motions to be made.

The resistance of the parts of a machine includes the proportions of the main framing, as well as the various shafts, gear-wheels, pulleys, connecting rods, and other parts, in most of which the actual strength is of less importance than the stiffness, or rigidity, and the mass necessary to furnish satisfactory solidity to absorb vibrations and shocks. The resistance also includes the proportions of the various fastenings, such as bolts, rivets, keys, pins, etc.

The work which the machine is compelled to do is the basis upon which the dimensions and form of the resistant parts are determined, and this is usually taken from the resistance opposed to the motion by the material to be cut, the weight to be lifted or propelled, or, in general, the opposing forces to be overcome.

The determinate motions involve some of the most intricate problems in machine design, and properly form the subject of a distinct science,—that of Kinematics, the science of controlled motion, considered apart from the magnitude and character of the forces involved.

It is impossible to do more here than to give the results of accepted modern practice with regard to these various elements of machine design, with such suggestions as experience may indicate for use with the special requirements of each case.

Usually, the determinate motions demand the first attention. The form of the work to be done must be considered at the one end of the system, and the nature of the motion from which it is to be effected at the other, the machine standing between them and effecting the transformation. Thus, in the case of the steam engine, the flowing steam, passing through a pipe, is converted into the rotary motion of the shaft, fly-wheel, and pulley. In like manner, the rotary motion communicated to the shaft of a Jacquard loom is transformed into all the complicated sequence of intermittent, yet determinate movements, which, through the medium of cards, heddles, shuttles, beams, etc., produce the elaborate woven fabric.

Having laid out the movements, and thus determined the positions of the various centres and connections, the forces to be transmitted must be considered and the dimensions of the actual pieces computed.

In many portions of a machine the dimensions of the parts may be based upon the direct knowledge of the strength of the materials, but in other cases empirical rules, based upon the results of experience, must be employed. Both methods will here be given, according to current practice, and whenever a rational method, used the direct strength of the material, is practicable, it will be given.

FASTENINGS.

Riveting.

Rivets are used to secure structures of sheet metal, and may be employed solely for strength, as in structural iron or steel work; or for strength and tightness combined, as in tank and boiler construction.

For strength alone the following method may be used for proportioning riveted connections:*

For any given thickness, δ, of plate it is impracticable to make the riveted joint the same strength as the plate itself, but the ratio between the strength of the plate and the strength of the joint can be made a maximum. This will best be attained, with the assumption of a sufficient margin, when the strength of the rivets and the strength of the remainder of the metal between the rivet-holes are equal to each other,—*i.e.*, when

* Reuleaux, "Constructor," § 55, *et seq.*

they reach their limit of elasticity at the same time. If the rivets and plate are of the same material, we have the stress in the cross-section of the rivets as 0.8 that of the plate. From this we derive the following formulæ, in which the friction of the joint is neglected as being of uncertain value:

Let

δ = the thickness of the plate, in inches;
d = the diameter of rivet, in inches;
a = the pitch of rivets,—*i.e.*, the distance from centre to centre of adjacent rivets, in inches;
n = the number of rows of rivets;
ϕ = the efficiency of the joint, being the ratio of the resistance of the joint to that of the full plate;

then the highest efficiency will be attained when we have, for lap-joint riveting,

$$\frac{a}{\delta} = n\frac{\pi}{5}\left(\frac{d}{\delta}\right)^2 + \frac{d}{\delta},$$

which gives

$$\phi = 1 - \frac{d}{a} = \frac{1}{1 + \frac{1}{n} \cdot \frac{5}{\pi} \cdot \frac{\delta}{d}};$$

or for butt-joint riveting,

$$\frac{a}{\delta} = 2n\frac{\pi}{5}\left(\frac{d}{\delta}\right)^2 + \frac{d}{\delta},$$

which gives

$$\phi = 1 - \frac{d}{a} = \frac{1}{1 + \frac{1}{2n} \cdot \frac{5}{\pi} \cdot \frac{\delta}{d}}.$$

The overlap of the plate is subjected both to shearing and bending. For the former conditions call the lap b', and for the latter b'', measuring in both cases from the centre of the rivets to the edge of the joint. To obtain the same resistance in the lap as in the perforated portion of the plate, we have, for lap-joint riveting,

$$\frac{b'}{\delta} = \tfrac{5}{8}\frac{a - d}{n\delta} = \frac{\pi}{8}\left(\frac{d}{\delta}\right)^2,$$

$$\frac{b''}{\delta} = \left(0.5 + 0.56\sqrt{\frac{d}{\delta}}\right);$$

for butt-joint riveting,

$$\frac{b'}{\delta} = \tfrac{5}{8}\frac{a - d}{n\delta} = \frac{\pi}{4}\left(\frac{d}{\delta}\right),$$

$$\frac{b''}{\delta} = \left(0.5 + 0.79\sqrt{\frac{d}{\delta}}\right)\frac{d}{\delta}.$$

In both cases a good value of b, in practice, giving sufficient room for rivet-heads, will be secured by making

$$b = 1.5d, \text{ or } \frac{b}{\delta} = 1.5\frac{d}{\delta}.$$

A point of interest is the superficial pressure, p, which exists between the body of the rivet and the cylindrical surface of the rivet-hole. If S_2 is the stress in the punched plate, we have, for lap-riveted joints,

$$\frac{p}{S_2} = 0.2\pi\frac{d}{\delta};$$

for butt-riveted joints,

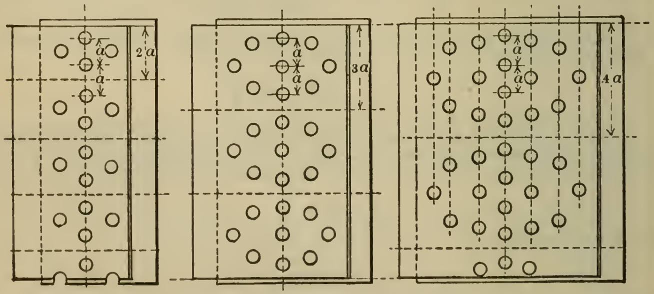
$$\frac{p}{S_2} = 0.4\pi\frac{d}{\delta}.$$

The following table will serve to reduce the numerical labor of these calculations:

Proportions for Riveted Joints.

	$\frac{d}{\delta} =$	1		1.5		2		2.5		3		4	
	$n =$	1	2	1	2	1	2	1	2	1	2	1	2
Lap joint.	$\frac{a}{\delta} =$	1.63	2.22	2.92	4.33	4.52	7.04	6.43	10.37	8.67	14.33	14.07	24.14
	$\frac{b'}{\delta} =$	0.39	0.39	0.88	0.88	1.57	1.57	2.54	2.54	3.53	3.53	6.28	6.28
	$\frac{b''}{\delta} =$	1.06	1.06	1.78	1.78	2.58	2.58	3.46	3.46	4.31	4.31	6.48	6.48
	$\phi =$	0.39	0.55	0.49	0.65	0.56	0.72	0.61	0.76	0.65	0.79	0.72	0.83
	$\frac{p}{S_2} =$	0.63	0.63	0.94	0.94	1.26	1.26	1.57	1.57	1.88	1.88	2.51	2.51
Butt joint.	$\frac{a}{\delta} =$	2.26	3.52	4.33	7.15	7.04	12.05	10.37	18.21	14.33	25.61	24.14	44.21
	$\frac{b'}{\delta} =$	0.79	0.79	0.96	0.96	3.14	3.14	4.91	4.91	7.07	7.07	12.56	12.56
	$\frac{b''}{\delta} =$	1.29	1.29	2.20	2.20	3.24	3.24	4.37	4.37	5.60	5.60	8.32	8.32
	$\phi =$	0.56	0.72	0.65	0.79	0.72	0.83	0.76	0.86	0.79	0.90	0.83	0.94
	$\frac{p}{S_2} =$	1.26	1.26	1.88	1.88	2.51	2.51	3.14	3.14	3.77	3.77	5.03	5.03

An examination of the preceding table shows that the higher efficiencies require the use of inconveniently large rivets. This may be avoided if more than two rows of rivets can be used, since they may then be disposed in groups. In this arrangement each row in a group has one less rivet than the preceding row, as shown in the illustration.



Thus, the rivets are arranged according to a certain pitch, a, for the middle row of a joint, and 2, 3, 4, or 5 rivets are selected as the base of a group. The next row on each side will have a wider pitch, and the next still wider, and so on.

If, as before, we take

δ = thickness of plate, in inches;
d = diameter of rivet, in inches;
a = pitch of rivets, in inches;
ϕ = efficiency of joint; and
m = number of rivets in the middle row of each group;

we have

Table for Group Riveting.

$m =$	2	3	4	5
$\frac{d}{\delta} =$	1.6	1.6	1.6	1.6
$\frac{a}{d} =$	2.5	3.33	4.25	5.2
$\frac{a}{\delta} =$	4.0	5.32	6.80	8.32
$\phi =$	.8	.90	.94	.96

The rivet-holes, in all cases, are made $\frac{1}{16}$ larger than the diameter of the rivet.

Boiler Riveting.

The necessity for making a tight joint has necessitated modifications of the proportions of joints based solely upon considerations of strength. The following tables represent standard practice:

Table of Proportions for Riveted Joints with Iron Plates and Rivets.

Thickness of plate...............	$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$
Diameter of rivet	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
Diameter of rivet-hole...........	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$	$\frac{15}{16}''$
Pitch—single riveting	2″	$2\frac{1}{16}''$	$2\frac{1}{8}''$	$2\frac{3}{16}''$	$2\frac{1}{4}''$
Pitch—double riveting	3″	$3\frac{1}{8}''$	$3\frac{1}{4}''$	$3\frac{3}{8}''$	$3\frac{1}{2}''$
Efficiency—single riveting.......	.66%	.64%	.62%	.60%	.58%
Efficiency—double riveting......	.77%	.76%	.75%	.74%	.73%

Table of Proportions for Riveted Joints in Steel Plates with Iron Rivets.

	Inch.	Inch.	Inch.	Inch.	Inch.
Thickness of plate...............	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$
Diameter of rivet................	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$
Diameter of rivet-hole...........	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1
Pitch—single riveting	2	$2\frac{1}{16}$	$2\frac{1}{8}$	$2\frac{3}{16}$	$2\frac{1}{4}$
Pitch—double riveting	3	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{1}{2}$

Bolts.

The dimensions of standard bolts and nuts will be found on pages 304 and 305, the United States standard being used in America and the Whitworth standard in Great Britain and on the Continent.

Bolts are usually made of wrought-iron or mild steel. Fibre stresses of 10,000 to 15,000 pounds per square inch might be permitted under normal conditions of loading, but very often a heavy initial stress is put upon a bolt by reason of the tension applied when the nut is screwed up. A source of weakness is also found in the varying cross-section of the bolt at different points, thus preventing uniform stretching. The result is frequent breakages at the root of the thread, especially at the point where the thread merges into the full body of the bolt, since the change in section at this point localizes the stretch. By drilling a central hole from the head to the beginning of the thread, and thus making the cross-section of the main body of the bolt the same as at the bottom of the thread, the stretch may be distributed. For ordinary joints the fibre stress on bolts may be taken as 8000 pounds for iron and 11,000 pounds for mild steel.

Bolted flange-joints to resist steam-, air-, or water-pressure must be designed for *tightness* rather than for strength. Here it is the initial tension upon the bolts which holds the faces of the joint together. The sum of the initial tensions of all the bolts in a flange-joint must be greater than the force acting to separate the joint, or it will open and leakage will occur. Under such conditions the maximum stress which should be put upon the bolts is about 6000 pounds per square inch, and lower stresses, down to 3000 pounds, are preferable. Instead of using larger bolts, the stresses should be reduced by using more of them. Recommended spacings for bolts on pipe flanges will be found in the table of standard flanges on page 329.

For pipe flanges the number of bolts is always made a multiple of four, in order to permit any member to be rotated 90° in making connections. For other bolted work the distance between bolt-centres for steam-tight work should not be less than $6d$, in which d is the diameter of the bolt, while for heavy pressures a spacing of $4d$ is to be preferred.

For steam cylinders, or similar situations, the number of bolts may be determined by the following formula:

$$N = \frac{p}{2400}\left(\frac{D}{d}\right)^2,$$

in which

N = number of bolts;
D = diameter of cylinder, in inches;
d = diameter of bolts;
p = pressure, in pounds per square inch.

Thus, for a cylinder 36 inches in diameter, with a pressure of 100 pounds, we have, for 1¼-inch bolts,

$$N = \frac{100}{2400}\left(\frac{36}{1.25}\right)^2 = 35 \text{ bolts}.$$

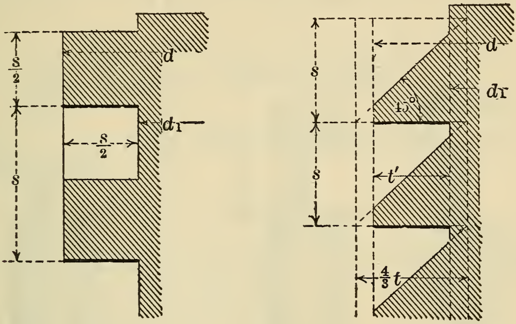
For moderate pressures, say under 50 pounds, the number of bolts may be taken as

$$N = 2 + \frac{D}{2};$$

the diameter of the bolts then being chosen so as to keep the fibre stress within the predetermined limit.

For all general purposes the proportions of bolts are made according to the standard sizes, the United States standard, page 304, being used in America, and the Whitworth standard in Great Britain and on the Continent of Europe. For some purposes, however, special threads are advisable. In such cases the proportions should be directly designed for the

existing conditions. The forms most generally used are the square thread, suited to receive pressure in either direction, and the trapezoidal thread, flat on one face and inclined on the other, to sustain heavy pressures in one direction, as in screw-presses and similar work.



Referring to the figures,

d = outside diameter of screw;
d_1 = bottom diameter of thread;
s = pitch;
P = total load on screw.

For a fibre stress of 3000 pounds the diameter, d_1, at the bottom of the thread is obtained from

$$d_1 = 0.02\sqrt{P}\ ;\ P = 2360d_1^2;$$

or for a fibre stress of 6000 pounds per square inch,

$$d_1 = 0.0145\sqrt{P}\ ;\ P = 4720d_1^2.$$

The depth of thread, both for square and trapezoidal threads, is

$$t = \frac{d}{10} = \frac{d_1}{8};$$

and for square threads

$$s = \frac{d}{5} = \frac{d_1}{4},$$

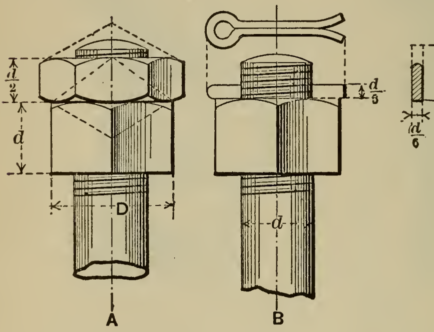
and for trapezoidal threads

$$s = \frac{2}{15}d = \frac{d_1}{6}.$$

For such threads the nut should be made deeper than for ordinary bolts; from $1\frac{1}{2}$ to 2 times the outside diameter of the screw being a proportion found in practice. This insures a sufficient number of threads in the nut and provides for wear.

In important structures, or where much vibration is expected, some form of nut-lock is used to prevent the bolt from working loose.

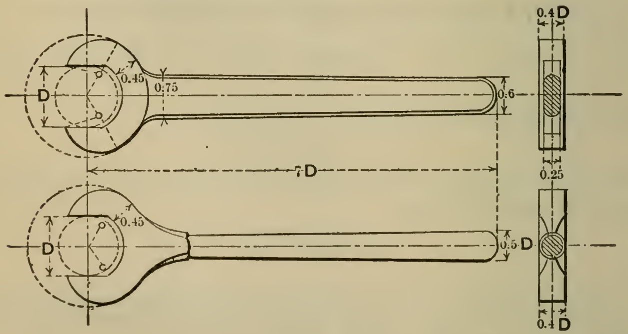
One of the oldest and most useful forms is the jam-nut shown at *A*. Both nuts should be truly faced, so that they will bear fairly upon each other. The thin nut is frequently placed under the thicker one, but this is immaterial, since a nut of a thickness of 0.45 to 0.4*d* is as strong as the bolt thread. At *B* is shown a split pin, often used in connection with a jam-nut. At *C* is shown an arrangement with a key upon the nut, making



a very convenient and secure combination. In all three cases the action is such as to tighten the nut upon the thread.

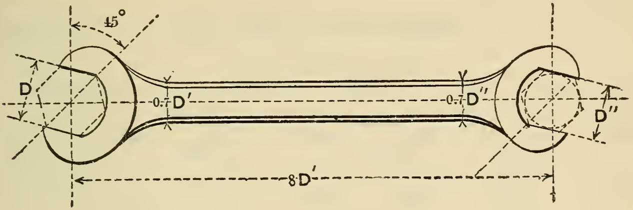
Numerous patented devices have been made to secure bolts and nuts, but the design necessarily varies according to the conditions. In some cases a washer is arranged to be bent up against the face of the nut, or a wedge is driven between the nut and some portion of the structure; these methods being used in rail-joints. In other cases set-screws are used, or special nuts with ratchets are employed, as in heavy steam-engine work. These forms are modified in many ways, according to the ingenuity of the designer. Numerous examples will be found in Reuleaux's "Constructor" and in Unwin's "Machine Design."

Wrenches.



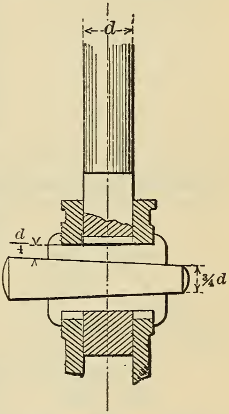
For most purposes wrenches to fit the standard nuts are to be had as a regular article of trade, being drop-forged to standard sizes. If special

wrenches are desired, the following proportions may be used, the unit being the diameter, D, of the hexagon nut across the flat.



Keyed Fastenings.

For many purposes, where the amount of movement is slight, and where the parts may be required to be readily disconnected, keys or cotters may be used. The principal proportions of such cotters have been determined empirically. The depth of a cotter is made equal to the diameter of the rod to be secured, and the thickness is made one-fourth of the depth. The taper varies from 1 in 30 to 1 in 100. For many purposes ⅛ inch in the foot is taken = 1 in 96. A greater taper than 1 in 30 is apt to cause the cotter to fly back. The general proportions of a gib and cotter connection are shown in the figure.



The use of gibs, as shown in the figure, increases the bearing surface of the cotter, and such gibs should always be used when the parts are to be frequently disconnected.

If

d = diameter of rod;
h = mean depth of cotter;
t = thickness of cotter;

we have

$$h = d; \; t = 0.25d.$$

The tip of the cotter should not be less than $\frac{3}{4}d$.

Keys are used to secure the hubs of pulleys, wheels, levers, etc., to shafts, to prevent rotation of one piece upon another.

If the shaft to which a hub is to be keyed is proportioned to stand a certain load, the dimensions of the key may be based upon the diameter of the shaft.

Let

d = diameter of shaft;
s = breadth of key;
s^1 = depth of key.

Then, according to Reuleaux,

$$s = \frac{d}{5} + 0.16 \text{ inch}; \; s^1 = \frac{d}{10} + 0.16 \text{ inch}.$$

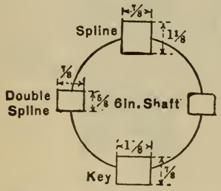
Feathers or splines are keys upon which a sleeve or collar may slide in a direction parallel to the axis of the shaft, while compelled to rotate with it. The proportions of a feather may be taken as a key placed on edge,—*i.e.*, with the greater dimension of the cross-section upon the radius of the shaft.

The following table, while giving dimensions differing slightly from those determined by the formula of Reuleaux, correspond with American machine shop practice.

Standard Keys, Splines, Etc.

Diameter of shaft.	Key.		Spline.		Double spline.	
	Wide.	Deep.	Wide.	Deep.	Wide.	Deep.
Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
3/4	3/16	5/32	5/32	3/16	1/8	5/32
1	1/4	3/16	3/16	1/4	1/8	5/32
1 1/4	5/16	1/4	1/4	5/16	5/32	3/16
1 1/2	3/8	5/16	5/16	3/8	3/16	1/4
2	7/16	3/8	3/8	7/16	1/4	5/16
2 1/2	9/16	7/16	7/16	9/16	5/16	3/8
3	5/8	1/2	1/2	5/8	3/8	7/16
3 1/2	3/4	9/16	9/16	3/4	7/16	9/16
4	7/8	5/8	5/8	7/8	1/2	5/8
5	1	3/4	3/4	1	9/16	3/4
6	1 1/8	7/8	7/8	1 1/8	5/8	7/8

Double splines are set opposite to each other, and their sizes are taken from the last two columns of the table. For sizes of shafts not tabulated take the sizes of keys for shafts of the next smaller size. Thus, for a 4½-inch shaft take sizes for 4-inch shaft.



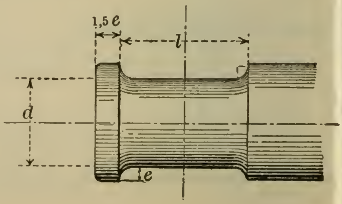
JOURNALS.

The most important form of journal is the overhung form shown in the illustration, and from its computed dimensions other forms may be proportioned. The ratio of length, l, to diameter, d, varies according to the service and the character of the bearing. For rigid bearings, such as pillow-blocks, with the pressure constant in one direction, $\frac{l}{d} = 1.5$ to 2, while for crank pins and similar locations, in which the pressure is alternating in direction, $\frac{l}{d} = 1$ to 1.3.

When ball and socket bearings are used, as in shafting hangers, etc., $\frac{l}{d} = 4$ in general practice.

Referring to the figure, let

d = diameter of journal;
l = length of journal;
e = shoulder of collar;
p = pressure per square inch of projected area;
P = total load on journal;
S = fibre stress on material;
n = revolutions per minute.



We then have

$$d = \sqrt{\frac{16}{\pi S}\left(\frac{l}{d}\right)}\sqrt{P}.$$

For speeds up to 150 revolutions per minute the following values may be used:

Constant Pressure.

	Wrought-iron.	Cast-iron.	Steel.
$p =$	700	360	700
$S =$	8500	4260	14000
$\frac{l}{d} =$	1.5	1.5	2
$d =$	$0.03\sqrt{P}$	$0.043\sqrt{P}$	$0.027\sqrt{P}$

Intermittent Pressure.

	Wrought-iron.	Cast-iron.	Steel.
$p =$	1400	700	1400
$S =$	7000	3500	12000
$\frac{l}{d} =$	1	1	1.3
$d =$	$0.027\sqrt{P}$	$0.037\sqrt{P}$	$0.024\sqrt{P}$

When the speeds become higher than 150 revolutions per minute, the ratio, $\frac{l}{d}$, should be determined from the speed, according to the following formulas:

Constant Pressure.

	Wrought-iron.	Steel.
$S =$	8500	14000
$\frac{l}{d} =$	$0.13\sqrt{n}$	$0.17\sqrt{n}$
$d =$	$0.0244\sqrt{\frac{l}{d}}\sqrt{P}$	$0.019\sqrt{\frac{l}{d}}\sqrt{P}$

Intermittent Pressure.

	Wrought-iron.	Steel.
$S =$	7000	12000
$\frac{l}{d} =$	$0.08\sqrt{n}$	$0.10\sqrt{n}$
$d =$	$0.0273\sqrt{\frac{l}{d}}\sqrt{P}$	$0.02\sqrt{\frac{l}{d}}\sqrt{P}$

The value of $\frac{l}{d}$ is first computed, and then substituted in the following formula to find the value of d.

The depth of shoulder, e, is obtained from the diameter of the journal.

$$e = 0.07d + \tfrac{1}{8}\text{ inch}.$$

The following table gives the diameters of journals for various pressures for speeds not exceeding 150 revolutions. For higher speeds the formulas should be used.

Table of Journal Proportions.

Total pressure, P, pounds.

Diameter of journal, d.	Direction of pressure, constant.		Direction of pressure, alternating.	
	Wrought-iron. $\frac{l}{d} = 1.5.$	Steel. $\frac{l}{d} = 2.$	Wrought-iron. $\frac{l}{d} = 1.$	Steel. $\frac{l}{d} = 1.3.$
Inch.	Lb.	Lb.	Lb.	Lb.
1.00	1 100	1 400	1 400	1 800
1.25	1 700	2 200	2 200	2 200
1.50	2 500	3 200	3 200	4 100
1.75	3 400	4 300	4 300	5 200
2.00	4 500	5 700	5 700	7 300
2.25	5 700	6 800	6 800	9 300
2.50	7 000	8 900	8 900	11 400
2.75	8 500	10 700	10 700	13 800
3.00	10 000	13 000	13 000	16 500
3.25	11 800	15 000	15 000	19 300
3.50	13 700	17 300	17 300	22 400
3.75	15 800	19 800	19 800	25 000
4.00	17 900	22 700	22 700	29 300
4.25	20 000	25 600	25 600	33 100
4.50	23 000	28 700	28 700	37 100
4.75	25 000	32 000	32 000	41 300
5.0	28 000	35 500	35 500	45 800
5.5	34 000	43 000	43 000	55 400
6.0	40 000	51 000	51 000	66 000
6.5	47 000	60 000	60 000	79 200
7.0	55 000	69 500	69 500	89 800
7.5	63 000	80 000	80 000	103 000
8.0	72 000	91 000	91 000	117 000
8.5	81 000	102 000	102 000	132 000
9.0	91 000	115 000	115 000	148 000
9.5	101 000	128 000	128 000	165 000
10.0	112 000	142 000	142 000	183 000
10.5	124 000	156 000	156 000	202 000
11.0	135 000	172 000	172 000	222 000
11.5	148 000	188 000	188 000	242 000
12.0	160 000	204 000	204 000	264 000

The use of the table is apparent.

When the diameter of the journal is given, the load which may be put upon it is found. When a given load is to be put upon a journal, the nearest value in the proper column is found and the corresponding diameter of journal taken.

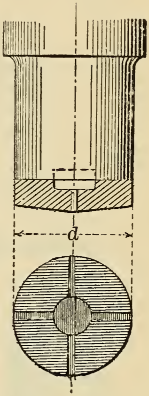
Necked journals, formed in the body of a shaft, are naturally stronger than overhung journals, but the diameter in this case is determined by the duty to be performed by the shaft, which will generally make it larger than would be required for an overhung journal.

PIVOTS.

The bearing end of a vertical shaft or spindle is termed a pivot. Such pivot bearings are usually made with a recess in the middle, with cross oil channels. Taking the diameter of the recess as $\frac{1}{3}$ the diameter of the shaft, we may make the oil channels $\frac{1}{12}$ the diameter in width.

Let

P = total vertical pressure on pivot, in pounds;
p = pressure, in pounds, per square inch;
d = diameter of pivot, in inches;
n = number of revolutions per minute.



We then have the following relations, according to Reuleaux:

Formulas for Pivots.

	Wrought-iron or steel on bronze.	Cast-iron on bronze.	Iron or steel on lignum vitæ.
Slow-moving pivots	$p = 1422$	700	....
	$d = 0.035\sqrt{P}$	$0.05\sqrt{P}$	....
$n =$ or < 150	$p = 700$	350	1422
	$d = 0.05\sqrt{P}$	$0.07\sqrt{P}$	$0.035\sqrt{P}$
$n > 150$	....	....	$p = 1422$
	$d = 0.004\sqrt{Pn}$	....	$d = 0.035\sqrt{P}$

$d =$	$0.035\sqrt{P}$	$0.05\sqrt{P}$	$0.07\sqrt{P}$
	P	P	P
1.00	816	398	204
1.25	1275	622	319
1.50	1836	895	459
1.75	2500	1219	625
2.00	3265	1592	816
2.25	4132	2016	1033
2.50	5102	2488	1275
2.75	6173	3011	1543
3.00	7347	3494	1836
3.25	8622	4205	2155
3.50	10000	4877	2500
3.75	11479	5599	2869
4.00	13061	6370	3265
4.25	14745	7192	3686
4.50	16530	8063	4132
4.75	18418	8983	4604
5.00	20498	9954	5102
5.25	22140	10974	5535
5.50	24694	12044	6673
5.75	26990	13164	6747
6.00	29388	14334	7344
6.25	31890	15630	7972
6.50	34490	16900	8623
6.75	37190	18220	9298
7.00	41690	19600	10000

The three columns headed P give the total pressures permissible for wrought-iron or steel on bronze, cast-iron on bronze, and wrought-iron or steel on lignum vitæ, respectively. If the load is given, find the nearest value in the proper column and take the corresponding diameter.

The frictional resistance of a flat pivot bearing may be determined as follows:

Let

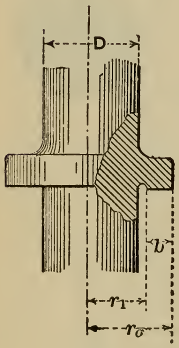
F = the tangential frictional resistance, in pounds, at the periphery of the pivot;
r_0 = the radius of shaft, in inches;
r_1 = the radius of recess, in inches;
P = total load on shaft, in pounds;
f = coefficient of friction.

Then

$$F = \frac{f}{2} P \left(1 + \frac{r_1}{r_0}\right).$$

If we take, as indicated above, $r_1 = \frac{1}{3} r_0$,

$$F = \tfrac{2}{3} f P.$$



These formulas apply also to collar bearings of the form here shown.

For very heavy pressures, as in the thrust bearings of screw-propeller shafts, the thrust is taken upon a number of collars. Good practice limits the pressure upon such collars from 40 to 80 pounds per square inch.

If

n = the number of collars;
d = diameter of shaft;
D = outside diameter of collars;
P = total thrust;

we have, according to Seaton,

$$D = \sqrt{d^2 + \frac{P}{47n}}.$$

This provides for a pressure of 60 pounds per square inch on the collars. The thickness of each collar is made $= 0.4(D - d)$, and the space between the collars may be $0.75(D - d)$.

SHAFTING.

In determining the dimensions of shafting there are two principal elements to be considered: the strength and the stiffness. Generally, the load acting upon the shaft is given in either one of two forms,—as horse-power to be transmitted at a given number of revolutions per minute, or as a twisting moment, or torque, expressed in a certain force acting at the end of a lever of a given length. In the latter case, the torque is here considered to be in inch-pounds. Thus, a belt pulling 100 pounds over a 20-inch pulley would give 100 pounds at a lever arm of 10 inches, or 1000 inch-pounds, etc.

In order that satisfactory results may be secured, a shaft should be so proportioned that it may not be subjected to a fibre stress at the circumference greater than the predetermined limit; and also that it may not be twisted through a greater angle than has been established as satisfactory. It is, therefore, necessary to compute the diameter by two methods, one for strength and the other for stiffness, and use the result which gives the greatest size.

In the formulas the following symbols are

$P =$ the force acting to rotate the shaft;
$R =$ the lever arm at which it acts;
$N =$ the horse-power transmitted;
$n =$ the number of revolutions per minute;
$d =$ the diameter of the shaft;
$L =$ the length of shaft, in feet;
$\vartheta =$ the angle of torsion, in degrees;
$S =$ the fibre stress at the circumference;
$G =$ the modulus of torsion of the material $= \frac{2}{5}$ of the modulus of elasticity.

We then have, for strength,

$$d = \sqrt[3]{\frac{16}{\pi S} PR},$$

and for stiffness,

$$d = \sqrt[4]{\frac{32}{\pi G} \cdot \frac{12 \, . \, L}{\vartheta^{\circ}} \cdot \frac{360}{2\pi} PR.}$$

Taking the fibre stress, $S = 8500$ pounds, we have for wrought-iron shafts, for strength,

$$d = 0.091 \sqrt[3]{PR} = 3.33 \sqrt[3]{\frac{N}{n}}.$$

In taking the torsion of shafting into consideration, the greatest allowable twist in degrees should not be over 0.075° per foot in length of shafting,—that is, $\vartheta^{\circ} = 0.075L$, which gives for stiffness, against torsion,

$$d = 0.3 \sqrt[4]{PR} = 4.7 \sqrt[4]{\frac{N}{n}}.$$

The quotient of effect, $\frac{N}{n}$, is obtained from the relation to the statical moment, PR, as follows:

$$PR = \frac{33000 \times 12}{2\pi} \cdot \frac{N}{n} = 63025 \frac{N}{n}.$$

From these formulas the following table for round wrought-iron shafts has been calculated. An inspection of the table will show that it is quite possible for a shaft to be strong enough to resist permanent deformation and yet be so light as to be liable to spring under its load. For example, a shaft 26 feet long, with a twisting force of 220 pounds applied at one end, and acting with a lever arm of 20 inches, gives a turning moment, $PR =$ 4400 inch-pounds, which would require a shaft only $1\frac{1}{2}$ inches diameter (see column 2). This, however, would permit far too much torsion, and in order that the angular deflection should not exceed the limit of 0.075° per foot, a corresponding value of PR, in column 4, must be found, and against it, in column 1, will be given the diameter,—in this case about $2\frac{3}{8}$ inches,—which, by comparison with column 2, gives about five-fold strength.

For short shafts this examination of angular deflection is unnecessary, as, for example, in the short lengths between two gear-wheels, for here the value of ϑ will be small enough in any case. With longer shafts, and in all special constructions, it is important to consider the angular deflection and keep it within the given limit.

For steel shafts, whose modulus of resistance is $\frac{4}{3}$ greater than wrought-

iron, the diameters in both cases may be taken as $\sqrt[3]{0.6}$,—that is, 0.84 times that of correspondingly-loaded wrought-iron shafts.

Shafting which is subjected to sudden and violent shocks, as in rolling mills, etc., must be made much stronger than the preceding formulas require, and these must be classed with the special cases which occur in every branch of construction.

Wrought-iron Shafting.

d.	For strength.		For stiffness (torsional).	
	PR.	$\frac{N}{n}$.	PR.	$\frac{N}{n}$.
Inch.				
1	1 327	.021	123	.0019
1¼	2 591	.052	301	.0048
1½	4 479	.071	625	.0099
1¾	7 112	.114	1 157	.0183
2	10 616	.168	1 975	.0313
2¼	15 115	.239	3 164	.0502
2½	20 730	.329	4 822	.0765
2¾	27 600	.438	7 061	.1120
3	35 830	.568	10 000	.1587
3½	56 890	.902	18 520	.2941
4	84 930	1.347	31 600	.5015
4½	120 900	1.919	50 620	.8032
5	165 800	2.632	77 160	1.2240
5½	220 800	3.503	111 000	1.7920
6	286 600	4.548	160 000	2.5390
6½	364 400	5.784	220 300	3.4960
7	455 200	7.222	296 400	4.7040
7½	559 800	8.883	390 600	6.2000
8	679 400	10.780	505 700	8.0240
8½	815 000	12.930	644 400	10.2300
9	967 400	15.350	810 000	12.8600
9½	1 138 000	18.050	982 700	15.6000
10	1 327 000	21.050	1 230 000	19.5900
10½	1 536 000	24.380	1 501 000	23.8100
11	1 766 000	28.020	1 808 000	28.6800
11½	2 018 000	32.020	2 159 000	34.2600
12	2 293 000	36.390	2 560 000	40.6200

d = diameter of shaft, in inches;
R = lever arm of torque, in inches (as radius of pulley or gear-wheel);
P = force on lever arm, in pounds;
N = actual horse-power transmitted;
n = revolutions per minute.

Find the nearest value for PR or $\frac{N}{n}$, both for strength and for stiffness, and take the largest diameter of shaft corresponding. For steel shafts multiply this diameter by 0.84.

For any given shaft the angle of torsion, ϑ, for a given statical moment, PR, may be found from the following formulas:

$$\vartheta = 0.00062\frac{PRL}{d^4},$$

$$= 0.0001208S\frac{L}{d},$$

in which L is the length of shaft, in feet, and d the diameter, in inches,—S being the fibre stress at the point of application on the shaft.

When the force is applied at one end of the shaft and taken off at the other, L is the whole length of the shaft. When the twisting forces are applied over the whole length of the shaft uniformly, L may be taken as one-half the length of the shaft; and when the twisting forces diminish uniformly from one end to the other, L is taken as one-third the length.

For a number of twisting forces applied at various points along the shaft, multiply the horse-power at each point by its distance from the end of the shaft, add the several products together, and divide by the total horse-power transmitted. The quotient may be used as the mean value of L in the formula.

Since the modulus of elasticity is practically the same for iron and steel, these formulas are good for either material.

Since $PR = 63025\frac{N}{n}$, the above formulas can easily be used when the load is given in horse-power for a given number of revolutions instead of torque.

Thus, suppose a shaft 164 feet long transmitting 70 horse-power at 100 revolutions, the power being taken off by machines uniformly distributed along its length. The effective length, L, may then be taken as $\frac{164}{2} = 82$ feet. We also have $\frac{N}{n} = \frac{70}{100} = 0.7$, and from the preceding table, under the column for torsional stiffness, we find the values 0.5015, corresponding to 4 inches diameter, and 0.8032, corresponding to $4\frac{1}{2}$ inches diameter, so that we make the shaft $4\frac{1}{4}$ inches diameter. We have, also,

$$PR = 63025\frac{N}{n} = 63025 \times 0.7 = 44117.$$

The angular deflection will then be

$$\vartheta = 0.00062\frac{44117 \times 82}{(4.25)^4} = 6.88,$$

or 6° 53′.

Hollow Shafts.

Since the metal close to the axis of a shaft is of much less value in resisting stresses than the portion near the perimeter, there is a manifest advantage in using hollow or tubular shafting. Such shafts are very generally used for screw-propeller engines.

If

d = the diameter of a solid shaft;
d' = the outside diameter of a hollow shaft of equal strength;
d_0 = diameter of hole through hollow shaft;
$\psi = \frac{d_0}{d_1}$ = ratio of external to internal diameter of hollow shaft;

then

$$d_1 = \sqrt[3]{\frac{d^3}{1 - \psi 4}}.$$

For d = unity we have, for the following values of ψ, the corresponding values of d_1:

ψ = 0.3	0.4	0.5	0.6	0.7	0.8	0.9
d_1 = 1.002	1.000	1.021	1.047	1.096	1.192	1.427

For any diameter solid shaft, therefore, we have simply to multiply by the value for d_1, for the chosen ratio of external to internal diameters, to get the external diameter of a hollow shaft of equal strength.

In marine practice the ratio, ψ, of diameter to bore is generally 0.5, the bore being one-half the external diameter. The external diameter will then be 1.2 times that of an equivalent solid shaft, or only 2 per cent. greater, and, at the same time, the shaft will be 25 per cent. lighter.

The power which a hollow shaft will transmit may be obtained by finding the capacity of the corresponding solid shaft.

Thus, for

$\psi = 0.3$	0.4	0.5	0.6	0.7	0.8	0.9
$d = 0.998$	0.992	0.979	0.955	0.912	0.839	0.7

Thus, a shaft 10 inches in diameter, with a 5-inch hole through it, will transmit as much power as a solid shaft $10 \times 0\ 979 = 9.79$ inches diameter.

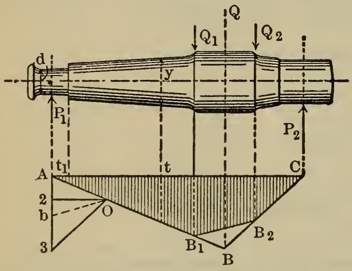
Deflection of Shafts.

In proportioning a shaft to carry a given load the practical method is to determine the pressures upon the journals and proportion them according to the methods already given. The rest of the shaft can then be proportioned according to the statical moments at various points, determined graphically, as follows:

Draw the line, A–C, equal in length to the distance between centres of journals, and upon it construct any triangle, ABC, whose apex lies on the line of the load, Q. Draw A–3 normal to A–C, making A–$3 = Q$; draw 3–O parallel to B–C, and 2–O parallel to A–C; then A–$2 = P_1$, 2–$3 = P_2$.

The two journals may then be proportioned for these pressures.

By dropping the perpendiculars from the ends of the hub-seat we may divide Q into two forces, Q_1 and Q_2, shown in the force polygon by O–b, parallel to B_1B_2, giving A–$b = Q_1$, b–$3 = Q_2$. The vertical ordinate, t, at any point of the surface of moments is proportional to the statical moment, M_y, at its point of intersection with the axis, as, for example, the ordinate, t_1, at the base of the journal for P_1. We have, in any case,



$$y^3 = \frac{32}{\pi S} M_y, \quad d_1{}^3 = \frac{32}{\pi S} M_1,$$

S being the fibre stress; and hence

$$\frac{y}{d_1} = \sqrt[3]{\frac{M_y}{M_1}}, \text{ or } = \sqrt[3]{\frac{t}{t_1}},$$

from which y can readily be obtained.

A similar diagram may be drawn for any loading, and the relative proportions of the various parts of the shaft or axle determined. The graphical method has the further advantage of showing the distribution of stresses on the axle at one time; and even if a straight axle is used the points of greatest stress are thus clearly seen.

The practice of mounting heavy fly-wheels, or the rotor members of large electric generators, upon engine shafts, renders it necessary to consider the influence of bending and twisting moments combined. This may be done by uniting the two moments into an equivalent or "ideal" bending moment, such that the proportions of the shaft may be computed from it directly.

Let

M_d = the twisting moment for a given shaft section;
M_b = the bending moment for the same section.

Then the ideal moment, combining them both, will be

$$M = \tfrac{3}{8} M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_d^2}.$$

The application of this formula is best seen by an example.

If we have a wheel subjected to a force of 6000 pounds, at a radius of 12 inches, on a shaft 100 inches long, being 20 inches from one end and 80 inches from the other, we have a bending moment, M_b^1, of

$$6000 \times \frac{80}{100} \times 20 = 96000 \text{ inch-pounds.}$$

We have, also, the twisting moment,

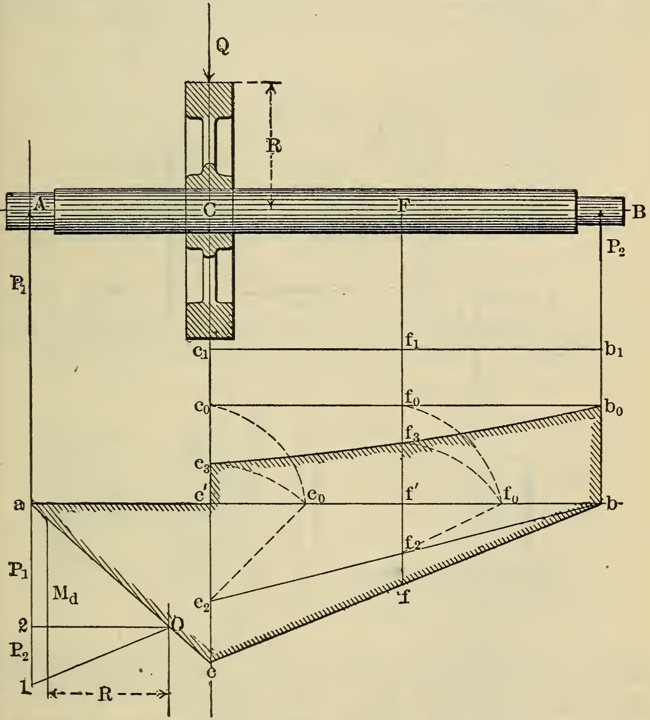
$$M_d = PR = 6000 \times 12 = 72000 \text{ inch-pounds.}$$

The combined moment will then be

$$\begin{aligned} M &= \tfrac{3}{8} \cdot 96000 + \tfrac{5}{8}\sqrt{72000^2 + 96000^2} \\ &= 36000 + 75000 = 111\,000 \text{ inch-pounds;} \end{aligned}$$

and the corresponding shaft diameter from the table, page 430, is about $4\frac{3}{8}$ inches. For the twisting moment of 72,000 inch-pounds alone the diameter would have been about $3\frac{3}{4}$ inches.

The graphical method may be applied to this problem very effectively.



Referring to the figure, first construct the link polygon, *abc*, for the bending moments, and the force polygon, *a*10, giving the forces, P_1 and P_2, and also *acc'*, the surface of moments for the shank, *AC*.

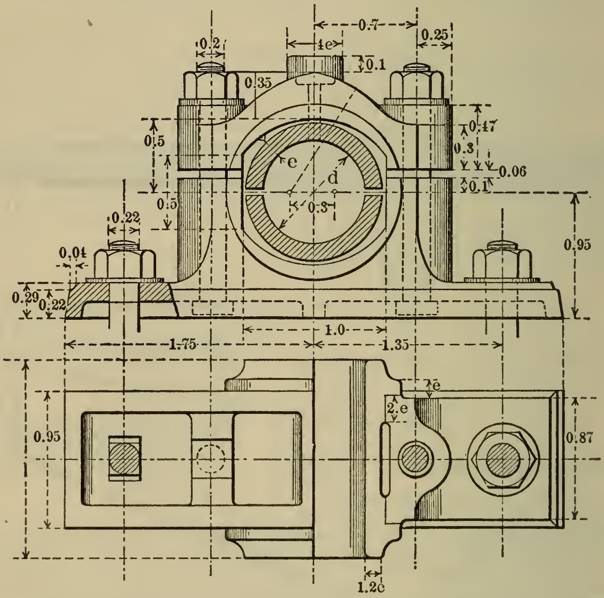
The moment, M_d, is yet to be determined. In the force polygon, with a distance, *R*, from the pole, *O*, draw a vertical ordinate; this will be M_d. Lay its value off at $c'c_1$ and bb_1, and five-eighths of these values give $c'c_0b_0b$ for the parallelogram of torsion for the shank, *CB*.

The combination of the bending and twisting moments may then be made according to the formula. Make $cc_2 = \frac{3}{8}cc'$, and join *cb*; then at

any point of the polygon, as, for example, at f, the distance, $ff_2 = \frac{3}{8}ff'$. Now transfer $c'c_0$ to ab, at $c'c_0'$; then will the hypothenuse of the triangle, $c_2c'c_0'$, divided by $c_2c_0' = \sqrt{(\frac{3}{8}cc')^2 + (\frac{5}{8}c_1c')^2}$, and the sum, $cc_2 + c_2c_0' = cc_2 + c_2c_3$, the desired moment, $(M_b)_i$, for the point, C. In the same manner we obtain $ff_2 + f_2f_0' = ff_2 + f_2f_3$, the moment, $(M_b)_i$, for the point, F. The line, $c_3f_3b_0$, is a curve (hyperbola), which may be taken approximately with sufficient accuracy as a straight line, c_3b_{02}. The various dimensions may be obtained from the polygon, $acbb_0c_3c'$, in a similar manner, as shown in the discussion of axles.

BEARINGS.

The form and shape of bearings in which journals are carried vary much with the service demanded. For line shafting the most satisfactory results are obtained with cast-iron boxes, usually made four diameters in length, and supported on spherical seats in adjustable screw-plugs, permitting a limited adjustment and enabling the box to align itself to the shaft. Such boxes are used in drop-hangers, pillow-blocks, and wall-



brackets, the whole being a general article of manufacture. When the bearing is made rigid, the length is rarely more than one to two diameters, as it is difficult to maintain good alignment for greater lengths, and heating and cutting are apt to follow.

The proportions of a standard pillow-block bearing are given in the illustration, this being a form used in heavy mill shafting, in the outboard bearings of steam engines, and similar work.

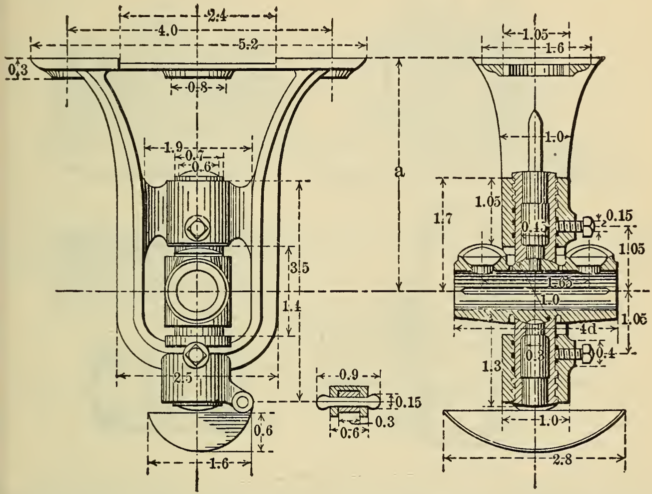
The proportions of the pillow-block are in terms of a modulus, $d_1 = 1.15d + 0.4$ inches, in which d is the diameter of the journal to run in the bearing.

The dimensions of the brasses are in terms of the modulus, $e = 0.07d + 0.125$ inch.

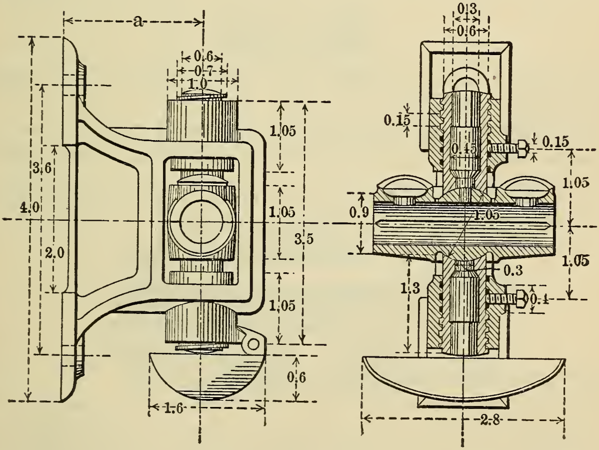
The main bearings of steam engines are similar in design, but are gen-

erally made with side brasses and adjustments to take up wear both horizontally and vertically, the casting of the pillow-block forming a portion of the engine bed-plate. For various designs of such bearings see Reuleaux's "Constructor" and Unwin's "Machine Design."

The proportions of hangers and pillow-blocks for shafting are given in the following illustrations.



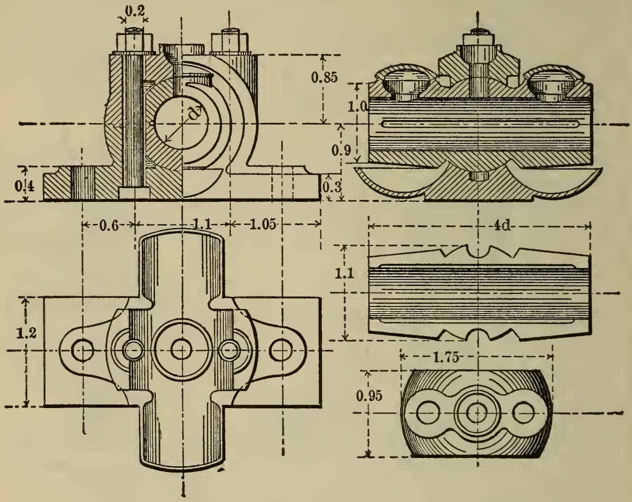
The dimensions of the hanger are in terms of the modulus, $d' = 1.4d + 0.25$ inches. The drop, a, varies according to local conditions. The screw-



plugs are of cast-iron, as are also the boxes. Lateral adjustment is made by providing slotted bolt-holes in the base of the hanger.

The wall-bracket is based upon the same modulus as the hanger, $d_1 = 1.4d + 0.25$ inches.

The pillow-block differs from the hangers in having no vertical adjustment, the spherical sockets being cast in the base and cap, as shown in

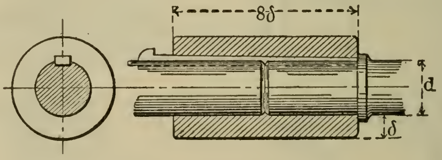


the illustration. The modulus is $d_1 = 1.4d + 0.25$ inches. These designs are originally due to Wm. Sellers & Co., of Philadelphia.

COUPLINGS.

The simplest form of coupling is the plain cylindrical muff coupling shown in the illustration.

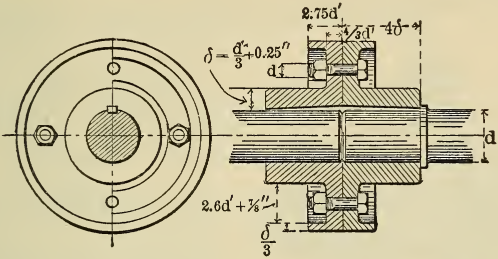
The thickness $\delta = \frac{d}{3} + 0.25$ inch, d being the diameter of the shaft; the length being 8δ. This coupling is cheap, and serves for light work.



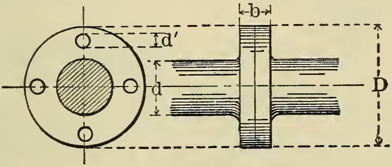
For heavier shafts the plate coupling is used.

The thickness of the hub $\delta = \frac{d}{3} + 0.25$ inch, and the length of hub on each plate $= 4\delta$. The other dimensions are based on the modulus, $d' = 0.125d + \frac{5}{16}$ inch, this being the diameter of the bolts. The number of bolts, $N = 2 + 0.8d$.

The two halves of a plate coupling should be forced on their respective shafts and keyed fast as well, and should then be turned up and faced off



on the same centres as were used in turning the shafts. All pulleys, gears, etc., should then be put on the shaft in halves, the plates of the coupling not being removed.



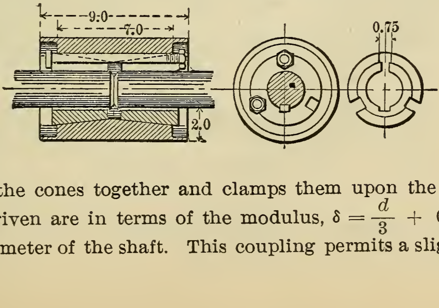
For screw-propeller shafts the plates are forged on the shafts, as shown in the illustration.

According to Seaton, the diameter, d', of the bolts in such couplings should be:

For 4 bolts, $d' = 0.32d$;
For 5 bolts, $d' = 0.28d$;
For 6 bolts, $d' = 0.25d$;
For 8 bolts, $d' = 0.20d$;
For 10 bolts, $d' = 0.18d$;
Thickness of plate $= \frac{b}{2} = 0.3d$;
Diameter of bolt circle $= 1.6d$;
Outside diameter, $D = 1.6d + 2\frac{1}{4}d'$.

Number of bolts, one for every two inches diameter of shaft.

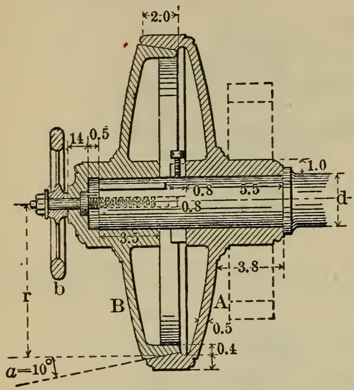
For ordinary line shafting the double-cone coupling of Sellers' has been very extensively used. As shown in the illustration, the tightening of the



bolts draws the cones together and clamps them upon the shafts. The dimensions given are in terms of the modulus, $\delta = \frac{d}{3} + 0.25$ inch, d being the diameter of the shaft. This coupling permits a slight variation

in the diameters of the shafts, and, unlike the plate coupling, it may be removed and replaced satisfactorily to permit the placing of pulleys upon the shaft.

When it is desired to disconnect portions of a transmission, various forms of clutch couplings are used. A great variety of these have been designed, and, as an example, the cone clutch is given.

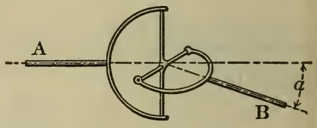


The general proportions of a cone clutch are given in the illustration, based on the modulus, $\delta = \frac{d}{3} + 0.25$ inch. The angle of bevel, α, is made not less than 10°, or it is found difficult to disengage the parts. If PR is the turning moment, or torque, to be transmitted, the axial pressure, Q, to hold the parts in engagement will be

$$Q = \frac{PR}{r}\left(\frac{\sin\alpha}{f} + \cos\alpha\right),$$

in which r is the radius of the cone bearing and f the coefficient of friction. For iron on iron, f may be taken at 0.15, and r should not be made less than $3d$,—preferably greater.

For connecting shafts which are placed at an angle with each other the universal joint is employed, shown in skeleton in the illustration. While this is convenient, it must be remembered that the angular velocity transmitted is not uniform. If α is the angle between the shafts, and ω and ω_1 the angular movements of the two shafts, respectively, then



$$\tan\omega_1 = \tan\omega\cos\alpha.$$

The variation thus has a period of 180°.

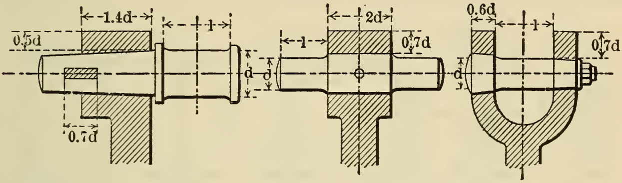
The following table gives the values of ω_1 for successive values of ω, for various angles, α:

ω	$\alpha = 10°$		20°		30°		40°	
Deg.	Deg.	Min.	Deg.	Min.	Deg.	Min.	Deg.	Min.
30	29	38	28	29	26	34	23	51
45	44	34	43	12	40	54	37	27
60	59	34	58	26	56	22	53	4
90	90	..	90	..	90	..	90	..
120	120	26	121	34	123	38	126	56
135	135	26	136	48	139	6	142	33
150	150	22	151	31	153	26	156	1
180	180	..	180	..	180	..	180	..

Where this variation is injurious it may be avoided by using two universal joints which correct each other.

LEVERS.

The proportions of the various forms of lever arms used in machine design are dependent upon various considerations. Thus, the ends are determined by the diameters of the pins to be inserted.



In the above forms the proportions are given in terms of the diameter of the pin. These dimensions are for wrought-iron; for cast-iron they should be doubled.

The calculations of the dimensions of simple lever arms of rectangular section are made upon the assumption that the force, P, acts in a plane, passing through the middle of the arm and in a direction normal to the arm.

If we let

h = width of the arm at the axis;
b = thickness of the arm at the axis;
S = the maximum permissible fibre stress;

$$b = 6\frac{PR}{Sh^2}.$$

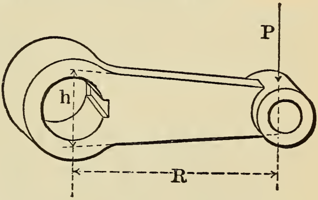
Taking S for wrought-iron = 8500, and for cast-iron = 4250, we have,

$$\text{for wrought-iron, } b = 0.00072\frac{PR}{h^2}\text{; for cast-iron, } 0.00144\frac{PR}{h^2}.$$

These formulas are adapted for the determination of b when h has been selected, the latter being most conveniently chosen with regard to the other condition.

Example. Let P = 4400 pounds, R = 24 inches for a lever arm of wrought-iron, and $h = 7\frac{1}{8}$ inches, we have

$$b = 0.00072\frac{4400 \times 24}{(7.125)^2} = 1\tfrac{1}{2} \text{ inches.}$$



If b is kept constant for the whole length of the arm, the width at the small end may be $0.5h$, while if a constant ratio of $b : h$ is kept, the small end = $\frac{2}{3}h$.

If, as often occurs, the force, P, does not act in the middle plane, then there must exist a combined bending and twisting stress on the arm. We may then derive a combined stress whose bending moment will give an ideal arm, R'.

If the plane in which the force, P, acts is distant from the middle of the arm by an amount, a, we may make, approximately,

$$R' = \tfrac{3}{8}R + \tfrac{5}{8}\sqrt{R^2 + a^2}.$$

Thus, if the lever in the preceding example was acted upon by the same force with an overhang, a, of 15 inches, we have

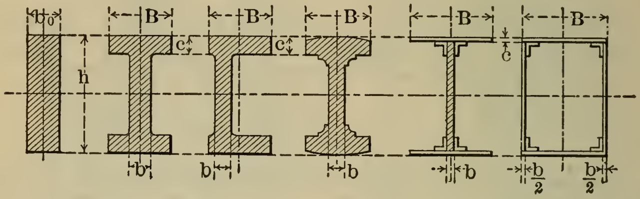
$$R' = \tfrac{3}{8} \cdot 24 + \tfrac{5}{8}\sqrt{24^2 + 15^2}$$
$$= 9 + \tfrac{5}{8} \cdot 28.3 = 26.7;$$

whence

$$b = 0.00072\frac{4400 \times 26.7}{(7.125)^2} = 1.66 \text{ inches.}$$

Sometimes it is desired to make a lever arm of ribbed or I-section to secure lightness or economy of material. The dimensions may then best be obtained by computing the rectangular section and transforming this into the section desired.

Let h_0 be the width and b_0 the thickness of the rectangular section as found in the preceding method, and let h and b be the corresponding dimensions in the section selected from among those given in the illustration.



Then we have
$$\frac{b}{b_0} = \frac{1}{1 + a},$$
in which
$$a = \left(\frac{B}{b} - 1\right)\left[6\,\frac{c}{h} - 12\left(\frac{c}{h}\right)^2\right].$$

These formulas permit a choice of the ratios, $\frac{B}{b}$ and $\frac{c}{h}$, which may be left to the judgment of the designer.

In the structural built-up sections the value of the angle-irons has been neglected, as it may be considered as making up for the weakening effect of the rivet-holes.

The following table of values of $\frac{1}{1+a}$ enables the transformation to be readily effected.

Table for Transforming Arm Sections.

$\frac{h}{c}$	Values of $\frac{1}{1+a}$.									
	$\frac{B}{b}$ = 2.5	3	3.5	4	4.5	5	6	7	8	10
6	.50	.43	.38	.33	.30	.27	.23	.20	.18	.14
7	.52	.45	.40	.35	.32	.29	.25	.21	.19	.15
8	.54	.47	.42	.37	.34	.31	.26	.23	.20	.16
9	.56	.49	.44	.39	.36	.33	.28	.24	.22	.18
10	.58	.51	.46	.41	.37	.34	.29	.26	.23	.19
11	.60	.53	.48	.43	.39	.36	.31	.27	.24	.20
12	.62	.55	.50	.44	.41	.37	.32	.29	.26	.21
14	.64	.58	.52	.47	.44	.40	.35	.31	.28	.23
16	.67	.60	.55	.50	.47	.43	.38	.34	.30	.25
18	.69	.63	.57	.52	.49	.46	.40	.36	.33	.27
20	.71	.65	.60	.55	.52	.48	.42	.38	.34	.29
22	.73	.67	.62	.57	.53	.50	.45	.40	.37	.31
24	.75	.68	.64	.59	.56	.52	.47	.42	.38	.33
27	.76	.71	.66	.62	.58	.55	.50	.45	.41	.35
30	.78	.73	.68	.64	.61	.57	.52	.47	.43	.37
33	.79	.75	.70	.66	.63	.60	.54	.50	.45	.39
36	.81	.76	.72	.68	.65	.61	.56	.52	.48	.41
40	.83	.78	.74	.70	.67	.64	.58	.54	.50	.44
45	.84	.80	.76	.72	.69	.66	.61	.57	.53	.47
50	.85	.81	.78	.74	.71	.68	.63	.59	.56	.49

Example. A lever arm has a length, $R = 78.75$ inches, and the journal pressure at the end $= P = 5500$ pounds. It is to be of cast-iron of double T-section with a height, $h_0 = 12\frac{5}{8}$ inches. We have, for a rectangular section,

$$b_0 = 0.00144 \frac{5500 \times 78.75}{(12.625)^2} = 3.9 \text{ inches.}$$

With this the I-section may be compared. Here we may take $c : h = 1 : 12$, $B : b = 4$, and we get from the table $\frac{1}{1 + a} = 0.44$ and $b = 0.44 b_0 = 1.71$ inches, and the flange breadth, $B = 1.71 \times 4 = 6.84$ inches, the flange thickness $= c = \frac{1}{12} h = \frac{12.625}{12} = 1.05$ inches, all of which are practical dimensions. It may be found desirable to have $c = b$ or any reasonable ratio, or $B : b$, and $c : h$ be chosen.

Example. A wrought-iron arm has been found to require $b_0 = 2\frac{3}{8}$ inches, $h = 12\frac{5}{8}$ inches. It is desired to make $\frac{b}{b_0} = 0.25$, and in column 10 we find 0.25 opposite $\frac{h}{c} = 16$; hence, $b = 0.57$ inch and $B = 10 \times 0.59 = 5.90$ inches, and $c = \frac{12.625}{16} = 0.8$ inch.

This table may be used for transforming sections for many other purposes, such as beams, crane booms, struts, etc.

CRANKS.

The general proportions of engine cranks are obtained from the methods already given. The diameter and length of pin are found, as are those of a journal subjected to alternating stresses, according to the table on page 425, and the shaft is determined by the values of the twisting and bending moments upon it. The thickness of metal about the hub and eye of the crank is then proportioned according to the diameters of the shaft and pin, as shown on page 439.

For important structures it is desirable to make a graphical analysis of all the stresses upon the crank and its shaft. Starting with a pin proportioned to resist the maximum effort of the connecting rod upon it, the following graphostatic analysis will enable all the parts to be equal in strength to the pin, according to Reuleaux:

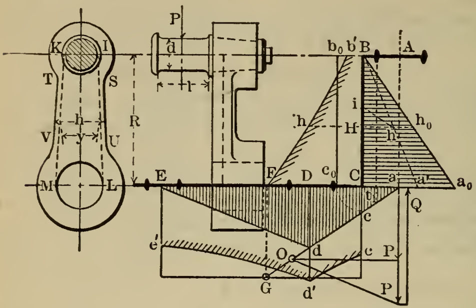
The Crank Axle.—Having calculated d and l, draw the skeleton diagram of the crank,—that is, the neutral axis, $ABCDE$, in which BC represents the axis of the crank arm, which in this case lies normal to the axis of the shaft, and is placed in its proportional distance from the centre of the crank pin, A, and from the bearing, D. Then lay off the force, P, from a, normal to Ea, choose the pole, O, of the force polygon (this being best placed upon a line passing through the end of P and parallel to the axis, Ea), draw the ray, adO, and line, dE, also the ray, OP_1, parallel to dE · then adE will represent the cord polygon for the bending which P produces upon the axle, aCE, and PP_1 represents the force upon the journal, E, and P_1a the force upon the journal, D. Also make aF equal to the crank radius, R, and draw FG; this latter will be the twisting moment which P exerts upon the axis. This moment, M_d, may be combined with the bending moment, M_b, to give for each point an ideal bending moment,

$$M_i = \tfrac{3}{8} M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_d^2},$$

from which the polygon curve, $c'd'e'$, and surface of moments, $Cc'd'e'E$, are obtained. From the latter, in combination with the pin diameter, d, and ordinate, t, of the base of the pin, the diameter of the shaft may be obtained according to formula

$$\frac{y}{d_1} = \sqrt[3]{\frac{t}{t_1}}.$$

The Crank Arm.—Prolong Ea to a_0, and transfer the cord polygon, Dad, to the base line, BC,—that is, make the angle $a_0BC =$ the angle Dad, and then will Ba_0C be, with horizontal ordinates, the surface of moments for the bending of the crank arm due to the force, P. Also make $Cc_0 = Bb_0 = Cc$, then will the horizontal ordinates of the torsion rectangle, Bb_0c_0C, be the moments with which P acts to twist the crank arm about the axis, BC. This moment may again be combined with the bending moment to give an ideal moment, as before; ($a_0a' = \frac{3}{8}a_0C$, draw Ba', make at any point, H, the space, $Hi = \frac{5}{8}Bb_0$, and make $Hh = h_0h' + h'i$), which gives the surface of moments, $Bb'hFC$, for the crank arm. From this and from the diameter, d, and ordinate, t, we can construct the conoidal form of the arm, $IKLM$, according to formula



$$\frac{y}{d_1} = \sqrt[3]{\frac{t}{t_1}}.$$

From this, again, the profile, $STUV$, of an arm of rectangular section may be derived, the width, h, being assumed for any point, and the corresponding thickness, b, obtained from the value, y, of the conoid, according to the formula

$$\frac{b}{y} = 0.6\left(\frac{y}{h}\right)^2.$$

If the position of the axis, BC, does not give satisfactory results, the operation must be repeated with a better relation of parts. By proceeding in this manner the dimensions of a crank and axle may be so determined that they will be equal in strength to the pin upon which the power is exerted.

For a similar treatment of other forms of cranks and cranked axles see Reuleaux's "Constructor."

CONNECTING RODS.

The body of a connecting rod may be made of wrought-iron, cast-iron, steel, or even of wood. In the latter case it is usually only subject to tension.

If the rod is of circular cross-section of diameter, D, and the force of tension be P, we have the following relations:

wrought-iron, $\frac{D}{\sqrt{P}} = 0.015$; cast-iron, $\frac{D}{\sqrt{P}} = 0.03$;

steel, $\frac{D}{\sqrt{P}} = 0.012$; oak, $\frac{D}{\sqrt{P}} = 0.06$.

These give stresses of 5600, 9500, 2800, and 400 pounds, respectively, or about two-thirds the value given for ordinary conditions.

For short connecting rods the formulas cited are all right for compression as well as for tension, but for long rods a greater diameter should be used to provide against buckling. Owing to the great variety of conditions, a factor of safety, m, must be introduced, and we have the following formulas, in which D is the diameter of the round rod; L, its length, in inches; and P, the total pressure, in pounds:

$$\text{wrought-iron or steel, } D = 0.0164\sqrt[4]{m}\sqrt{L\sqrt{P}};$$

$$\text{cast-iron, } D = 0.0195\sqrt[4]{m}\sqrt{L\sqrt{P}};$$

$$\text{wood, } D = 0.034\sqrt[4]{m}\sqrt{L\sqrt{P}}.$$

We have for

$m =$	1.5	2.0	3.0	4.0	6.0	8.0	10.0	15.0	20.0	25.0	30.0	40.0	50.0	60.0
$\sqrt[4]{m} =$	1.11	1.19	1.32	1.41	1.56	1.68	1.78	1.97	2.11	2.24	2.34	2.51	2.66	2.78

For various services the following values of m may be taken: locomotive engines, $m = 2$ to 5; high-speed stationary engines, $m = 10$; ordinary stationary engines, $m = 20$ to 25; marine engines, $m = 30$ to 40.

The above dimensions are for the middle of the rod. When the rod is tapered both ways, it is made $0.8D$ at the crank ends and $0.7D$ at the cross-head end. For high-speed engines the size is usually made greatest at the crank end, being about $1.7D$, the cross-head end being $0.7D$.

For rods of a rectangular cross-section, in which the depth of cross-section $= h$ and thickness $= b$, we have, for any given ratio of h to b,

$$h = 0.0144\sqrt[4]{m}\sqrt[4]{\left(\frac{h}{b}\right)^3}\sqrt{L\sqrt{P}}.$$

In order to simplify the use of this formula, the following table will be of use:

$\frac{h}{b} =$	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
$\sqrt[4]{\left(\frac{h}{b}\right)^3} =$	1.36	1.42	1.49	1.55	1.62	1.68	1.74	1.80	1.87	1.93	1.99

Example. Let $P = 30{,}000$ pounds, $L = 72$ inches, $\frac{h}{b} = 2.5$. Taking $m = 2$ for a locomotive engine, we have

$$\sqrt[4]{m} = 1.19,$$

and $$h = 0.0144 \times 1.19 \times 1.99\sqrt{72\sqrt{30000}} = 3.8 \text{ inches},$$

and $$b = \frac{3.8}{2.5} = 1.52 \text{ inches}.$$

Connecting-rod Ends.

The general proportions of a strap end for a connecting rod are given in the illustration. The dimensions are in terms of the modulus,

$$d_1 = d + 0.2 \text{ inch},$$

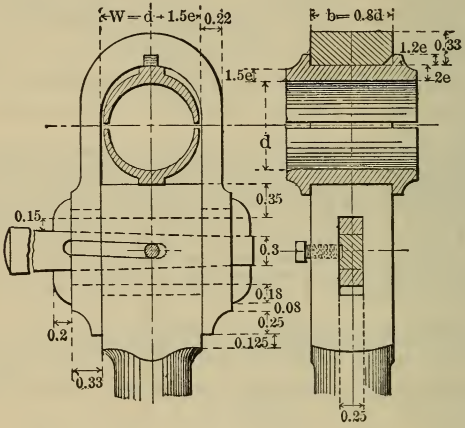
d being the diameter of the journal or crank pin. The dimensions of the brasses are in terms of the unit,

$$e = 0.07d + 0.125 \text{ inch}.$$

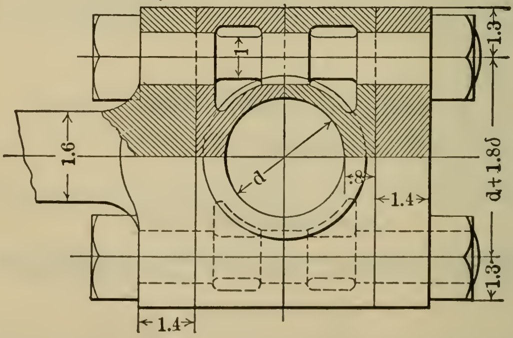
The modulus, d_1, is assumed on the basis of an ordinary overhung crank pin. For a cranked axle or an eccentric, however, the increased diameter would give unsuitable dimensions, and in such cases the modulus becomes

$$d'_1 = d_1 + \sqrt{\frac{b}{b^1}}\sqrt{\frac{d^1}{d}},$$

in which d_1 is the modulus for an overhung crank pin for the same pressure as the one under consideration; b and d being the corresponding values for the overhung pin and b^1 and d^1 those selected for the new one.



For heavy service the marine type of rod end is used, one form being shown in the illustration. Here the end of the rod is forged into a T, and the brasses, cored out as shown, form the bearing and the rod end, the bolts and steel cap forming the resistance to the driving stresses.



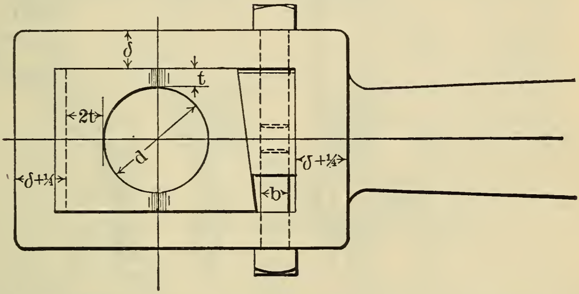
The dimensions here are based upon the diameter of the bolts at the bottom of the thread. The bolts are turned down, as shown, in order to distribute the strain and avoid breakage at the base of the thread.

Taking fibre stresses of 5000 pounds for wrought-iron and 6600 pounds for steel, we have, according to Unwin, for the diameter of the bolts at the bottom of the thread,

$$\delta = 0.02\sqrt{P} \text{ for wrought-iron,}$$
$$= 0.018\sqrt{P} \text{ for steel;}$$

the other dimensions being in terms of δ; P being the one-half maximum pressure upon the piston, or the pressure upon one bolt.

For rods of moderate size, where a closed end is permissible, the following type is convenient, compact, and inexpensive.



The proportions of the above type of stub end are based on the modulus,

$$\delta = 0.15d + 0.2 \text{ inch.}$$

The brasses are based on $t = 0.08d + \frac{1}{8}$ inch.

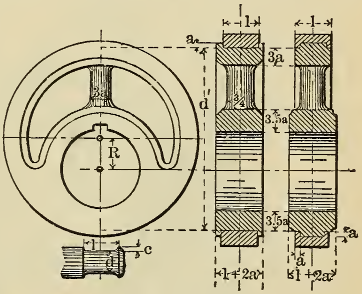
The diameter of the bolts is $0.02d = 0.25$ inch, taking the nearest even size.

The taper of the wedge is made $\frac{1}{8}$ inch to the inch.

The square end brass is made with a small lip on each side on the end only, to prevent lateral movement; the collar on the pin prevents the lateral movement of the bevelled brass.

The brasses should not be left open upon the pin, but either fitted close, and filed off when wear is to be taken up, or else a number of sheets of copper foil placed between them before boring the hole, forming slivers which can be taken out one at a time when necessary.

A variety of connecting-rod ends will be found in Reuleaux's "Constructor" and Unwin's "Machine Design."



ECCENTRICS.

Eccentrics may be considered as cranks in which the diameter of the pin is greater than the sum of the crank circle plus the shaft diameter.

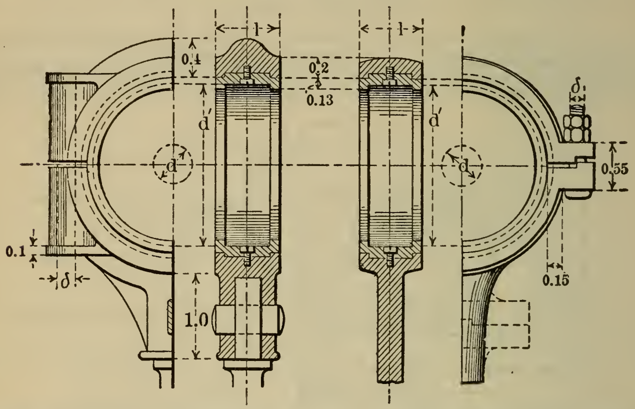
The breadth of the eccentric (properly the length of pin, l) is the same as that of the equivalent overhung journal subjected to the same pressure; for the depth of flange, a, we have

$$a = 1.5e = 0.07l + 0.2,$$

from which the other dimensions can be determined as in the illustrations.

For some forms of shafts with multiple cranks or other obstructions the eccentrics cannot be made as shown before, but must be in halves, bolted together.

The eccentric straps may be proportioned as in the illustrations, the modulus being derived from that for the equivalent overhung journal, as

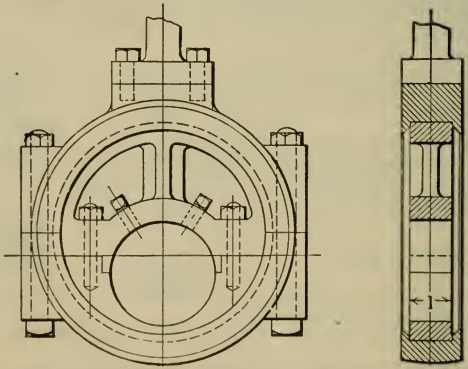


already described. The diameter of the bolts, δ, is found from the two moduli, as follows:

$$\delta = 0.33d_1 + 0.06d'_1.$$

The form on the left is intended to be made in cast-iron, and that on the right in wrought-iron. The most important feature in the operation of eccentrics is the maintenance of complete lubrication, as otherwise the high lineal velocity of the rubbing surfaces is apt to produce heating.

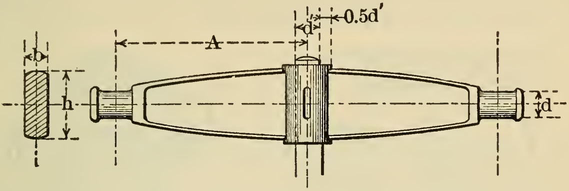
In the form shown in the illustration the strap is made wider than the eccentric, and the lip is bevelled as shown, thus forming a circular channel on each side, in which the oil collects and is distributed over the



rubbing surfaces. When this form of strap is to be used in the horizontal position the strap should be divided at an angle of 45° with the horizontal, otherwise the oil will run out at the joint, when standing. In practice, cast-iron on cast-iron is found to give excellent results as to wear and smooth running.

CROSS-HEADS.

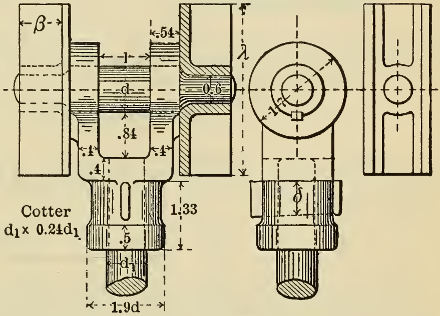
For a simple T cross-head the dimensions shown in the illustration may be used. If P is the maximum load upon the rod the journals are to



be computed for a load of $\frac{1}{2}P$, and the depth, h, in the middle may be made $= 2.5d + \frac{1}{14}A$. The thickness, b, may be determined from

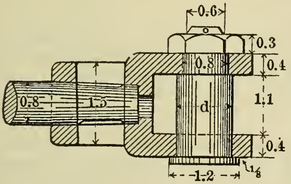
$$b = 0.00035\frac{PA}{h^2},$$

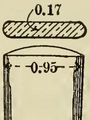
which corresponds to a fibre stress of 8500 pounds for wrought-iron.

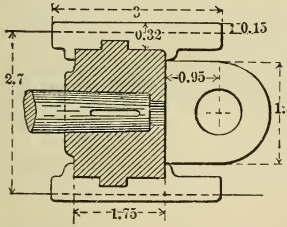


The arrangement of cross-heads for use in connection with guide bars depends very much upon the form of guides employed. Some examples will serve to indicate the general proportions.

For four-bar guides the following design may be employed, the proportions being those given by Unwin. The length and diameter of the pin are determined by the pressure upon the rod, the diameter usually being about 0.8 the size for the crank pin, and the length equal to the diameter. The dimensions of the other parts are in terms of the pin diameter, d. The cross-head itself is of wrought-iron, with cast-iron slide blocks.







When but two guide bars are used the form of cross-head shown here may be employed.

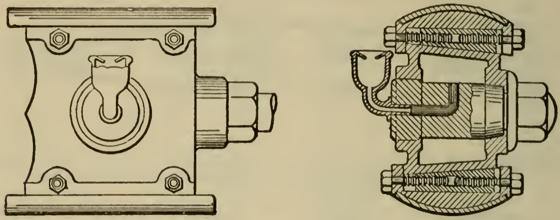
The dimensions are in terms of the pin diameter.

In these cross-heads the length and width, λ and β, of the slides should be so proportioned that the pressure should not be more than 40 to 60 pounds per square inch. For large engines the slides may be fitted with bronze shoes and with set-screws or wedges for adjustment. If, however, the area is made large, the wear is very slight, and no such provision is necessary.

In some engines of moderate size the guides are cast on the frame and

bored out in line with the cylinder. The cross-head shown below is an example designed for use with such guides. Numerous special forms of cross-heads will be found in Reuleaux's "Constructor" and Unwin's "Machine Design."

The pressure on the guides depends upon the total pressure on the piston-rod and upon the maximum angle which the connecting rod makes



with the line of the guides. Assuming that the greatest pressure occurs when the position of the crank is at right angles with the line of the guides, we have

$$P_1 = \frac{1}{\sqrt{n^2 - 1}} P,$$

in which P_1 is the pressure on the guide; P, the total pressure on the piston-rod; and n, the ratio of the length of the connecting rod to the radius of the crank. Thus, if the connecting rod be made 5 cranks in length, we have

$$P_1 = \frac{1}{\sqrt{25 = 1}} P = 0.204P.$$

If the pressure on the piston is 10,000 pounds, the greatest pressure on the guides will be 2040 pounds; and at 40 pounds per square inch a single slide-block should have an area of 51 square inches.

GEARING.

In the transmission of motion by toothed gearing it is necessary to know the number of teeth and their shape, as well as the dimensions of the cylinders, cones, or other figures upon which they are formed.

In all cases toothed gear-wheels are substitutes for smooth, rolling surfaces, the teeth being employed merely to obviate the slipping which might otherwise take place.

If we consider two spur-gears in engagement with each other, we can imagine the teeth being made smaller and smaller in size and, at the same time, greater and greater in number, until they become indefinitely small and the surfaces become practically smooth. Such rolling surfaces constitute the pitch surfaces of the gear-wheels, and the aim of toothed-gearing design is to shape the teeth so that the rolling action of the pitch surfaces may be maintained and, at the same time, forces of determinate magnitude transmitted without slip. In discussing gear-teeth, therefore, the pitch circles, of which the rolling action is to be reproduced, are the basis upon which the teeth are constructed.

Let

R = radius of pitch circle;
t = distance from centre to centre of adjacent teeth = circumferential pitch;
Z = number of teeth.

We then have

$$\frac{R}{t} = \frac{Z}{2\pi} = 0.15916Z.$$

When the gear-wheels are of large size and to be cast, made from wooden patterns, it is desirable to work to definite and convenient lineal pitch distances, in which case the pitch, t, is selected, and the corresponding radius, R, found for the given number of teeth, thus,

$$R = 0.15916Zt.$$

Thus, for a wheel of 75 teeth and 2½ inches pitch, we have

$$R = 0.15916 \times 187.5 = 29.85 \text{ inches},$$

and the pitch diameter $= 2R = 59.70$ inches.

In order to abridge the work of computation the following table may be used. It is only necessary to take out the number corresponding to the number of teeth and multiply it by the pitch to obtain the radius of the corresponding pitch circle. The pitch may be taken in any unit, inches, sixteenths of an inch, millimetres, etc., and the radius will be in the same unit.

Thus, for 75 teeth, we find opposite 70 and under 5 the number 11.94; and $11.94 \times 2.5 = 29.85$ inches, the same as before.

Table of Radii of Gear-wheels.

Multiply tabular number for given number of teeth by the circumferential pitch to obtain radius.

Z	0	1	2	3	4	5	6	7	8	9
0	....	.159	.318	.477	.637	.796	.955	1.114	1.273	1.432
10	1.59	1.75	1.91	2.07	2.23	2.39	2.55	2.71	2.86	3.02
20	3.18	3.34	3.50	3.66	3.82	3.98	4.14	4.30	4.46	4.62
30	4.77	4.93	5.09	5.25	5.41	5.57	5.73	5.89	6.05	6.21
40	6.37	6.53	6.68	6.84	7.00	7.16	7.32	7.48	7.64	7.80
50	7.96	8.12	8.28	8.44	8.59	8.75	8.91	9.07	9.23	9.39
60	9.55	9.71	9.87	10.03	10.19	10.35	10.50	10.66	10.82	10.98
70	11.14	11.30	11.46	11.62	11.78	11.94	12.10	12 25	12.41	12.57
80	12.73	12.89	13.05	13.21	13.37	13.53	13.69	13.85	14.01	14.16
90	14.32	14.48	14.64	14.80	14.96	15.12	15.28	15.44	15.60	15.76
100	15.92	16.07	16.23	16.39	16.55	16.71	16.87	17.03	17.19	17.35
110	17.51	17.67	17.83	17.98	18.14	18.30	18.46	18.62	18.78	18.94
120	19.10	19.26	19.42	19.58	19.73	19.89	20.05	20.21	20.37	20.53
130	20.69	20.85	21.01	21.17	21.33	21.49	21.65	21.80	21.96	22.12
140	22.28	22.44	22.60	22.76	22.92	23.08	23.24	23.40	23.55	23.71
150	23.87	24.03	24.19	24.35	24.51	24.67	24.83	24.99	25.15	25.31
160	25.46	25.62	25.78	25.94	26.10	26.26	26.42	26.58	26.74	26.90
170	27.06	27.21	27.37	27.53	27.69	27.85	28.01	28.17	28.33	28.49
180	28.65	28.81	28.97	29.13	29.28	29.44	29.60	29.76	29.92	30.08
190	30.24	30.40	30.56	30.72	30.88	31.04	31.19	31.35	31.51	31.67
200	31.83	31.99	32.15	32.31	32.47	32.63	32.79	32.95	33.10	33.26
210	33.42	33.58	33.74	33.90	34.06	34.22	34.38	34.54	34.70	34.85
220	35.01	35.17	35.33	35.49	35.65	35.81	35.97	36.13	36.29	36.45
230	36.61	36.76	36.92	37.08	37.24	37.40	37.56	37.72	37.88	38.04
240	38.20	38.36	38.51	38.67	38.83	38.99	39.15	39.31	39.47	39.63
250	39.79	39.95	40.11	40.27	40.42	40.58	40.74	40.90	41.06	41.22
260	41.38	41.54	41.70	41.86	42.02	42.18	42.34	42.49	42.65	42.81
270	42.97	43.13	43.29	43.45	43.61	43.77	43.93	44.09	44.25	44.40
280	44.56	44.72	44.88	45.04	45.20	45.36	45.52	45.68	45.84	46.00
290	46.15	46.31	46.47	46.63	46.79	46.95	47.11	47.27	47.43	47.59

For small pitches, especially for cut gearing, the so-called **Diametral Pitch** is much used.

Thus, we have, as before,

$$\frac{R}{t} = \frac{Z}{2\pi};\ \text{or}\ \frac{2R}{t} = \frac{Z}{\pi};$$

whence

$$Z = 2R\frac{\pi}{t}.$$

Or, the number of teeth is equal to the pitch diameter of the gear multiplied by π, divided by the circumferential pitch. By making the circumferential pitch an aliquot part of π, the relation of the number of teeth to the diameter may be very simply expressed. Thus, instead of making a gear of ½-inch pitch, the pitch may be made equal to

$$\frac{\pi}{6} = \frac{3.1416}{6} = 0.5236 \text{ inch},$$

and we have $$\frac{\pi}{t} = 6 \text{ and } Z = 2R \times 6,$$

so that for every wheel we have only to choose the diameter and multiply by 6 to obtain the number of teeth; or select the number of teeth and divide by 6 to obtain the diameter. In like manner we may choose pitches of π, $\frac{1}{2}\pi$, $\frac{1}{3}\pi$, $\frac{1}{4}\pi$, etc., or, as they are commonly called, one pitch, two pitch, three pitch, etc., these really meaning the number of teeth corresponding to each inch in diameter of the wheel; hence, the name, Diametral Pitch.

Since such gears are cut with standard cutters, the fractional circumferential pitch is provided for in the making of the cutter, and need not be further considered.

For convenience in selecting the approximate size of tooth required, the following table, showing the lineal value of diametral pitches, is given:

Diametral pitch.	Circumferential pitch.	Diametral pitch.	Circumferential pitch.
	Inch.		Inch.
1	3.1416	6	.5236
2	1.5708	7	.4488
3	1.0472	8	.3927
4	.7854	9	.3491
5	.6283	10	.3142

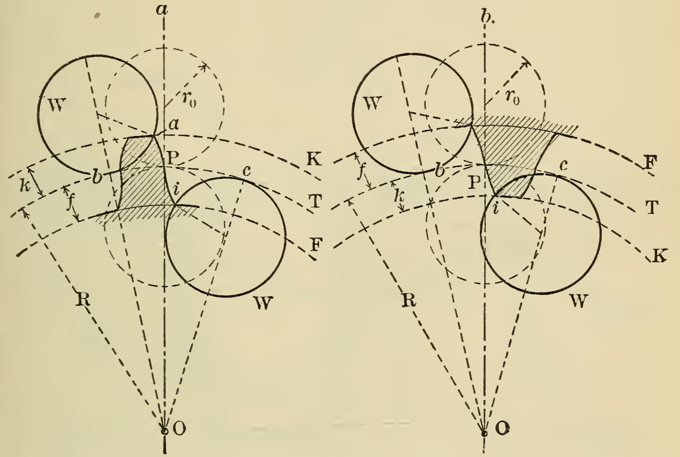
Thus, 3 diametral pitch is about equal to 1 inch circumferential pitch, 4 diametral pitch is a little more than ¾ inch, and so on. The diametral system simply throws the inconvenient fraction into the size of the gear-cutter, and thus simplifies all the succeeding work.

The form of gear teeth is a subject to which much study has been given. Formerly, when each establishment made its own gear-cutters and designed its own tooth outlines, the question was of more practical importance than at the present time, when accurately-formed cutters are regular articles of merchandise, and when it is only necessary to indicate the diametral pitch and the number of teeth to enable the proper cutter to be selected.

Two systems are in general use, the epicycloidal and the involute, and their respective merits have been actively discussed. Practice has shown, however, that there is little real difference between them, but the facility with which the involute system adapts itself to the design of machines for automatically generating tooth outlines gives it practical advantages.

Epicycloidal teeth are generated in the following manner:

External Teeth (Fig. *a*).—Given the number of teeth, Z, and pitch, t, or ratio, $\frac{t}{\pi}$, of the wheel. Make $OP = R = \frac{Zt}{2\pi} = \frac{1}{2}Z\left(\frac{t}{\pi}\right)$, and the radius, r_0, of the rolling circle, $W = 0.875t$, or $= 2.75\frac{t}{\pi}$; draw the outside circle of the teeth, K, with a radius $= R + 0.3t$, and the inside circle, F, with radius $= R - 0.4t$, and make the thickness of tooth $= \frac{19}{40}t$. Arc Sb = arc



ab; arc Sc = arc ic. Sa, the face curve, is generated by the rolling of W upon T; Si, the flank curve, by the rolling of W in T. For pinions of eleven teeth, Si becomes a straight line and radial. Pinions with as few as seven teeth can be made to work on this system, for although the flanks are undercut, they are still within the limits of the theoretical flank profile, as shown in the following illustration, where a 7-tooth pinion is shown with a rack tooth. The backlash is $\frac{1}{10}t$.

Internal Teeth (Fig. *b*).—The generation of internal teeth is similar to the preceding. The radius of base circle is $-R$, and the length of tooth above and below the pitch circle is $0.3t$ and $0.4t$, as before; $r_0 = 0.875t = 2.75\frac{t}{\pi}$, and the thickness of tooth $= \frac{19}{40}t$. The flank, Sa, is generated by rolling W upon T, and the face, Si, by rolling W inside of T.

In the case of a rack, $R = \infty$, Sa and Si then become similar portions of the common cycloid.

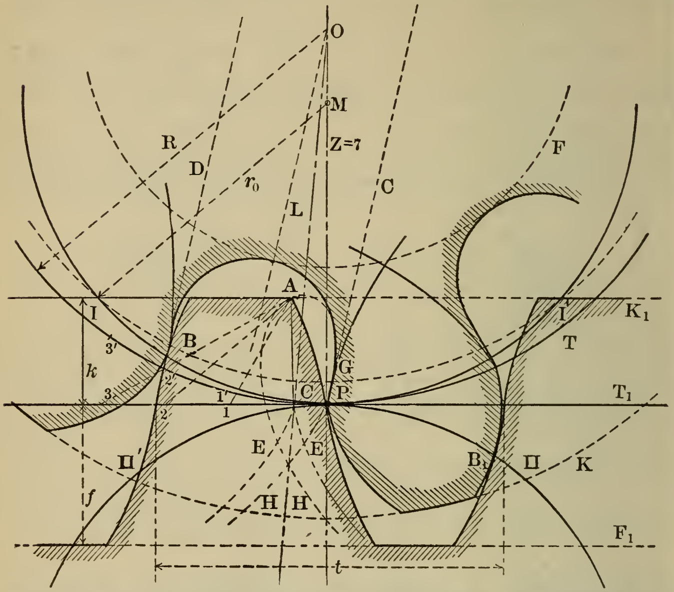
In teeth of this form the line of action coincides with the rolling circles, the portion included being = arc ba + the corresponding arc, b_1a_1, of the opposing wheel, when both are external gears, and + the arc, ci, for an internal gear working with a spur gear. The duration of action, e, varies between 1.22 and 1.60.

Involute teeth are generated as follows:

The curve is developed by unwrapping a line from a base circle, which is concentric with and bears a definite relation to the pitch circle.

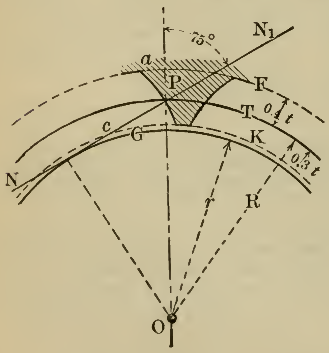
External and Internal Teeth.—Given the number of teeth, Z, and pitch, t, or ratio, $\frac{t}{\pi}$, for the required wheel. Make $OP = R = \frac{Zt}{2\pi} = \frac{1}{2}Z\left(\frac{t}{\pi}\right)$, and draw the outer and inner circles, giving the distances, $f = 0.4t$, $k = 0.3t$, above and below the pitch circle; also make the thickness of the tooth $= \frac{19}{40}t$.

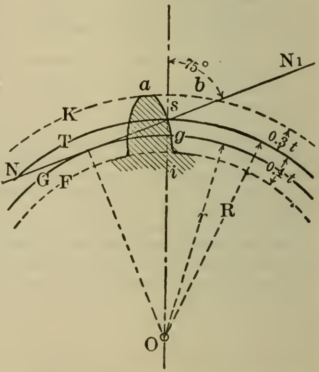
Draw the line, NPN_1, at an angle of 75° with OP, and it will be tangent to the base circle, G, the radius of which $= r = 0.966R = 0.154Zt =$



Limiting case of epicycloidal gear teeth, showing 7-tooth pinion engaged with rack.

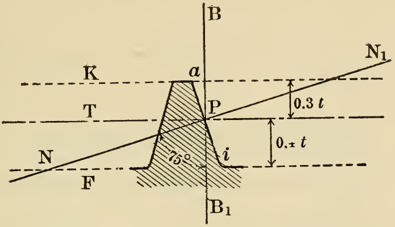
$0.483Z\left(\frac{t}{\pi}\right)$. If, now, we unwrap the line, NP, upon the circle, G, from P outward to a and inward to g, the path, aPg, of the point, P, will be the





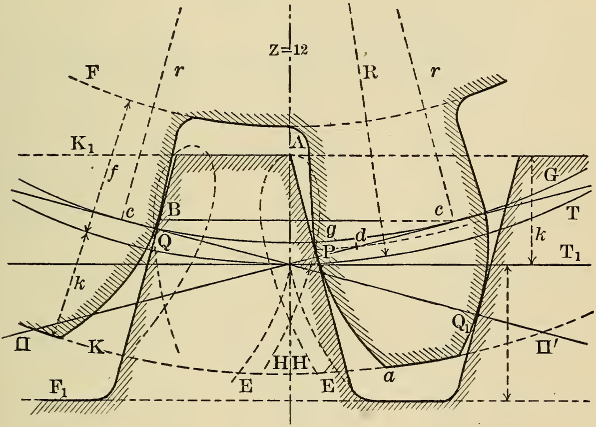
required tooth outline, which for wheels of fewer than 55 teeth may be prolonged by a radial line to reach the bottom circle.

For two equal wheels of 14 teeth, e is only a little greater than unity; it varies between 1 and 2.5.



Rack Teeth.—The profile, aPi, is straight and makes an angle of 75° with the pitch line, T. The angle 75° can readily be laid off by using the drawing triangles of 45° and 30° together.

For low-numbered pinions on the involute system care must be taken to avoid interference. Thus, in the illustration below, in which a 12-tooth



pinion is shown engaged with a rack, it will be seen that the radial flanks of the pinion are crossed by the dotted line of action of the rack teeth, as at Ag. This may be remedied by reducing the length of the rack teeth, or by rounding off their points.

Diametral Pitch Formulas.

Brown & Sharpe Manufacturing Company.

Let

$P =$ the diametral pitch;

$$\left.\begin{array}{l}\left.\begin{array}{l} D' = \text{the pitch diameter;} \\ D = \text{the outside diameter;} \\ N = \text{the number of teeth;} \\ V = \text{the velocity ratio;} \end{array}\right\}\text{Gear.} \\ \left.\begin{array}{l} d' = \text{the pitch diameter;} \\ d = \text{the outside diameter;} \\ n = \text{the number of teeth;} \\ v = \text{the velocity ratio;} \end{array}\right\}\text{Pinion.}\end{array}\right\}\text{These wheels run together.}$$

$a =$ distance between the centres of the two wheels;

$b =$ the number of teeth in both wheels.

Formulas.

$$b = 2aP; \qquad n = \frac{PD'V}{v}; \qquad d = \frac{2a(n+2)}{b};$$

$$n = \frac{bV}{v+V}; \qquad V = \frac{nv}{N}; \qquad a = \frac{b}{2P};$$

$$N = \frac{nv}{V}; \qquad v = \frac{NV}{n}; \qquad D' = \frac{2av}{v+V};$$

$$n = \frac{NV}{v}; \qquad v = \frac{PD'V}{n}; \qquad d' = \frac{2aV}{v+V};$$

$$N = \frac{bv}{v+V}; \qquad D = \frac{2a(N+2)}{b}; \qquad a = \frac{D'+d'}{2}.$$

Circular Pitch.

With its Equivalent in Diametral Pitch, Depth of Space, and Thickness of Tooth.

Circular pitch.	Diametral pitch.	Thickness of tooth on pitch line.	Depth to be cut in gear.	Addendum.
Inch.		Inch.	Inch.	Inch.
6	.5236	3.0000	4.1196	1.9098
5	.6283	2.5000	3.4330	1.5915
4	.7854	2.0000	2.7464	1.2732
3½	.8976	1.7500	2.4031	1.1140
3	1.0472	1.5000	2.0598	.9550
2¾	1.1424	1.3750	1.8882	.8754
2½	1.2566	1.2500	1.7165	.7958
2¼	1.3963	1.1250	1.5449	.7162
2	1.5708	1.0000	1.3732	.6366
1⅞	1.6755	.9375	1.2874	.5968
1¾	1.7952	.8750	1.2016	.5570
1⅝	1.9333	.8125	1.1158	.5173
1½	2.0944	.7500	1.0299	.4775
1⅜	2.2848	.6875	.9441	.4377
1¼	2.5133	.6250	.8583	.3979
1⅛	2.7925	.5625	.7724	.3581
1	3.1416	.5000	.6866	.3183
$\frac{15}{16}$	3.3510	.4687	.6437	.2984
⅞	3.5904	.4375	.6007	.2785
$\frac{13}{16}$	3.8666	.4062	.5579	.2586
¾	4.1888	.3750	.5150	.2387
$\frac{11}{16}$	4.5696	.3437	.4720	.2189
⅝	5.0265	.3125	.4291	.1989
$\frac{9}{16}$	5.5851	.2812	.3862	.1790
½	6.2832	.2500	.3433	.1592
$\frac{7}{16}$	7.1808	.2187	.3003	.1393
⅜	8.3776	.1875	.2575	.1194
$\frac{5}{16}$	10.0531	.1562	.2146	.0995
¼	12.5664	.1250	.1716	.0796
⅛	25.1327	.0625	.0858	.0398
$\frac{1}{16}$	50.2655	.0312	.0429	.0199

Diametral Pitch.

With its Equivalent in Circular Pitch, Depth of Space, and Thickness of Tooth.

Diametral pitch.	Circular pitch.	Thickness of tooth on pitch line.	Depth to be cut in gear.	Addendum.
	Inch.	Inch.	Inch.	Inch.
½	6.2832	3.1416	4.3142	2.0000
¾	4.1888	2.0944	2.8761	1.3333
1	3.1416	1.5708	2.1571	1.0000
1¼	2.5133	1.2566	1.7257	.8000
1½	2.0944	1.0472	1.4381	.6666
1¾	1.7952	.8976	1.2326	.5714
2	1.5708	.7854	1.0785	.5000
2¼	1.3963	.6981	.9587	.4444
2½	1.2566	.6283	.8628	.4000
2¾	1.1424	.5712	.7844	.3636
3	1.0472	.5236	.7190	.3333
3½	.8976	.4488	.6163	.2857
4	.7854	.3927	.5393	.2500
5	.6283	.3142	.4314	.2000
6	.5236	.2618	.3595	.1666
7	.4488	.2244	.3081	.1429
8	.3927	.1963	.2696	.1250
9	.3491	.1745	.2397	.1111
10	.3142	.1571	.2157	.1000
11	.2856	.1428	.1961	.0909
12	.2618	.1309	.1798	.0833
14	.2244	.1122	.1541	.0714
16	.1963	.0982	.1348	.0625
18	.1745	.0873	.1198	.0555
20	.1571	.0785	.1079	.0500
22	.1428	.0714	.0980	.0455
24	.1309	.0654	.0898	.0417
26	.1208	.0604	.0829	.0385
28	.1122	.0561	.0770	.0357
30	.1047	.0524	.0719	.0333
32	.0982	.0491	.0674	.0312
36	.0873	.0436	.0599	.0278
40	.0785	.0393	.0539	.0250
48	.0654	.0327	.0449	.0208

Strength of Gear Teeth.

(Lewis.)

W = load transmitted, in pounds;
p = circular pitch;
f = face;
y = factor for different number and forms of teeth;
s = safe working stress of material.

$$W = spfy.$$

Number of teeth.	Value of factor, y. Involute 20°.	Involute 15° cycloidal.	Radial flanks.	Number of teeth.	Value of factor, y. Involute 20°.	Involute 15° cycloidal.	Radial flanks.
12	.078	.067	.052	27	.111	.100	.064
13	.083	.070	.053	30	.114	.102	.065
14	.088	.072	.054	34	.118	.104	.066
15	.092	.075	.055	38	.122	.107	.067
16	.094	.077	.056	43	.126	.110	.068
17	.096	.080	.057	50	.130	.112	.069
18	.098	.083	.058	60	.134	.114	.070
19	.100	.087	.059	75	.138	.116	.071
20	.102	.090	.060	100	.142	.118	.072
21	.104	.092	.061	150	.146	.120	.073
23	.106	.094	.062	300	.150	.122	.074
25	.108	.097	.063	Rack	.154	.124	.075

Safe Working Stress, s, for Different Speeds.

Material.	Speed of teeth, in feet, per minute. 100 or less.	200	300	600	900	1200	1800	2400
Cast-iron	8000	6000	4800	4000	3000	2400	2000	1700
Steel	20000	15000	12000	10000	7500	6000	5000	4300

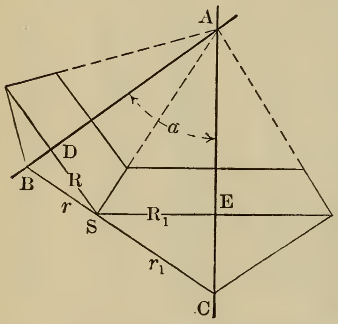
When great strength is required, and the pressure is always in one direction only, the teeth may be shaped with a much greater angle of curvature on the back than on the working faces, it being only necessary that the back outlines clear each other properly. This may be done by making the working faces of the teeth according to the epicycloidal form, as already described, and the backs of the teeth of the involute curve, using an angle of 53° instead of 75°, as in the usual method. This is equivalent to the use of a generating circle for the involute of a diameter of 0.8 times the pitch diameter of the gear-wheel. The so-called "thumb-shaped" teeth thus derived are sharp on the point and thick at the base, and are much stronger than the ordinary form of teeth.

There has been much controversy as to the relative merits of epicycloidal and involute teeth, but in actual practice there is little difference. With wheels properly proportioned to their work, and especially with the correct relations of the axes firmly maintained, either form answers all practical requirements fully. The greater convenience with which involute teeth may be made, especially in the machines in which the tooth profile is automatically generated, gives it advantages in construction which in most cases far outweigh any points of superiority which have been advanced for the epicycloidal system.

Bevel Gears.

When the axes between which motion is to be transmitted are not parallel, but intersect each other, the gear teeth must be formed upon conical surfaces. Such gears are broadly called bevel gears, and when the shafts form a right angle with each other and the wheels are equal to each other in diameter they are called mitre gears.

The geometrical figures, which are formed by one cone rolling upon another, require that both cones should have a common apex. The surface thus developed is called a spherical cycloid. Of these there are five particular forms, as with the plane cycloids, the latter being really those for a cone with an apex angle of 180°. The spherical cycloid is very similar in form to the plane cycloid, as are also the corresponding evolutes.



The use of the spherical cycloid for the formation of bevel gear teeth would involve many difficulties. In order to construct such teeth it is, therefore, common to use the method (first devised by Tredgold) of auxiliary circles, based upon the supplementary cones, and enabling the teeth to be laid out in a similar manner to those of spur gears. The auxiliary circles for the bevel gears, R and R_1, are those of the spur gears having the same pitch, their radii being respectively r and r_1, the elements BS and CS of the supplementary cones.

For any given angle, a, between the axes, the radius, r, and the number of teeth, 3, for the auxiliary circle can be determined from the radii, R and R_1, and tooth numbers, Z and Z_1, by the following formula:

$$\frac{r}{R} = \frac{\sqrt{R^2 + R_1^2 + 2RR_1 \cos a}}{R_1 + R \cos a},$$

$$\frac{z}{Z} = \frac{\sqrt{Z^2 + Z_1^2 + 2ZZ_1 \cos a}}{Z_1 + Z \cos a}.$$

If the axes are at right angles we have

$$\frac{r}{R} = \frac{\sqrt{R^2 + R_1^2}}{R_1}, \qquad \frac{z}{Z} = \frac{\sqrt{Z^2 + Z_1^2}}{Z_1}, \qquad \frac{r}{r_1} = \left(\frac{n_1}{n}\right)^2.$$

Example. A pair of bevel gears have 30 and 50 teeth, and an angle between axes $a = 60°$; hence, $\cos a = ½$, and we have for the auxiliary circle of the 30-tooth gear

$$z = 30\frac{\sqrt{30^2 + 50^2 + 2 \,.\, 30 \,.\, 50 \,.\, 0.5}}{50 + 30 \,.\, 0.5} = 6\frac{\sqrt{4900}}{13} = 32.3, \text{ say } 32.$$

For the 50-tooth gear we have, also,

$$z_1 = 50\frac{\sqrt{4900}}{30 + 50 \,.\, 0.5} = 64.$$

From these numbers and the given pitch the auxiliary circles can be laid off and the teeth drawn.

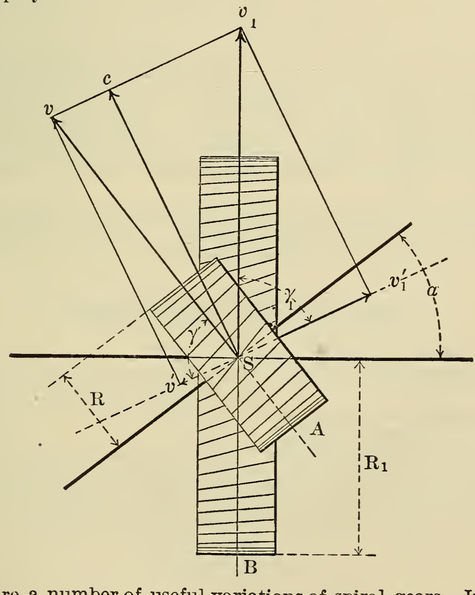
Low tooth numbers are not available for bevel gears, since the errors which are involved in the method of auxiliary circles become disproportionately great. By using not fewer than 24 teeth for the bevel gear a minimum of 28 for the auxiliary circle is obtained, and the evolute system can be used to advantage. This form of tooth is best adapted for this purpose, on account of its simplicity of form, notwithstanding the minor defects which have already been noticed.

Owing to the fact that the form and shape of teeth on bevel gears vary along the face of the tooth, such gears cannot be cut theoretically correct by rotating cutters. When such cutters are used an approximate form is obtained, and filing must be resorted to in order to correct the shape of the teeth. At the present time, large bevel gears are usually made by planing the teeth, the tool being guided by a former, while small gears are cut on machines in which the tooth outlines are generated by the movement of the gear blank under the cutter, according to the method first devised by Professor Hermann, of Aix-la-Chapelle, in 1877, and employed in the ingenious machines of Bilgram and of Warren.

The whole subject of the form and action of gear teeth is thoroughly discussed in Grant's "Handbook on the Teeth of Gears," Beale's "Practical Treatise on the Teeth of Gears," Reuleaux's "Constructor," Unwin's "Machine Design," and numerous other standard works.

Spiral Gears.

When the axes between which motion is to be transmitted are not parallel to each other, and yet do not intersect, gears with spiral teeth are usually employed.



There are a number of useful variations of spiral gears. In the illustration is shown a pair of wheels, A and B, both with left-hand spirals and corresponding tooth profiles. The pitch angles, γ and γ_1, are so chosen that at the point of contact the pitch cylinders have a common tangent, so that if α be the angle of inclination of the axes, $\gamma + \gamma_1 + \alpha = 180°$. If

we indicate by v and v_1 the circumferential velocity in the direction of the tangent and normal, respectively, we have

$$\frac{v_1}{v} = \frac{\sin \gamma}{\sin \gamma_1}, \text{ whence } \frac{n_1}{n} = \frac{R \sin \gamma}{R_1 \sin \gamma_1} = \frac{Z}{Z_1}.$$

The normal pitches, $\tau = t \sin \gamma$ and $\tau_1 = t_1 \sin \gamma_1$, must be equal to each other, whence $\frac{t}{t_1} = \frac{\sin \gamma_1}{\sin \gamma}$.

As indicated by the components of velocity, v' and v_1', there is an end, long-sliding action of the teeth upon each other, with a velocity

$$c' = v' + v_1' = c(\cot \gamma + \cot \gamma_1).$$

This sliding consumes power and causes wear, and will be at a minimum when v' and v_1' are equally great,—that is, when $\gamma = \gamma_1$.

With regard to the choice of γ and γ_1, the conditions may be so taken that the position of the coinciding tangents of the two spirals shall be slightly before or slightly after the actual line of contact, but as close as may be possible. The position of the line of contact may be stated as follows:

$$\frac{R}{R_1} = \frac{\cot \gamma}{\cot \gamma_1} = \frac{\frac{n_1}{n} + \cos \alpha}{\frac{n}{n_1} + \cos \alpha};$$

as also

$$\cot \gamma = \frac{\sin \alpha}{\frac{n}{n_1} + \cos \alpha}.$$

For $\alpha = 90°$ we have $\cot \gamma = \frac{n_1}{n}$. Such spiral wheels, when the teeth are well made, transmit motion very smoothly, but the surface of working contact is very small. One of the most important applications is that of the worm and worm-wheel. In this case $\alpha = 90°$ and $Z = 1$, the teeth of the wheel, R_1, being inclined at an angle, γ, with the edge of the wheel; whence $\tan \gamma = \frac{t}{2\pi R} = 0.15916 \frac{t}{R}$. The velocity ratio of transmission is $n_1 : n = Z : Z_1$.

The subject of spiral gears is extensively discussed in Reuleaux's "Constructor" and in Halsey's "Spiral Gearing." See, also, "Transactions of the American Society of Mechanical Engineers," 1886, Vol. VII., p. 273.

Proportions of Gear-wheels.

Gear-wheels may be divided into two classes:

Hoisting Gears, such as are used in cranes and similar machinery, and

Transmission Gears, used to transmit power continuously at a determinate velocity.

We may include under the term Hoisting Gears all those having a linear velocity at the pitch circle of not more than 100 feet per minute, and under Transmission Gears all those running at a higher velocity.

For a pitch, t, face, b, length of teeth, l, and base thickness of tooth, h, we have for a tooth pressure, P, corresponding to a stress, S, the general formula:

$$bt = 6\frac{P}{S}\left(\frac{l}{t}\right)\left(\frac{t}{h}\right)^2;$$

and for a length of $0.7t$ and a thickness of $0.5t$ we have

$$bt = 16.8\frac{P}{S}.$$

This assumes that the resistance of the teeth is proportional to their cross-section, which is also equally true for those which have the same ratio of b to t to each other, a condition which is often of much service in practice.

For a hoisting gear of cast-iron let

(PR) = the statical moment of the driving force;
Z = the number of teeth;
R = its previously-determined pitch radius, in inches;
t = the pitch.

We have for the given dimensions

$$t = 0.230\sqrt[3]{\frac{(PR)}{Z}}, \qquad \frac{t}{\pi} = 0.0730\sqrt[3]{\frac{(PR)}{Z}};$$

$$t = 0.045\sqrt{\frac{(PR)}{R}}, \qquad \frac{t}{\pi} = 0.0145\sqrt{\frac{(PR)}{R}};$$

the face, b, being made

$$b = 2t.$$

These are intended to give a fibre stress, S, of about 4200 pounds. The actual stress is properly somewhat less, because the thickness of the tooth at the base is usually more than $\frac{1}{2}t$.

Since the value of $\frac{PR}{R}$ is the same as the pressure, P, we can use the above formulas in cases in which P only is given, as for rack teeth.

In proportioning transmission gears, in which the velocity is greater than 100 feet per minute, the greater liability to shock with increased speed renders it desirable to assume a lower working fibre stress, S, as the circumferential velocity, v, increases.

For cast-iron we may take

$$S = \frac{9\,600\,000}{v + 2164},$$

in which v is the lineal velocity, in feet, per minute. For steel, S may be taken $3\frac{1}{3}$ times, and for wood, $\frac{6}{10}$ times the value thus obtained. For

Material.	v = 100	200	400	600	800	1000	1500	2000	2500
Cast-iron	S = 4240	4060	3744	3473	3238	3034	2620	2302	2068
Steel.........	S = 14112	13520	12467	11565	10782	10103	8725	7665	6886
Wood	S = 2544	2436	2246	2083	1943	1820	1572	1381	1240

The velocity, v, may be obtained when n and R (the latter in inches) are given, by the following formula:

$$v = \frac{2\pi Rn}{12} = 0.5236Rn.$$

It is also found that the breadth of face, b, should increase with the increase of P. Tredgold states that the pressure per inch of face, $\frac{P}{b}$, should not exceed 400 pounds. This, however, is not to be followed implicitly, since pressures as high as 1400 pounds have been successfully used in practice. It is better, however, to consider the question of wear from the product of $\frac{P}{b}$ into n, which should not exceed a predetermined maximum. It is found that if $\frac{P}{b} \times n$ exceeds 67,000, the wear becomes excessive. In a

pair of wheels where the teeth of both are made of iron, the greatest wear comes upon the teeth of the smaller wheel. In this case we may make

$$\frac{Pn}{b} = \text{not more than } 28{,}000,$$

and, if possible, it should be taken at less than this value. For smaller forces this constant, which we may call the coefficient of wear and designate as A, may readily be made as low as 12,000, and even 6000, without obtaining inconvenient dimensions. When the teeth are of wood and iron the wear upon the iron may be neglected, as the wear comes almost entirely upon the wooden teeth. For wooden teeth the value of A should not exceed 28,000, and is better made about 15,000 to 20.000.

It must be remembered that the different values of A do not appreciably affect the strength, but rather control the rapidity of wear. When sufficient space is available, and a low value can be given to the coefficient of wear, it is advisable to do so; if this cannot be done, the coefficient which is selected will give an indication of the proportional amount of wear which may be expected.

In cases where a number of wheels gear into one other wheel it is better to take, instead of the number of revolutions of the common wheel, the number of tooth contacts.—that is, the product of the revolutions and number of wheels in the group.

If R is given, as is often the case with water-wheels, fly-wheels, etc., P is also known; and since A can be chosen, we have, taking N to be the horse-power transmitted,

$$b = \frac{Pn}{A} = \frac{63000}{A} \cdot \frac{N}{R};$$

hence,

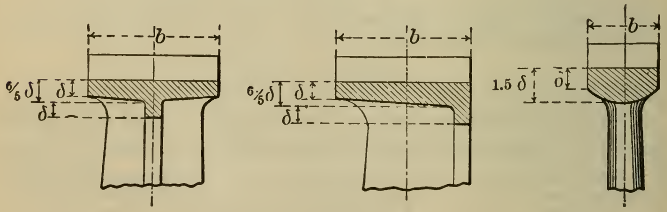
$$t = \frac{16.8P}{Sb} = \frac{16.8A}{Sn}.$$

If, however, as occurs in many cases, R is not previously determined, the choice of the number of teeth, Z, is unrestricted. In such cases we have for the width of face, b,

$$b = \frac{396\,000}{A} \cdot \frac{N}{Zt}.$$

For transmission gears the minimum number of teeth should not be fewer than 20, in order that the unavoidable errors of construction shall not cause excessive wear; for quick-running gears it is desirable to have still more teeth. The gear-wheels on high-speed turbines seldom have fewer than 40, and often as many as 80 teeth. When wood and iron teeth are used the least wear is produced when the wooden teeth are on the driver, because the action begins at the base of the tooth and passes towards the point, while on the driven gear the action is reversed.

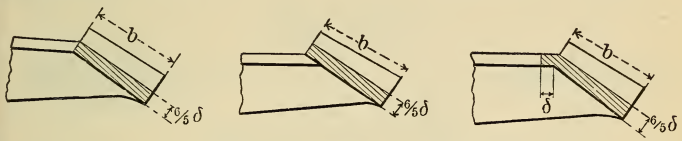
Proportions of Gear-wheel Parts.



The Rim.—The ring of metal upon which the teeth of a gear-wheel are placed is called the rim. For cast-iron spur gears the thickness of the rim is given by the formula

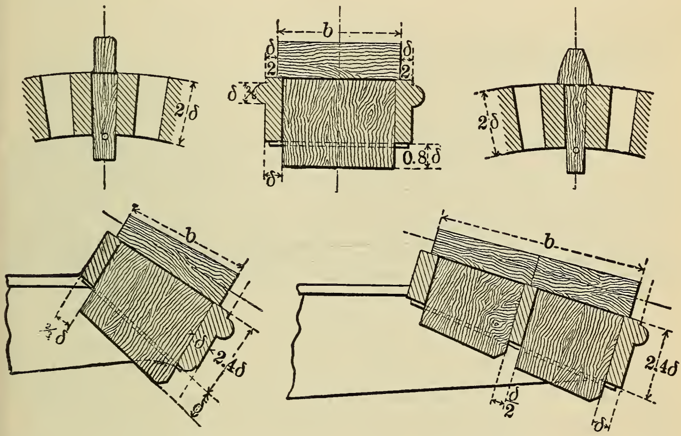
$$\delta = 0.4t + 0.125 \text{ inch.}$$

The rim is thickened in the middle, or at one edge, to $\frac{6}{5}\delta$, and also stiffened by a rib, and for gears of fine pitch the section of the rim is curved, which harmonizes well with arms of oval section. Accordingly, a pitch of 1 inch would give a rim thickness $\delta = 0.4$ inch $+ 0.125$ inch $= 0.525$ inch, or a little over $\frac{1}{2}$ inch; and for a pitch of $\frac{1}{2}$ inch, $\delta = 0.325$ inch.

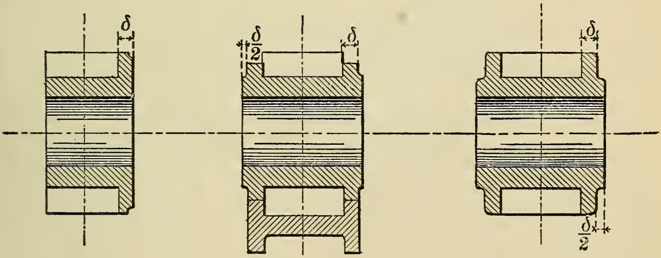


For bevel gears of cast-iron the rim is made $\frac{6}{5}\delta$ thick at the outer edge, and of the various forms shown in the illustrations.

For wooden teeth it is necessary to have a deeper and stronger rim, the dimensions being dependent somewhat upon the method of inserting the



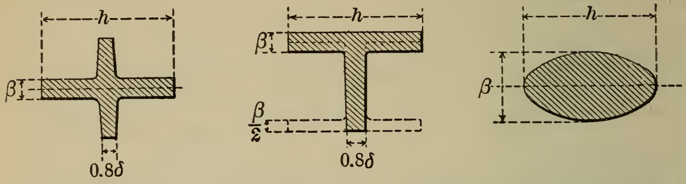
teeth. The proportions are shown in the illustrations. For very wide faces the wooden teeth are made in two pieces and a stay bar cast in the mortise.



Small pinions are often cast solid, and when subjected to heavy pressures are strengthened by shrouding, and sometimes this shrouding is turned down to the pitch line.

Wheel Arms.—The arms of gear-wheels are made according to the following forms, dependent upon the kind of rim used.

Ribbed sections are made sometimes as shown in the dotted lines, as may be most convenient in moulding. Oval sections have the thickness,



β, of the arm generally made one-half the width, h. A good proportion for the arms is obtained when their number, A, is made as follows:

$$A = 0.55\sqrt{Z\sqrt[4]{t}}.$$

From these we obtain the following:

A	$= 3$	4	5	6	7	8	10	12
$Z\sqrt{t}$	$= 30$	53	83	119	162	211	330	475

Example. For a gear-wheel of 50 teeth and 2-inch pitch we have $Z\sqrt{t} = 50\sqrt{2} = 50 \times 1.414 = 70$, and this lies between 53 and 83; being nearer the latter, we give the wheel five arms. If the pitch had been ¾ inch, and the same number of teeth, $Z\sqrt{t} = 50\sqrt{0.75} = 50 \times 0.866 = 43.3$, or between three and four arms, the latter number being used in practice.

The width of arm, h, in the plane of the wheel is somewhat a matter of judgment, but may suitably be made according to the ratio, $h = 2$ to $2.5t$, when the thickness, β, may be obtained from the following formula:

$$\frac{\beta}{b} = 0.07\frac{Z}{A}\left(\frac{t}{h}\right)^2.$$

Should this formula give a thickness either too great or too small for convenience in casting, another value for $\frac{h}{t}$ must be taken and the calculation repeated. The following table will assist in this operation.

Table of Gear-wheel Arms.

$\frac{h}{t}$	Value of $\frac{\beta}{b}$, when								
	$\frac{Z}{A} = 7$	9	12	16	20	25	30	35	40
1.50	.20	.28	.37	.50	.62	.78	.93	1.08	1.24
1.75	.16	.21	.27	.37	.46	.57	.69	.80	.91
2.00	.12	.16	.21	.28	.35	.44	.53	.61	.70
2.25	.10	.12	.17	.22	.28	.35	.41	.48	.55
2.50	.08	.10	.13	.18	.22	.28	.34	.39	.45
2.75	.06	.08	.11	.15	.18	.23	.28	.32	.37
3.00	.05	.07	.09	.12	.16	.19	.23	.27	.31

The taper of the arms may be made as follows: the ribs at the rim are made slightly narrower than the breadth of face, b, and at the hub equal to, or slightly greater, than b. For arms of oval section, h may be made equal $2t$ at the centre of the wheel, tapering to two-thirds this width at the rim.

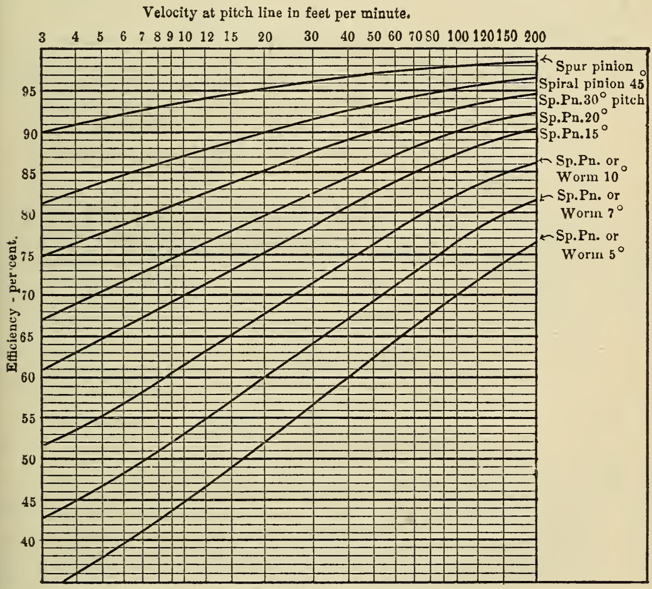
Hub.—The thickness, w, of the hub may be made

$$w = 0.4h + 0.4 \text{ inch.}$$

The above proportions are those recommended by Reuleaux.

Efficiency of Gearing.

The efficiency of spur gearing depends upon the lineal speed at the pitch line, while for spiral and worm gearing the angle of the teeth must also be taken into account.



The accompanying diagram, from experiments by William Sellers & Co., Incorporated, gives the efficiencies for practical cases.

For all ordinary calculations the following efficiencies may be used:

Cut spur gears	0.96
Cast spur gears	0.94
Cut bevel gears	0.95
Cast bevel gears	0.92

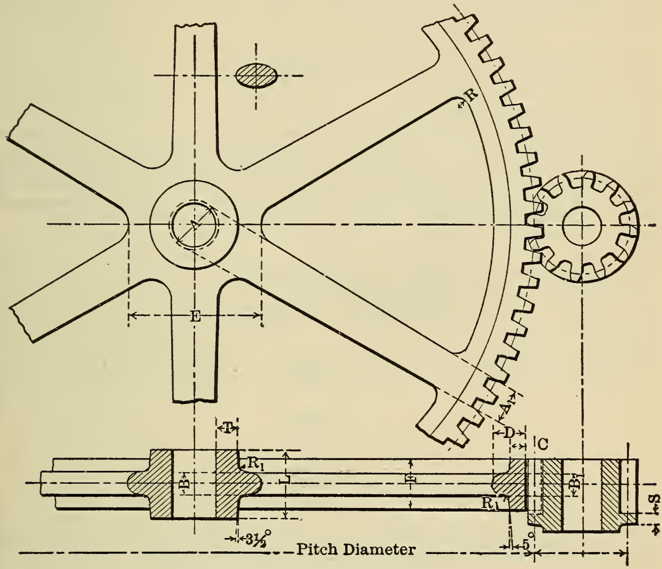
Table of Proportions for Gear-wheels. The Yale & Towne Manufacturing Company.

Pitch.		Dimensions of																
		Teeth.			Rim.					Arms.					Hub.			
Circular.	Diametral.	Addendum.	Flank.	Thickness.	C.	D.	F.	R.	R_1.	A.	A_1.	B.	B_1.	Web.	S.	T.	L.	
.79	4.00	.25	.32	.37	15/32	31/32	1 9/16	3/8	5/32	1 9/16	1 3/32	25/32	9/16	15/32	11/32	$T = 0.16 \text{ pitch } \sqrt[3]{N} + 0.18$	$L = F + \frac{\text{Diametral pitch}}{40}$	
.90	3.50	.29	.36	.42	17/32	1 3/16	1 25/32	7/16	3/16	1 25/32	1 1/4	7/8	5/8	17/32	13/32			
1.00	3.14	.32	.40	.47	9/16	1 9/32	2	1/2	3/16	2	1 13/32	1	11/16	9/16	7/16			
1.05	3.00	.33	.42	.50	19/32	1 11/32	2 3/32	17/32	7/32	2 3/32	1 15/32	1 1/16	3/4	19/32	7/16			
1.14	2.75	.36	.45	.54	5/8	1 7/16	2 9/32	9/16	7/32	2 9/32	1 19/32	1 1/8	13/16	5/8	15/32			
1.25	2.50	.40	.49	.60	11/16	1 9/16	2 1/2	5/8	1/4	2 1/2	1 3/4	1 1/4	7/8	11/16	17/32			
1.38	2.28	.44	.54	.65	23/32	1 11/16	2 3/4	11/16	9/32	2 3/4	1 29/32	1 3/8	31/32	23/32	17/32			
1.40	2.25	.44	.55	.67	3/4	1 11/16	2 25/32	11/16	9/32	2 25/32	1 15/16	1 13/32	31/32	3/4	9/16			
1.50	2.08	.48	.58	.72	13/16	1 13/16	3	3/4	5/16	3	2 3/32	1 1/2	1 1/16	13/16	19/32			
1.57	2.00	.50	.61	.75	27/32	1 7/8	3 1/8	25/32	7/16	3 1/8	2 7/32	1 9/16	1 1/8	27/32	5/8			
1.68	1.88	.53	.65	.80	7/8	1 31/32	3 11/32	27/32	11/32	3 11/32	2 11/32	1 11/16	1 3/16	7/8	21/32			
1.75	1.80	.56	.67	.84	29/32	2 1/16	3 1/2	7/8	11/32	3 1/2	2 7/16	1 3/4	1 7/32	29/32	11/16			
1.80	1.75	.57	.69	.86	15/16	2 3/32	3 19/32	29/32	3/8	3 19/32	2 1/2	1 13/16	1 1/4	15/16	11/16			
2.00	1.57	.64	.77	.96	1 1/32	2 5/16	4	1	13/32	4	2 13/16	2	1 13/32	1 1/32	3/4			
2.09	1.50	.67	.80	1.00	1 1/16	2 13/32	4 3/16	1 1/16	13/32	4 3/16	2 15/16	2 3/32	1 15/32	1 1/16	13/16			
2.25	1.40	.72	.86	1.08	1 1/8	2 9/16	4 1/2	1 1/8	7/16	4 1/2	3 5/32	2 1/4	1 19/32	1 1/8	27/32			
2.50	1.25	.80	.95	1.20	1 1/4	2 13/16	5	1 1/4	1/2	5	3 1/2	2 1/2	1 3/4	1 1/4	15/16			
3.00	1.05	.95	1.13	1.44	1 15/32	3 5/16	6	1 1/2	19/32	6	4 3/16	3	2 3/32	1 15/32	1 1/8			
3.14	1.00	1.00	1.18	1.51	1 17/32	3 15/32	6 9/32	1 9/16	5/8	6 9/32	4 13/32	3 1/8	2 3/16	1 17/32	1 5/32			
3.25	.97	1.04	1.22	1.56	1 19/32	3 9/16	6 1/2	1 5/8	21/32	6 1/2	4 9/16	3 1/4	2 9/32	1 19/32	1 9/16			
3.50	.90	1.11	1.31	1.68	1 11/16	3 27/32	7	1 3/4	11/16	7	4 29/32	3 1/2	2 7/16	1 11/16	1 9/32			
3.75	.84	1.19	1.40	1.81	1 13/16	4 3/32	7 1/2	1 7/8	3/4	7 1/2	5 1/4	3 3/4	2 5/8	1 13/16	1 3/8			
4.00	.79	1.27	1.49	1.93	1 15/16	4 11/32	8	2	13/16	8	5 19/32	4	2 13/16	1 15/16	1 7/16			

Pitch.		Maximum number of teeth, N, and dimensions, E, for different numbers of arms.										
			4		5		6		7		8	
Circular.	Diametral.	Web.	N.	E.	N.	E.	N.	E.	N.	E.	N.	E.
.79	4.00	39	64	....	96	....	134	....	178	....	228	....
.90	3.50	36	60	....	90	....	125	....	166	....	213	....
1.00	3.14	34	56	....	85	....	118	....	157	....	201	....
1.05	3.00	34	56	....	84	4.5	117	5.5	155	6.5	195	....
1.14	2.75	32	53	....	79	....	110	....	146	....	187	....
1.25	2.50	30	51	....	76	5.5	106	6.5	141	7.5	181	....
1.38	2.28	29	48	....	71	....	100	...	133	....	170	....
1.40	2.25	29	47	....	70	....	99	....	132	....	169	....
1.50	2.08	27	45	....	68	....	95	....	126	....	163	....
1.57	2.00	27	44	....	67	6.5	93	7.5	124	9.0	160	10.8
1.68	1.88	26	43	....	65	....	90	....	120	....	155	....
1.75	1.80	26	42	....	63	....	88	....	117	....	151	....
1.80	1.75	25	42	....	62	....	87	....	116	....	149	....
2.00	1.57	24	39	....	59	....	83	....	110	....	142	....
2.09	1.50	23	39	7	58	9.0	81	10.5	108	12.5	138	....
2.25	1.40	23	38	....	55	....	78	....	104	....	133	....
2.50	1.25	21	35	...	52	10.0	74	12.0	98	14.5	126	....
3.00	1.05	19	32	11	48	12.5	67	14.5	89	17.0	115	....
3.14	1.00	19	32	....	47	....	66	....	88	....	113	....
3.25	.97	19	31	....	46	....	64	....	86	....	110	....
3.50	.90	18	30	....	44	...	62	....	83	....	107	....
3.75	.84	18	29	....	43	....	61	....	81	....	103	....
4.00	.79	16	28	....	41	....	58	....	77	....	99	....

Proportions of Gear-wheels.

The following proportions are those of the Yale & Towne Manufacturing Company:



See table opposite.

BELTS AND PULLEYS.

Where an exact velocity ratio of transmission is not essential, and when the distance between shafts is too great for a positive means of transmission, belting and pulleys are extensively employed. The question of the transmitting capacity of belting is one upon which many discussions have been held, and the differences of opinion which have been found serve to emphasize the fact that the conditions under which belts are used are too varying to permit absolute rules and formulas to be employed. For a full discussion of the elements which enter into the problems of belt transmission reference must be had to such works as Reuleaux's "Constructor," Unwin's "Machine Design," and especially to the valuable practical paper of Mr. J. W. Taylor in the "Transactions of the American Society of Mechanical Engineers," Vol. XV., p. 204. We shall here give the general working principles, which will serve to guide practical installations.

The power which can be transmitted by a belt is measured by the pull and by the lineal velocity at which the belt travels. The pull is limited by the strength of the belt and by the friction upon the pulleys, while the lineal velocity is dependent upon the revolving speed of the pulleys and upon their diameter. If it is attempted to increase the strength by increasing the thickness, it is possible that the stiffness of the belt will prevent it from wrapping closely about the pulley, and hence the friction

will be reduced. If the speed is made too high the centrifugal force will act to throw the belt out of close contact with the pulley, and the friction will again be reduced. There are, therefore, several practical limits within which satisfactory belt transmissions should be kept.

The tension which can be maintained in actual practice ranges from about 30 to 60 pounds per inch of width. If a high tension is put upon a belt transmission when it is installed it will gradually diminish, owing to stretch, and, unless some tightening device is employed, the belt will, before long, slacken until the stress upon it becomes low enough to check further stretching. If this tension is sufficient to transmit the power the transmission will run well and give but little trouble, while if the load is too heavy the belt will slip, and it must either be tightened or a change made in width or speed.

If the power to be transmitted is given in horse-power, we have 33,000 foot-pounds per minute to consider. If the belt tension is to be 30 pounds per inch of width, we must, therefore, have a speed of 1100 feet per minute. If the speed is one-half as much, the width must be twice as great, and so the given elements must be taken and the others found. Usually, the speed and the power are given and the width required.

If

w = width, in inches;
s = speed, in feet, per minute;
N = horse-power;
t = tension, per inch width of belt;

we have

$$N = \frac{tws}{33000};$$

$$w = \frac{33000N}{ts};$$

$$s = \frac{33000N}{tw}.$$

Or, if we have given the width, speed, and horse-power, the minimum tension which can be reached before slipping will occur is

$$t = \frac{33000N}{ws}.$$

Thus, if a belt 10 inches wide, running at 4000 feet per minute, is transmitting 50 horse-power, the tension is $\frac{33000 \times 50}{10 \times 4000} = 41.25$ pounds.

The tension available for transmitting power is really the difference between the tensions of the tight and slack sides, since there must always be tension enough on the slack side to secure sufficient friction on the pulley to keep the belt from slipping.

If we take the formula

$$N = \frac{tws}{33000},$$

and write it

$$N = \frac{t \times 12}{33000} \times \frac{ws}{12},$$

the last term will represent square feet per minute passing a given point. By substituting any value for t, and making $N = 1$, we can thus find how many square feet per minute will transmit a horse-power. Good, practical belting rules are: For single belts, 60 square feet per minute equals 1 horse-power; and for double belts, 40 square feet per minute equals 1 horse-power. These correspond to 45 pounds and 68 pounds tension per inch of width, respectively,—tensions which are readily maintained in practice.

These values are based on the assumption that the belt embraces 180° of each pulley. If the arc of contact is less, the power transmitted may be taken in the following proportions:

Percentage of Efficiency for Various Arcs of Contact.

90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
0.65	0.70	0.75	0.79	0.83	0.87	0.91	0.94	0.97	1.00

The power for 180° is to be multiplied by the percentage coefficient for other arcs. Thus, for 130°, only 83 per cent. as much power is transmitted as with 180°.

Pulleys.

The function of a pulley is to enable the rotary motion of the shaft on which it is mounted to be translated into the lineal motion of the belt, and *vice versa.* This is accomplished by the frictional contact of the wrapping connection—be it belt, rope, or wire cable—with the perimeter of the pulley. The following general discussion, from Reuleaux, will enable special computations to be made for any given conditions :

When a tension organ, which is loaded at both ends, is passed over a curved surface there is produced between the tension organ and the surface a very considerable sliding friction. The curved surface over which the cord is passed is the pulley, and the motion of the cord takes place in the plane of the pulley. If the tension, T, on the driving side of the cord is to overcome the cord friction, F, as well as the tension, t, of the driven side, we have, for the value of the friction, $F = T - t$. It is dependent upon the magnitude of the angle of contact, α, and upon the coefficient of friction, f, but is independent of the radius, R, of the pulley; it is also dependent upon the influence of centrifugal force. For these conditions we have

$$T = te^{f\alpha(1-z)},$$

$$F = t\left(e^{f\alpha(1-z)} - 1\right).$$

In these e is the base of the natural system of logarithms = 2.71828, and $z = 12\frac{\gamma v^2}{gS}$, v being the velocity of the tension organ, in feet, per second; S, the stress in its cross-section; γ, the weight of a cubic inch of the material; and g, the acceleration of gravity = 32.2.

The influence of centrifugal force becomes important at high speeds and when the tension organ is under small stress. For hemp or cotton rope, or for leather belting, we may take $\gamma = 0.035$, and for wire rope about 9 times as great.

The value of S in the formula, $z = 12\frac{\gamma v^2}{gS}$, is properly considered a function of α, and we may therefore assume a constant value for the arc, α, and thus calculate the following table for the values of $1 - z$.

S.	Value of coefficient, $1 - z$, for centrifugal force.					*S.*
Hempen rope.	Velocity of rope, in feet, per second.					Wire rope.
	20	**40**	**60**	**80**	**100**	
Lb.						Lb.
400	.987	.948	.882	.791	.674	3600
600	.991	.965	.922	.861	.783	5400
800	.993	.974	.941	.896	.837	7200
1000	.995	.980	.953	.916	.870	9000
1200	.996	.982	.961	.930	.892	10800
1400	.996	.985	.966	.940	.907	12600

This table serves both for hemp and for wire rope by taking the ninefold value of S in the right-hand column for wire rope. It should be observed that the velocities are in feet per second. It will be seen that for high speeds a high stress in the tension organ is necessary, in order to oppose the action of the centrifugal force.

In order to simplify practical calculations we may substitute for the exponent, $f\alpha(1-z)$, in each case the form, $f'\alpha$,—that is, instead of using the actual coefficient of friction, f, taking another one, f', which is equal to $(1-z)f$. If it is a transmission system which is under consideration, the friction of the cord, belt, chain, etc., must at least equal the transmitted force, P; hence, also, must the stress be that of a cord friction $\geqq P$, which gives, for a minimum value of T,

$$\frac{T}{P} = \tau = \frac{e^{f'\alpha}}{e^{f'\alpha}-1} = \frac{\rho}{\rho-1},$$

whence

$$\frac{T}{t} = \rho = e^{f'\alpha}.$$

Both of these values are absolute numbers. The ratio, $\frac{T}{P}$, indicates the amount of stress which must be given to the tension organ, and hence may be called the stress modulus, and is designated as τ. The ratio, $\frac{T}{t}$, we may, in like manner, call the modulus of cord friction, this being understood to apply to any wrapping connector, and indicate as ρ.

A series of values for ρ and τ are given in the following table:

Moduli for Cord Friction and Stress.

$f\alpha$.	$\tau = \frac{T}{P}$.	$\rho = \frac{T}{t}$.	$f\alpha$.	$\tau = \frac{T}{P}$.	$\rho = \frac{T}{t}$.
.1	10.41	1.11	1.6	1.25	4.95
.2	5.52	1.22	1.7	1.22	5.47
.3	3.86	1.35	1.8	1.20	6.05
.4	3.03	1.49	1.9	1.18	6.69
.5	2.54	1.65	2.0	1.16	7.39
.6	2.22	1.82	2.2	1.13	9.03
.7	1.99	2.01	2.4	1.10	11.02
.8	1.86	2.23	2.6	1.08	13.46
.9	1.69	2.46	2.8	1.07	16.44
1.0	1.58	2.72	3.0	1.05	20.09
1.1	1.50	3.00	3.2	1.04	24.53
1.2	1.43	3.32	3.4	1.03	29.96
1.3	1.37	3.67	3.6	1.03	36.60
1.4	1.33	4.06	3.8	1.02	44.70
1.5	1.29	4.48	4.0	1.02	54.60

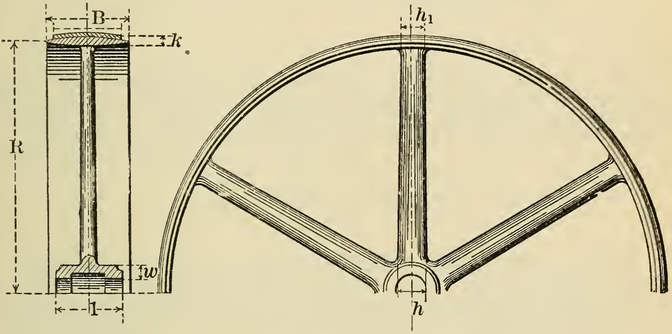
The superficial pressure, p, of the tension organ upon the circumference of the pulley increases as the belt or cord passes from the slack to the tight side. It is equal to $\frac{Qd\alpha}{b'Rd\alpha}$, in which b' is the breadth of the surface

of contact of the belt. Now, for any cross-section, q, the force $Q = qS$; hence, we have

$$\frac{p}{S} = \frac{q}{b'R},$$

from which it will be seen that the pressure, p, can easily be kept within moderate limits.

Within the limits of injurious action of centrifugal force it is desirable that the lineal speed of belts or cords be kept as high as practicable, since the power transmitted is directly proportioned to the speed. Belt transmissions are therefore best designed with pulleys of large diameter, and small pulleys employed only when the rotative speeds are such as to make their use imperative.



The general dimensions of belt pulleys may be taken as follows:

Let A = the number of arms, and let the other dimensions be as in the figure, then

$$A = \tfrac{1}{2}\left(5 + \frac{R}{b}\right),$$

which gives, for

$\frac{R}{b} =$	1	2	3	4	5	6	7	8	9	10	11	12	13
$A =$	3		4		5		6		7		8		9

The width, h, of the arm, if prolonged to the middle of the hub, may be obtained from

$$h = 0.25 \text{ inch} + \frac{b}{4} + \frac{1}{10}\frac{R}{A}.$$

The width, h_1, of the arm at the rim is equal to $0.8h$, and the corresponding thicknesses are $e = \tfrac{1}{2}h$ and $e_1 = \tfrac{1}{2}h_1$.

Pulleys with two or three sets of arms may be considered as two or three separate pulleys combined in one, except that the proportions of the arms should be 0.8 or 0.7 times that of single-arm pulleys, or in the proportion of $\sqrt[3]{\tfrac{1}{2}}$ and $\sqrt[3]{\tfrac{1}{3}}$.

The thickness of the rim may be made $k = \tfrac{1}{5}$ to $\tfrac{1}{4}h$, this being frequently turned much thinner. The width of face should be from $\tfrac{9}{8}$ to $\tfrac{5}{4}$ the width of the belt.

The thickness of metal in the hub may be made $W = h$ to $\tfrac{3}{4}h$. The length of hub may $= b$ for single-arm pulleys, and $2b$ for double-arm pulleys.

In order to cause the belt to run in the middle of the pulley, the face should be made crowning or rounded, the rise being about $\tfrac{1}{20}$ of the width of the face. Tight and loose pulleys should be made with rounded faces, and the wide face pulley from which they are driven made with straight face.

Three causes of loss exist in belt transmissions,—viz., journal friction, belt stiffness, and belt creeping. For horizontal belting we have for the journal friction expressed at the circumference of the pulley a loss, E_z, when $T = 2.5P$, $t = 1.5P$:

$$\frac{F'}{P} = E_z = \frac{T+t}{P} \cdot \frac{4}{\pi} f\left(\frac{d}{2R} + \frac{d_1}{2R_1}\right) = \frac{8}{\pi} f\left(\frac{d}{R} + \frac{d_1}{R_1}\right),$$

in which d and d_1 are the journal diameters and f the coefficient of journal friction. This loss is doubtless the greatest of the three. According to Eytelwein, the coefficient of stiffness, s, for force, S', which includes both pulleys, is

$$\frac{S'}{P} = E^s = s\frac{T+t}{P}\left(\frac{\delta^2}{R} + \frac{\delta^2}{R_1}\right) = 4s\left(\frac{\delta^2}{R} + \frac{\delta^2}{R_1}\right),$$

in which $s = 0.009\frac{4}{\pi} = 0.012$.

The loss from creep is due to the fact that the greater stress on the driving pulley over that on the driven requires for a given volume of belt a longer arc of contact. For the expenditure of force, G', for creep on both pulleys, we have for a stress, S_1, on the leading side of the belt,

$$\frac{G'}{P} = E_s = \frac{1 - \frac{t}{T}}{1 + \frac{E}{S_1}} = \frac{0.4S_1}{E + S_1}.$$

In this E is the modulus of elasticity of the belt, which for leather is 20,000 to 30,000 pounds. The losses from stiffness and creep are small.

Example. Let d and $d_1 = 4$ inches, $R = R_1 = 20$ inches, $\delta = 0.2$, $f = 0.08$, $S = 0.012$, $E = 28{,}440$, $S_1 = 425$, we have

$$F^1 = P\frac{8 \times 0.08}{\pi} \times 0.4 = 0.08P;$$

also, $$S^2 = P(0.048 \times 2)\frac{0.2}{20} = 0.0048P,$$

and $$G^1 = P\frac{0.4 \times 425}{28440 + 425} = 0.0059P.$$

The total loss is, therefore, $0.08 + 0.0048 + 0.0059 = 9.1$ per cent.

Cone Pulleys.

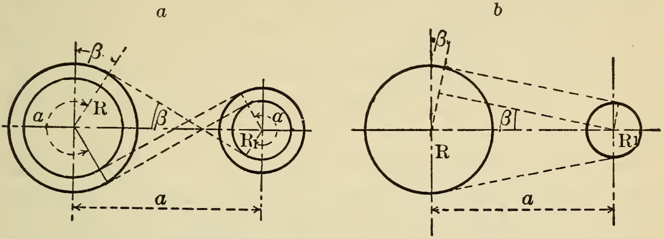
When a number of pulleys are placed side by side in order to enable varied speeds to be obtained with belt transmission, and are united together in one member, we obtain what is called a cone pulley, such pulley being used in pairs. This construction involves the problem of determining the proper radii for the various pulleys, so that the same belt shall serve for all the changes,—*i.e.*, so that the length of the belt shall be the same for each pair of pulleys in the set. The problem may be solved as follows:

Crossed Belts (Fig. *a*).—The belt makes the angle, β, with the centre line of the pulleys, R and R_1; and the half length of the belt, $l = R\left(\frac{\pi}{2} + \beta\right) + R_1\left(\frac{\pi}{2} + \beta\right) + a\cos\beta$, a being the distance from centre to centre of the pulley. We then have

$$l = (R + R_1)\left(\frac{\pi}{2} + \beta\right) + a\sqrt{1 - \frac{(R+R_1)^2}{a^2}}.$$

This value is constant when $R + R_1$ is constant,—that is, when the increase to the radius of one pulley is equal to the decrease in the radius of

the other. Crossed belts are seldom used for this service, however, because of the injurious friction between the rubbing parts of the belt.



Open belts (Fig. b).—In this case we have

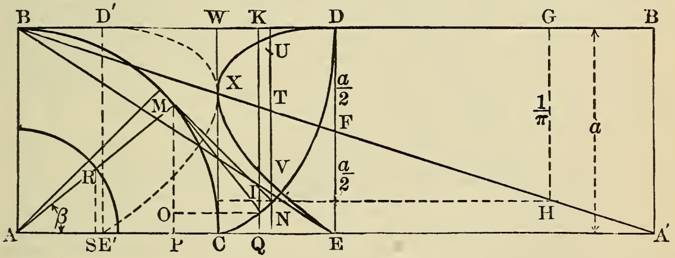
$$l = (R + R_1)\frac{\pi}{2} + (R - R_1)\beta + a\cos\beta,$$

and, also, $a \sin \beta = R - R_1$, which gives

$$R = \frac{l}{\pi} - \frac{a}{\pi}(\beta \sin \beta + \cos \beta) + \frac{a}{2}\sin\beta,$$

$$R_1 = \frac{l}{\pi} - \frac{a}{\pi}(\beta \sin \beta + \cos \beta) - \frac{a}{2}\sin\beta.$$

This function is transcendental, but may be graphically represented in the following manner: in the rectangle, $ABB'A'$, with a radius, $AB = a$, strike the quadrant, BMC, about the centre, A. Within this arc will fall all the values of β which can occur. For any value of $\beta = CAM$, draw



MN perpendicular to MA and make $MN =$ the arc, $MC = a\beta$. Drop the perpendicular, MP, to AC, and draw NO perpendicular to MP. NO will then $= a\beta \sin \beta$. Through N draw QNK parallel to AB, and we have $AQ = PQ + AP = a(\beta \sin \beta + \cos \beta)$. By taking successively all the values of β between 0° and 90° in this manner, we can determine the path of the point, N, which will be the evolute of a circle, CND, BD being equal to the length of the arc, $BMC = \frac{\pi}{2}a$. If we now draw DE parallel to BA, and take its middle point, F, we have $DF = EF = \frac{a}{2}$, and hence the proportion:

$$DF : DB = \frac{a}{2} : \frac{\pi}{2}a = a : \pi, \text{ and by similar triangles:}$$

$$TK = \frac{a}{\pi}QA = \frac{a}{\pi}(\beta \sin \beta + \cos \beta).$$

This value is dependent upon $\frac{l}{\pi}$. If we prolong BF until it intersects AC prolonged, the resulting length, $AA' = BB'$, will bear to $A'B'$ the ratio, $\frac{\pi}{1}$. By then working $BG = l$, and drawing GH parallel to $A'B'$, we have $GH = \frac{l}{\pi}$. This length being transferred to IK gives $IT = \frac{l}{\pi} - \frac{a}{\pi}(\beta \sin \beta + \cos \beta)$. We then have only to use $\pm \frac{a}{2} \sin \beta$ to solve the problem.

Make $AR = \frac{a}{2}$, and we have the perpendicular, $RS = \frac{a}{2} \sin \beta$. By laying this length off above and below T on QK we obtain the points, U and V, and this finally gives IU for the radius, R, of the larger cone pulley, and $IV = R_1$, the radius of the corresponding smaller cone pulley.

By solutions for successive values of β we obtain the curve, $DUXVE$, which can be used for the determination of the radii of any desired pair of pulleys, each pair of ordinates measured from HI belonging to corresponding pulley on each cone.

In practice it is usual to find one of the cone pulleys given and the dimensions of the other required. In this case VU may be taken as the difference, $R - R_1$, between the radii, were the steps uniform. By taking this difference, $R - R_1$, in the dividers, and finding the equivalent ordinate, UV, on the curve, and then adding $VI = R_1$, the axis, HI, is found.

In order to use the curve conveniently, it may also be laid off left-handed, as shown in the dotted lines, $D'XE'$.

The use of the diagram will be rendered still more convenient if we omit the unnecessary value, l. This enables us to distort the curve in the direction of the abscissas to any desired extent. This has been done in the proportional diagram on page 475, due to Professor Reuleaux.

The method of using the diagram is as follows:

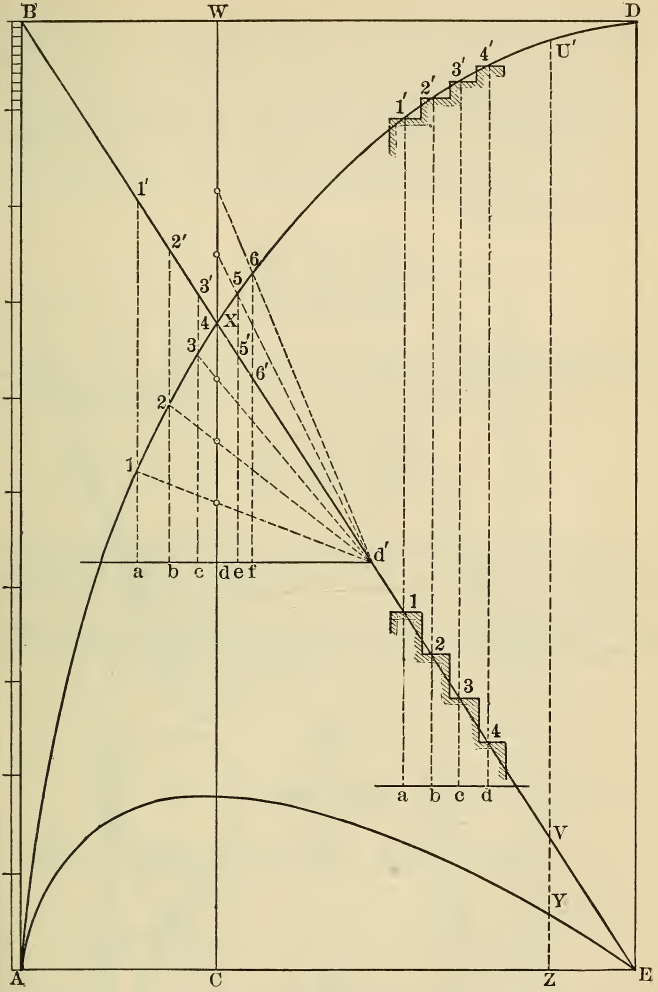
The sides, AB and DE, of the rectangle represent the distance, a, between the centres of the pulleys; all radii are given in proportional parts of a, for which reason AB is subdivided, the size of the diagram being selected so that AB = 18 to 20 inches. If, then, $1a$ and $1'a$ are two given radii for a pair of pulleys on a pair of cones, we take the vertical chord of the curve which $= 1'a - 1a$, prolong the chord downward until its length $= 1a$, and draw the axis, $abcd$, parallel to AE. Then, for the other pairs of pulleys on the cones, we have $b2$ and $b2'$, $c3$ and $c3'$, etc., which can be taken directly from the diagram with the dividers. If the given pair of radii to which the cones are to be made are equal, the chord $R - R_1 = 0$, and the axis will pass through X at right angles to CX.

If it is desired to construct a pair of cone pulleys to any given speed ratio, this can readily be done. If, for example, the given ratio is 1 : 1, we lay off toward C the corresponding radius, Xd, and prolong the axial line, dd', to its intersection, d, with BE. Then lay off the given geometric ratio on CX, considering Xd as 1 (shown in the diagram by the small circles for the ratios $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$), and draw rays from d' through the points of division, and these rays will intersect the curve at the corresponding points for the pulley radii, R_1. We then have for the radii,

$a1$ and $a1'$ for the ratio 1 : 4;
$b2$ and $b2'$ for the ratio 2 : 4;
$c3$ and $c3'$ for the ratio 3 : 4;
dX and dX' for the ratio 4 : 4;
$e5$ and $e5'$ for the ratio 5 : 4;
$e6$ and $e6'$ for the ratio 6 : 4;

If the slowest and fastest speeds for any set of cone pulleys be given in revolutions per minute, as n and n_x, x being the number of speed changes, or steps of the cone, we have for a, the geometric ratio of the series,

$$a = \sqrt[x-1]{\frac{n_x}{n}}.$$



Thus, for a cone of four steps, with an entire speed ratio of four to one, we have $x = 4$ and $x - 1 = 3$; hence,

$$a = \sqrt[3]{\frac{4}{1}} = \sqrt[3]{4} = 1.58.$$

Then, if the first speed be 100 revolutions, the succeeding speeds will be $100 \times 1.58 = 158$; $158 \times 1.58 = 249.6$; and $249.6 \times 1.58 = 394$, or say 400.

When, as in many lathes, a back-gear system is introduced, it is desirable that the gear ratio should be so arranged that the speeds may proceed

in a geometric ratio throughout all the changes. This is readily done according to the same principle. The introduction of the back gear simply doubles the number of speed changes; in the above case it converts a lathe with a 4-step cone and four speed changes into one with eight changes. The speed ratio of the back gear, therefore, corresponds to the next term in the series, or at $a^4 = 1.58^4 = 6.25$.

If, to take another example, we have a lathe with a 5-step cone, with back gear, the whole should give ten changes. If these are to range from 100 to 600, we have

$$a = \sqrt[9]{6} = 1.22.$$

The series will then be

$$\begin{aligned} 100 \times 1.22^0 &= 100; \\ 100 \times 1.22^1 &= 122; \\ 100 \times 1.22^2 &= 149; \\ 100 \times 1.22^3 &= 181; \\ 100 \times 1.22^4 &= 221; \end{aligned}$$

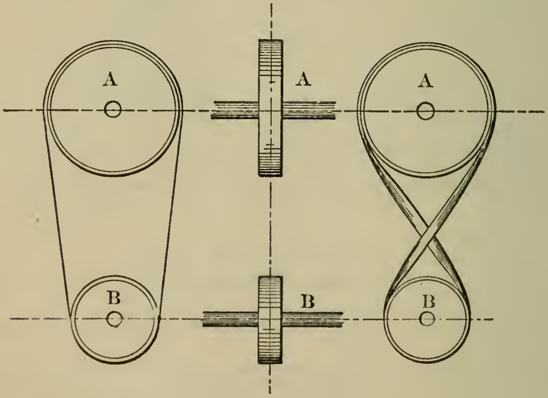
for the cone acting direct.

The back-gear ratio will then give the next term in the series, or $1.22^5 = 2.70$, which, starting with the first step in the cone again, gives

$$\begin{aligned} 100 \times 1.22^5 &= 270; \\ 100 \times 1.22^6 &= 330; \\ 100 \times 1.22^7 &= 403; \\ 100 \times 1.22^8 &= 492; \\ 100 \times 1.22^9 &= 600. \end{aligned}$$

When a lathe is not carefully proportioned in this manner it may have what is termed a "lump" in its speed, the change produced by throwing in the back gear not conforming to the regular geometric ratio of the steps of the cone.

The simplest and most usual arrangements of belting are the plain open and the crossed belts. In these, as in all belt transmissions, the velocity ratio is inversely as the diameter of the pulleys.

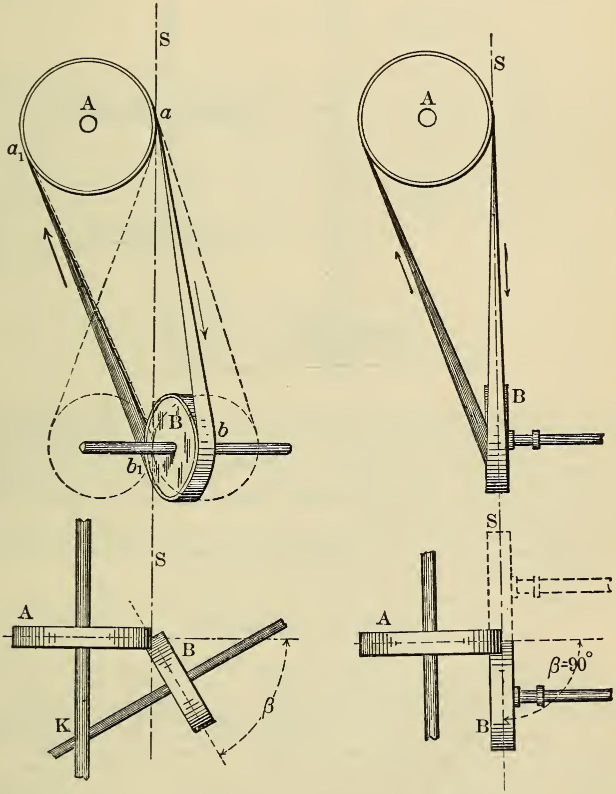


For these simple arrangements the belts are self-guiding, the only requirements being that the shafts shall be truly parallel to each other and one or both pulleys be made with crowning face.

For inclined and intersecting axes self-guiding belts are not suitable,

except in the case of inclined axes, in which the trace, *SS*, of the intersection of the planes of the two pulleys passes through the points at which the belt leaves the pulleys. The leading line then falls in the middle plane of each pulley, but the following side of the belt does not; hence, such systems can only be run in one direction. The leaving points in the figures are at a and b_1. The arrangement gives an open belt when the angle, β, between the planes of the pulleys $= 0°$, and a crossed belt when $\beta = 180°$. In the intermediate positions a partial crossing of the belt is produced. If $\beta = 90°$, the belt is half crossed (or, as commonly called, quarter twist); if $\beta = 45°$, it is quarter crossed.

The leading-off angle may be made as much as 25°, which occurs when the distance between the axes is equal to twice the diameter of the largest



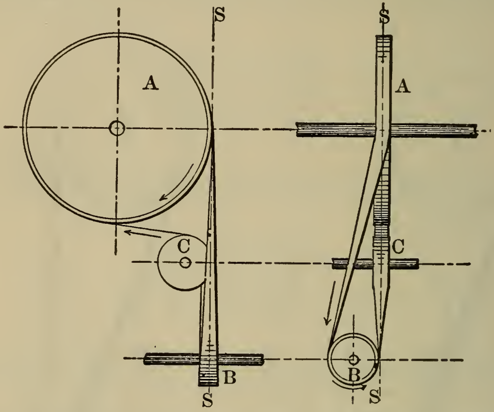
pulley. Another rule for the minimum distance between shafts for quarter-twist belts is to make the distance never less than $\sqrt{bD}$.

In general, the rule to be observed for any such arrangement of belting is that each part of the belt must lie in the plane of the pulley toward which it is moving.

It is evident that if such a system has its motion reversed the belt will leave the pulleys. Under such conditions guide pulleys are introduced, as shown in the illustration.

The introduction of electric driving of machinery is rendering quarter-twist belts and similar contrivances of minor importance in connection with the transmission of power, but such belts will probably continue to be used in connection with machines themselves, and hence care must be taken in their application.

In arranging belt transmissions the direction of motion should be made, when possible, so as to bring the slack side of the belt on the upper part for belts in the horizontal or inclined positions. This brings the sag of the belt in such a position as to increase the arc of contact about the pulleys and diminishes the probability of slipping. When practicable, machines should be so placed with regard to the line shaft that belts on adjacent



pulleys pull in opposing directions, as in that manner much of the pressure due to belt pull may be taken off of the bearings of the shaft, the pulls of the belts neutralizing each other. If possible, one pulley should never be placed vertically over another, since the weight of the belt acts to diminish the contact with the lower pulley. When such an arrangement must be employed a tightening pulley may be found necessary, placed upon the slack side of the belt.

Belts are usually joined by lacings, but whenever possible they should be scarfed and cemented, this making a much neater joint and rendering the joint as effective as any other portion of the belt

Rope Transmission.

The transmission of power over longer distances than are practicable for belting may be accomplished by use of rope running at high velocities, and hence requiring but small diameters. This form of transmission was at one time thought to offer great possibilities for long-distance transmission, but the development of electrical transmission has caused it to be superseded. For many purposes, however, for spans of not less than 70 or more than 400 feet, wire-rope transmission may be used with success. The complete computations for wire-rope transmission are to be found in Reuleaux's "Constructor," but for general purposes the practical rules of Messrs. John A. Roebling's Sons Company may be employed.

The rope used for transmission purposes may be either 6-strand, of seven wires each, or with nineteen wires to the strand. For the 7-wire rope the diameter of the sheaves should be not less than 100 times the diameter of the rope, and for a 19-wire rope the minimum diameter of sheave is 60 times the rope diameter. The wheels are made with a deep V-groove, the bottom of the groove on which the rope runs being provided with a filling composed of alternate blocks of leather and rubber.

The tension upon the rope in a transmission is that due to the weight of the rope itself, and since this is the measure of the power transmitted for a given speed it is entirely practicable to provide such a sag or deflection to the rope as will give the tension desired in practice. According to Messrs. Roebling, the sag of both parts of a horizontal transmission should be $\frac{1}{36}$ part of the span when the rope is stationary. When the rope is running the deflection of the upper part will become about $\frac{1}{25}$ of the span, and that of the lower part about $\frac{1}{50}$ of the span. Under such conditions the difference of tension, T, or pull on the tight side of the rope, will be *three times the weight* of a single portion of rope between the sheaves. If V is the velocity of the rope, in feet, per minute, the horse-power transmitted will be

$$HP = \frac{TV}{33000}.$$

The rope diameters used range from $\frac{3}{8}$ inch to 1 inch, and the weights will be found in the tables on pages 342–344.

For Manila-rope driving the formulas of Mr. C. W. Hunt may be used to advantage. He recommends ropes of 1 to 2 inches in diameter, and estimates the strength of good Manila ropes for driving as about 7000 pounds per square inch. The working stress, however, should be only about 200 pounds per square inch, this making allowance for wear and for the reduction in strength at the splice.

The power transmitted by ropes depends upon the tension and the speed, the power increasing with the speed until the influence of centrifugal force begins to preponderate.

Let

T = tension on driving side of rope;
t = tension on slack side of rope;
F = tension due to centrifugal force;
v = velocity of rope, in feet, per minute;
W = weight of rope, in pounds, per foot;
g = acceleration of gravity.

The value of W, the weight per foot for a rope of diameter, D, or circumference, C, is

$$W = 0.3D^2 = 0.032C^2.$$

We have for the tension due to centrifugal force

$$F = \frac{Wv^2}{g}.$$

Assuming that the tension on the slack side necessary for giving adhesion is equal to one-half the force doing useful work on the driving side and calling this available tension for useful work R, we have

$$R = \tfrac{2}{3}(T - F).$$

The tension on the slack side to give the required adhesion will, therefore, be equal to $\frac{1}{3}(T - F)$, whence we have

$$t = \tfrac{1}{3}(T - F) + F.$$

Since F increases with the square of the velocity, there are, with increasing speeds, a decreasing useful force and an increasing tension, t, on the slack side. The horse-power may, therefore, be obtained from the following formula:

$$HP = \frac{2v(T - F)}{3 \times 33000}.$$

The following table has been computed from this formula.

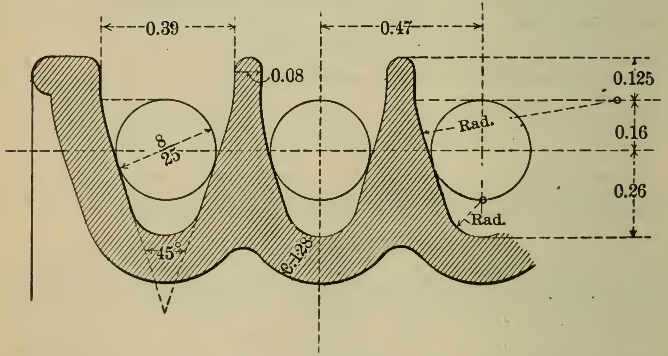
Horse-power of Manila-rope Transmission.

C. W. Hunt.

Diameter of ropes.	Speed of the rope, in feet, per minute.											Smallest diameter of pulleys.
	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000	8000	
Inch.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	Inch.
½	1.45	1.9	2.3	2.7	3.0	3.2	3.4	3.4	3.1	2.2	0	20
⅝	2.3	3.2	3.6	4.2	4.6	5.0	5.3	5.3	4.9	3.4	0	24
¾	3.3	4.3	5.2	5.8	6.7	7.2	7.7	7.7	7.1	4.9	0	30
⅞	4.5	5.9	7.0	8.2	9.1	9.8	10.8	10.8	9.3	6.9	0	36
1	5.8	7.7	9.2	10.7	11.9	12.8	13.6	13.7	12.5	8.8	0	42
1¼	9.2	12.1	14.3	16.8	18.6	20.0	21.2	21.4	19.5	13.8	0	54
1½	13.1	17.4	20.7	23.1	26.8	28.8	30.6	30.8	28.2	19.8	0	60
1¾	18.0	23.7	28.2	32.8	36.4	39.2	41.5	41.8	37.4	27.6	0	72
2	23.2	30.8	36.8	42.8	47.6	51.2	54.4	54.8	50.0	35.2	0	84

Where large amounts of power are to be transmitted a number of ropes are used. In English practice separate ropes are generally employed, but in the United States the rope is made endless, passing around the grooves in the pulleys as many times as may be necessary, and finally over an idler guide pulley supported in a tension carriage, the required initial tension being secured by weighting. In the American system ropes of small diameter are generally employed.

The form of grooves employed for rope driving, according to Unwin, are given in the illustration. The unit for the proportional figures is γ, the



girth of the rope. If the pulley is a guide pulley merely, the rope should rest on the bottom of the groove. The sides of the groove are usually inclined at 45°.

Mr. Spencer Miller has proposed that the angle of the sides of the grooves should be varied to suit the difference in the diameters of the pulleys, the angles being equal only when both pulleys are of the same diameter. This may well be done when the pulleys are made to order, but it is impracticable if pulleys are to be carried in stock.

The following table gives the transmitting power of cotton driving ropes, according to good British practice.

Horse-power of Cotton-rope Transmission.

Speed, in feet, per minute.	Diameter of ropes, in inches.								
	1	1⅛	1¼	1⅜	1½	1⅝	1¾	1⅞	2
	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.	H.-P.
2500	10.8	13.4	16.7	20.5	24.3	28.5	33.2	38.1	43.4
2600	11.1	13.9	17.2	20.8	25.0	29.4	34.1	39.4	44.7
2700	11.4	14.3	17.7	21.7	25.7	30.2	35.3	40.6	46.0
2800	11.8	14.7	18.2	22.3	26.4	31.0	36.2	41.7	47.3
2900	12.1	15.1	18.7	22.9	27.1	31.9	37.2	42.8	48.6
3000	12.3	15.4	19.1	23.4	27.8	32.6	38.1	43.8	49.5
3100	12.5	15.7	19.5	24.0	28.4	33.4	39.0	44.8	50.6
3200	12.9	16.1	19.9	24.5	29.0	34.0	39.9	45.8	52.0
3300	13.2	16.5	20.3	25.0	29.6	34.8	40.8	46.8	53.2
3400	13.4	16.7	20.6	25.5	30.1	35.4	41.6	47.7	54.3
3500	13.6	16.9	20.9	26.0	30.6	36.2	42.3	48.6	55.2
3600	13.9	17.1	21.2	26.4	31.1	36.5	43.0	49.5	56.0
3700	14.1	17.3	21.5	26.8	31.5	37.1	43.6	50.2	56.8
3800	14.2	17.5	21.7	27.0	31.9	37.5	44.2	50.8	57.6
3900	14.4	17.7	21.9	27.3	32.2	37.9	44.8	51.4	58.2
4000	14.5	17.8	22.1	27.5	32.6	38.4	45.3	51.9	58.9
4100	14.6	17.9	22.3	27.8	32.9	38.7	45.8	52.4	59.6
4200	14.7	18.0	22.5	28.0	33.1	39.0	46.3	52.8	60.3
4300	14.8	18.0	22.6	28.1	33.3	39.3	46.6	53.2	60.6
4400	14.9	18.1	22.7	28.2	33.4	39.6	46.8	53.5	60.9
4500	15.0	18.1	22.7	28.3	33.5	39.7	47.0	53.8	61.2
4600	15.1	18.1	22.7	28.4	33.6	39.7	47.2	54.0	61.4
4700	15.1	18.1	22.6	28.4	33.7	39.8	47.4	54.2	61.5
4800	15.1	18.0	22.6	28.5	33.7	39.8	47.5	54.2	61.5
4900	15.0	18.0	22.5	28.5	33.7	39.9	47.6	54.3	61.6
5000	15.0	17.9	22.4	28.4	33.6	39.8	47.5	54.3	61.5
5100	14.9	17.8	22.3	28.3	33.4	39.6	47.4	54.0	61.3
5200	14.8	17.6	22.0	28.2	33.2	39.3	47.2	53.8	61.1
5300	14.7	17.4	21.8	28.0	33.0	39.0	47.0	53.6	60.9
5400	14.6	17.2	21.6	27.7	32.7	38.6	46.8	53.3	60.4
5500	14.5	17.0	21.3	27.3	32.3	38.2	46.1	52.8	59.8

HEAT.

Heat is defined as a form of molecular energy which is manifested by the changes which it produces in the form or state of the bodies upon which it acts. The most readily observed effect of heat is that of the expansion of the bodies to which it is applied; and this effect is used both for the measurement of quantities of heat and for its useful application by conversion into mechanical work.

Heat can be transferred from one body to another, the hotter body parting with heat to the body which is less hot. The scale of quantities upon which such transfers are compared is called the scale of *temperatures*. When there is no tendency for the transfer of heat from one body to another the two bodies are said to be at the same temperature.

There are, in nature, certain temperatures which can be identified by

positive phenomena which occur with them. Among them are the melting-point of ice and the boiling-point of water, these being considered as occurring at the average atmospheric pressure of 14.7 pounds to the square inch, corresponding to 29.922 inches, or 760 millimetres of mercury on the barometer. Having these, or certain other standards of temperature, it is practicable to make scales by which other temperatures may be compared.

The practical method of making instruments for the measurement of temperatures is to use the expansive effect of heat upon certain liquids or upon a gas. For temperatures within the range of its freezing and boiling points mercury is generally used.

There are three forms of mercurial thermometers, or temperature-indicating instruments, in use. These all consist of sealed tubes of fine bore, there being a bulb at one end containing mercury. The expansion or contraction of the mercury in the bulb causes the portion in the tube to move, the extent of this movement indicating the changes in temperature. The three thermometers differ from each other only in the graduation and numbering of the scales upon the tube.

In the *Centigrade* thermometer the position of the mercury at the melting-point of ice is taken as the zero of the scale, while the boiling-point of water is called 100, the space between being divided into 100 equal parts, called degrees.

The *Fahrenheit* thermometer was originally designed to range between two altogether different standard points, one of these being the temperature of pounded ice and salt, the other the normal temperature of the human body. The space between these was divided duodecimally, and these large divisions subdivided by repeated bisection into halves, quarters, and eighths, thus making 96 divisions. Owing to the erroneous measurements made by Fahrenheit in constructing his early instruments the temperature of the human body was taken too low, and it is really equal to 98 degrees above the Fahrenheit zero. This scale, if prolonged upward to the boiling-point of water, reaches that temperature at 212 degrees, and it is often erroneously stated that Fahrenheit's scale was originally derived between those points.

The remarkable uniformity of the early thermometers made by Fahrenheit caused his instruments to be used for work involving scientific accuracy, and it is still the scale most extensively used in steam engineering in English-speaking countries.

The *Reaumur* scale has its zero at the melting-point of ice, as in the centigrade scale, and the graduations were intended to correspond to the expansion of the mercury in the bulb by $\frac{1}{1000}$ of its original volume for each degree. Upon this scale the boiling-point of water is reached at 80 degrees above zero, and the scale is generally so defined. The Reaumur scale is now rarely used; but many old measurements of importance are recorded in it, and hence it is valuable for purposes of comparison.

In converting the several scales from one to the other the following formulas are used:

$$\text{Zero Fahr.} = -17.77^\circ \text{ Cent.} = -14.22^\circ \text{ Reau.}$$

Melting-point of Ice.

$$\text{Zero Cent.} = 32 \text{ Fahr.} = \text{zero Reau.}$$

Boiling-point of Water.

$$212^\circ \text{ Fahr.} = 100^\circ \text{ Cent.} = 80^\circ \text{ Reau.}$$
$$9^\circ \text{ Fahr.} = 5^\circ \text{ Cent.} = 4^\circ \text{ Reau.}$$

Formulas.

$$\text{Cent.} = \tfrac{5}{9}(\text{Fahr.} \mp 32) = \tfrac{5}{4} \text{ Reau.}$$
$$\text{Fahr.} = \tfrac{9}{5} \text{ Cent.} \pm 32 = \tfrac{9}{4} \text{ Reau.} \pm 32.$$
$$\text{Reau.} = \tfrac{4}{5} \text{ Cent.} = \tfrac{4}{9}(\text{Fahr.} \mp 32).$$

In the accompanying tables the corresponding values of Fahrenheit and Centigrade degrees are given for the temperatures generally used in engineering. The main tables give the values for even degrees, and by means of the supplementary tables the values for tenths of a degree may be taken out.

Fahrenheit to Centigrade.

Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.
—5	—20.55	57	13.88	119	48.33	181	82.77	243	117.22
—4	—20.00	58	14.44	120	48.88	182	83.33	244	117.77
—3	—19.44	59	15.00	121	49.44	183	83.88	245	118.33
—2	—18.88	60	15.55	122	50.00	184	84.44	246	118.88
—1	—18.33	61	16.11	123	50.55	185	85.00	247	119.44
Zero.	—17.77	62	16.66	124	51.11	186	85.55	248	120.00
+1	—17.22	63	17.22	125	51.66	187	86.11	249	120.55
2	—16.66	64	17.77	126	52.22	188	86.66	250	121.11
3	—16.11	65	18.33	127	52.77	189	87.22	251	121.66
4	—15.55	66	18.88	128	53.33	190	87.77	252	122.22
5	—15.00	67	19.44	129	53.88	191	88.33	253	122.77
6	—14.44	68	20.00	130	54.44	192	88.88	254	123.33
7	—13.88	69	20.55	131	55.00	193	89.44	255	123.88
8	—13.33	70	21.11	132	55.55	194	90.00	256	124.44
9	—12.77	71	21.66	133	56.11	195	90.55	257	125.00
10	—12.22	72	22.22	134	56.66	196	91.11	258	125.55
11	—11.66	73	22.77	135	57.22	197	91.66	259	126.11
12	—11.11	74	23.33	136	57.77	198	92.22	260	126.66
13	—10.55	75	23.88	137	58.33	199	92.77	261	127.22
14	—10.00	76	24.44	138	58.88	200	93.33	262	127.77
15	— 9.44	77	25.00	139	59.44	201	93.88	263	128.33
16	— 8.88	78	25.55	140	60.00	202	94.44	264	128.88
17	— 8.33	79	26.11	141	60.55	203	95.00	265	129.44
18	— 7.77	80	26.66	142	61.11	204	95.55	266	130.00
19	— 7.22	81	27.22	143	61.66	205	96.11	267	130.55
20	— 6.66	82	27.77	144	62.22	206	96.66	268	131.11
21	— 6.11	83	28.33	145	62.77	207	97.22	269	131.66
22	— 5.55	84	28.88	146	63.33	208	97.77	270	132.22
23	— 5.00	85	29.44	147	63.88	209	98.33	271	132.77
24	— 4.44	86	30.00	148	64.44	210	98.88	272	133.33
25	— 3.88	87	30.55	149	65.00	211	99.44	273	133.88
26	— 3.33	88	31.11	150	65.55	212	100.00	274	134.44
27	— 2.77	89	31.66	151	66.11	213	100.55	275	135.00
28	— 2.22	90	32.22	152	66.66	214	101.11	276	135.55
29	— 1.66	91	32.77	153	67.22	215	101.66	277	136.11
30	— 1.11	92	33.33	154	67.77	216	102.22	278	136.66
31	— .55	93	33.88	155	68.33	217	102.77	279	137.22
32	Zero.	94	34.44	156	68.88	218	103.33	280	137.77
33	+ .55	95	35.00	157	69.44	219	103.88	281	138.33
34	1.11	96	35.55	158	70.00	220	104.44	282	138.88
35	1.66	97	36.11	159	70.55	221	105.00	283	139.44
36	2.22	98	36.66	160	71.11	222	105.55	284	140.00
37	2.77	99	37.22	161	71.66	223	106.11	285	140.55
38	3.33	100	37.77	162	72.22	224	106.66	286	141.11
39	3.88	101	38.33	163	72.77	225	107.22	287	141.66
40	4.44	102	38.88	164	73.33	226	107.77	288	142.22
41	5.00	103	39.44	165	73.88	227	108.33	289	142.77
42	5.55	104	40.00	166	74.44	228	108.88	290	143.33
43	6.11	105	40.55	167	75.00	229	109.44	291	143.88
44	6.66	106	41.11	168	75.55	230	110.00	292	144.44
45	7.22	107	41.66	169	76.11	231	110.55	293	145.00
46	7.77	108	42.22	170	76.66	232	111.11	294	145.55
47	8.33	109	42.77	171	77.22	233	111.66	295	146.11
48	8.88	110	43.33	172	77.77	234	112.22	296	146.66
49	9.44	111	43.88	173	78.33	235	112.77	297	147.22
50	10.00	112	44.44	174	78.88	236	113.33	298	147.77
51	10.55	113	45.00	175	79.44	237	113.88	299	148.33
52	11.11	114	45.55	176	80.00	238	114.44	300	148.88
53	11.66	115	46.11	177	80.55	239	115.00	400	204.44
54	12.22	116	46.66	178	81.11	240	115.55	600	315.55
55	12.77	117	47.22	179	81.66	241	116.11	800	433.33
56	13.33	118	47.77	180	82.22	242	116.66	1000	537.77

For Supplementary Tables, see page 485.

Centigrade to Fahrenheit.

Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.	Cent.	Fahr.
—273.00	—460.7	16	60.8	330	626	950	1742	1570	2858
—260.00	—436.0	17	62.6	340	644	960	1760	1580	2876
—250.00	—418.0	18	64.4	350	662	970	1778	1590	2894
—240.00	—400.0	19	66.2	360	680	980	1796	1600	2912
—230.00	—382.0	20	68.0	370	698	990	1814	1610	2930
—220.00	—364.0	21	69.8	380	716	1000	1832	1620	2948
—210.00	—346.0	22	71.6	390	734	1010	1850	1630	2966
—200.00	—328.0	23	73.4	400	752	1020	1868	1640	2984
—190.00	—310.0	24	75.2	410	770	1030	1886	1650	3002
—180.00	—292.0	25	77.0	420	788	1040	1904	1660	3020
—170.00	—274.0	26	78.8	430	806	1050	1922	1670	3038
—160.00	—256.0	27	80.6	440	824	1060	1940	1680	3056
—150.00	—238.0	28	82.4	450	842	1070	1958	1690	3074
—140.00	—220.0	29	84.2	460	860	1080	1976	1700	3092
—130.00	—202.0	30	86.0	470	878	1090	1994	1710	3110
—120.00	—184.0	31	87.8	480	896	1100	2012	1720	3128
—110.00	—166.0	32	89.6	490	914	1110	2030	1730	3146
—100.00	—148.0	33	91.4	500	932	1120	2048	1740	3164
— 90.00	—130.0	34	93.2	510	950	1130	2066	1750	3182
— 80.00	—112.0	35	95.0	520	968	1140	2084	1760	3200
— 70.00	— 94.0	36	96.8	530	986	1150	2102	1770	3218
— 60.00	— 76.0	37	98.6	540	1004	1160	2120	1780	3236
— 50.00	— 58.0	38	100.4	550	1022	1170	2138	1790	3254
— 40.00	— 40.0	39	102.2	560	1040	1180	2156	1800	3272
— 30.00	— 22.0	40	104.0	570	1058	1190	2174	1810	3290
— 20.00	— 4.0	41	105.8	580	1076	1200	2192	1820	3308
— 19.00	— 2.2	42	107.6	590	1094	1210	2210	1830	3326
— 18.00	— 0.4	43	109.4	600	1112	1220	2228	1840	3344
— 17.77	Zero.	44	111.2	610	1130	1230	2246	1850	3362
— 17.00	+ 1.4	45	113.0	620	1148	1240	2264	1860	3380
— 16.00	+ 3.2	46	114.8	630	1166	1250	2282	1870	3398
— 15.00	+ 5.0	47	116.6	640	1184	1260	2300	1880	3416
— 14.00	+ 6.8	48	118.4	650	1202	1270	2318	1890	3434
— 13.00	+ 8.6	49	120.2	660	1220	1280	2336	1900	3452
— 12.00	+ 10.4	50	122.0	670	1238	1290	2354	1910	3470
— 11.00	+ 12.2	60	140.0	680	1256	1300	2372	1920	3488
— 10.00	+ 14.0	70	158.0	690	1274	1310	2390	1930	3506
— 9.00	+ 15.8	80	176.0	700	1292	1320	2408	1940	3524
— 8.00	+ 17.6	90	194.0	710	1310	1330	2426	1950	3542
— 7.00	+ 19.4	100	212.0	720	1328	1340	2444	1960	3560
— 6.00	+ 21.2	110	230.0	730	1346	1350	2462	1970	3578
— 5.00	+ 23.0	120	248.0	740	1364	1360	2480	1980	3596
— 4.00	+ 24.8	130	266.0	750	1382	1370	2498	1990	3614
— 3.00	+ 26.6	140	284.0	760	1400	1380	2516	2000	3632
— 2.00	+ 28.4	150	302.0	770	1418	1390	2534	2010	3650
— 1.00	+ 30.2	160	320.0	780	1436	1400	2552	2020	3668
Zero.	+ 32.0	170	338.0	790	1454	1410	2570	2030	3686
+ 1	+ 33.8	180	356.0	800	1472	1420	2588	2040	3704
2	35.6	190	374.0	810	1490	1430	2606	2050	3722
3	37.4	200	392.0	820	1508	1440	2624	2060	3740
4	39.2	210	410.0	830	1526	1450	2642	2070	3758
5	41.0	220	428.0	840	1544	1460	2660	2080	3776
6	42.8	230	446.0	850	1562	1470	2678	2090	3794
7	44.6	240	464.0	860	1580	1480	2696	2100	3812
8	46.4	250	482.0	870	1598	1490	2714	2110	3830
9	48.2	260	500.0	880	1616	1500	2732	2120	3848
10	50.0	270	518.0	890	1634	1510	2750	2130	3866
11	51.8	280	536.0	900	1652	1520	2768	2140	3884
12	53.6	290	554.0	910	1670	1530	2786	2150	3902
13	55.4	300	572.0	920	1688	1540	2804	2160	3920
14	57.2	310	590.0	930	1706	1550	2822	2180	3956
15	59.0	320	608.0	940	1724	1560	2840	2200	3992

For Supplementary Tables, see page 485.

SUPPLEMENTARY TABLES.

Number of Degrees Cent. = Number of Degrees Fahr.

Degrees Cent.	Tenths of a degree—Centigrade scale.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.	Fahr.
0	.00	.18	.36	.54	.72	.90	1.08	1.26	1.44	1.62
1	1.80	1.98	2.16	2.34	2.52	2.70	2.88	3.06	3.24	3.42
2	3.60	3.78	3.96	4.14	4.32	4.50	4.68	4.86	5.04	5.22
3	5.40	5.58	5.76	5.94	6.12	6.30	6.48	6.66	6.84	7.02
4	7.20	7.38	7.56	7.74	7.92	8.10	8.28	8.46	8.64	8.82
5	9.00	9.18	9.36	9.54	9.72	9.90	10.08	10.26	10.44	10.62
6	10.80	10.98	11.16	11.34	11.52	11.70	11.88	12.06	12.24	12.42
7	12.60	12.78	12.96	13.14	13.32	13.50	13.68	13.86	14.04	14.22
8	14.40	14.58	14.76	14.94	15.12	15.30	15.48	15.66	15.84	16.02
9	16.20	16.38	16.56	16.74	16.92	17.10	17.28	17.46	17.64	17.82

Number of Degrees Fahr. = Number of Degrees Cent.

Degrees Fahr.	Tenths of a degree—Fahrenheit scale.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.	Cent.
0	.00	.06	.11	.17	.22	.28	.33	.39	.44	.50
1	.56	.61	.67	.72	.78	.83	.89	.94	1.00	1.06
2	1.11	1.17	1.22	1.28	1.33	1.39	1.44	1.50	1.56	1.61
3	1.67	1.72	1.78	1.83	1.89	1.94	2.00	2.06	2.11	2.17
4	2.22	2.28	2.33	2.39	2.44	2.50	2.56	2.61	2.67	2.72
5	2.78	2.83	2.89	2.94	3.00	3.06	3.11	3.17	3.22	3.28
6	3.33	3.39	3.44	3.50	3.56	3.61	3.67	3.72	3.78	3.83
7	3.89	3.94	4.00	4.06	4.11	4.17	4.22	4.28	4.33	4.39
8	4.44	4.50	4.56	4.61	4.67	4.72	4.78	4.83	4.89	4.94
9	5.00	5.06	5.11	5.17	5.22	5.28	5.33	5.39	5.44	5.50

By the use of the above tables any value may be obtained in connection with the preceding tables. Thus, to convert 1375.4° C. to Fahrenheit we have

$$\begin{array}{rcr} 1370.0^\circ\text{ C.} & = & 2498.00^\circ\text{ F.} \\ 5.0^\circ\text{ C.} & = & 9.00^\circ\text{ F.} \\ 0.4^\circ\text{ C.} & = & 0.72^\circ\text{ F.} \\ \hline 1375.4^\circ\text{ C.} & = & 2507.72^\circ\text{ F.} \end{array}$$

Coefficients of Expansion.

Per Degree of Fahrenheit Scale.

Temperatures.	Solids.	Linear.	Surface.	Volume.
Degrees.				
32 to 212	Glass	.000 00478	.000 00956	.000 01434
212 to 392		.000 00546	.000 01093	.000 01639
392 to 572		.000 00660	.000 01320	.000 01980
32 to 212	Wrought-iron	.000 00656	.000 01312	.000 01968
32 to 572		.000 00895	.000 01790	.000 02686
32 to 212	Soft iron	.000 00680	.000 01360	.000 02040
32 to 212	Cast-iron	.000 00618	.000 01236	.000 01854
32 to 212	Cast-steel	.000 00600	.000 01200	.000 01800
32 to 212	Hardened steel	.000 00689	.000 01378	.000 02067
32 to 212	Copper	.000 00955	.000 01910	.000 02865
32 to 572		.000 01092	.000 02184	.000 03276
32 to 212	Lead	.000 01580	.000 03160	.000 04740
32 to 212	Gold, pure	.000 00815	.000 01630	.000 02445
32 to 212	Gold, hammered	.000 00830	.000 01660	.000 02490
32 to 212	Silver, pure	.000 01060	.000 02120	.000 03180
32 to 212	Silver, hammered	.000 01116	.000 02232	.000 03348
32 to 212	Brass, common cast	.000 01043	.000 02086	.000 03129
32 to 212	Brass, wire or sheet	.000 01075	.000 02150	.000 03225
32 to 212	Platinum, pure	.000 00491	.000 00982	.000 01473
32 to 572		.000 00520	.000 01040	.000 01560
32 to 212	Platinum, hammered	.000 00530	.000 01060	.000 01590
32 to 212	Palladium	.000 00555	.000 01110	.000 01665
32 to 212	Roman cement	.000 00797	.000 01594	.000 02391
32 to 212	Zinc, pure or cast	.000 01633	.000 03266	.000 04899
32 to 212	Zinc, hammered	.000 01722	.000 03444	.000 05166
32 to 212	Tin, cast	.000 01207	.000 02414	.000 03621
32 to 212	Tin, hammered	.000 01500	.000 03000	.000 04500
32 to 212	Fire-brick	.000 00235	.000 00470	.000 00705
32 to 212	Good red brick	.000 00305	.000 00610	.000 00915
32 to 212	Marble	.000 00613	.000 01226	.000 01839
32 to 212	Granite	.000 00438	.000 00876	.000 01314
32 to 212	Bismuth	.000 00773	.000 01546	.000 02319
32 to 212	Antimony	.000 00602	.000 01204	.000 01806
32 to 212	Mercury	.000 03333	.000 06666	.000 10000
212 to 392		.000 03416	.000 06833	.000 10250
392 to 572		.000 03500	.000 07000	.000 10500
32 to 212	Water	.000 08806	.000 17612	.000 26420
212 to 392		.000 17066	.000 34133	.000 51198
392 to 572		.000 18904	.000 37808	.000 56713
32 to 212	Salt, dissolved	.000 09250	.000 18500	.000 27750
32 to 212	Sulphuric acid	.000 11111	.000 22222	.000 33333
32 to 212	Turpentine and ether	.000 12966	.000 25933	.000 38900
32 to 212	Oil, common	.000 14814	.000 29629	.000 44443
32 to 212	Alcohol and nitric acid	.000 15151	.000 30302	.000 45453
32 to 212	All permanent gases	.000 69416	.001 38832	.002 08250

According to the investigations of M. Guillaume, an alloy of nickel-steel, containing 36 per cent. of nickel, has a coefficient of expansion only 1/15 that of platinum, or about 0.0000003 for 1° F. Wires made of this alloy have been used for the measurement of geodetic base lines, without requiring any temperature correction.

Coefficients of Expansion.

Per Degree of the Centigrade Scale.

Substance.	Linear.	Surface.	Volume.
Aluminum	.000 0231	.000 0462	.000 0693
Brass, cast	.000 0187	.000 0374	.000 0561
Brass wire	.000 0193	.000 0386	.000 0579
Bronze	.000 0184	.000 0368	.000 0552
Carbon, gas	.000 0054	.000 0108	.000 0162
Carbon, graphite	.000 0077	.000 0154	.000 0231
Copper	.000 0168	.000 0336	.000 0504
German silver	.000 0184	.000 0368	.000 0552
Gold	.000 0144	.000 0288	.000 0432
Glass, crown	.000 0090	.000 0180	.000 0270
Glass, flint	.000 0079	.000 0158	.000 0237
Iron, cast	.000 0106	.000 0212	.000 0318
Iron, wrought	.000 0114	.000 0228	.000 0342
Steel, hard	.000 0132	.000 0264	.000 0396
Steel, soft	.000 0109	.000 0218	.000 0327
Lead	.000 0292	.000 0584	.000 0876
Nickel	.000 0128	.000 0256	.000 0384
Platinum	.000 0090	.000 0180	.000 0270
Silver	.000 0192	.000 0384	.000 0576
Tin	.000 0223	.000 0446	.000 0669
Zinc	.000 0292	.000 0584	.000 0876

Linear Expansion or Contraction, in Inches, of Cast-iron.

Lengths in Feet.

Length.	Difference in temperature.—Fahrenheit.								
	100°	150°	200°	250°	300°	400°	500°	600°	800°
Feet.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.	Inch.
1	.0072	.0110	.0150	.0192	.0237	.0336	.0444	.0561	.0787
2	.0144	.0220	.0300	.0384	.0474	.0632	.0885	.1123	.1574
3	.0216	.0330	.0450	.0576	.0711	.1008	.1332	.1684	.2361
4	.0288	.0440	.0600	.0768	.0948	.1344	.1776	.2246	.3148
5	.0360	.0550	.0750	.0960	.1185	.1680	.2220	.2805	.3935
6	.0432	.0660	.0900	.1152	.1422	.2016	.2664	.3368	.4722
7	.0504	.0770	.1050	.1344	.1659	.2352	.3108	.3929	.5509
8	.0576	.0880	.1200	.1536	.1896	.2688	.3552	.4496	.6396
9	.0648	.0990	.1350	.1728	.2133	.3024	.3996	.5052	.7083
10	.0720	.1102	.1502	.1926	.2376	.3360	.4440	.5616	.7872
11	.0792	.1214	.1652	.2125	.2615	.3696	.4884	.6177	.8659
12	.0864	.1316	.1802	.2318	.2853	.4032	.5328	.6739	.9446
13	.0936	.1417	.1952	.2510	.3090	.4368	.5772	.7300	1.0233
14	.1008	.1519	.2102	.2703	.3328	.4704	.6216	.7862	1.1020
15	.1080	.1620	.2253	.2895	.3565	.5040	.6660	.8423	1.1808
16	.1152	.1722	.2403	.3088	.3803	.5376	.7104	.8985	1.2595
17	.1224	.1823	.2553	.3280	.4040	.5712	.7548	.9546	1.3382
18	.1296	.1925	.2703	.3472	.4278	.6048	.7992	1.0108	1.4169
19	.1368	.2026	.2853	.3665	.4515	.6384	.8436	1.0669	1.4956
20	.1440	.2203	.3005	.3852	.4752	.6720	.8880	1.1232	1.5744
21	.1512	.2305	.3155	.4045	.4995	.7056	.9324	1.1793	1.6531
22	.1584	.2407	.3305	.4238	.5228	.7392	.9768	1.2394	1.7318
23	.1656	.2508	.3455	.4430	.5465	.7728	1.0212	1.2915	1.8105
24	.1728	.2610	.3606	.4623	.5703	.8064	1.0656	1.3477	1.8892
25	.1800	.2711	.3756	.4815	.5940	.8400	1.1100	1.4038	1.9679
26	.1872	.2813	.3906	.5008	.6179	.8736	1.1544	1.4600	2.0467
27	.1944	.2914	.4056	.5200	.6415	.9072	1.1988	1.5161	2.1254
28	.2016	.3016	.4206	.5393	.6553	.9408	1.2432	1.5723	2.2041
29	.2088	.3117	.4356	.5585	.6890	.9744	1.2876	1.6284	2.2829
30	.2160	.3304	.4507	.5778	.7128	1.0080	1.3320	1.6848	2.3616
31	.2232	.3405	.4657	.5970	.7365	1.0416	1.3764	1.7409	2.4403
32	.2304	.3507	.4807	.6163	.7603	1.0752	1.4208	1.7971	2.5190
33	.2376	.3608	.4957	.6355	.7841	1.1088	1.4652	1.8533	2.5977
34	.2448	.3710	.5107	.6548	.8078	1.1424	1.5096	1.9094	2.6764
35	.2520	.3811	.5258	.6740	.8316	1.1760	1.5540	1.9656	2.7552
36	.2592	.3913	.5408	.6933	.8553	1.2096	1.5984	2.0217	2.8339
37	.2664	.4014	.5558	.7125	.8791	1.2432	1.6428	2.0779	2.9126
38	.2736	.4116	.5708	.7298	.9028	1.2768	1.6872	2.1340	2.9913
39	.2808	.4217	.5858	.7490	.9266	1.3104	1.7316	2.1902	3.0701
40	.288	.4406	.6009	.7704	.9504	1.344	1.776	2.2464	3.1488
45	.324	.4957	.6760	.8667	1.0692	1.512	1.998	2.5272	3.5424
50	.360	.5508	.7512	.9630	1.1880	1.680	2.220	2.8080	3.9360
55	.396	.6059	.8263	1.0593	1.3068	1.848	2.442	3.0888	4.3296
60	.432	.6610	.9014	1.1556	1.4256	2.016	2.664	3.3696	4.7132
65	.468	.7150	.9765	1.2519	1.5444	2.184	2.886	3.6540	5.1068
70	.504	.7711	1.0517	1.3482	1.6632	2.352	3.108	3.9312	5.5104
75	.540	.8262	1.1268	1.4445	1.7820	2.520	3.330	4.2120	5.9040
80	.576	.8813	1.2019	1.5408	1.9008	2.688	3.552	4.4948	6.2976
85	.612	.9364	1.2770	1.6371	2.0196	2.856	3.774	4.7756	6.6912
90	.648	.9914	1.3521	1.7334	2.1384	3.024	3.996	5.0544	7.0848
95	.684	1.0465	1.4272	1.8297	2.2572	3.192	4.218	5.3352	7.4784
100	.720	1.1016	1.5024	1.9260	2.3760	3.360	4.440	5.6160	7.8720
	.000 006	**612**	**626**	**642**	**660**	**700**	**740**	**780**	**820**

Expansion per degree.—Fahrenheit.

Multiply by 1.1, for wrought-iron; 1.5, for copper; 1.6, for brass; 2.6, for zinc.

For the determination of temperatures above the boiling-point of mercury various forms of pyrometers are used. The most reliable for practical use is the thermo-electric pyrometer of Le Chatelier, composed of a thermo-electric couple, one member of which is of pure platinum and the other of platinum alloyed with 10 per cent. of rhodium. For a full description of this and other methods of measurement of high temperatures reference may be made to the work entitled "High Temperature Measurements," by Le Chatelier and Boudouard, and translated by George K. Burgess.*

The following table gives the melting-points of the elements used in engineering.

Fusing-points.

Substance.	Fahr.	Cent.	Substance.	Fahr.	Cent.
	Degrees.	Degrees.		Degrees.	Degrees.
Aluminum	1213	657	Glass	1832	1000
Antimony	815	435	Glass, lead free	2192	1200
Copper	1949	1065	Delta metal	1742	950
Gold	1947	1064			
Iron, pure	2975	1635	**Fusible Metals.**		
Iron, white pig	1967	1075	3 Tin, 5 Lead, 8 Bismuth	212	100
Iron, gray pig	2192	1200			
Steel	2507	1375			
Lead	621	327	4 Tin, 4 Lead, 1 Bismuth	263	128
Manganese	3452	1900			
Nickel	2732	1500			
Platinum	3452	1900	3 Tin, 2 Lead	275	135
Silver	1751	955			
Tin	446	230	1 Tin, 1 Lead	304	151
Zinc	787	419			
Brass	1859	1015	1 Tin, 2 Lead	361	183
Bronze	1652	900			

Expansion of Gases.

All perfect gases, so called, expand and contract alike under the action of heat. That is to say, every substance, when in the gaseous state, and not near its point of liquefaction, has the same coefficient of expansion, this coefficient being $\frac{1}{273}$ of its volume, or 0.003665 for each degree Centigrade, or $\frac{1}{491.4}$ part = 0.002035 for each degree Fahrenheit.

Since a gas contracts $\frac{1}{273}$ part of its volume when its temperature is lowered 1° C., such a rate of contraction would theoretically reduce its volume to zero at a temperature of —273° C. = —459.4° F. Since all gases reach their liquefying point before this low temperature is attained, however, no such contraction exists. At the same time, it may be said that if heat is considered as a motion of the molecules of a substance, that motion is to be considered as having ceased when the temperature has reached —273° C.

This temperature of —273° C. = —459.4° F. is, therefore, called the *absolute zero*, and from it all temperatures should properly be reckoned. When-

* New York, John Wiley & Sons.

ever a temperature is mentioned as being in degrees *absolute*, either in the Centigrade or the Fahrenheit scale, it is understood to be counted from the absolute zero, and therefore is equal to the observed temperature plus 273 or 459.4, as the case may be.

The lowest temperature which has thus far been attained is that produced by the evaporation of liquid hydrogen by Dewar, = —252° C.

Heat Units.

In expressing quantities of heat the temperature alone is not sufficient, since the substance in which the change of temperature is produced must be considered. The substance chosen as a standard is pure water, at or near its point of greatest density.

Two heat units are in general use.

The **British Thermal Unit,** abbreviated B. T. U., is the quantity of heat required to raise the temperature of 1 pound of water 1° F., at or near the temperature of 39.1°.

The limitation of the part of the scale, at or near which the measurement should be made, need be considered only for very precise physical work, since the variation in the quantity of heat corresponding to an interval of one degree in a given weight of water varies but slightly for different parts of the scale.

In the metric system the kilogramme of water is taken, and the degree on the Centigrade scale. The unit is the **Calorie**, being the quantity of heat required to raise 1 kilogramme of water 1° C., at or near the temperature of 4° C. In French the calorie is sometimes abbreviated Cal., and in German it is written W. E. (Wärme Einheit).

1 B. T. U = 0.252 calorie.
1 calorie = 3.968 B. T. U.

When the effect of the application of a given number of British thermal units or calories upon a given weight of any substance is under consideration, care must be taken to take into account the relation of the weights in making the conversion, or errors may be made. Thus, 1 calorie *per kilogramme* is only 1.8 times greater than 1 British thermal unit *per pound*, since the calorie is considered in connection with a weight equal to 2.2 pounds, and, conversely, 1 British thermal unit per pound is equal to 0.555 calories per kilogramme.

The following tables will be found convenient for transforming quantities in one kind of heat units to another.

Conversion of British Thermal Units into Calories.

B. T. U.	Calories.	B. T. U.	Calories.	B. T. U.	Calories.
1	.252	34	8.568	67	16.884
2	.504	35	8.820	68	17.136
3	.756	36	9.072	69	17.388
4	1.008	37	9.324	70	17.640
5	1.260	38	9.576	71	17.892
6	1.512	39	9.828	72	18.144
7	1.764	40	10.080	73	18.396
8	2.016	41	10.332	74	18.648
9	2.268	42	10.584	75	18.900
10	2.520	43	10.836	76	19.152
11	2.772	44	11.088	77	19.404
12	3.024	45	11.340	78	19.656
13	3.276	46	11.592	79	19.908
14	3.528	47	11.844	80	20.160
15	3.780	48	12.096	81	20.412
16	4.032	49	12.348	82	20.664
17	4.284	50	12.600	83	20.916
18	4.536	51	12.852	84	21.168
19	4.788	52	13.104	85	21.420
20	5.040	53	13.356	86	21.672
21	5.292	54	13.608	87	21.924
22	5.544	55	13.860	88	22.176
23	5.796	56	14.112	89	22.428
24	6.048	57	14.364	90	22.680
25	6.300	58	14.616	91	22.932
26	6.552	59	14.868	92	23.184
27	6.804	60	15.120	93	23.436
28	7.056	61	15.372	94	23.688
29	7.308	62	15.624	95	23.940
30	7.560	63	15.876	96	24.192
31	7.812	64	16.128	97	24.444
32	8.064	65	16.380	98	24.696
33	8.316	66	16.632	99	24.948

Conversion of Calories into British Thermal Units.

Calories.	B. T. U.	Calories.	B. T. U.	Calories.	B. T. U.
1	3.97	34	134.92	67	265.88
2	7.94	35	138.89	68	269.85
3	11.90	36	142.86	69	273.81
4	15.87	37	146.83	70	277.78
5	19.84	38	150.80	71	281.75
6	23.81	39	154.76	72	285.72
7	27.78	40	158.73	73	289.69
8	31.75	41	162.70	74	293.66
9	35.71	42	166.67	75	297.62
10	39.68	43	170.64	76	301.59
11	43.65	44	174.61	77	305.56
12	47.62	45	178.57	78	309.53
13	51.59	46	182.54	79	313.50
14	55.56	47	186.51	80	317.47
15	59.52	48	190.48	81	321.43
16	63.49	49	194.45	82	325.40
17	67.46	50	198.42	83	329.37
18	71.43	51	202.38	84	333.34
19	75.40	52	206.35	85	337.31
20	79.37	53	210.32	86	341.28
21	83.33	54	214.29	87	345.24
22	87.30	55	218.26	88	349.21
23	91.27	56	222.23	89	353.18
24	95.24	57	226.19	90	357.15
25	99.21	58	230.16	91	361.12
26	103.18	59	234.13	92	365.09
27	107.14	60	238.10	93	369.05
28	111.11	61	242.07	94	373.02
29	115.08	62	246.04	95	376.99
30	119.05	63	250.00	96	380.96
31	123.02	64	253.97	97	384.93
32	126.99	65	257.94	98	388.90
33	130.95	66	261.91	99	392.86

Conversion of Calories into Foot-pounds.

Calories.	Foot-pounds.	Calories.	Foot-pounds.	Calories.	Foot-pounds.
1	3 091	34	105 106	67	207 121
2	6 183	35	108 198	68	210 212
3	9 274	36	111 289	69	213 304
4	12 365	37	114 380	70	216 395
5	15 457	38	117 472	71	219 487
6	18 548	39	120 563	72	222 578
7	21 640	40	123 654	73	225 669
8	24 731	41	126 746	74	228 761
9	27 822	42	129 837	75	231 852
10	30 914	43	132 928	76	234 943
11	34 005	44	136 020	77	238 035
12	37 096	45	139 111	78	241 126
13	40 188	46	142 203	79	244 217
14	43 279	47	145 294	80	247 309
15	46 370	48	148 387	81	250 400
16	49 462	49	151 477	82	253 492
17	52 553	50	154 568	83	256 583
18	55 644	51	157 659	84	259 674
19	58 736	52	160 751	85	262 766
20	61 827	53	163 824	86	265 857
21	64 919	54	166 933	87	268 948
22	68 010	55	170 025	88	272 040
23	71 101	56	173 116	89	275 131
24	74 193	57	176 208	90	278 222
25	77 284	58	179 299	91	281 314
26	80 375	59	182 390	92	284 405
27	83 467	60	185 482	93	287 496
28	86 558	61	188 573	94	290 588
29	89 649	62	191 664	95	293 679
30	92 741	63	194 756	96	296 771
31	95 835	64	197 847	97	299 862
32	98 924	65	200 938	98	302 953
33	102 015	66	204 030	99	306 045

Conversion of Foot-pounds into Calories.

Foot-pounds.	Calories.	Foot-pounds.	Calories.	Foot-pounds.	Calories.
1	.000 323	34	.010 998	67	.021 676
2	.000 647	35	.011 322	68	.021 997
3	.000 970	36	.011 645	69	.022 320
4	.001 294	37	.011 969	70	.022 644
5	.001 617	38	.012 292	71	.022 967
6	.001 941	39	.012 616	72	.023 291
7	.002 264	40	.012 939	73	.023 614
8	.002 588	41	.013 263	74	.023 938
9	.002 911	42	.013 586	75	.024 261
10	.003 235	43	.013 910	76	.024 584
11	.003 558	44	.014 233	77	.024 908
12	.003 882	45	.014 557	78	.025 231
13	.004 205	46	.014 880	79	.025 555
14	.004 529	47	.015 204	80	.025 878
15	.004 852	48	.015 527	81	.026 202
16	.005 176	49	.015 851	82	.026 525
17	.005 499	50	.016 174	83	.026 849
18	.005 823	51	.016 497	84	.027 172
19	.006 146	52	.016 821	85	.027 496
20	.006 470	53	.017 144	86	.027 819
21	.006 793	54	.017 468	87	.028 143
22	.007 117	55	.017 791	88	.028 466
23	.007 440	56	.018 115	89	.028 790
24	.007 764	57	.018 438	90	.029 113
25	.008 087	58	.018 762	91	.029 437
26	.008 410	59	.019 085	92	.029 760
27	.008 734	60	.019 409	93	.030 084
28	.009 057	61	.019 732	94	.030 407
29	.009 381	62	.020 056	95	.030 731
30	.009 704	63	.020 379	96	.031 054
31	.010 028	64	.020 703	97	.031 378
32	.010 351	65	.021 026	98	.031 701
33	.010 675	66	.021 350	99	.032 025

Mechanical Equivalent of Heat.

In the conversion of heat into mechanical energy there is always a definite amount of work produced for a definite quantity of heat. One British thermal unit is equal to 778 foot-pounds, and one calorie is equal to 428 kilogrammetres. The maximum amount of energy which can be obtained for any given number of heat units is, therefore, found by multiplying by 778. Conversely, 1 foot-pound $= \frac{1}{778} = 0.001285$ heat unit.

Specific Heat.

We have seen that heat requires for its determination the production of a determinate change in temperature of a definite weight of a given substance. For the purpose of establishing a unit, water has been chosen as the standard substance. The quantity of heat required to raise the temperature of other substances is different from that required for water. The ratio of the quantity of heat required for any substance by that required for water is called the **Specific Heat** of the substance. Thus, it is found that it takes only about one-ninth as much heat to raise the temperature of a pound of iron one degree that it does to raise a pound of water; hence, the specific heat of iron is one-ninth, or, more precisely, = 0.1138.

The methods of measuring specific heats vary according to the character of the substances. For metals the most convenient is the method of mixtures, in which a known weight of the metal is raised to a definite temperature and then plunged into a given weight of water at a known temperature. The rise in temperature of the water gives the number of heat units which have been imparted to it, and these have obviously been derived from the metal which has been cooled. We then have, if

x = specific heat of metal required;
T = fall in temperature of metal;
t = rise in temperature of water;
W = weight of metal;
w = weight of water;

$$x = \frac{wt}{Wt}.$$

The specific heats of various substances are not constant, but gradually increase with the temperature. The following table gives the mean between 10° C. and 100° C., the usual working temperatures. Fuller tables for various ranges of temperatures are to be found in the Smithsonian Physical Tables.

Table of Specific Heats.

Solids (mean specific heat between 10° C. and 100° C.).

Copper	.0951	Antimony	.0508
Silver	.0570	Brass	.0939
Iron	.1138	Magnesium	.2499
Zinc	.0955	Aluminum	.2143
Tin	.0562	Glass	.1877
Lead	.0314	Ice	.5040
Gold	.0324	Sulphur	.1777
Platinum	.0324	Graphite	.2008
Bismuth	.0308	Diamond	.1469

Liquids.

Mercury	.0333	Alcohol	.615
Sulphuric acid	.3430	Oil of turpentine	.462
Ether	.5030	Acetic acid	.659

Gases (at constant pressure).

Air	.2374	Nitrogen	.2438
Hydrogen	3.4090	Carbonic anhydride	.2163
Oxygen	.2175	Carbonic oxide	.2450
Chlorine	.1210	Steam	.4805

The above specific heats represent the quantity of heat, in British thermal units, required to raise the temperature of 1 pound of the substance 1° F., or, in calories, required to raise the temperature of 1 kilogramme of the substance 1° C.

Latent Heat.

The phenomena of expansion follow the simple rule of direct relation to the temperature only when the substance does not suffer any change of state. Thus, there are determinate coefficients for solids, for liquids, and for gases. When, however, a substance under observation passes from the solid to the liquid state, and from the liquid to the gaseous state, certain amounts of heat are absorbed which do not raise the temperature, this heat being expended in molecular work, separating the molecules of the substance. The heat thus absorbed is said to be rendered *latent*—every substance having a *latent heat of fusion*, required to convert it from a solid to a liquid, and another latent heat of vaporization.

Thus, a pound of ice may be heated and its temperature will rise until the melting-point, 32° F. or 0° C., has been reached, when further application of heat, however intense, will cause no further rise in temperature until the ice has been entirely melted. Experiments have shown that 142.6 British thermal units are required to convert a pound of ice at 32° into a pound of water at 32°, and hence the latent heat of fusion of water is said to be 142.6°. Further application of heat causes a rise in temperature directly proportional to the quantity of heat supplied, 180 thermal units raising it to the boiling-point, 212° F. Here, again, the rise in temperature ceases until all the pound of water at 212° has been converted into steam at 212°. This operation requires 966 heat units, so that the latent heat of vaporization of water is 966°.

The following table gives the latent heats of various substances.

Substance.	B. T. U. per pound.		Calories per kilogramme.	
	Fusion.	Vaporization.	Fusion.	Vaporization.
Water	142.60	966.6	79.24	537
Alcohol, ethyl	....	371.0	....	205
Alcohol, methyl	....	481.0	....	267
Ammonia	....	529.0	....	294
Bisulphide of carbon	....	162.0	....	90
Sulphur dioxide	....	164.0	....	91
Turpentine	....	133.0	....	74
Iron, gray	41.40	....	23.00	....
Iron, white	59.40	....	33.00	....
Lead	10.55	....	5.86	....
Mercury	5.08	....	2.82	....
Silver	37.92	....	21.07	....
Zinc	50.63	....	28.13	....

Coefficients for Heat Transmission.

Substance.	Metric.	British.	Substance.	Metric.	British.
Aluminum	.00036	.00203	Lead	.00008	.00045
Antimony......	.00004	.00022	Mercury.......	.00002	.00011
Brass, yellow...	.00025	.00142	Steel, hard.....	.00006	.00034
Brass, red	.00028	.00157	Steel, soft	.00011	.00062
Copper	.00072	.00404	Silver	.00109	.00610
German silver..	.00009	.00050	Tin	.00015	.00084
Iron............	.00016	.00089	Zinc	.00030	.00170

In the above table the metric coefficients give the quantity of heat, in calories, transmitted per second through a plate 1 centimetre thick, per square centimetre of surface, for a difference of 1° C., at a temperature of 100° C.

The British coefficients give the quantity of heat transmitted, in British thermal units, per second through a plate 1 inch thick, per square inch of surface, for a difference of 1° F., at a temperature of 212° F.

The coefficients vary somewhat with the temperature, but the above will serve in practice.

Radiation.

For moderate differences in temperature the loss of heat by radiation may be taken as dependent upon the character of the surface, the area, and the difference in temperature.

The coefficients of radiation, as determined by Péclet, give the number of heat units emitted per hour, per square foot of surface, for 1° F., or the number of calories emitted per hour, per square metre, per 1° C., as below:

Coefficients of Radiation.

Surface.	B. T. U., per 1° F., per square foot, per hour.	Calories, per 1° C., per square metre, per hour.
Silver, polished	.02657	.13
Copper, polished	.03270	.16
Tin, polished..................	.04395	.22
Tinned iron, polished...... ...	.08585	.42
Iron, sheet-, polished..........	.0920	.45
Iron, ordinary	.5662	2.77
Glass	.5948	2.91
Cast-iron, new	.6480	3.17
Cast-iron, rusted	.6868	3.36
Sawdust........................	.7215	3.53
Sand, fine	.7400	3.62
Water	1.0853	5.31
Oil	1.4800	7.24

The number of heat units radiated from any surface per hour may, therefore, be computed by multiplying the area by the difference in temperature between the hot surface and the surrounding air, and by the coefficient corresponding to the character of the surface. It will be seen from the table that ordinary cast-iron is about six times as good a radiating

surface as polished sheet-iron, and about twenty-five times as effective as polished silver.

The coefficients in the preceding table are sufficiently correct for use when the difference in temperature is not great. When, however, there is a considerable difference in the temperature of the heated body and the surrounding air, the rate of cooling becomes more rapid. The following tables give the ratio of increase in the rate of cooling for larger differences in temperature.

Ratio of Increase in Radiation for Temperatures from 10° F. to 450° F.

Temperatures of Air = 70° F.

Difference in temperature, Fahr.	Ratio.	Difference in temperature, Fahr.	Ratio.	Difference in temperature, Fahr.	Ratio.
Degrees.		Degrees.		Degrees.	
10	1.15	160	1.61	310	2.34
20	1.18	170	1.65	320	2.40
30	1.20	180	1.68	330	2.47
40	1.23	190	1.73	340	2.54
50	1.25	200	1.78	350	2.60
60	1.27	210	1.82	360	2.68
70	1.32	220	1.86	370	2.77
80	1.35	230	1.90	380	2.84
90	1.38	240	1.95	390	2.93
100	1.40	250	2.00	400	3.02
110	1.44	260	2.05	410	3.10
120	1.47	270	2.10	420	3.20
130	1.50	280	2.16	430	3.30
140	1.54	290	2.21	440	3.40
150	1.57	300	2.27	450	3.50

Ratio of Increase in Radiation for Temperatures from 10° C. to 240° C.

Temperatures of Air = 20° C.

Difference in temperature, Cent.	Ratio.	Difference in temperature, Cent.	Ratio.	Difference in temperature, Cent.	Ratio.
Degrees.		Degrees.		Degrees.	
10	1.16	90	1.60	170	2.31
20	1.21	100	1.68	180	2.42
30	1.25	110	1.75	190	2.54
40	1.30	120	1.83	200	2.66
50	1.36	130	1.90	210	2.79
60	1.42	140	2.00	220	2.93
70	1.48	150	2.09	230	3.07
80	1.54	160	2.20	240	3.23

In computing the number of heat units radiated from a given area and material the result should first be calculated by the coefficients of radiation, given on page 497, and the value thus obtained, multiplied by the ratio,

corresponding to the difference in temperature, as given in the preceding tables.

Heating Pipes (Iron).

Mean temperature of pipes, Fahr.	Units of heat (B. T. U.) emitted, per square foot, per hour. Temperature of air = 70° F.				
	By convection.		By radiation alone.	By convection and radiation, combined.	
	Air, still.	Air, moving.		Air, still.	Air, moving.
Degrees.					
80	5.04	8.40	7.43	12.47	15.83
90	11.84	19.73	15.31	27.15	35.04
100	19.53	32.55	23.47	43.00	56.02
110	27.86	46.43	31.93	57.79	78.36
120	36.66	61.10	40.82	77.48	101.92
130	45.90	76.50	50.00	95.90	126.50
140	55.51	92.52	59.63	115.14	152.15
150	65.45	109.18	69.69	135.14	178.87
160	75.68	126.13	80.19	155.87	206.32
170	86.18	143.30	91.12	177.30	234.42
180	96.93	161.55	102.50	199.43	264.05
190	107.90	179.83	114.45	222.35	294.28
200	119.13	198.55	127.00	246.13	325.55
210	130 49	217.48	139.96	270.49	357.48
220	142.20	237.00	155.27	297.47	392.27
230	153.95	256.58	169.56	323.51	426.14
240	165.90	279.83	184.58	350.48	464.41
250	178.00	296.66	200.18	378.18	496.84
260	189.90	316.50	214.36	404.26	530.86
270	202.70	337.83	233.42	436.12	571.25
280	215.30	358.85	251.21	466.51	610.06
290	228.55	380.91	267.73	496.28	648.64
300	240.85	401.41	279.12	519.97	680.53

Loss of Heat Through Walls.

Loss, in British Thermal Units, per Square Foot, per Hour, for 1° F. Difference.

Thickness, in inches.	Brick.	Stone.	Thickness, in inches.	Brick.	Stone.
4	.273	.330	24	.129	.255
8	.223	.312	28	.116	.244
12	.188	.295	32	.106	.234
16	.163	.280	36	.097	.224
20	.144	.267	40	.090	.216

AIR.

Air is composed of a mixture of oxygen and nitrogen, in the proportion of 21 of oxygen to 79 of nitrogen, by volume; or 23 of oxygen to 77 of nitrogen, by weight, with an average of 0.04 per cent. of carbonic acid.

A cubic metre of dry air, at 0° C. and a pressure of 760 millimetres of mercury, weighs 1.29305 kilogrammes. A cubic foot of dry air, at 32° F. and a pressure of 29.92 inches of mercury, weighs 0.08072 pound. Above its critical temperature of —140° C. air may be considered as a permanent gas, expanding $\frac{1}{273}$ of its volume for each degree Centigrade increase in temperature, and $\frac{1}{461}$ of its volume for an increase of 1° Fahrenheit.

Taking the volume at freezing-point as unity, the weights and volumes at other temperatures are given in the following table.

Volume and Weight of Dry Air at Different Temperatures.

Under a Constant Atmospheric Pressure of 29.92 Inches of Mercury, the Volume at 32° F. being 1.

Temperature, Fahr.	Volume.	Weight of a cubic foot.	Temperature, Fahr.	Volume.	Weight of a cubic foot.
Degrees.		Lb.	Degrees.		Lb.
0	.935	.0864	500	1.954	.0413
12	.960	.0842	552	2.056	.0385
22	.980	.0824	600	2.150	.0376
32	1.000	.0807	650	2.260	.0357
42	1.020	.0791	700	2.362	.0338
52	1.041	.0776	750	2.465	.0328
62	1.061	.0761	800	2.566	.0315
72	1.082	.0747	850	2.668	.0303
82	1.102	.0733	900	2.770	.0292
92	1.122	.0720	950	2.871	.0281
102	1.143	.0707	1000	2.974	.0268
112	1.163	.0694	1100	3.177	.0254
122	1.184	.0682	1200	3.381	.0239
132	1.204	.0671	1300	3.584	.0225
142	1.224	.0659	1400	3.788	.0213
152	1.245	.0649	1500	3.993	.0202
162	1.265	.0638	1600	4.196	.0192
172	1.285	.0628	1700	4.402	.0183
182	1.306	.0618	1800	4.605	.0175
192	1.326	.0609	1900	4.808	.0168
202	1.347	.0600	2000	5.012	.0161
212	1.367	.0591	2100	5.217	.0155
230	1.404	.0575	2200	5.420	.0149
250	1.444	.0559	2300	5.625	.0142
275	1.495	.0540	2400	5.827	.0138
300	1.546	.0522	2500	6.032	.0133
325	1.597	.0506	2600	6.236	.0130
350	1.648	.0490	2700	6.440	.0125
375	1.689	.0477	2800	6.644	.0121
400	1.750	.0461	2900	6.847	.0118
450	1.852	.0436	3000	7.051	.0114

On the Compression and Expansion of a Definite Weight of Air Enclosed in a Vessel.

In this treatment no heat must be lost or gained by radiation from the sides of the vessel in which the air is enclosed. Let D and d represent the degrees of absolute temperatures of volumes, v and V, of the air to be experimented upon.

The absolute zero is 461° below Fahr. zero and 273° Cent. below the freezing-point of water. $D = 461 + T$, $d = 461 + t$, and $D - d = T - t$, Fahr. scale.

Volume and Temperature.

$$\frac{V}{v} = \left(\frac{D}{d}\right)^{2.45}, \quad \text{and} \quad \frac{v}{V} = \left(\frac{d}{D}\right)^{2.45}.$$

Expansion, $V = v\left(\frac{D}{d}\right)^{2.45}$; compression, $v = \left(\frac{d}{D}\right)^{2.45}$.

Compression, $D = d\sqrt[2.45]{\frac{V}{v}}$; expansion, $d = D\sqrt[2.45]{\frac{V}{v}}$.

Example. To what fraction must air of $t = 65°$ be compressed, in order to fire tinder at a temperature of $T = 550°$, $d = 461 + 65 = 526°$, $D = 550 + 461 = 1011°$?

Formula. $\frac{v}{V} = \left(\frac{526}{1011}\right)^{2.45} = 0.20$, the answer.

Example. How much must air of $T = 80°$ be expanded to reduce the temperature to $t = 32°$, or freezing-point of water?

Formula. $\frac{V}{v} = \left(\frac{541}{493}\right)^{2.45} = 1.3308$ times, the answer.

Example. $v = 360$ cubic inches of air, of temperature, $T = 380°$ or $D = 841°$, is to be expanded until the temperature becomes $t = 80°$ or $d = 541°$. Required the volume, V, corresponding to that temperature?

Formula. $V = 360\left(\frac{821}{541}\right)^{2.45} = 1025.9$ cubic feet.

Example. $V = 20$ cubic feet of air, of $t = 32°$ or $d = 493°$, is to be compressed to $v = 12$ cubic feet. Required the temperature, t, of compression?

Formula. $D = 493\sqrt[2.45]{\frac{20}{12}} = 607.29°$ or $T = 146.29°$.

Pressure and Temperature.

$$\frac{P}{p} = \left(\frac{D}{d}\right)^{3.42}, \quad \text{and} \quad \frac{p}{P} = \left(\frac{D}{d}\right)^{3.42}.$$

Compression, $P = p\left(\frac{D}{d}\right)^{3.42}$; expansion, $p = P\left(\frac{d}{D}\right)^{3.42}$.

$$\text{Compression, } D = d\sqrt[3.42]{\frac{P}{p}}\,; \qquad \text{expansion, } p = D\sqrt[3.42]{\frac{p}{P}}.$$

Example. A volume of air, of pressure, $p = 15$ pounds to the square inch, and of temperature, $t = 62°$, is to be compressed until the temperature becomes $T = 120°$. Required the pressure, P, per square inch, at $T = 120°$?

$$d = 461 + 62 = 523, \quad \text{and} \quad D = 461 + 120 + 581.$$

Formula. $$P = 15\left(\frac{581}{523}\right)^{3.42} = 21.49 \text{ pounds per square inch.}$$

Example. A volume of air, of pressure, $P = 45$ pounds to the square inch, and of temperature, $T = 250°$ or $D = 711°$, is to be expanded to a pressure of $p = 25$ pounds. Required the temperature, t, of the expanded air?

Formula. $$d = 711\sqrt[3.42]{\frac{25}{45}} = 598.72°, \text{ and}$$

$$t = 598.72 - 461 = 137.72°, \text{ the temperature required.}$$

Pressure and Volume.

$$\sqrt[.41]{\frac{V}{v}} = \sqrt[.29]{\frac{p}{P}}, \quad \text{and} \quad \sqrt[.41]{\frac{v}{V}} = \sqrt[.29]{\frac{p}{P}}.$$

$$\text{Expansion, } \quad V = v\sqrt[1.4]{\frac{P}{p}}\,; \qquad \text{compression, } v = V\sqrt[1.4]{\frac{p}{P}}.$$

$$\text{Compression, } P = p\left(\frac{V}{v}\right)^{1.4}; \quad \text{expansion, } \quad p = P\left(\frac{v}{V}\right)^{1.4}.$$

Example. A volume, $v = 50$ cubic inches, and of pressure, $P = 80$ pounds per square inch, is to be expanded until the pressure becomes $p = 15$ pounds. Required the expanded volume, V?

Formula. $$V = 50\sqrt[1.4]{\frac{80}{15}} = 165 \text{ cubic inches.}$$

Example. What will be the pressure of a volume of air expanded 1.3308 times?

Formula. $$p = \left(\frac{1}{1.3308}\right)^{1.4} = 0.5324 \text{ of the primitive pressure.}$$

In the compression and expansion of air, as given in the following table, it is supposed that no heat is transmitted to or from the air operated upon. In compression, the temperature of the air rises; and if the heat is allowed to be conducted through the sides of the vessel enclosing the air, the pressure will not correspond with the table. In expanding the air the temperature is lowered, as seen in the table. The primitive volume is assumed to be at 32° F.

Compression and Expansion of Air.

Compression of air.				Expansion of air.			
Volume. $v = 1.$	Temperature, Fahr. Degrees.	Pressure.		Volume. $v = 1.$	Temperature, Fahr. Degrees.	Pressure.	
		Atmosphere.	Pounds per square inch.			Atmosphere.	Pounds per square inch.
V	T	A	P	V	T	A	P
1.000	32.00	1.0000	14.700	1.00	+ 32.00	1.00000	14.7000
.950	42.43	1.0297	15.137	1.10	+ 13.20	.87510	12.8640
.900	53.66	1.159	17.036	1.20	— 3.30	.77470	11.3930
.850	65.81	1.255	18.456	1.30	— 18.06	.69260	10.1810
.800	79.01	1.366	20.090	1.40	— 31.26	.62430	9.1778
.750	93.43	1.496	21.991	1.50	— 39.65	.58354	8.5780
.700	109.26	1.647	24.215	1.60	— 54.06	.5179	7.6130
.650	126.77	1.828	26.561	1.70	— 64.00	.4757	6.9934
.600	146.30	2.044	30.054	1.80	— 73.16	.4391	6.4556
.550	168.25	2.309	33.948	1.90	— 82.34	.4083	6.0020
.500	193.20	2.639	38.792	2.00	— 89.47	.3789	5.5700
.450	221.96	3.058	44.547	2.25	—106.90	.3213	4.7235
.400	245.70	3.607	53.020	2.50	—121.83	.2779	4.0851
.350	295.73	4.348	63.917	2.75	—134.77	.2426	3.5666
.330	314.10	4.721	69.406	3.00	—146.15	.2148	3.1576
.300	344.87	5.396	79.313	3.25	—156.27	.1920	2.8228
.250	407.13	6.964	102.38	3.50	—167.29	.1731	2.5446
.200	489.91	9.518	139.92	3.75	—173.57	.1572	2.3103
.150	606.4	14.240	209.31	4.0	—181.00	.1436	2.1111
.125	691.0	18.380	270.17	4.5	—194.18	.1218	1.7900
.10	800.9	25.120	369.24	5.0	—205.40	.1051	1.5444
.05	1213.5	66.289	974.45	6.0	—223.74	.0813	1.1965
.04	1373.2	90.60	1331.8	7.0	—238.20	.0656	.9642
.03	1601.7	135.53	1992.3	8.0	—250.03	.0544	.7998
.02	1973.0	239.09	3514.6	9.0	—259.92	.0461	.6782
.01	4469.0	794.33	11676.0	10.0	—268.39	.0355	.5216

The above table shows the necessity for taking into account the heat produced in air compressors. If the cylinders and valve chests are not sufficiently cooled there is danger of explosion from the air carburetted by the lubricant. High compression in gas engines is limited by the production of a temperature sufficient to cause a premature ignition of the charge. In the Diesel motor the air is compressed to about 30 atmospheres, this giving a temperature of about 875° F., supposing no cooling to occur. This is ample to ignite the heaviest oil injected into the cylinder.

Table of Volumes of Air Transmitted, in Cubic Feet, per Minute in Pipes of Various Dimensions.

Velocity of flow, in feet, per second.	Actual diameter of pipe, in inches.											
	1	2	3	4	5	6	8	10	12	16	20	24
1	.33	1.31	2.95	5.2	8.2	11.8	20.9	32.7	47.1	83.8	131	188
2	.65	2.62	5.89	10.5	16.4	23.6	41.9	65.4	94.2	167.5	262	377
3	.98	3.93	8.84	15.7	24.5	35.3	62.8	98.2	141.4	251.3	393	565
4	1.31	5.24	11.78	20.9	32.7	47.1	83.8	131.0	188.0	335.0	523	754
5	1.64	6.55	14.7	26.2	41.0	59.0	104.0	163.0	235.0	419.0	654	942
6	1.96	7.85	17.7	31.4	49.1	70.7	125.0	196.0	283.0	502.0	785	1131
7	2.29	9.16	20.6	36.6	57.2	82.4	146.0	229.0	330.0	586.0	916	1319
8	2.62	10.50	23.5	41.9	65.4	94.0	167.0	262.0	377.0	670.0	1047	1508
9	2.95	11.78	26.5	47.0	73.0	106.0	188.0	294.0	424.0	754.0	1178	1696
10	3.27	13.1	29.4	52.0	82.0	118.0	209.0	327.0	471.0	838.0	1309	1885
12	3.93	15.7	35.3	63.0	99.0	141.0	251.0	393.0	565.0	1005.0	1571	2262
15	4.91	19.6	44.2	78.0	122.0	177.0	314.0	491.0	707.0	1256.0	1963	2827
18	5.89	23.5	53.0	94.0	147.0	212.0	377.0	589.0	848.0	1508.0	2356	3393
20	6.55	26.2	59.0	105.0	164.0	235.0	419.0	654.0	942.0	1675.0	2618	3770
24	7.86	31.4	71.0	125.0	196.0	283.0	502.0	785.0	1131.0	2010.0	3141	4524
25	8.18	32.7	73.0	131.0	204.0	294.0	523.0	818.0	1178.0	2094.0	3272	4712
28	9.16	36.6	82.0	146.0	229.0	330.0	586.0	916.0	1319.0	2346.0	3665	5278
30	9.80	39.3	88.0	157.0	245.0	353.0	628.0	982.0	1414.0	2513.0	3927	5655

Velocity of Escaping Compressed Air.

(Hiscox.)

Pressure, in atmospheres.	Pressure, in inches, of mercury.	Pressure, in pounds, per square inch.	Theoretical velocity, in feet, per second.	Pressure, in atmospheres.	Pressure, in inches, of mercury.	Pressure, in pounds, per square inch.	Theoretical velocity, in feet, per second.
.010	.30	.147	94.4	.680	20.40	10.0	780
.066	2.10	1.00	246.0	.809	24.28	12.0	855
.100	3.00	1.47	299.0	1.0	30.0	14.7	946
.136	4.08	2.00	348.0	2.0	60.0	29.4	1094
.204	6.12	3.00	472.0	5.0	150.0	73.5	1219
.272	8.16	4.00	493.0	10.0	300.0	147.0	1275
.340	10.20	5.00	552.0	20.0	600.0	294.0	1304
.408	12.24	6.00	604.0	40.0	1200.0	588.0	1323
.500	15.00	7.35	673.0	100.0	3000.0	1470.0	1331
.544	16.32	8.0	697.0	200.0	6000.0	2940.0	1334
.611	18.34	9.0	741.0				

The theoretical velocities of efflux of compressed air. as given in the above table, are to be reduced by multiplying by the coefficient of actual

discharge, the coefficient varying according to the nature of the orifice and the air pressure.

The following coefficients will serve in practice:

Coefficients of Air Discharge.

	Pressures, in atmospheres.						
	.01	.1	.5	1	5	10	100
Orifice in thin plate	.65	.64	.57	.54	.45	.436	.423
Orifice in short tube	.834	.82	.71	.67	.53	.51	.487

Thus, for a pressure of 5 atmospheres, or 73.5 pounds per square inch, the theoretical efflux would be 1219 feet per second. The actual efflux through a hole in a thin plate would be

$$1219 \times 0.45 = 548.55 \text{ feet per second};$$

and through a short tube,

$$1219 \times 0.53 = 646.07 \text{ feet per second.}$$

The work required to compress a cubic foot of air to any desired pressure may be obtained as follows:

Let

$P =$ the initial pressure, usually 14.7 pounds;
$p =$ final pressure required;
$S =$ initial pressure per square foot (for 14.7 pounds per square inch = 2116.8 pounds per square foot);
$W =$ required work of compression, in foot-pounds;

$$W = S \text{ hyp. log. } \frac{p}{P}.$$

This is true for isothermal compression only, in which the heat of compression is removed as rapidly as produced, so that a constant temperature is maintained. For adiabatic compression, in which all the heat is retained, the work is much greater. In actual practice the power is about midway between the two.

Foot-pounds of Work Required to Compress Air.

(Hiscox.)

Initial pressure = 1 atmosphere.

Pressure, in pounds, per square inch.	Foot-pounds per cubic foot.			Pressure, in pounds, per square inch.	Foot-pounds per cubic foot.		
	Isothermal.	Adiabatic.	Actual.		Isothermal.	Adiabatic.	Actual.
5	619.6	649.5	637.5	55	3393.7	4188.9	3870.8
10	1098.2	1192.0	1154.6	60	3440.4	4422.8	4029.8
15	1488.3	1661.2	1592.0	65	3577.6	4645.4	4218.2
20	1817.7	2074.0	1971.4	70	3706.3	4859.6	4398.1
25	2102.6	2451.6	2312.0	75	3828.0	5063.9	4569.5
30	2353.6	2794.0	2617.8	80	3942.9	5259.7	4732.9
35	2578.0	3111.0	2897.8	85	4051.5	5450.0	4890.1
40	2780.8	3405.5	3155.6	90	4155.7	5633.1	5042.1
45	2966.0	3681.7	3395.4	95	4254.3	5819.3	5187.3
50	3136.2	3942.3	3619.8	100	4348.1	5981.2	5327.9

Compressor Efficiencies at Different Altitudes.

70 Pounds Pressure per Square Inch.

Altitude. Feet.	Barometric pressure.		Volumetric efficiency of compressor. Per cent.	Loss of capacity. Per cent.	Decreased power. Per cent.
	Inches of mercury.	Pounds per square inch.			
Sea-level.	30.00	14.75	100	....	....
1000	28.88	14.20	97	3	1.8
2000	27.80	13.67	93	7	3.5
3000	26.76	13.16	90	10	5.2
4000	25.76	12.67	87	13	6.9
5000	24.79	12.20	84	16	8.5
6000	23.86	11.73	81	19	10.1
7000	22.97	11.30	78	22	11.6
8000	22.11	10.87	76	24	13.1
9000	21.29	10.46	73	27	14.6
10000	20.49	10.07	70	30	16.1
11000	19.72	9.70	68	32	17.6
12000	18.98	9.34	65	35	19.1
13000	18.27	8.98	63	37	20.6
14000	17.59	8.65	60	40	22.1
15000	16.93	8.32	58	42	23.5

The above table is computed for a delivery of air compressed to 70 pounds per square inch. For pressures above 70 pounds the volumetric efficiency of the compressor may be decreased 3 per cent. for each 10 pounds, and the power required diminished 10 per cent.

Cubic Feet of Air Required, per Indicated Horse-power, in Motors.

(Hiscox.)

Point of cut-off.	Gauge pressures.										
	30	40	50	60	70	80	90	100	110	125	150
1	23.30	21.3	20.2	19.40	18.80	18.42	18.10	17.80	17.62	17.40	17.05
¾	18.70	17.1	16.1	15.47	15.00	14.60	14.35	14.15	13.98	13.78	13.50
⅔	17.85	16.2	15.2	14.50	14.20	13.75	13.47	13.28	13.08	12.90	12.60
½	16.4	14.5	13.5	12.80	12.30	11.93	11.70	11.48	11.30	11.10	10.85
⅓	17.5	15.2	12.9	11.85	11.26	10.80	10.50	10.21	10.02	9.78	9.5
¼	20.6	15.6	13.4	13.3	11.4	10.72	10.31	10.0	75	9.42	9.1

In applying this table the amount must be increased to provide for the clearance of the cylinder, this depending upon the construction of the motor. When the air is reheated the amount required will be diminished. The economy due to reheating will be proportional to the increase in absolute temperature. Thus, if T be the initial temperature of the air, and T' the reheated temperature, we have the amount of air required, equal to the tabular amount, multiplied by

$$\frac{T + 461}{T' + 461}.$$

Thus, if the air be reheated from 60° to 300° F., the tabular value should be multiplied by

$$\frac{60 + 461}{300 + 461} = 0.684,$$

showing a gain of nearly 32 per cent., due to reheating.

The flow of compressed air in pipes may be computed from the formula:

$$Q = c\sqrt{\frac{pd^5}{wL}},$$

in which

Q = flow, in cubic feet, per minute;
p = difference in pressure, in pounds, per square inch, by which the flow is caused;
d = the diameter of the pipe, in inches;
L = the length, in feet;
w = the density of the entering air, in pounds, per cubic foot; and
c = a constant coefficient. According to Halsey, the value of c may be taken as = 58.

Table of Head or Additional Pressure Required to Deliver Air at 80 Pounds Gauge Pressure through 1000 Feet of Pipe of Various Sizes.

Pipe Sizes are Inside Diameters.

Velocity, in feet, per second.	Cubic feet of free air per minute.	Additional pressure.	Velocity, in feet, per second.	Cubic feet of free air per minute.	Additional pressure.	Velocity, in feet, per second.	Cubic feet of free air per minute.	Additional pressure.
	1 inch.			4 inches.			12 inches.	
3.07	6	.337	3.07	88	.031	3.07	799	.0067
6.14	12	1.348	6.14	176	.124	6.14	1598	.0268
9.20	18	3.033	9.20	264	.279	9.20	2397	.0603
12.27	24	5.392	12.27	352	.495	12.27	3196	.1072
15.34	30	8.425	15.34	440	.775	15.34	3995	.1675
18.41	36	12.132	18.41	528	1.116	18.41	4794	.2412
24.54	48	21.568	24.54	704	1.984	24.54	6392	.4288
30.68	60	33.700	30.68	880	3.100	30.68	7990	.6700
	1½ inches.			5 inches.			14 inches.	
3.07	14	.134	3.07	141	.022	3.07	1087	.0055
6.14	29	.536	6.14	282	.088	6.14	2174	.0220
9.20	43	1.206	9.20	423	.198	9.20	3261	.0495
12.27	57	2.144	12.27	564	.352	12.27	4348	.0880
15.34	72	3.350	15.34	705	.550	15.34	5435	.1375
18.41	86	4.824	18.41	846	.792	18.41	6522	.1980
24.54	115	8.576	24.54	1128	1.408	24.54	8696	.3520
30.68	144	13.400	30.68	1410	2.200	30.68	10870	.5500
	2 inches.			6 inches.			16 inches.	
3.07	23	.100	3.07	204	.018	3.07	1420	.0047
6.14	47	.400	6.14	408	.072	6.14	2840	.0188
9.20	70	.900	9.20	612	.162	9.20	4260	.0423
12.27	94	1.600	12.27	816	.288	12.27	5680	.0752
15.34	118	2.500	15.34	1020	.450	15.34	7100	.1175
18.41	141	3.600	18.41	1224	.648	18.41	8520	.1692
24.54	188	7.200	24.54	1632	1.152	24.54	11360	.3009
30.68	235	10.000	30.68	2040	1.800	30.68	14200	.4700
	2½ inches.			8 inches.			20 inches.	
3.07	33	.058	3.07	353	.011	3.07	2219	.0036
6.14	67	.232	6.14	706	.044	6.14	4438	.0144
9.20	100	.522	9.20	1059	.099	9.20	6657	.0324
12.27	134	.928	12.27	1412	.176	12.27	8876	.0576
15.34	168	1.450	15.34	1765	.275	15.34	11095	.0900
18.41	201	2.088	18.41	2118	.336	18.41	13314	.1296
24.54	268	3.712	24.54	2824	.704	24.54	17752	.2304
30.68	335	5.800	30.68	3530	1.100	30.68	22190	.3600
	3 inches.			10 inches.			24 inches.	
3.07	52	.050	3.07	566	.0087	3.07	3194	.0029
6.14	104	.200	6.14	1132	.0348	6.14	6388	.0116
9.20	156	.450	9.20	1698	.0783	9.20	9582	.0261
12.27	208	.800	12.27	2264	.1392	12.27	12776	.0464
15.34	260	1.250	15.34	2830	.2175	15.34	15970	.0725
18.41	312	1.800	18.41	3396	.3132	18.41	19164	.1044
24.54	416	3.200	24.54	4528	.5568	24.54	25552	.1856
30.68	520	5.000	30.68	5660	.8700	30.68	31940	.2900

Movement of Air.

When large volumes of air are to be moved at low pressures, as in ventilation, mechanical draft, etc., the following formulas, derived from Weisbach, by Snow, for the B. F. Sturtevant Company, may be used:

Let

d = diameter of pipe, in inches;
l = length of pipe, in feet;
v = velocity, in feet, per second;
p = loss of pressure, in ounces, per square inch, by friction.

Then

$$p = \frac{lv^2}{25000d},$$

$$l = \frac{25000dp}{v^2},$$

$$v = \frac{\sqrt{25000dp}}{l},$$

$$d = \frac{lv^2}{25000p}.$$

If we call the area of the pipe = A, and take the weight of a cubic foot of air as 0.08 pound, we have, for the loss in horse-power by friction in a length of 100 feet,

$$HP = \frac{pAv}{8800}.$$

From these formulas the following tables have been computed for pipes of various diameters, all 100 feet long, the losses being directly proportioned for pipes of other lengths. Since the loss in pressure varies as the square of the velocity, the advantage of using large pipes and reducing the velocity is apparent.

The whole subject of the compression, utilization, and movement of air is a most extensive one. For further details of the different departments of the subject reference may be had to the following works:

"Mechanical Draft." By Walter B. Snow.
"Compressed Air and its Applications." By Gardner D. Hiscox.
"Compressed Air Information." By W. L. Saunders.
"Compressed Air." By Frank Richards.

The monthly periodical, "Compressed Air," also contains current information of value and interest.

Pressure and Horse-power Lost by Friction of Air in Pipes 100 Feet Long.

Velocity of air, in feet, per minute.	Diameter of pipes.													
	1 inch.		2 inches.		3 inches.		4 inches.		5 inches.		6 inches.		7 inches.	
	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.
600	.400	.0004	.200	.0007	.133	.0011	.100	.0014	.080	.0018	.067	.0021	.057	.0025
700	.544	.0006	.272	.0011	.181	.0017	.138	.0023	.109	.0028	.091	.0034	.078	.0040
800	.711	.0008	.356	.0017	.237	.0025	.178	.0034	.142	.0042	.119	.0051	.102	.0059
900	.900	.0012	.450	.0024	.300	.0036	.225	.0048	.180	.0060	.150	.0072	.129	.0084
1000	1.111	.0017	.556	.0033	.370	.0049	.278	.0066	.222	.0083	.185	.0099	.159	.0116
1100	1.344	.0022	.672	.0044	.448	.0066	.338	.0088	.269	.0110	.224	.0132	.192	.0164
1200	1.600	.0029	.800	.0057	.533	.0086	.400	.0114	.320	.0143	.267	.0171	.229	.0200
1300	1.878	.0036	.939	.0073	.626	.0109	.469	.0145	.376	.0182	.313	.0218	.268	.0254
1400	2.178	.0045	1.089	.0090	.726	.0136	.544	.0181	.436	.0227	.363	.0272	.311	.0317
1500	2.500	.0056	1.250	.0112	.833	.0167	.625	.0223	.500	.0279	.417	.0335	.357	.0390
1600	2.844	.0068	1.422	.0135	.948	.0203	.711	.0271	.569	.0338	.474	.0406	.406	.0473
1700	3.211	.0081	1.605	.0162	1.070	.0224	.803	.0325	.642	.0406	.535	.0487	.459	.0568
1800	3.600	.0096	1.800	.0192	1.200	.0289	.900	.0385	.720	.0482	.600	.0578	.514	.0674
1900	4.011	.0113	2.006	.0227	1.337	.0340	1.003	.0453	.802	.0567	.669	.0680	.573	.0793
2000	4.444	.0132	2.222	.0264	1.481	.0397	1.111	.0529	.889	.0661	.741	.0793	.635	.0925
2200	5.378	.0176	2.689	.0352	1.793	.0528	1.344	.0704	1.076	.0880	.896	.1161	.683	.1232
2400	6.400	.0228	3.200	.0457	2.133	.0685	1.600	.0914	1.280	.1142	1.067	.1371	.914	.1600
2600	7.511	.0290	3.756	.0581	2.504	.0871	1.877	.1162	1.502	.1452	1.252	.1743	1.073	.2033
2800	8.711	.0363	4.356	.0726	2.904	.1088	2.178	.1451	1.742	.1814	1.452	.2177	1.244	.2539
3000	10.000	.0446	5.000	.0892	3.333	.1339	2.500	.1785	2.000	.2231	1.667	.2677	1.429	.3123
3200	11.378	.0541	5.689	.1083	3.792	.1625	2.844	.2166	2.276	.2708	1.896	.3249	1.625	.3790
3400	12.844	.0649	6.422	.1299	4.281	.1949	3.211	.2598	2.569	.3247	2.141	.3897	1.835	.4546
3600	14.400	.0770	7.200	.1542	4.800	.2313	3.600	.3084	2.880	.3855	2.400	.4626	2.057	.5397
3800	16.044	.0906	8.022	.1814	5.349	.2720	4.011	.3627	3.209	.4534	2.674	.5441	2.292	.6347
4000	17.778	.1058	8.889	.2115	5.926	.3173	4.444	.4230	3.556	.5288	2.963	.6346	2.540	.7403

Pressure and Horse-power Lost by Friction of Air in Pipes 100 Feet Long.—*Continued.*

Velocity of air, in feet, per minute.	8 inches. Loss of pressure, in ounces, per square inch.	8 inches. Horse-power lost in friction.	9 inches. Loss of pressure, in ounces, per square inch.	9 inches. Horse-power lost in friction.	10 inches. Loss of pressure, in ounces, per square inch.	10 inches. Horse-power lost in friction.	11 inches. Loss of pressure, in ounces, per square inch.	11 inches. Horse-power lost in friction.	12 inches. Loss of pressure, in ounces, per square inch.	12 inches. Horse-power lost in friction.	14 inches. Loss of pressure, in ounces, per square inch.	14 inches. Horse-power lost in friction.	16 inches. Loss of pressure, in ounces, per square inch.	16 inches. Horse-power lost in friction.
600	.050	.0029	.044	.0032	.040	.0036	.036	.0039	.033	.0043	.029	.0050	.025	.0057
700	.068	.0045	.060	.0051	.054	.0057	.049	.0062	.045	.0068	.039	.0079	.034	.0091
800	.089	.0067	.079	.0076	.071	.0085	.064	.0093	.059	.0102	.051	.0118	.044	.0135
900	.112	.0096	.100	.0108	.090	.0120	.081	.0132	.075	.0145	.064	.0169	.056	.0193
1000	.139	.0132	.123	.0148	.111	.0165	.101	.0182	.092	.0198	.079	.0231	.069	.0264
1100	.168	.0176	.149	.0198	.134	.0220	.122	.0242	.112	.0264	.096	.0308	.084	.0352
1200	.200	.0230	.178	.0256	.160	.0286	.145	.0314	.133	.0343	.114	.0400	.100	.0457
1300	.235	.0290	.209	.0327	.188	.0363	.170	.0399	.156	.0437	.134	.0508	.117	.0581
1400	.272	.0363	.242	.0408	.218	.0453	.198	.0499	.181	.0544	.156	.0635	.136	.0726
1500	.312	.0446	.278	.0502	.250	.0558	.227	.0613	.208	.0669	.179	.0781	.156	.0892
1600	.356	.0541	.316	.0609	.284	.0677	.259	.0735	.237	.0812	.203	.0948	.178	.1083
1700	.401	.0649	.357	.0731	.321	.0812	.292	.0893	.268	.0974	.229	.1137	.207	.1299
1800	.450	.0771	.400	.0867	.360	.0964	.327	.1060	.300	.1156	.257	.1351	.225	.1542
1900	.501	.0907	.446	.1020	.401	.1133	.365	.1247	.334	.1360	.287	.1587	.251	.1814
2000	.556	.1058	.493	.1190	.444	.1322	.404	.1454	.370	.1586	.317	.1851	.278	.2115
2200	.672	.1408	.597	.1583	.538	.1760	.489	.1936	.448	.2111	.384	.2463	.336	.2815
2400	.800	.1728	.711	.2056	.640	.2284	.582	.2513	.533	.2741	.457	.3198	.400	.3455
2600	.939	.2324	.835	.2614	.751	.2904	.683	.3195	.626	.3485	.537	.4066	.468	.4647
2800	1.089	.2902	.968	.3265	.871	.3628	.792	.3990	.726	.4353	.622	.5079	.544	.5804
3000	1.250	.3569	1.111	.4016	1.000	.4462	.909	.4908	.833	.5354	.714	.6245	.625	.7140
3200	1.422	.4332	1.263	.4873	1.138	.5415	1.034	.5956	.948	.6498	.813	.7581	.711	.8664
3400	1.606	.5237	1.427	.5845	1.284	.6495	1.168	.7144	1.070	.7794	.917	.9093	.827	1.0475
3600	1.800	.6168	1.600	.7039	1.440	.7710	1.309	.8481	1.200	.9252	1.029	1.0800	.900	1.2335
3800	2.006	.7254	1.783	.8171	1.604	.9068	1.459	.9974	1.337	1.0871	1.146	1.2695	1.003	1.4508
4000	2.222	.8461	1.975	.9518	1.778	1.0576	1.616	1.1734	1.481	1.2691	1.270	1.4807	1.111	1.6922

Pressure and Horse-power Lost by Friction of Air in Pipes 100 Feet Long.—*Continued.*

Velocity of air, in feet, per minute.	Diameter of pipes.													
	18 inches.		20 inches.		22 inches.		24 inches.		26 inches.		28 inches.		32 inches.	
	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.
600	.022	.0064	.020	.0071	.018	.0079	.017	.0086	.015	.0093	.014	.0100	.012	.0114
700	.030	.0102	.027	.0113	.025	.0125	.023	.0136	.021	.0147	.019	.0159	.017	.0181
800	.040	.0152	.036	.0169	.032	.0186	.029	.0203	.027	.0220	.025	.0237	.022	.0271
900	.050	.0217	.045	.0241	.041	.0265	.037	.0289	.035	.0313	.032	.0337	.028	.0385
1000	.062	.0297	.056	.0330	.051	.0364	.046	.0397	.043	.0430	.040	.0463	.035	.0569
1100	.075	.0396	.067	.0440	.061	.0484	.056	.0528	.052	.0572	.048	.0616	.042	.0704
1200	.089	.0512	.080	.0571	.073	.0628	.067	.0685	.062	.0742	.057	.0800	.050	.0914
1300	.104	.0654	.094	.0726	.085	.0799	.078	.0873	.072	.0944	.067	.1017	.058	.1162
1400	.121	.0816	.109	.0907	.099	.0988	.091	.1088	.084	.1179	.078	.1270	.068	.1451
1500	.139	.1004	.125	.1115	.114	.1227	.104	.1339	.096	.1450	.089	.1562	.078	.1785
1600	.158	.1218	.142	.1354	.129	.1469	.119	.1624	.109	.1760	.102	.1895	.089	.2166
1700	.178	.1461	.161	.1624	.146	.1786	.134	.1948	.124	.2110	.115	.2273	.103	.2598
1800	.200	.1735	.180	.1927	.164	.2120	.150	.2313	.139	.2506	.129	.2702	.112	.3084
1900	.223	.2040	.201	.2267	.182	.2494	.167	.2720	.154	.2945	.143	.3174	.125	.3627
2000	.247	.2380	.222	.2644	.202	.2908	.185	.3173	.171	.3437	.159	.3702	.139	.4230
2200	.299	.3167	.269	.3519	.244	.3871	.224	.4223	.207	.4575	.192	.4927	.168	.5631
2400	.356	.4112	.320	.4569	.291	.5026	.267	.5483	.246	.5939	.239	.6396	.200	.6910
2600	.417	.5228	.376	.5809	.341	.6390	.313	.6971	.289	.7552	.268	.8132	.234	.9294
2800	.484	.6530	.436	.7255	.396	.7981	.363	.8706	.335	.9432	.311	1.0157	.272	1.1608
3000	.556	.8031	.500	.8923	.455	.9816	.417	1.0708	.385	1.1601	.357	1.2490	.312	1.4280
3200	.632	.9747	.569	1.0830	.517	1.1913	.474	1.2996	.438	1.4079	.406	1.5162	.356	1.7328
3400	.714	1.1690	.642	1.2990	.584	1.4289	.535	1.5588	.494	1.6882	.459	1.8186	.413	2.0950
3600	.800	1.4078	.720	1.5420	.655	1.6962	.600	1.8504	.557	2.0046	.514	2.1620	.450	2.4671
3800	.891	1.6342	.802	1.8135	.729	1.9949	.669	2.1742	.617	2.3760	.573	2.5389	.501	2.9016
4000	.988	1.9037	.889	2.1152	.808	2.3467	.741	2.5382	.684	2.7498	.635	2.9613	.556	3.3843

Pressure and Horse-power Lost by Friction of Air in Pipes 100 Feet Long.—*Continued.*

Velocity of air, in feet, per minute.	Diameter of pipes. 36 inches.		40 inches.		44 inches.		48 inches.		52 inches.		56 inches.		60 inches.	
	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.	Loss of pressure, in ounces, per square inch.	Horse-power lost in friction.
600	.011	.0128	.010	.0143	.009	.0157	.008	.0171	.008	.0186	.007	.0200	.007	.0214
700	.015	.0204	.014	.0227	.012	.0249	.011	.0272	.010	.0295	.010	.0317	.009	.0340
800	.020	.0305	.018	.0339	.016	.0372	.015	.0406	.014	.0440	.013	.0474	.012	.0508
900	.025	.0434	.022	.0482	.020	.0530	.019	.0578	.017	.0626	.016	.0675	.015	.0723
1000	.031	.0595	.028	.0661	.025	.0727	.023	.0793	.021	.0859	.020	.0952	.019	.0991
1100	.035	.0792	.034	.0880	.030	.0968	.028	.1055	.026	.1144	.024	.1232	.022	.1320
1200	.044	.1024	.040	.1142	.036	.1256	.033	.1371	.031	.1485	.029	.1599	.027	.1713
1300	.052	.1307	.047	.1452	.043	.1597	.039	.1746	.036	.1888	.034	.2033	.031	.2178
1400	.060	.1632	.054	.1814	.049	.1995	.045	.2177	.042	.2360	.039	.2539	.036	.2721
1500	.069	.2008	.062	.2231	.057	.2454	.052	.2677	.048	.2900	.045	.3123	.042	.3346
1600	.079	.2437	.071	.2707	.069	.2938	.059	.3249	.055	.3520	.051	.3790	.047	.4061
1700	.089	.2923	.080	.3247	.073	.3572	.067	.3897	.062	.4220	.057	.4546	.054	.4871
1800	.100	.3469	.090	.3855	.082	.4240	.075	.4626	.069	.5011	.064	.5406	.060	.5782
1900	.111	.4080	.103	.4534	.091	.4987	.084	.5440	.077	.5890	.072	.6347	.067	.6800
2000	.123	.4759	.111	.5288	.101	.5817	.093	.6346	.085	.6874	.079	.7403	.074	.7932
2200	.149	.6334	.134	.7038	.122	.7742	.112	.8446	.103	.9150	.096	.9854	.090	1.1607
2400	.178	.8224	.160	.9138	.145	1.0051	.133	1.0965	.123	1.1879	.119	1.2793	.107	1.3706
2600	.209	1.0456	.188	1.1618	.171	1.2779	.156	1.3941	.144	1.5103	.134	1.6265	.125	1.7427
2800	.242	1.3059	.218	1.4510	.198	1.5961	.181	1.7412	.168	1.8864	.156	2.0314	.145	2.1765
3000	.278	1.6062	.250	1.7847	.227	1.9632	.208	2.1416	.192	2.3201	.179	2.4979	.167	2.6771
3200	.316	1.9494	.284	2.1660	.259	2.3826	.237	2.5992	.219	2.8157	.203	3.0324	.190	3.2489
3400	.357	2.3382	.321	2.5980	.292	2.8578	.268	3.1176	.247	3.3764	.229	3.6372	.214	3.8970
3600	.400	2.8156	.360	3.0840	.327	3.3924	.300	3.7007	.279	4.0091	.257	4.3239	.240	4.6259
3800	.446	3.2683	.401	3.6270	.365	3.9897	.334	4.3484	.309	4.7519	.287	5.0779	.267	5.4406
4000	.494	3.8074	.444	4.2304	.404	4.6934	.370	5.0765	.342	5.4995	.317	5.9226	.296	6.3456

Determination of Difference in Level by Difference in Atmospheric Pressure.

According to the law of Mariotte, also called Boyle's law, the volume of a given quantity of any gas varies inversely as the pressure which it bears, the temperature remaining constant. The average pressure of the atmosphere at the sea-level is 1.033 kilogrammes per square centimetre, corresponding to a column of mercury 760 millimetres in height. This is the same as 14.7 pounds per square inch, or a mercury column of 29.92 inches. In English-speaking countries an *atmosphere* of pressure is understood to mean 14.7 pounds per square inch, but in countries in which the metric system is used an atmosphere means a pressure of 1 kilogramme per square centimetre = 14.22 pounds per square inch. This is sometimes called a *metric atmosphere*, and pressure gauges in France, Germany, and elsewhere on the Continent are generally graduated in *metric atmospheres* and tenths.

In obedience to the law of Mariotte, the density of the air diminishes as we ascend, and the law of this reduction in pressure has been found to be proportional to the logarithms of the pressures at any two points under consideration. Thus, if b be the height of the barometer at any given point, and b' the height of the barometer at another point, and h the difference in altitude between them, we have

$$h = C \log. \frac{b}{b'},$$

C being a constant.

For a difference of height, in metres, we have

$$h = 18429.1 \log. \frac{b}{b'},$$

and for feet,

$$h = 60463.4 \log. \frac{b}{b'}.$$

It is necessary to correct results obtained by these formulas for the effects of varying temperatures and other atmospheric conditions. If we assume the mean temperature of the air between two stations to be the half sum of the temperatures at these stations, we may take the coefficient of expansion of air, $\frac{1}{273} = 0.00366$ per degree centigrade, and hence have for a temperature correction factor,

$$0.00366 \frac{t + t'}{2} = 0.00183(t + t'),$$

in which t and t' are the temperatures of the two stations, in degrees centigrade. For the Fahrenheit thermometer, taking the coefficient of expansion at $\frac{1}{491} = 0.002036$, and remembering that the freezing-point is 32° above zero, we have

$$0.002036 \frac{(t - 32) + (t' - 32)}{2} = 0.00102(t + t' - 64).$$

If we take the sea-level as a base station we may compute the altitudes for various barometric readings, and thus enable altitudes to be readily and accurately measured. With such tables the altitude of each station above sea-level may be computed separately, and their difference taken.

Barometric Table A.

Metric.

Normal Heights. 0° C.

$$18429.1 \log. \frac{760}{b}.$$

B. mm.	H. metres.	Difference.	B. mm.	H. metres.	Difference.
400	5137.2	—19.8	600	1892.0	—13.2
410	4939.6	—19.3	610	1759.8	—13.0
420	4746.7	—18.8	620	1629.6	—12.8
430	4558.3	—18.4	630	1501.5	—12.6
440	4374.4	—18.0	640	1375.5	—12.6
450	4294.5	—17.6	650	1251.4	—12.2
460	4018.6	—17.2	660	1129.1	—12.0
470	3846.4	—16.8	670	1008.8	—11.9
480	3677.9	—16.5	680	890.2	—11.7
490	3512.8	—16.2	690	773.3	—11.5
500	3351.2	—15.9	700	658.2	—11.4
510	3192.7	—15.5	710	544.6	—11.3
520	3037.3	—15.2	720	432.7	—11.0
530	2884.9	—15.0	730	322.4	—10.9
540	2735.2	—14.7	740	213.4	—10.7
550	2588.4	—14.4	750	105.9	—10.6
560	2444.2	—14.2	760	0.0	—10.5
570	2302.6	—13.9	770	—104.8	—10.3
580	2163.3	—13.7	780	—207.9	—10.2
590	2026.5	—13.3			

In Table A the first column contains the reading of the barometer, in millimetres, for every 10 millimetres, and in the second column the corresponding heights above sea-level. The third column, headed "Difference," contains the difference for every millimetre, so that the height can be obtained very correctly for barometer readings as close as the hundredth of a millimetre.

Suppose, now, that we have at one station a reading of 765 millimetres, and at another 732 millimetres, the air being at 0° C. We find in Table A, for a barometric weight of 760 millimetres, an altitude of 0.0 metres and $5 \times -10.5 = -52.5$ metres. Also, for 732 millimetres, we have 730 millimetres = 322.4 metres; and $2 \times -10.9 = -21.8$ metres, so that for 732 millimetres we have $322.4 - 21.8 = 300.6$ metres; hence, one station is 300.6 metres above sea-level, and the other is 52.5 metres below, and the difference in altitude is 353.1 metres. It will be noticed that the differences are negative, because the altitude diminishes as the height of the barometer increases, and hence we multiply the number of millimetres in excess of the reading in the first column by the tabular difference and *subtract* the product from the tabular altitude.

If the air had been at some other temperature than 0° a correction would have been necessary, and this may be readily applied by the following table.

Barometric Table B.

Temperature Correction Factors.

Centigrade.

$1 + 0.00183\,(t + t')$.

$t + t'$.	Factor.	$t + t'$.	Factor.	$t + t'$.	Factor.	$t + t'$.	Factor.
1	1.0018	15	1.0275	29	1.0531	43	1.0787
2	1.0037	16	1.0293	30	1.0549	44	1.0805
3	1.0055	17	1.0311	31	1.0567	45	1.0823
4	1.0073	18	1.0329	32	1.0586	46	1.0842
5	1.0091	19	1.0348	33	1.0604	47	1.0860
6	1.0110	20	1.0366	34	1.0622	48	1.0878
7	1.0128	21	1.0384	35	1.0640	49	1.0897
8	1.0146	22	1.0403	36	1.0659	50	1.0915
9	1.0164	23	1.0421	37	1.0677	51	1.0933
10	1.0183	24	1.0439	38	1.0696	52	1.0952
11	1.0201	25	1.0458	39	1.0714	53	1.0970
12	1.0220	26	1.0476	40	1.0732	54	1.0988
13	1.0238	27	1.0495	41	1.0750	55	1.1006
14	1.0257	28	1.0513	42	1.0769	56	1.1025

In this table t and t' are the temperatures of the two stations. By taking the factor opposite their sum and multiplying by it the result obtained from Table A, the corrected difference in altitude between the two stations will be obtained. Thus, in the example just given, suppose that the temperature at the lower station had been 22° C., and at the upper station 16° C., we have $22 + 16 = 38$; and opposite 38, in Table B, we find 1.0696. The corrected altitude will then be

$$353 \times 1.0696 = 377.5 \text{ metres.}$$

Table C is computed for English measures, the barometer readings being given in inches and tenths and the corresponding heights in feet above sea-level, the sea-level reading of the barometer being assumed as 30 inches. The column of differences here gives the differences in altitude for every hundredth of an inch, and so, if the difference be multiplied by the hundredths and thousandths, for any reading, and the product subtracted from the tabular altitude for the inches and tenths, it will give the precise altitude. An example will make this more readily understood.

Suppose one reading to be 29.832 inches, and the other 26.636 inches, we have, from Table C, for 29.8 inches, 176 feet, and the difference —8.8 multiplied by 32, the hundredths and thousandths, $= -8.8 \times 32 = -28.16$, and $176 - 28.16 = 147.84$ feet. Likewise, we have for 26.5 inches, from the table, 3257 feet, and the difference —9.9 multiplied by $36 = -35.64$, whence $3257 - 35.64 = 3221.36$ feet, and the difference in altitude between the two stations is

$$3221.36 - 147.84 = 3073.36 \text{ feet.}$$

Barometric Table C.

English.

Normal Heights. 32° F.

$$60463.4 \log. \frac{30}{b}.$$

B. inches.	H. feet.	Difference.	B. inches.	H. feet.	Difference.	B. inches.	H. feet.	Difference.
15.0	18201	—17.4	20.2	10386	—13.0	25.4	4371	—10.3
15.1	18027	—17.4	20.3	10256	—12.9	25.5	4268	—10.3
15.2	17853	—17.2	20.4	10127	—12.9	25.6	4165	—10.3
15.3	17681	—17.1	20.5	9998	—12.8	25.7	4062	—10.2
15.4	17510	—17.0	20.6	9871	—12.7	25.8	3960	—10.1
15.5	17340	—16.9	20.7	9744	—12.7	25.9	3859	—10.1
15.6	17171	—16.8	20.8	9617	—12.6	26.0	3758	—10.1
15.7	17003	—16.7	20.9	9491	—12 6	26.1	3657	—10.1
15.8	16836	—16.6	21.0	9366	—12.5	26.2	3556	—10.0
15.9	16670	—16.4	21.1	9241	—12.4	26.3	3456	—10.0
16.0	16506	—16.3	21.2	9117	—12.4	26.4	3356	— 9.9
16.1	16343	—16.3	21.3	8993	—12.3	26.5	3257	— 9.9
16.2	16180	—16.2	21.4	8869	—12.2	26.6	3158	— 9.9
16.3	16019	—16.1	21.5	8747	—12.1	26.7	3060	— 9.8
16.4	15858	—16.0	21.6	8626	—12.1	26.8	2962	— 9.8
16.5	15698	—15.8	21.7	8505	—12.1	26.9	2864	— 9.7
16.6	15540	—15.8	21.8	8384	—12.0	27.0	2767	— 9.7
16.7	15382	—15.7	21.9	8264	—12.0	27.1	2670	— 9.7
16.8	15225	—15.6	22.0	8144	—11.9	27.2	2573	— 9.6
16.9	15069	—15.5	22.1	8025	—11.9	27.3	2476	— 9.6
17.0	14914	—15.3	22.2	7906	—11.8	27.4	2380	— 9.5
17.1	14761	—15.3	22.3	7788	—11.7	27.5	2285	— 9.5
17.2	14607	—15.2	22.4	7671	—11.7	27.6	2190	— 9.5
17.3	14455	—15.1	22.5	7554	—11.7	27.7	2095	— 9.5
17.4	14304	—15.1	22.6	7438	—11.6	27.8	2000	— 9.4
17.5	14153	—15.0	22.7	7322	—11.6	27.9	1906	— 9.4
17.6	14004	—14.9	22.8	7206	—11.5	28.0	1812	— 9.4
17.7	13855	—14.8	22.9	7091	—11.5	28.1	1718	— 9.3
17.8	13707	—14.7	23.0	6977	—11.4	28.2	1625	— 9.3
17.9	13560	—14.7	23.1	6863	—11.4	28.3	1532	— 9.2
18.0	13413	—14.6	23.2	6750	—11.3	28.4	1439	— 9.2
18.1	13267	—14.5	23.3	6637	—11.3	28.5	1347	— 9.2
18.2	13123	—14.4	23.4	6524	—11.2	28.6	1255	— 9.2
18.3	12979	—14.3	23.5	6412	—11.1	28.7	1163	— 9.1
18.4	12836	—14.2	23.6	6301	—11.1	28.8	1072	— 9.1
18.5	12694	—14.2	23.7	6190	—11.1	28.9	981	— 9.1
18.6	12552	—14.1	23.8	6079	—11.0	29.0	890	— 9.0
18.7	12411	—14.0	23.9	5969	—11.0	29.1	800	— 9.0
18.8	12271	—13.9	24.0	5859	—10.9	29.2	710	— 9.0
18.9	12132	—13.8	24.1	5750	—10.9	29.3	620	— 9.0
19.0	11994	—13.8	24.2	5641	—10.8	29.4	530	— 8.9
19.1	11856	—13.7	24.3	5533	—10.8	29.5	441	— 8.9
19.2	11719	—13.7	24.4	5425	—10.8	29.6	352	— 8.8
19.3	11582	—13.6	24.5	5318	—10.7	29.7	264	— 8.8
19.4	11446	—13.4	24.6	5211	—10.6	29.8	176	— 8.8
19.5	11312	—13.4	24.7	5105	—10.6	29.9	88	— 8.8
19.6	11177	—13.3	24.8	4999	—10.6	30.0	0	— 8.7
19.7	11044	—13.3	24.9	4893	—10.6	30.1	— 87	— 8.7
19.8	10911	—13.2	25.0	4787	—10.5	30.2	—174	— 8.7
19.9	10779	—13.1	25.1	4683	—10.5	30.3	—261	— 8.7
20.0	10648	—13.1	25.2	4578	—10.4	30.4	—348	— 8.6
20.1	10516	—13.0	25.3	4474	—10.4	30.5	—434	

For the temperature correction the following may be used:

Barometric Table D.

Temperature Correction Factors.

Fahrenheit.

$$1 + 0.00102\,(t + t' - 64).$$

$t + t'$.	Factor.	$t + t'$.	Factor.	$t + t'$.	Factor.	$t + t'$.	Factor.
32	.9673	68	1.0041	102	1.0388	136	1.0735
34	.9697	70	1.0061	104	1.0408	138	1.0755
36	.9714	72	1.0082	106	1.0429	140	1.0776
38	.9735	74	1.0102	108	1.0450	142	1.0796
40	.9755	76	1.0122	110	1.0470	144	1.0817
42	.9776	78	1.0143	112	1.0490	146	1.0837
44	.9796	80	1.0163	114	1.0511	148	1.0858
46	.9816	82	1.0183	116	1.0531	150	1.0878
48	.9837	84	1.0204	118	1.0552	152	1.0898
50	.9857	86	1.0224	120	1.0572	154	1.0919
52	.9878	88	1.0245	122	1.0592	156	1.0939
54	9898	90	1.0265	124	1.0612	158	1.0960
56	.9918	92	1.0286	126	1.0633	160	1.0980
58	.9938	94	1.0306	128	1.0653	162	1.1000
60	.9959	96	1.0326	130	1.0674	164	1.1019
62	.9980	98	1.0347	132	1.0694	166	1.1039
64	1.0000	100	1.0368	134	1.0714	168	1.1060
66	1.0021						

Thus, if in the preceding example the temperatures at the two stations had been 65° F. and 43° F., we have 65 + 43 = 108, and opposite 108, in Table D, we find the correction factor, 1.045.

The corrected altitude will then be

$$3073.36 \times 1.045 = 3211.66,$$

an increase of more than 38 feet.

When observations are taken simultaneously, the preceding tables will enable altitudes to be computed with much accuracy. When but single observations are possible, the date and hour of the day should always be noted, as the simultaneous reading of the nearest weather bureau station may then be subsequently obtained, as well as its altitude, and the desired height thus computed.

For field work the aneroid barometer is undoubtedly the best. It should be carefully compared with the standard at the base station, both on leaving and returning, and the mean of the difference used as a base correction.

Aneroids are often marked "compensated, meaning that they are so constructed as to be unaffected by changes in their own temperature. This is rarely perfectly accomplished, as may be seen by warming or cooling the instrument. The best plan is to set the instrument, by means of the adjusting-screw at the back, so that it agrees with a standard mercurial barometer at 32°, and then warm the aneroid carefully up to about 70°, taking readings at every 10°. A correction table can then be prepared for use on subsequent occasions.

The complete barometric formula of Laplace includes corrections for atmospheric humidity and for the variations in the action of gravity, but these need be considered only in precise work for great differences in altitude. Full details of this work will be found in the Smithsonian Meteorological Tables.

The altitude scales engraved on the dials of some aneroid barometers are of little use, except for rough approximate work, and their use has done much to bring the barometric method into undeserved discredit.

WATER.

Water is composed of 1 part of hydrogen combined with 8 parts of oxygen, or more nearly, according to the determinations of Morley and of Rayleigh, its composition by weight is

Hydrogen, 2 atoms............................	2.00	or	11.186
Oxygen, 1 atom	15.88	or	88.814
	17.88		100.000

This gives 17.88 for the molecular weight in the gaseous state, but in the liquid state it is probably a multiple of this.

In the production of 1 kilogramme of water by the burning of hydrogen and oxygen 3830 calories are evolved.

Its specific heat is taken as unity, being the basis upon which the specific heats of solids and liquids are computed; but this specific heat is not constant, but varies with the temperature.

According to Dieterici, the specific heat at various temperatures, taking the specific heat at 0° C. as unity, varies as follows:

Specific Heat of Water.

0° C.	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
1.000	0.9943	0.9893	0.9872	0.9934	0.9995	1.0057	1.0120	1.0182	1.0244	1.0306

In regard to the density of water at various temperatures there has been a material difference of opinion among various authorities. The temperature of maximum density is 4° C. or 39.1° F., but the actual weight of a unit of volume at this temperature ranges, according to different authorities, from 62.379 pounds to 62.425 pounds. The tables on pages 520–523 are those computed by Nystrom from the experiments of Kopp, and may be accepted as being as accurate as any. In the metric system the litre is usually made equal to a kilogramme of water by weighing, thus practically determining the volume from the weight.

Temp. Fahr.	Volume 1 at 39°.	Units of heat. Per pound.	Units of heat. Per cubic foot.	Pounds per cubic foot.	Cubic feet per pound.	Temp. Cent.
32	1.000 109	.000 000 000	.000	62.381 000	.016 03046	.000
33	1.000 077	1.000 000 867	62.383	62.383 000	.016 02994	.555
34	1.000 055	2.000 000 545	124.77	62.384 000	.016 02956	1.111
35	1.000 035	3.000 001 609	187.16	62.385 871	.016 02927	1.666
36	1.000 020	4.000 034 680	249.55	62.386 791	.016 02904	2.222
37	1.000 009	5.000 062 940	311.99	62.387 493	.016 02886	2.777
38	1.000 002	6.000 102 410	374.33	62.387 930	.016 02874	3.333
39	1.000 000	7.000 154 550	436.72	62.388 055	.016 02871	3.888
40	1.000 002	8.000 220 760	499.12	62.387 930	.016 02874	4.444
41	1.000 009	9.000 302 340	561.51	62.387 493	.016 02886	5.000
42	1.000 019	10.000 400 560	623.89	62.386 869	.016 02902	5.555
43	1.000 034	11.000 516 630	686.28	62.385 933	.016 02926	6.111
44	1.000 053	12.000 651 750	748.66	62.384 748	.016 02956	6.666
45	1.000 077	13.000 807 040	811.03	62.383 251	.016 02994	7.222
46	1.000 104	14.000 983 620	873.40	62.381 567	.016 03038	7.777
47	1.000 136	15.001 326 000	935.70	62.379 571	.016 03088	8.333
48	1.000 171	16.001 405 000	997.77	62.377 388	.016 03146	8.888
49	1.000 211	17.001 651 800	1060.0	62.374 893	.016 03210	9.444
50	1.000 254	18.001 924 200	1122.8	62.372 212	.016 03278	10.000
51	1.000 302	19.002 223 000	1185.1	62.369 219	.016 03355	10.555
52	1.000 353	20.002 549 300	1248.0	62.366 039	.016 03437	11.111
53	1.000 408	21.002 924 100	1310.1	62.362 611	.016 03525	11.666
54	1.000 468	22.003 288 000	1372.3	62.358 871	.016 03621	12.222
55	1.000 531	23.003 702 400	1434.3	62.354 944	.016 03723	12.777
56	1.000 597	24.004 147 900	1496.4	62.350 831	.016 03828	13.333
57	1.000 668	25.004 625 600	1558.6	62.346 407	.016 03942	13.888
58	1.000 740	26.005 136 200	1620.9	62.341 921	.016 04057	14.444
59	1.000 819	27.005 680 800	1683.2	62.337 000	.016 04184	15.000
60	1.000 901	28.006 260 000	1745.5	62.331 893	.016 04316	15.555
61	1.000 986	29.006 874 900	1807.8	62.326 620	.016 04451	16.111
62	1.001 075	30.007 526 300	1870.1	62.321 059	.016 04594	16.666
63	1.001 167	31.008 214 900	1932.4	62.315 333	.016 04741	17.222
64	1.001 262	32.008 941 600	1994.4	62.309 420	.016 04894	17.777
65	1.001 362	33.009 707 300	2056.6	62.303 198	.016 05054	18.333
66	1.001 464	34.010 513	2118.7	62.296 852	.016 05218	18.888
67	1.001 570	35.011 359	2180.8	62.290 259	.016 05388	19.444
68	1.001 680	36.012 246	2242.9	62.283 418	.016 05564	20.000
69	1.001 793	37.013 175	2305.0	62.276 293	.016 05748	20.555
70	1.001 909	38.014 148	2367.1	62.269 183	.016 05921	21.111
71	1.002 028	39.015 164	2429.2	62.261 788	.016 06122	21.666
72	1.002 151	40.016 224	2491.2	62.254 146	.016 06318	22.222
73	1.002 277	41.017 330	2553.2	62.246 320	.016 06521	22.777
74	1.002 406	42.018 482	2615.2	62.238 309	.016 06728	23.333
75	1.002 539	43.019 680	2677.1	62.230 052	.016 06941	23.888
76	1.002 675	44.020 926	2739.2	62.221 612	.016 07158	24.444
77	1.002 814	45.022 220	2801.0	62.212 987	.016 07382	25.000
78	1.002 956	46.023 563	2862.8	62.204 179	.016 07610	25.555
79	1.003 101	47.024 956	2924.6	62.195 187	.016 07841	26.111
80	1.003 249	48.026 398	2985.4	62.186 012	.016 08078	26.666
81	1.003 400	49.027 893	3048.2	62.176 654	.016 08321	27.222
82	1.003 554	50.029 438	3111.0	62.167 113	.016 08567	27 777
83	1.003 711	51.031 039	3172.8	62.157 388	.016 08820	28.333
84	1.003 872	52.032 688	3234.4	62.147 420	.016 09077	28.888
85	1.004 035	53.034 394	3296.2	62.137 330	.016 09338	29.444
86	1.004 199	54.036 154	3358.2	62.127 182	.016 09601	30.000
87	1.004 370	55.037 969	3418.7	62.116 605	.016 09875	30.555
88	1.004 542	56.039 841	3480.4	62.105 969	.016 10151	31.111
89	1.004 717	57.041 769	3542.1	62.095 152	.016 10432	31.666
90	1.004 894	58.043 754	3603.8	62.084 214	.016 10715	32.222

Temp. Fahr.	Volume 1 at 39°.	Units of heat. Per pound.	Units of heat. Per cubic foot.	Pounds per cubic foot.	Cubic feet per pound.	Temp. Cent.
91	1.005 094	59.045 797	3665.0	62.071 860	.016 11036	32.777
92	1.005 258	60.047 899	3726.6	62.061 734	.016 11298	33.333
93	1.005 444	61.050 061	3788.2	62.050 252	.016 11597	33.888
94	1.005 633	62.052 282	3849.8	62.038 591	.016 11900	34.444
95	1.005 825	63.054 564	3911.2	62.026 749	.016 12208	35.000
96	1.006 019	64.056 907	3972.6	62.014 787	.016 12519	35.555
97	1.006 216	65.059 312	4033.9	62.002 646	.016 12834	36.111
98	1.006 415	66.061 780	4095.2	61.990 386	.016 13153	36.666
99	1.006 618	67.064 311	4156.5	61.977 885	.016 13478	37.222
100	1.006 822	68.066 906	4217.7	61.965 322	.016 13806	37.777
101	1.007 030	69.069 565	4278.9	61.952 528	.016 14140	38.333
102	1.007 240	70.072 290	4340.1	61.939 612	.016 14475	38.888
103	1.007 553	71.075 080	4401.3	61.920 370	.016 14944	39.444
104	1.007 668	72.077 937	4462.5	61.913 303	.016 15161	40.000
105	1.007 905	73.080 861	4523.0	61.898 745	.016 15541	40.555
106	1.008 106	74.083 852	4585.0	61.886 403	.016 15863	41.111
107	1.008 328	75.086 912	4645.9	61.872 778	.016 16220	41.666
108	1.008 554	76.090 044	4706.8	61.858 913	.016 16581	42.222
109	1.008 781	77.093 239	4767.7	61.844 994	.016 16946	42.777
110	1.009 032	78.096 509	4828.6	61.829 609	.016 17348	43.333
111	1.009 244	79.099 846	4889.5	61.816 622	.016 17677	43.888
112	1.009 479	80.103 255	4950.4	61.802 231	.016 18064	44.444
113	1.009 718	81.106 740	5011.3	61.787 602	.016 18447	45.000
114	1.009 956	82.110 290	5072.2	61.773 042	.016 18829	45.555
115	1.010 197	83.113 920	5133.0	61.758 305	.016 19216	46.111
116	1.010 442	84.117 620	5193.7	61.743 331	.016 19608	46.666
117	1.010 688	85.121 400	5254.3	61.728 302	.016 20003	47.222
118	1.010 938	86.125 250	5314.9	61.713 037	.016 20403	47.777
119	1.011 189	87.129 180	5375.5	61.697 719	.016 20806	48.333
120	1.011 442	88.133 180	5436.1	61.682 286	.016 21211	48.888
121	1.011 698	89.137 260	5496.6	61.666 678	.016 21621	49.444
122	1.011 956	90.141 410	5557.1	61.650 956	.016 22034	50.000
123	1.012 216	91.145 650	5617.6	61.635 123	.016 22451	50.555
124	1.012 478	92.149 960	5678.1	61.619 170	.016 22871	51.111
125	1.012 743	93.154 350	5738.6	61.603 047	.016 23296	51.666
126	1.013 010	94.158 820	5798.9	61.586 810	.016 23724	52.222
127	1.013 278	95.163 380	5859.2	61.570 516	.016 24153	52.777
128	1.013 550	96.168 010	5919.5	61.553 998	.016 24590	53.333
129	1.013 823	97.172 720	5979.7	61.537 423	.016 25027	53.888
130	1.014 098	98.177 520	6040.0	61.520 735	.016 25468	54.444
131	1.014 358	99.182 390	6100.2	61.504 966	.016 25884	55.000
135	1.015 505	103.202 740	6340.3	61.435 497	.016 27724	57.222
140	1.016 962	108.230 090	6639.6	61.347 282	.016 30064	60.000
145	1.018 468	113.259 650	6937.9	61.256 765	.016 32473	62.777
150	1.020 021	118.291 470	7215.1	61.163 500	.016 34961	65.555
155	1.021 619	123.325 620	7531.2	61.067 829	.016 37523	68.333
160	1.023 262	128.362 170	7826.2	60.969 776	.016 40156	71.111
165	1.024 947	133.401 190	8098.1	60.869 542	.016 42857	73.888
170	1.026 672	138.442 730	8412.8	60.767 270	.016 45623	76.666
175	1.028 438	143.486 870	8704.2	60.662 047	.016 48477	79.444
180	1.030 242	148.536 660	8994.9	60.556 699	.016 51345	82.222
185	1.032 083	153.583 160	9281.9	60.448 679	.016 54296	85.000
190	1.033 960	158.635 450	9571.6	60.338 944	.016 57305	87.777
195	1.035 873	163.690 570	9858.5	60.227 513	.016 60370	90.555
200	1.037 819	168.748 580	10318.0	60.114 581	.016 63489	93.333
205	1.039 798	173.809 560	10428.0	60.000 168	.016 66662	96.111
210	1.041 809	178.873 550	10712.0	59.884 350	.016 69885	98.888
212	1.042 622	180.900 000	10824.0	59.837 654	.016 71160	100.000

Indicated pressure.			Water.						
Atmos. excluded. Lb. per sq. in.	Atmos. excluded. Inches of mercury.	Temp., Fahr. scale.	Units of heat. Per cub. ft.	Units of heat. Per pound.	Bulk, cub. ft. per lb.	Weight, lbs., per cub. ft.	Volume wat. = 1 at 39°.	Temp., Cent. scale.
—14	—28.52	101.36	4301	69.430	.01617	61.848	1.0071	30.83
—13	—26.48	126.21	5631	94.369	.01624	61.583	1.0130	41.87
—12	—24.44	141.67	6583	109.91	.01630	61.317	1.0174	48.74
—11	—22.41	153.27	7331	121.58	.01637	61.101	1.0210	53.90
—10	—20.37	162.51	7974	130.89	.01638	60.920	1.0241	58.00
— 9	—18.33	170.25	8421	138.69	.01644	60.762	1.0267	61.44
— 8	—16.29	176.97	8812	145.46	.01647	60.657	1.0288	64.43
— 7	—14.26	182.96	9203	151.52	.01652	60.514	1.0309	67.09
— 6	—12.22	188.36	9531	156.97	.01656	60.372	1.0333	69.49
— 5	—10.18	193.20	9755	161.87	.01659	60.282	1.0359	71.64
— 4	— 8.149	197.60	9975	166.32	.01663	60.169	1.0369	73.60
— 3	— 6.111	201.90	10183	170.67	.01666	60.072	1.0385	75.51
— 2	— 4.074	205.77	10398	174.59	.01669	59.973	1.0401	77.23
— 1	— 2.037	209.55	10613	178.42	.01672	59.896	1.0416	78.91
0	.0000	212.00	10824	180.95	.01674	59.838	1.0426	100.00
.3125	.6365	213.04	10883	181.95	.01675	59.814	1.0430	100.58
+ 1	+ 2.037	216.33	11047	185.29	.01677	59.735	1.0444	102.45
+ 2	+ 4.074	219.45	11225	188.45	.01679	59.659	1.0457	104.36
+ 3	+ 6.111	222.40	11389	191.44	.01680	59.592	1.0469	105.78
+ 4	+ 8.149	225.25	11550	194.33	.01681	59.523	1.0481	107.35
+ 5	+10.18	227.95	11718	197.08	.01684	59.459	1.0492	108.86
+ 6	+12.22	230.60	11868	199.77	.01686	59.389	1.0503	110.33
+ 7	+14.26	233.10	12012	202.40	.01688	59.329	1.0514	111.50
+ 8	+16.29	235.49	12150	204.73	.01690	59.270	1.0524	113.05
+ 9	+18.33	237.81	12282	207.10	.01692	59.212	1.0534	114.00
+10	+20.37	240.07	12408	209.39	.01693	59.154	1.0545	115.59
+11	+22.41	242.24	12528	211.57	.01695	59.097	1.0555	116.80
+12	+24.44	244.32	12642	213.72	.01696	59.057	1.0564	117.95
+13	+26.48	246.35	12750	215.78	.01697	59.006	1.0573	119.08
+14	+28.52	248.33	12852	217.80	.01698	58.953	1.0589	120.18
+15	+30.55	250.26	12946	219.76	.01699	58.901	1.0590	121.25
+16	+32.59	252.13	13053	221.67	.01700	58.851	1.0599	122.29
+17	+34.63	253.98	13157	223.55	.01701	58.803	1.0607	123.32
+18	+36.67	255.77	13258	225.38	.01702	58.757	1.0615	124.32
+19	+38.71	257.52	13336	227.16	.01703	58.713	1.0623	125.29
+20	+40.74	259.22	13430	228.89	.01704	58.671	1.0631	126.23
+21	+42.78	260.88	13520	230.59	.01705	58.631	1.0639	127.15
+22	+44.82	262.50	13608	232.24	.01707	58.592	1.0646	128.05
+23	+46.85	264.09	13694	233.86	.01708	58.560	1.0654	128.94
+24	+48.89	265.65	13778	235.45	.01709	58.517	1.0661	129.80
+25	+50.93	267.17	13860	237.00	.01710	58.481	1.0668	130.65
+26	+52.97	268.66	13940	238.52	.01711	58.435	1.0675	131.48
+27	+55.00	270.12	14018	240.02	.01712	58.400	1.0684	132.29
+28	+57.04	271.55	14094	241.48	.01713	58.366	1.0688	133.05
+29	+59.08	272.96	14168	242.92	.01714	58.332	1.0695	133.86
+30	+61.11	274.33	14241	244.32	.01715	58.298	1.0701	134.63
+31	+63.15	275.68	14314	245.70	.01716	58.264	1.0708	135.38
+32	+65.19	277.01	14385	247.06	.01717	58.230	1.0714	136.12
+33	+67.23	278.32	14454	248.40	.01718	58.197	1.0720	136.84
+34	+69.20	279.62	14522	249.73	.01719	58.164	1.0726	137.56
+35	+71.30	280.89	14592	251.03	.01720	58.131	1.0732	138.27
+36	+73.34	282.14	14659	252.30	.01721	58.098	1.0738	138.96
+37	+75.38	283.39	14725	253.58	.01722	58.066	1.0744	139.66
+38	+77.41	284.58	14789	254.80	.01723	58.035	1.0750	140.33
+39	+79.45	285.76	14852	256.01	.01724	58.004	1.0756	140.98
+40	+81.49	286.96	14913	257.24	.01725	57.972	1.0761	141.64
+41	+83.52	288.06	14973	258.38	.01726	57.941	1.0767	142.27
+42	+85.56	289.24	15032	259.67	.01727	57.910	1.0773	142.91
+43	+87.61	290.37	15091	260.71	.01728	57.879	1.0778	143.54
+44	+89.64	291.48	15149	261.87	.01729	57.848	1.0783	144.15
+45	+91.67	292.58	15208	262.99	.01730	57.817	1.0789	144.76

Indicated pressure.			Water.						
Atmos. excluded. Lb. per sq. in.	Atmos. excluded. Inches of mercury.	Temp., Fahr. scale.	Units of heat. Per cub. ft.	Units of heat. Per pound.	Bulk, cub. ft. per lb.	Weight, lbs., per cub. ft.	Volume wat. = 1 at 39°.	Temp., Cent. scale.
+ 46	+ 93.71	293.66	15265	264.10	.01731	57.786	1.0794	145.37
+ 47	+ 95.75	294.73	15321	265.20	.01732	57.769	1.0799	145.96
+ 48	+ 97.78	295.78	15377	266.27	.01733	57.742	1.0804	146.54
+ 49	+ 99.82	296.82	15432	267.34	.01734	57.714	1.0809	147.12
+ 50	+101.8	297.84	15485	268.39	.01735	57.687	1.0814	147.69
+ 51	+103.9	298.85	15536	269.42	.01735	57.660	1.0820	148.25
+ 52	+105.9	299.85	15588	270.45	.01736	57.633	1.0825	148.80
+ 53	+108.0	300.84	15639	271.46	.01737	57.606	1.0830	149.34
+ 54	+110.0	301.81	15690	272.46	.01737	57.580	1.0835	149.89
+ 55	+112.0	302.77	15739	273.44	.01738	57.554	1.0840	150.43
+ 56	+114.1	303.72	15789	274.42	.01739	57.529	1.0844	150.95
+ 57	+116.1	304.69	15839	275.40	.01739	57.504	1.0849	151.48
+ 58	+118.1	305.60	15888	276.35	.01740	57.480	1.0854	152.00
+ 59	+120.2	306.52	15936	277.30	.01741	57.456	1.0859	152.51
+ 60	+122.2	307.42	15983	278.22	.01741	57.432	1.0863	153.01
+ 61	+124.3	308.38	16029	279.14	.01742	57.410	1.0867	153.51
+ 62	+126.3	309.22	16075	280.07	.01743	57.388	1.0871	154.01
+ 63	+128.3	310.11	16120	280.98	.01743	57.364	1.0875	154.50
+ 64	+130.4	310.99	16165	281.87	.01744	57.344	1.0880	154.99
+ 65	+132.4	311.86	16209	282.78	.01745	57.322	1.0884	155.48
+ 66	+134.4	312.72	16254	283.66	.01745	57.300	1.0888	155.95
+ 67	+136.5	313.57	16298	284.54	.01746	57.278	1.0892	156.42
+ 68	+138.5	314.42	16342	285.41	.01746	57.254	1.0897	156.90
+ 69	+140.5	315.25	16384	286.27	.01747	57.232	1.0901	157.36
+ 70	+142.6	316.08	16426	287.12	.01748	57.210	1.0905	157.82
+ 71	+144.6	316.90	16467	287.96	.01748	57.188	1.0909	158.28
+ 72	+146.7	317.71	16507	288.80	.01749	57.166	1.0913	158.73
+ 73	+148.7	318.51	16547	289.62	.01750	57.144	1.0918	159.17
+ 74	+150.7	319.31	16587	290.44	.01751	57.122	1.0921	159.62
+ 75	+152.8	320.10	16637	291.26	.01752	57.101	1.0926	160.05
+ 76	+154.8	320.88	16677	292.06	.01752	57.080	1.0929	160.49
+ 77	+156.8	321.66	16717	292.85	.01753	57.059	1.0935	160.92
+ 78	+158.9	322.42	16756	293.65	.01753	57.038	1.0937	161.34
+ 79	+160.9	323.18	16795	294.43	.01754	57.017	1.0941	161.76
+ 80	+163.0	323.94	16834	295.21	.01755	56.996	1.0945	162.17
+ 81	+165.0	324.67	16872	295.96	.01756	56.975	1.0949	162.59
+ 82	+167.0	325.43	16910	296.75	.01756	56.954	1.0953	163.02
+ 83	+169.1	326.17	16947	297.51	.01757	56.933	1.0956	163.43
+ 84	+171.1	326.90	16984	298.26	.01757	56.912	1.0960	163.83
+ 85	+173.1	327.63	17020	299.01	.01758	56.891	1.0964	164.24
+ 86	+175.2	328.35	17056	299.75	.01759	56.871	1.0968	164.64
+ 87	+177.2	329.07	17092	300.50	.01759	56.862	1.0972	165.04
+ 88	+179.2	329.78	17127	301.23	.01760	56.844	1.0975	165.43
+ 89	+181.3	330.48	17162	301.95	.01761	56.826	1.0979	165.82
+ 90	+183.3	331.18	17197	302.67	.01761	56.808	1.0982	166.21
+ 91	+185.4	331.87	17231	303.38	.01762	56.790	1.0986	166.59
+ 92	+187.4	332.56	17265	304.10	.01763	56.772	1.0989	166.98
+ 93	+189.4	333.24	17299	304.80	.01763	56.754	1.0993	167.35
+ 94	+191.5	333.92	17333	305.50	.01764	56.735	1.0996	167.77
+ 95	+193.5	334.59	17366	306.19	.01765	56.716	1.0999	168.10
+ 96	+195.5	335.26	17399	306.88	.01765	56.699	1.1003	168.47
+ 98	+199.6	336.58	17465	308.34	.01767	56.664	1.1010	169.21
+ 99	+201.6	337.23	17497	308.91	.01768	56.647	1.1013	169.57
+100	+203.7	337.89	17529	309.60	.01769	56.629	1.1017	169.94
+105	+213.9	341.0	17688	312.87	.01772	56.549	1.1035	171.70
+110	+224.1	344.1	17840	316.04	.01775	56.469	1.1050	173.40
+115	+234.2	347.1	17993	319.12	.01778	56.389	1.1065	175.06
+120	+244.4	350.0	18136	322.13	.01781	56.309	1.1080	176.68
+125	+254.6	352.8	18278	325.06	.01784	56.220	1.1095	178.25
+130	+264.8	355.6	18413	327.91	.01786	56.146	1.1110	179.78
+135	+275.0	358.4	18549	330.75	.01788	56.073	1.1124	181.35

Density and Volume of Water.

Centigrade Temperatures.

Temp. Cent.	Density.	Volume.	Temp. Cent.	Density.	Volume.	Temp. Cent.	Density.	Volume.
—10	.99814	1.00186	14	.99930	1.00070	38	.99310	1.00694
— 9	.99843	1.00157	15	.99916	1.00084	39	.99273	1.00732
— 8	.99868	1.00132	16	.99900	1.00100	40	.99235	1.00770
— 7	.99891	1.00109	17	.99884	1.00116	41	.99197	1.00809
— 6	.99912	1.00088	18	.99865	1.00135	42	.99158	1.00849
— 5	.99930	1.00070	19	.99846	1.00154	43	.99118	1.00889
— 4	.99945	1.00054	20	.99826	1.00174	44	.99078	1.00929
— 3	.99959	1.00041	21	.99805	1.00196	45	.99037	1.00971
— 2	.99970	1.00030	22	.99783	1.00218	46	.98996	1.01014
— 1	.99980	1.00020	23	.99760	1.00240	47	.98954	1.01057
0	.99987	1.00013	24	.99737	1.00264	48	.98910	1.01101
1	.99993	1.00007	25	.99712	1.00289	49	.98865	1.01148
2	.99997	1.00003	26	.99687	1.00314	50	.98820	1.01195
3	.99999	1.00001	27	.99660	1.00341	55	.98582	1.01439
4	1.00000	1.00000	28	.99633	1.00368	60	.98338	1.01691
5	.99999	1.00001	29	.99605	1.00396	65	.98074	1.01964
6	.99997	1.00003	30	.99577	1.00425	70	.97794	1.02256
7	.99993	1.00007	31	.99547	1.00455	75	.97498	1.02566
8	.99989	1.00011	32	.99517	1.00486	80	.97194	1.02887
9	.99983	1.00018	33	.99485	1.00518	85	.96879	1.03221
10	.99975	1.00025	34	.99452	1.00551	90	.96556	1.03567
11	.99965	1.00034	35	.99418	1.00586	95	.96219	1.03931
12	.99955	1.00045	36	.99383	1.00621	100	.95865	1.04312
13	.99943	1.00057	37	.99347	1.00657			

Table of Water-heads, Equivalent Pressures, Work, and Horse-power.

Pelton Water-wheel Company.

Head, in feet.	Equivalent pressure, in pounds, per square inch.	Foot-pounds of work when raising 100 gallons per minute against corresponding heads.	Corresponding water horse-power.	Head, in feet.	Equivalent pressure, in pounds, per square inch.	Foot-pounds of work when raising 100 gallons per minute against corresponding heads.	Corresponding water horse-power.
1	.43	834	.03	500	216.50	417 000	12.64
2	.87	1 668	.05	525	227.33	437 850	13.27
3	1.30	2 502	.08	550	238.15	458 700	13.90
4	1.73	3 336	.10	575	248.98	479 550	14.53
5	2.17	4 170	.13				
6	2.60	5 004	.15	600	259.80	500 400	15.16
7	3.03	5 838	.18	625	270.63	521 250	15.79
8	3.46	6 672	.20	650	281.45	542 100	16.42
9	3.90	7 506	.23	675	292.28	562 950	17.05
10	4.33	8 340	.25	700	303.10	583 800	17.68
11	4.76	9 174	.28	725	313.93	604 650	18.31
12	5.20	10 008	.30	750	324.75	625 500	18.95
13	5.63	10 842	.33	775	335.58	646 350	19.58
14	6.06	11 676	.35				
15	6.50	12 510	.38	800	346.40	667 200	20.20
16	6.93	13 344	.40	825	357.23	688 050	20.85
17	7.36	14 178	.43	850	368.05	708 900	21.48
18	7.79	15 012	.46	875	378.88	729 750	22.11
19	8.23	15 846	.48				
				900	389.70	750 600	22.74
20	8.66	16 680	.50	925	400.53	771 450	23.38
30	12.99	25 020	.76	950	411.35	792 300	24.01
40	17.32	33 360	1.01	975	422.18	813 150	24.64
50	21.65	41 700	1.26				
60	25.98	50 040	1.52	1000	433.00	834 000	25.27
70	30.31	58 380	1.77	1025	443.83	854 850	25.90
80	34.64	66 720	2.02	1050	454.65	875 700	26.53
90	38.97	75 060	2.27	1075	465.48	896 550	27.17
100	43.30	83 400	2.53	1100	476.30	917 400	27.80
125	54.13	104 250	3.16	1125	487.13	938 250	28.43
150	64.95	125 100	3.79	1150	497.95	959 100	29.06
175	75.78	145 950	4.42	1175	508.78	979 950	29.69
200	86.60	166 800	5.05	1200	519.60	1 000 800	30.33
225	97.43	187 650	5.68	1225	530.43	1 021 650	30.96
250	108.25	208 500	6.31	1250	541.25	1 042 500	31.59
275	119.08	229 350	6.94	1275	552.08	1 063 350	32.23
300	129.90	250 200	7.57	1300	562.90	1 084 200	32.86
325	140.73	271 050	8.22	1325	573.73	1 105 050	33.49
350	151.55	291 900	8.85	1350	584.55	1 125 900	34.12
375	162.38	312 750	9.48	1375	595.38	1 146 750	34.75
400	173.20	333 600	10.11	1400	606.20	1 167 600	35.38
425	184.03	354 450	10.74	1425	617.03	1 188 450	36.01
450	194.85	375 300	11.38	1450	627.85	1 209 300	36.64
475	205.68	396 150	12.01	1475	638.68	1 230 150	37.28

Table of Water-heads, etc.—*Continued.*

Head, in feet.	Equivalent pressure, in pounds, per square inch.	Foot-pounds of work when raising 100 gallons per minute against corresponding heads.	Corresponding water horse-power.	Head, in feet.	Equivalent pressure, in pounds, per square inch.	Foot-pounds of work when raising 100 gallons per minute against corresponding heads.	Corresponding water horse-power.
1500	649.50	1 251 000	37.91	2300	995.90	1 918 200	58.12
1525	660.33	1 271 850	38.54	2325	1006.73	1 939 050	58.75
1550	671.15	1 292 700	39.17	2350	1017.55	1 959 900	59.39
1575	681.98	1 313 550	39.80	2375	1028.38	1 980 750	60.02
1600	692.80	1 334 400	40.44	2400	1039.20	2 001 600	60.65
1625	703.63	1 355 250	41.07	2425	1050.03	2 022 450	61.28
1650	714.45	1 376 100	41.70	2450	1060.85	2 043 300	61.91
1675	725.28	1 396 950	42.33	2475	1071.68	2 064 150	62.55
1700	736.10	1 417 800	42.96	2500	1082.50	2 085 000	63.18
1725	746.93	1 438 650	43.59	2525	1093.33	2 105 850	63.81
1750	757.75	1 459 500	44.22	2550	1104.15	2 126 700	64.44
1775	768.58	1 480 350	44.85	2575	1114.98	2 147 550	65.07
1800	779.40	1 501 200	45.49	2600	1125.80	2 168 400	65.70
1825	790.23	1 522 050	46.13	2625	1136.63	2 189 250	66.34
1850	801.05	1 542 900	46.76	2650	1147.45	2 210 100	66.97
1875	811.88	1 563 750	47.39	2675	1158.28	2 230 950	67.60
1900	822.70	1 584 600	48.02	2700	1169.10	2 251 800	68.23
1925	833.53	1 605 450	48.65	2725	1179.93	2 272 650	68.85
1950	844.35	1 626 300	49.29	2750	1190.75	2 293 500	69.49
1975	855.18	1 647 150	49.92	2775	1201.58	2 314 350	70.12
2000	866.00	1 668 000	50.55	2800	1212.40	2 335 200	70.75
2025	876.83	1 688 850	51.18	2825	1223.23	2 356 050	71.39
2050	887.65	1 709 700	51.81	2850	1234.05	2 376 900	72.02
2075	898.48	1 730 550	52.44	2875	1244.88	2 397 750	72.65
2100	909.30	1 751 400	53.07	2900	1255.70	2 418 600	73.28
2125	920.13	1 772 250	53.70	2925	1266.53	2 439 450	73.92
2150	930.95	1 793 100	54.33	2950	1277.35	2 460 300	74.55
2175	941.78	1 813 950	54.96	2975	1288.18	2 481 150	75.18
2200	952.60	1 834 800	55.60	3000	1299.00	2 502 000	75.82
2225	963.43	1 855 650	56.23				
2250	974.25	1 876 500	56.86				
2275	985.08	1 897 350	57.49				

The head,—vertical distance to which water is pumped above level of supply. Constant used for equivalent pressure = 0.433, which is the pressure per square inch of 1 foot-head of water at 62° F.

1 gallon of water at 62° F. weighs 8.34 pounds.

1 horse-power = 33,000 foot-pounds per minute.

If equivalents of heads that are not tabulated are desired, divide the head into heads that are given, and add their equivalents.

E.g., to find the equivalent pressure for a head of 129 feet, 129 = 125 + 4.

$$\left.\begin{array}{l} \text{125 feet} = \text{54.13 pounds} \\ \text{4 feet} = \text{1.74 pounds} \end{array}\right\} \text{sum 55.86 = pounds = equivalent pressure of 129 feet.}$$

The pressure of 1 foot-head of water, taking the density at the average temperature of 62° F., is 0.433 pound per square inch. The head corresponding to a pressure of 1 pound per square inch is 2.3095 feet.

The pressure within a vessel is the same upon every square inch of its surface, regardless of the shape or size of the vessel, and is that due to the head of water upon it. The horizontal pressure against a wall or dam varies as the square of the height. If h be the height of the dam, and w the weight of a cubic foot of water, the pressure per foot-width will be $\frac{1}{2}wh^2$, and its point of application will be two-thirds of the distance from the top.

The theoretical velocity of issuing from an orifice is the same as that which would be acquired by a body falling from the height of the head of water above the orifice. This is

$$V = \sqrt{2gh},$$

in which h is the head of water; g, the acceleration of gravity = 32.2; and V, the velocity, in feet, per second. In practice, this theoretical velocity is not attained, owing to various resistances, but the principle should always be borne in mind. If the water is under a pressure other than that due to its own weight, the head corresponding to that pressure may be found, taking 2.3095 feet to the pound, and this value used in the formula.

If a be the area of a jet, in square inches; v, its velocity, in feet, per second; and w, the weight of a cubic foot of water, the energy, in foot-pounds, per second will be

$$K = \frac{wav^2}{2g}.$$

The *coefficient of discharge* of a jet of water is the proportion of the full theoretical discharge which is realized in practice. As a result of many experiments this coefficient may be given a mean value of 0.61. If, therefore, the area of an opening be multiplied by the theoretical velocity = $\sqrt{2gh}$, and 61 per cent. of this taken, the actual discharge will be found. This is true for orifices in the comparatively thin wall or bottom of the vessel containing the water; the area of the orifice being small compared with the size of the reservoir, and the edges having a definite square corner.

When, instead of a mere orifice, a short tube or nozzle is used, having a length of about three times its diameter, the coefficient of discharge is about 80 per cent. of the theoretical. By using smooth conveying nozzles, with the inner edges rounded, the coefficient may be raised to about 97 per cent.

In computing the flow of water through long pipes the principal loss to be provided for is that due to friction between the water and the surface of the pipe. The resistance due to friction may be computed in terms of feet of head,—that is, the number of feet of head necessary to overcome the resistance of friction may be found and deducted from the actual total head available.

Theoretical Velocity of Water Due to Given Heads.

Head of water, in feet.	Theoretical velocity, in feet, per second.	Theoretical velocity, in feet, per minute.	Head of water, in feet.	Theoretical velocity, in feet, per second.	Theoretical velocity, in feet, per minute.	Head of water, in feet.	Theoretical velocity, in feet, per second.	Theoretical velocity, in feet, per minute.
1	8.205	481.5	51	57.309	3438.5	105	82.231	4933.9
2	11.345	681.7	52	57.869	3472.1	110	84.166	5050.0
3	13.899	833.9	53	58.422	3505.3	115	86.058	5163.5
4	16.050	963.0	54	58.971	3538.2	120	87.909	5274.5
5	17.944	1076.6	55	59.515	3570.9	125	89.722	5383.3
6	19.657	1179.4	56	60.053	3603.2	130	91.499	5489.9
7	21.232	1273.6	57	60.587	3635.2	135	93.242	5594.5
8	22.698	1361.8	58	61.116	3666.9	140	94.953	5697.2
9	24.075	1444.5	59	61.641	3698.4	145	96.633	5798.0
10	25.377	1522.6	60	62.161	3729.6	150	98.285	5897.1
11	26.615	1596.9	61	62.677	3760.6	155	99.909	5994.5
12	27.799	1667.9	62	63.188	3791.3	160	101.50	6090.5
13	28.934	1736.0	63	63.696	3821.7	165	103.08	6184.9
14	30.026	1801.6	64	64.200	3852.0	170	104.63	6277.9
15	31.080	1864.8	65	64.699	3881.9	175	106.16	6369.6
16	32.100	1926.0	66	65.195	3911.7	180	107.66	6460.0
17	33.087	1985.2	67	65.687	3941.2	185	109.15	6549.1
18	34.047	2042.8	68	66.175	3970.3	190	110.61	6637.0
19	34.980	2098.8	69	66.660	3999.6	195	112.06	6723.7
20	35.888	2153.3	70	67.141	4028.5	200	113.49	6809.4
21	36.775	2206.5	71	67.619	4057.1	205	114.90	6894.0
22	37.640	2258.4	72	68.094	4085.6	210	116.29	6977.6
23	38.486	2309.1	73	68.565	4113.9	215	117.66	7060.1
24	39.314	2358.8	74	69.033	4142.0	220	119.03	7141.8
25	40.125	2407.5	75	69.498	4169.9	225	120.00	7222.5
26	40.919	2455.1	76	69.960	4197.6	230	121.70	7302.3
27	41.699	2501.9	77	70.419	4225.1	235	123.02	7381.2
28	42.464	2547.8	78	70.874	4252.4	240	124.32	7459.3
29	43.215	2592.9	79	71.327	4279.6	245	125.60	7536.6
30	43.954	2637.2	80	71.777	4306.6	250	126.88	7613.1
31	44.681	2680.8	81	72.225	4333.5	255	128.15	7648.8
32	45.396	2723.7	82	72.673	4360.4	260	129.39	7763.9
33	46.100	2766.0	83	73.111	4386.6	265	130.63	7837.6
34	46.793	2783.0	84	73.550	4413.0	270	131.86	7911.8
35	47.476	2848.5	85	73.986	4439.2	275	133.08	7984.8
36	48.150	2889.0	86	74.420	4465.2	280	134.28	8057.0
37	48.814	2928.8	87	74.852	4491.1	285	135.48	8128.6
38	49.469	2968.1	88	75.281	4516.8	290	136.66	8199.6
39	50.116	3006.9	89	75.707	4542.4	295	137.83	8270.1
40	50.754	3045.2	90	76.131	4567.9	300	138.99	8339.8
41	51.385	3083.1	91	76.553	4593.2	305	140.15	8409.0
42	52.007	3120.4	92	76.973	4618.3	310	141.29	8477.6
43	52.623	3157.4	93	77.390	4643.4	315	142.42	8545.6
44	53.231	3193.9	94	77.805	4668.3	320	143.55	8613.3
45	53.833	3229.9	95	78.217	4693.0	325	144.67	8690.4
46	54.427	3265.6	96	78.628	4717.7	330	145.78	8760.9
47	55.016	3301.0	97	79.037	4742.2	335	146.88	8812.9
48	55.598	3335.8	98	79.443	4766.6	340	147.97	8878.4
49	56.175	3370.5	99	79.847	4790.8	345	149.06	8943.5
50	56.745	3404.7	100	80.250	4815.0	350	150.13	9007.9

Flow of Water Through Pipes.

The quantity of water which flows through a pipe is measured by the product of the area of its cross-section and by the velocity of the flow.

The velocity is not uniform over the entire cross-section, but a mean velocity may be computed which will serve for purposes of computation. In order to compute the velocity two elements must be given: the *slope* and the *hydraulic radius.* The slope is the sine of the angle of inclination of the pipe, or the head divided by the length; the hydraulic radius is the area divided by the wetted perimeter. The slope is called s, and the hydraulic radius r. For pipes of circular cross-section running full, $r = \frac{\text{area}}{4}$, the same being true when half-full.

The first attempt to express the relations between these elements was that of Chézy, in 1775, his formula being

$$v = C\sqrt{rs},$$

v being the velocity, in feet or metres, per second, and C being a constant coefficient. A vast number of experiments have been made to determine the value of the coefficient, C, with the result of showing it to vary with different slopes and diameters of pipes. In 1896 Tutton collected the results of more than 1000 experiments and suggested a modification of the formula, which appears to be the most reliable one available, and which we shall use in preference to any other.

Instead of placing the two quantities, r and s, under the radical sign, Tutton gives them independent exponents, writing the formula

$$v = C r^x s^y.$$

By comparing the results of many experiments it appears that if the exponents are made $x = \frac{2}{3}$, $y = \frac{1}{2}$, the coefficient, C, remains practically constant for any one kind of pipe, regardless of slope or diameter. The formula then reads,

$$v = C r^{\frac{2}{3}} s^{\frac{1}{2}},$$

so that the cube root of the square of the hydraulic radius is taken and the square root of the slope, and the product of these, by a constant depending only upon the character of the pipe, gives the velocity.

The following values for C are given for different surfaces:

Values of C for Pipe Flow.

	C
Wrought-iron pipe	160
Cast-iron pipe, new	130
Cast-iron pipe, in service	104
Lap riveted pipe	115
Wrought-iron pipe, asphalted	170
Wood-stave pipe	125
Tuberculated pipe	30 to 80
Brick conduits	110

In order to facilitate the use of the formula the following tables are appended, giving values of $r^{\frac{2}{3}}$ and $s^{\frac{1}{2}}$. Other values may be taken from the tables of power and roots, the $\frac{2}{3}$ power being the square of the cube root, and the $\frac{1}{2}$ power being the square root.

Values of $r^{\frac{2}{3}}$ from 0.01 to 1.

r	$r^{\frac{2}{3}}$	r	$r^{\frac{2}{3}}$	r	$r^{\frac{2}{3}}$	r	$r^{\frac{2}{3}}$	r	$r^{\frac{2}{3}}$
.01	.0464	.21	.3533	.41	.5519	.61	.7193	.81	.8690
.02	.0737	.22	.3644	.42	.5608	.62	.7271	.82	.8761
.03	.0965	.23	.3754	.43	.5697	.63	.7349	.83	.8832
.04	.1169	.24	.3861	.44	.5785	.64	.7427	.84	.8902
.05	.1357	.25	.3969	.45	.5872	.65	.7503	.85	.8974
.06	.1533	.26	.4073	.46	.5958	.66	.7581	.86	.9044
.07	.1698	.27	.4177	.47	.6045	.67	.7656	.87	.9111
.08	.1857	.28	.4280	.48	.6131	.68	.7733	.88	.9183
.09	.2008	.29	.4381	.49	.6216	.69	.7809	.89	.9252
.10	.2155	.30	.4481	.50	.6300	.70	.7884	.90	.9322
.11	.2295	.31	.4580	.51	.6384	.71	.7958	.91	.9390
.12	.2432	.32	.4679	.52	.6465	.72	.8033	.92	.9459
.13	.2566	.33	.4775	.53	.6550	.73	.8107	.93	.9528
.14	.2696	.34	.4871	.54	.6631	.74	.8181	.94	.9596
.15	.2823	.35	.4966	.55	.6712	.75	.8255	.95	.9663
.16	.2947	.36	.5061	.56	.6795	.76	.8328	.96	.9732
.17	.3069	.37	.5154	.57	.6874	.77	.8401	.97	.9799
.18	.3188	.38	.5246	.58	.6955	.78	.8473	.98	.9866
.19	.3305	.39	.5338	.59	.7034	.79	.8545	.99	.9932
.20	.3420	.40	.5429	.60	.7113	.80	.8617	1.00	1.0000

Values of $s^{\frac{1}{2}}$ for Slopes from .000 025 to 1.

s	$s^{\frac{1}{2}}$	s	$s^{\frac{1}{2}}$	s	$s^{\frac{1}{2}}$
.000 025	.00500	.000 275	.01658	.006	.07746
.000 030	.00547	.000 300	.01732	.007	.08366
.000 035	.00592	.000 325	.01803	.008	.08944
.000 040	.00632	.000 350	.01871	.009	.09487
.000 045	.00671	.000 375	.01936	.01	.1000
.000 050	.00707	.000 400	.02000	.02	.1414
.000 055	.00742	.000 450	.02121	.03	.1732
.000 060	.00775	.000 500	.02236	.04	.2000
.000 065	.00806	.000 550	.02345	.05	.2236
.000 070	.00837	.000 600	.02449	.06	.2449
.000 075	.00866	.000 650	.02549	.07	.2646
.000 080	.00894	.000 700	.02646	.08	.2828
.000 085	.00921	.000 750	.02739	.09	.3000
.000 090	.00949	.000 800	.02828	.1	.3162
.000 095	.00975	.000 850	.02915	.2	.4472
.000 100	.01000	.000 900	.03000	.3	.5477
.000 125	.01118	.000 950	.03082	.4	.6324
.000 150	.01225	.001	.03162	.5	.7071
.000 175	.01323	.002	.04472	.6	.7746
.000 200	.01414	.003	.05477	.7	.8367
.000 225	.01500	.004	.06324	.8	.8944
.000 250	.01581	.005	.07071	.9	.9487

Example. A wrought-iron pipe 3 inches diameter, = 0.25 foot, and 1000 feet long, has a head of water of 20 feet. Required the velocity?

We have

$$r = \frac{0.25}{4} = 0.06;$$

$$s = 0.02;$$

and the formula

$$v = Cr^{\frac{2}{3}}s^{\frac{1}{2}}$$

becomes $v = 160 \times 0.1533 \times 0.1414 = 3.47$ feet per second.

Again: A brick conduit is 7.5 feet in diameter, with a slope, $s = 0.00058$. Required the velocity?

Here

$$r = \frac{7.5}{4} = 1.875,$$

and we have

$$v = Cr^{\frac{2}{3}}s^{\frac{1}{2}}$$

$$= 110 \times 1.52 \times 0.024 = 4.01 \text{ feet per second.}$$

The measured velocity in this conduit was 3.929 feet per second.

Discharge of Water from *Smooth* Wrought-iron Pipes.

$v = 160r^{\frac{2}{3}}s^{\frac{1}{2}}$, times area.

Cubic Feet per Second.

For Cast-iron, multiply by 0.81; Lap Riveted, 0.72; Wood Stave, 0.78; Brick, 0.68.

Diameter, in Inches.

Slope = head/length	2	4	6	8	10	12	14
.001	.014	.083	.249	.532	.970	1.591	2.390
.002	.019	.119	.351	.750	1.370	2.240	3.375
.003	.023	.145	.429	.936	1.680	2.752	4.135
.004	.026	.167	.494	1.062	1.935	3.180	4.780
.005	.029	.186	.549	1.179	2.175	3.555	5.341
.006	.032	.205	.612	1.280	2.372	3.900	5.850
.007	.035	.222	.657	1.405	2.565	4.200	6.320
.008	.037	.236	.700	1.500	2.740	4.500	6.755
.009	.040	.252	.742	1.590	2.910	4.770	7.170
.01	.043	.265	.784	1.675	3.061	5.026	7.544
.02	.059	.375	1.080	2.375	4.330	7.110	10.670
.03	.069	.458	1.357	2.812	5.310	8.720	13.100
.04	.081	.530	1.567	3.350	6.122	10.052	15.088
.05	.094	.593	1.735	3.650	6.850	11.250	16.870
.06	.103	.648	1.920	4.110	7.490	12.310	18.500
.07	.111	.695	2.072	4.340	8.100	13.300	20.000
.08	.118	.750	2.220	4.740	8.660	14.230	21.700
.09	.125	.795	2.350	5.022	9.183	15.078	22.632
.1	.133	.838	2.480	5.287	9.70	15.920	23.90
.2	.187	1.185	3.505	7.50	13.71	22.510	33.80
.3	.230	1.453	4.290	8.78	16.77	27.530	41.35
.4	.265	1.680	4.805	10.50	19.38	31.820	47.80
.5	.293	1.875	5.523	11.87	21.65	35.054	53.20
.6	.325	2.055	6.08	13.1	23.42	38.95	58.35

Discharge of Water from *Smooth* Wrought-iron Pipes.

$v = 160 r^{\frac{2}{3}} s^{\frac{1}{2}}$, times area.

Cubic Feet per Second.

For Cast-iron multiply by 0.81; Lap Riveted, 0.72; Wood Stave, 0.78; Brick. 0.68.

Diameter, in Inches.

Slope = head/length.	16	18	20	22	24	26	28
.001	3.390	4.650	6.230	7.935	10.000	12.420	15.130
.002	4.790	6.575	8.800	11.240	14.075	17.550	21.390
.003	5.865	8.045	10.770	13.740	17.250	21.470	26.170
.004	6.780	9.310	12.450	15.875	20.000	24.750	30.250
.005	7.580	10.400	13.920	17.760	22.700	26.300	33.750
.006	8.310	11.400	15.350	19.450	24.500	30.400	37.000
.007	8.970	12.300	16.460	21.000	26.475	32.750	39.900
.008	9.590	13.150	17.600	22.450	28.60	35.050	42.700
.009	10.175	13.950	18.670	23.800	30.00	37.200	44.300
.01	10.721	14.701	19.669	25.091	31.67	39.163	47.746
.02	15.150	20.770	27.80	35.450	44.70	55.400	67.40
.03	18.575	25.470	34.05	43.450	54.80	67.900	82.70
.04	21.442	29.402	39.34	50.182	63.34	78.326	95.50
.05	23.950	32.870	43.95	56.100	70.50	87.50	106.75
.06	26.230	36.000	48.15	61.400	77.40	94.90	117.00
.07	28.350	38.850	52.05	66.400	83.70	103.60	126.50
.08	30.300	41.500	55.60	70.950	89.40	110.75	135.00
.09	32.163	44.103	59.01	75.274	95.00	115.50	143.24
.1	33.800	46.500	62.05	79.35	100.00	124.20	151.3
.2	47.950	65.750	88.10	112.40	140.75	175.50	213.9
.3	58.700	80.500	107.75	137.40	172.50	214.70	261.7
.4	67.800	93.000	124.70	158.75	200.00	247.50	302.5
.5	75.800	104.000	138.25	177.60	227.00	263.00	337.5
.6	83.100	113.900	152.50	194.50	245.00	304.00	370.0

	30	32	34	36	38	40	42
.001	18.175	21.650	25.35	29.52	34.10	39.15	42.4
.002	25.70	30.650	35.85	41.75	48.20	55.30	60.1
.003	31.45	37.475	43.80	51.11	59.00	67.75	73.5
.004	36.70	43.35	50.70	59.00	68.20	78.3	85.0
.005	40.60	48.40	57.65	66.00	76.20	87.5	95.0
.006	44.30	53.00	62.10	72.30	83.50	95.9	104.1
.007	48.10	57.25	67.00	78.10	90.20	103.5	112.3
.008	51.40	61.25	71.60	83.50	96.40	110.6	120.1
.009	54.50	65.00	76.00	87.60	102.25	117.5	127.4
.01	57.43	68.15	80.13	93.35	107.75	123.7	134.2
.02	81.20	96.80	113.25	132.00	152.50	175.0	190.0
.03	99.50	110.86	138.75	161.75	186.75	214.5	232.5
.04	114.85	136.30	160.26	186.70	215.50	247.4	268.4
.05	128.50	153.20	178.00	208.70	241.00	276.5	300.0
.06	140.75	167.50	196.25	228.50	264.00	303.0	328.5
.07	152.00	181.20	212.00	247.00	286.20	327.0	355.0
.08	162.50	193.70	220.65	263.70	304.70	349.5	379.5
.09	172.28	204.45	240.4	280.05	323.25	371.0	402.6
.1	181.75	216.50	253.5	295.2	341.0	391.5	424.0
.2	257.0	306.50	358.5	417.5	482.0	553.0	601.0
.3	314.5	374.75	438.0	511.1	590.0	677.5	735.0
.4	367.0	433.5	507.0	590.0	682.0	783.0	850.0
.5	406.0	484.0	576.5	660.0	762.0	875.0	950.0
.6	443.0	530.0	621.0	723.0	835.0	959.0	1041.0

Loss of Head, in Pipe, by Friction.

Pelton Water-wheel Company.

The following tables show the loss of head by friction in each 100 feet in length of different diameters of pipe, when discharging the following quantities of water per minute:

Inside Diameter of Pipe, in Inches.

Velocity, in feet, per second.	1		2		3		4		5		6		7		8		9		10		11		12	
	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.
2.0	2.37	.65	1.185	2.62	.791	5.89	.593	10.4	.474	16.3	.395	23.5	.338	32.0	.296	41.9	.264	53.0	.237	65.4	.216	79.2	.198	94.2
2.2	2.80	.73	1.404	2.88	.936	6.48	.702	11.5	.561	18.0	.468	25.9	.401	35.3	.351	46.1	.312	58.3	.281	72.0	.255	87.1	.234	103.0
2.4	3.27	.79	1.639	3.14	1.093	7.07	.819	12.5	.650	19.6	.547	28.2	.468	38.5	.410	50.2	.365	63.6	.327	78.5	.297	95.0	.273	113.0
2.6	3.78	.86	1.891	3.40	1.26	7.65	.945	13.6	.757	21.3	.631	30.6	.540	41.7	.473	54.4	.420	68.9	.378	85.1	.344	103.0	.315	122.0
2.8	4.32	.92	2.16	3.66	1.44	8.24	1.08	14.6	.864	22.9	.720	32.9	.617	44.9	.540	58.6	.480	74.2	.432	91.6	.392	111.0	.360	132.0
3.0	4.89	.99	2.44	3.92	1.62	8.83	1.22	15.7	.978	24.5	.815	35.3	.698	48.1	.611	62.8	.544	79.5	.488	98.2	.444	119.0	.407	141.0
3.2	5.47	1.06	2.73	4.18	1.82	9.42	1.37	16.7	1.098	26.2	.915	37.7	.785	51.3	.686	67.0	.609	84.8	.549	105.0	.499	127.0	.457	151.0
3.4	6.09	1.12	3.05	4.45	2.04	10.0	1.52	17.8	1.22	27.8	1.021	40.0	.875	54.5	.765	71.2	.680	90.1	.612	111.0	.557	134.0	.510	160.0
3.6	6.76	1.19	3.38	4.71	2.26	10.6	1.69	18.8	1.35	29.4	1.131	42.4	.969	57.7	.848	75.4	.755	95.4	.679	118.0	.617	142.0	.566	169.0
3.8	7.48	1.26	3.74	4.97	2.49	11.2	1.87	19.9	1.49	31.0	1.25	44.7	1.070	60.9	.936	79.6	.831	101.0	.749	124.0	.680	150.0	.624	179.0
4.0	8.20	1.32	4.10	5.23	2.73	11.8	2.05	20.9	1.64	32.7	1.37	47.1	1.175	64.1	1.027	83.7	.913	106.0	.822	131.0	.747	158.0	.685	188.0
4.2	8.97	1.39	4.49	5.49	2.98	12.3	2.24	22.0	1.79	34.3	1.49	49.5	1.28	67.3	1.122	87.9	.998	111.0	.897	137.0	.816	166.0	.749	198.0
4.4	9.77	1.45	4.89	5.76	3.25	12.9	2.43	23.0	1.95	36.0	1.62	51.8	1.39	70.5	1.22	92.1	1.086	116.0	.977	144.0	.888	174.0	.815	207.0
4.6	10.60	1.52	5.30	6.02	3.53	13.5	2.64	24.0	2.11	37.6	1.76	54.1	1.51	73.7	1.32	96.3	1.177	122.0	1.059	150.0	.963	182.0	.883	217.0
4.8	11.45	1.58	5.72	6.28	3.81	14.1	2.85	25.1	2.27	39.2	1.90	56.5	1.63	76.9	1.43	100.0	1.27	127.0	1.145	157.0	1.040	190.0	.954	226.0
5.0	12.33	1.65	6.17	6.54	4.11	14.7	3.08	26.2	2.46	40.9	2.05	58.9	1.76	80.2	1.54	105.0	1.37	132.0	1.23	163.0	1.122	198.0	1.028	235.0
5.2	13.24	1.72	6.62	6.80	4.41	15.3	3.31	27.2	2.65	42.5	2.21	61.2	1.89	83.3	1.65	109.0	1.47	138.0	1.32	170.0	1.20	206.0	1.104	245.0
5.4	14.20	1.78	7.10	7.06	4.73	15.9	3.55	28.2	2.84	44.2	2.37	63.6	2.03	86.6	1.77	113.0	1.57	143.0	1.41	177.0	1.28	214.0	1.183	254.0
5.6	15.16	1.85	7.58	7.32	5.06	16.5	3.79	29.3	3.03	45.8	2.53	65.9	2.17	89.8	1.89	117.0	1.68	148.0	1.51	183.0	1.37	222.0	1.26	264.0
5.8	16.17	1.91	8.09	7.58	5.40	17.1	4.04	30.3	3.24	47.4	2.70	68.3	2.31	93.0	2.01	121.0	1.80	154.0	1.61	190.0	1.46	229.0	1.34	273.0
6.0	17.23	1.98	8.61	7.85	5.74	17.7	4.31	31.4	3.45	49.1	2.87	70.7	2.46	96.2	2.15	125.0	1.92	159.0	1.71	196.0	1.56	237.0	1.43	283.0
7.0	22.89	2.31	11.45	9.16	7.62	20.6	5.72	36.6	4.57	57.2	3.81	82.4	3.26	112.0	2.85	146.0	2.52	185.0	2.28	229.0	2.07	277.0	1.91	330.0

Loss of Head, in Pipe, by Friction.—*Continued.*

Inside Diameter of Pipe, in Inches.

Velocity, in feet, per second.	13		14		15		16		18		20		22		24		26		28		30		36	
	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.	Loss of head, in feet.	Cubic feet per minute.
2.0	.183	110	.169	128	.158	147	.147	167	.132	212	.119	262	.108	316	.098	377	.091	442	.084	513	.079	589	.066	848
2.2	.216	121	.200	141	.187	162	.175	184	.156	233	.140	288	.127	348	.116	414	.108	486	.099	564	.093	648	.078	933
2.4	.252	133	.234	154	.218	176	.205	201	.182	254	.164	314	.149	380	.136	452	.126	531	.116	616	.109	707	.091	1018
2.6	.290	144	.270	167	.252	191	.236	218	.210	275	.189	340	.171	412	.157	490	.145	575	.134	667	.126	766	.104	1100
2.8	.332	156	.308	179	.288	206	.270	234	.240	297	.216	366	.195	443	.180	528	.165	619	.153	718	.144	824	.119	1188
3.0	.375	166	.349	192	.325	221	.306	251	.271	318	.245	393	.222	475	.204	565	.188	663	.174	770	.163	883	.135	1273
3.2	.422	177	.392	205	.366	235	.343	268	.305	339	.275	419	.249	507	.220	603	.211	708	.195	821	.182	942	.152	1357
3.4	.471	188	.438	218	.408	250	.383	284	.339	360	.306	445	.278	538	.255	641	.235	752	.218	872	.204	1001	.169	1442
3.6	.522	199	.485	231	.452	265	.425	301	.377	382	.339	471	.308	570	.283	678	.261	796	.242	923	.226	1060	.188	1527
3.8	.576	210	.535	243	.499	280	.468	318	.416	403	.374	497	.340	601	.312	716	.288	840	.267	974	.249	1119	.207	1612
4.0	.632	221	.587	256	.548	294	.513	335	.456	424	.410	523	.373	633	.342	754	.315	885	.293	1026	.273	1178	.228	1697
4.2	.691	232	.641	269	.598	309	.561	352	.499	445	.449	550	.408	665	.374	791	.345	929	.320	1078	.299	1237	.249	1782
4.4	.751	243	.698	282	.651	324	.611	368	.542	466	.488	576	.444	697	.407	829	.375	973	.348	1129	.325	1296	.271	1866
4.6	.815	254	.757	295	.707	339	.662	385	.588	488	.529	602	.482	728	.441	867	.407	1017	.378	1180	.353	1355	.294	1951
4.8	.881	265	.818	308	.763	353	.715	402	.636	509	.572	628	.521	760	.476	905	.440	1062	.409	1231	.381	1414	.318	2036
5.0	.949	276	.881	321	.822	368	.770	419	.685	530	.617	654	.561	792	.513	942	.474	1106	.440	1283	.411	1472	.342	2121
5.2	1.020	287	.947	333	.883	383	.828	435	.736	551	.662	680	.602	823	.552	980	.510	1150	.473	1334	.441	1531	.368	2206
5.4	1.092	298	1.014	346	.947	397	.888	452	.788	572	.710	707	.645	855	.591	1018	.546	1194	.507	1385	.473	1590	.394	2291
5.6	1.167	309	1.083	359	1.011	412	.949	469	.843	594	.758	733	.690	887	.632	1055	.583	1239	.542	1437	.506	1649	.421	2376
5.8	1.245	321	1.155	372	1.078	427	1.011	486	.899	615	.809	759	.735	918	.674	1093	.622	1283	.578	1488	.540	1708	.450	2460
6.0	1.325	332	1.229	385	1.148	442	1.076	502	.957	636	.861	785	.782	950	.717	1131	.662	1327	.615	1539	.574	1767	.479	2545
7.0	1.75	387	1.63	449	1.52	515	1.43	586	1.27	742	1.143	916	1.040	1109	.953	1319	.879	1548	.817	1796	.762	2061	.636	2868

Example. Have 200-foot head and 600 feet of 11-inch pipe, carrying 119 cubic feet of water per minute. To find effective head: In right-hand column, under 11-inch pipe, find 119 cubic feet; opposite this will be found the coefficient of friction for this amount of water, which is .444. Multiply this by the number of hundred feet of pipe, which is 6, and you will have 2.66 feet, which is the loss of head. Therefore, the effective head is 200 — 2.66 = 197.34.

Flow of Water in Open Channels.

In computing the flow of water in channels, canals, rivers, ditches, etc., the form of the Chézy formula is retained, the various working formulas being arranged to permit the value of the coefficient, C, in the formula,

$$v = C\sqrt{rs},$$

to be determined for the various classes of channels.

In France the formula of Bazin is generally used, as follows:

$$v = \frac{157.6}{1 + \frac{\gamma}{\sqrt{r}}}\sqrt{rs}, \text{ for English measures;}$$

$$v = \frac{87}{1 + \frac{\gamma}{\sqrt{r}}}\sqrt{rs}, \text{ for metric measures.}$$

In these formulas γ is a coefficient dependent upon the character of the wetted surface; r is the hydraulic radius, or cross-section, divided by the wetted perimeter; and s is the slope, or sine of the angle of inclination.

Bazin divides channels into six classes, with a value of γ for each.

Class.	Character of wetted surface.	γ	
		Feet.	Metres.
I.	Smooth cement, planed wood	.109	.06
II.	Planks, bricks, cut masonry, etc.	.290	.16
III.	Rubble masonry	.833	.46
IV.	Earth, dry rubble, etc.	1.540	.85
V.	Earthen channels in ordinary condition	2.355	1.30
VI.	Earthen channels or rivers, with stony beds and grassy banks	3.170	1.75

Although the formula appears complicated, it is not difficult of application, and its use may be simplified by the use of the diagram on page 536, which is for the metric system, and is due to M. Soreau.

Diagram for Flow of Water.

Bazin's Formula.

Metric System.

$$v = \frac{87}{1 + \frac{\gamma}{\sqrt{r}}}\sqrt{rs}.$$

Join γ to r. Then draw a line parallel to this through s, and it will intersect v at the velocity value. In the diagram, $r = 4$, $\gamma = 1.30$, $s = 0.004$, and v is found to be 6.63 metres per second.

In Switzerland, Germany, and to some extent in the United States and in England, Kutter's formula is used. This is also in the Chézy form, and consists of a rather complicated expression for the value of the coefficient, C, in the formula,

$$v = C\sqrt{rs},$$

s being the slope of the stream, and r the hydraulic radius, or cross-section, divided by the wetted perimeter. In the English measure r is taken in

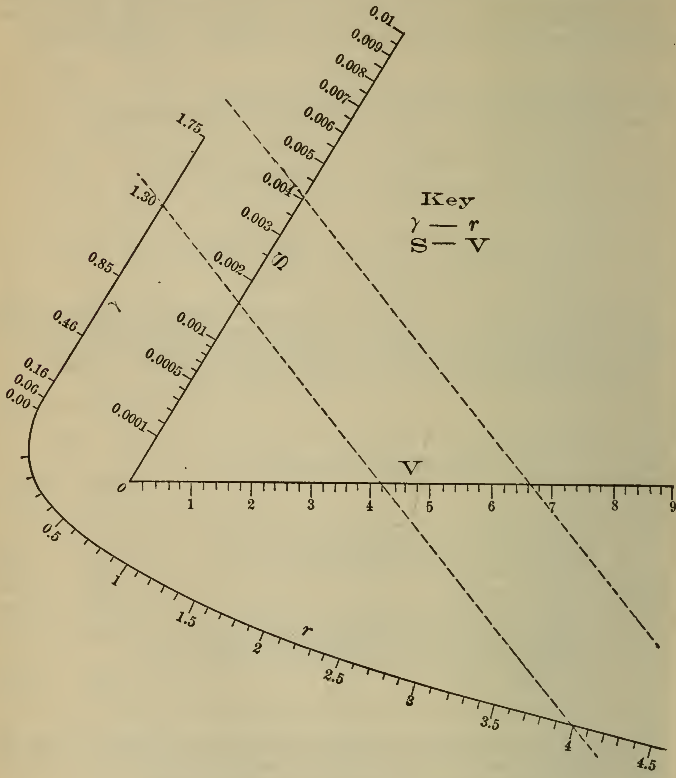


Diagram for Bazin Formula. (See page 535.)

feet,—*i.e.*, the cross-section in square feet is divided by the wetted perimeter in feet. In the metric system the cross-section is taken in square metres and the wetted perimeter in metres. The Kutter formula, then, is

$$C = \frac{41.6 + \dfrac{.00281}{s} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{.00281}{s}\right)n}{\sqrt{r}}}, \text{ English system;}$$

$$C = \frac{23 + \dfrac{.00155}{s} + \dfrac{1}{n}}{1 + \dfrac{\left(23 + \dfrac{.00155}{s}\right)n}{\sqrt{r}}}, \text{ metric system.}$$

In this formula the quantity, n, is a factor, the value of which depends upon the character of the channel. The value of n to be used in the formula may be taken from the following list:

Artificial Channels, Uniform Section.

Surface.	n
Planed boards	.009
Cement, neat	.010
Plaster, 3 cement, 1 sand	.011
Rough boards	.012
Ashlar, or brickwork	.013
Rubble	.017

Natural Channels.

Canals in very firm gravel	.020
Canals and rivers in fairly good order, free from stones and weeds	.025
Canals and rivers with occasional stones and weeds	.030
Streams in bad condition, with many stones and weeds	.035

The whole subject of the derivation and use of Kutter's formula, with many examples, are given in the book entitled, "A General Formula for the Uniform Flow of Water," by Ganguillet and Kutter, translated by Hering and Trautwine. In order to avoid the tedious computations with the formula, values of C are computed and tabulated; but sufficient precision may be attained by the use of a diagram, which is appended. This is a modification, by M. Soreau, of the original diagram by Kutter, the change being only to put it in more convenient form for the page. The diagram is for use in the metric system.

The use of the diagram will be best understood by an example. Taking the same data as were used with the Bazin formula, page 535, let $r = 4$ and $s = .004$. For a canal in fairly good condition take $n = .025$. We then join 4, on the horizontal line, r, with .025, on the curve. The intersection of the dotted line with the inclined scale gives the value, $C = 49$. We then have

$$v = 49\sqrt{4 \times .004} = 49\sqrt{.016} = 6.17 \text{ metres per second.}$$

Diagram for Flow of Water.

(See page 538.)

Kutter's Formula.

Metric System.

Join point on line, r, corresponding to given hydraulic radius, with point on the curves, corresponding to given values of s and n. The intersection with the inclined line gives the value of C in the formula,

$$v = C\sqrt{rs}.$$

The formula of Tutton, as used for pipes, may be modified for open channels, as follows:

$$v = \frac{1.54}{n} r^{\frac{2}{3}} s^{\frac{1}{2}},$$

in which n is the same as in Kutter's formula, English measures being used. This has the advantage of greater simplicity, and gives equally reliable results.

The difficulty with all these formulas lies in the fact that the flow depends to a great extent upon the condition of the channel, and therefore upon the selection of the coefficient of roughness, n.

Whenever possible, the actual velocity of the stream should be measured, computations based upon assumptions as to slope and condition of roughness being made only for canals and ditches prior to construction.

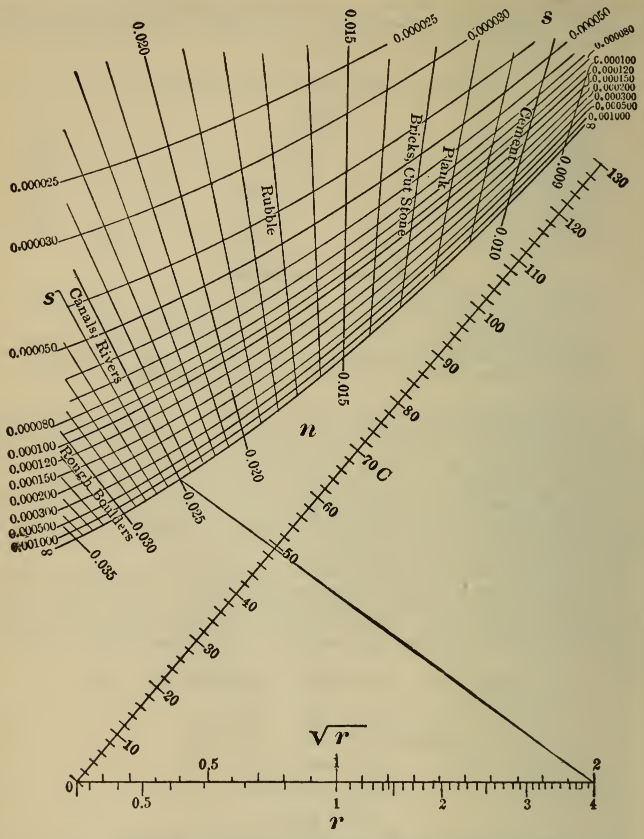


Diagram for Kutter's Formula. (See page 537.)

The following details of measurements represent the practice of the Pelton Water-wheel Company, and are based on large experience:

Select a stretch on the stream or ditch which will afford as straight and uniform a course as possible. If the water is at any point carried in a flume, it is better to measure at this point. Lay off a distance of, say, 300 feet; measure the width of flowing water at about six different places in this distance, and obtain the average width; likewise, at these same points, measure the depth of water at three or four places across the stream, and obtain the average depth. Next, drop a float in the water, noting the number of seconds it takes to travel the given distance. From this can be calculated the velocity of the water, in feet, per second. The quantity is the product obtained by multiplying the average width, in feet, by the average depth, in feet, by the velocity, which (if in feet per second) will give the flow of the stream, in cubic feet, per second. From the figures so obtained it is advisable to deduct about 20 per cent., as surface velocity of the water is in excess of the actual average velocity.

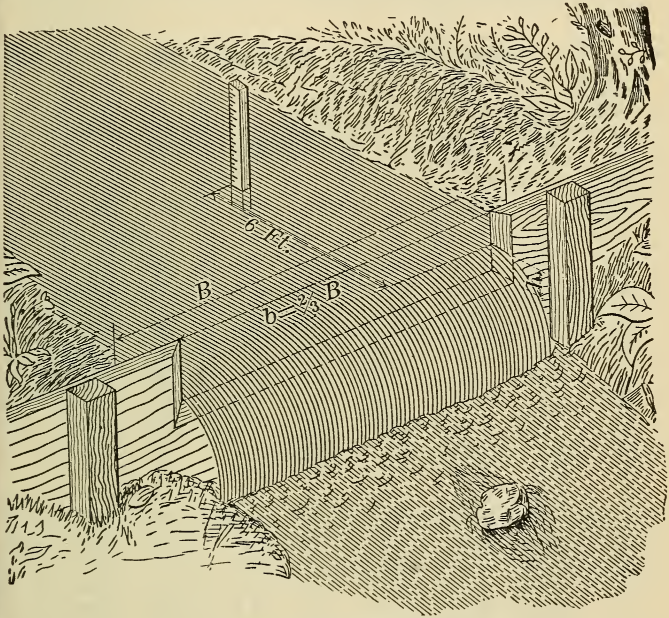
When the stream is of sufficient depth—say 3 feet or over—the average

velocity can be more closely obtained by using a pole, to one end of which is attached a stone or piece of lead of necessary weight to allow the pole to sink nearly to the bottom. In this way the velocities at the surface and bottom of the stream counteract one another, and a closer approximation of the average velocity is obtained.

The most accurate method of measuring the volume of water flowing in a stream is by the use of a *weir*.

The principle of the weir is that for a notch of given dimensions and determinate head of water the flow through it is constant and uniform. If, therefore, the flow of a stream can all be caused to pass through a notch of a certain shape, the volume can be determined from the size of the notch and the depth of the water.

The general arrangement of a weir will be seen in the illustration, the dimensions being determined by the volume of water flowing in the stream. The width of the notch can be carefully measured before it is set in place, and the depth of water measured afterwards.



General arrangement of Weir.

The instructions of the Pelton Water-wheel Company are as follows:

Place a board or plank in the stream, as shown in the drawing, at some point where a pond will form above. The length of the notch in the dam should be from two to four times its depth for small quantities, and longer for large quantities. The edges of the notch should be bevelled towards the intake side, as shown. The overfall below the notch should not be less than twice its depth,—that is, 12 inches, if the notch is 6 inches deep, and so on.

In the pond, about 6 feet above the dam, drive a stake, and then obstruct the water until it rises precisely to the bottom of the notch, and mark the stake at this level. Then complete the dam so as to cause all the water to flow through the notch, and, after allowing time for the water to

settle, mark the stake again for this new level. If preferred, the stake can be driven with its top precisely level with the bottom of the notch, and the depth of the water be measured with a rule after the water is flowing free; but the marks are preferable, in most cases.

The theoretical quantity of water passing over a weir is given by the formula,

$$Q = \tfrac{2}{3}\sqrt{2g}\,.\,bH^{\frac{3}{2}},$$

in which b is the width of the notch, or the length of the weir; H, the depth of water; g, the acceleration of gravity, $= 32.2$.

The actual quantity of water has been determined by numerous experiments. According to Francis, we may use

$$Q = 3.33bH^{\frac{3}{2}} = 3.33bH\sqrt{H},$$

H and b both being taken in feet.

The following table also may be used.

Table for Weir Measurement.

Pelton Water-wheel Company.

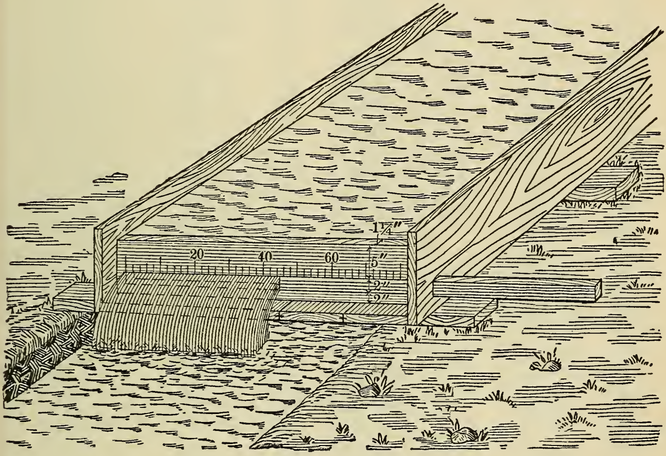
Giving Cubic Feet of Water per Minute that will Flow over a Weir 1 inch wide and from ⅛ to 20⅞ inches deep.

Inches.		⅛	¼	⅜	½	⅝	¾	⅞
0	.00	.01	.05	.09	.14	.19	.26	.32
1	.40	.47	.55	.64	.73	.82	.92	1.02
2	1.13	1.23	1.35	1.46	1.58	1.70	1.82	1.95
3	2.07	2.21	2.34	2.48	2.61	2.76	2.90	3.05
4	3.20	3.35	3.50	3.66	3.81	3.97	4.14	4.30
5	4.47	4.64	4.81	4.98	5.15	5.33	5.51	5.69
6	5.87	6.06	6.25	6.44	6.62	6.82	7.01	7.21
7	7.40	7.60	7.80	8.01	8.21	8.42	8.63	8.83
8	9.05	9.26	9.47	9.69	9.91	10.13	10.35	10.57
9	10.80	11.02	11.25	11.48	11.71	11.94	12.17	12.41
10	12.64	12.88	13.12	13.36	13.60	13.85	14.09	14.34
11	14.59	14.84	15.09	15.34	15.59	15.85	16.11	16.36
12	16.62	16.88	17.15	17.41	17.67	17.94	18.21	18.47
13	18.74	19.01	19.29	19.56	19.84	20.11	20.39	20.67
14	20.95	21.23	21.51	21.80	22.08	22.37	22.65	22.94
15	23.23	23.52	23.82	24.11	24.40	24.70	25.00	25.30
16	25.60	25.90	26.20	26.50	26.80	27.11	27.42	27.72
17	28.03	28.34	28.65	28.97	29.28	29.59	29.91	30.22
18	30.54	30.86	31.18	31.50	31.82	32.15	32.47	32.80
19	33.12	33.45	33.78	34.11	34.44	34.77	35.10	35.44
20	35.77	36.11	36.45	36.78	37.12	37.46	37.80	38.15

Example. Suppose the weir to be 66 inches long, and the depth of water on it to be 11⅝ inches. Follow down the left-hand column of the figures in the table until you come to 11 inches. Then run across the table on a line with the 11 until under ⅝, on top line, and you will find 15.85. This, multiplied by 66, the length of weir, gives 1046.10, the number of cubic feet of water passing per minute.

The Miner's Inch.

The term Miner's Inch is used in a number of Western States, being used in the measurement of water for mining and irrigation. The term is more or less indefinite, for the reason that the water companies do not all use the same head above the centre of the aperture, and the inch varies from 1.36 to 1.73 cubic feet per minute each; but the most common measure-



ment is through an aperture 2 inches high and whatever length is required, and through a plank 1¼ inches thick, as shown in the illustration. The lower edge of the aperture should be 2 inches above the bottom of the measuring box, and the plank 5 inches high above the aperture, thus making a 6-inch head above the centre of the stream. Each square inch of this opening represents a miner's inch, which is equal to a flow of 1½ cubic feet per minute.

The use of the miner's inch is to be discouraged, because of its indefinite value. In some States its legal value has been made 1.5 cubic feet per minute.

Tables for Calculating the Horse-power of Water.

Miner's Inch Table.

The following table gives the horse-power of 1 miner's inch of water under heads from 1 up to 1100 feet. This inch equals $1\frac{1}{2}$ cubic feet per minute.

Heads, in feet.	Horse-power.	Heads, in feet.	Horse-power.
1	.002 4147	320	.772 704
20	.048 2294	330	.796 851
30	.072 441	340	.820 998
40	.096 588	350	.845 145
50	.120 735	360	.869 292
60	.144 882	370	.893 439
70	.169 029	380	.917 586
80	.193 176	390	.941 733
90	.217 323	400	.965 880
100	.241 470	410	.990 027
110	.265 617	420	1.014 174
120	.289 764	430	1.038 321
130	.313 911	440	1.062 468
140	.338 058	450	1.086 615
150	.362 205	460	1.110 762
160	.386 352	470	1.134 909
170	.410 499	480	1.159 056
180	.434 646	490	1.183 206
190	.458 793	500	1.207 350
200	.482 940	520	1.255 644
210	.507 087	540	1.303 938
220	.531 234	560	1.352 232
230	.555 381	580	1.400 526
240	.579 528	600	1.448 820
250	.603 675	650	1.569 555
260	.627 822	700	1.690 290
270	.651 969	750	1.811 025
280	.676 116	800	1.931 760
290	.700 263	900	2.173 230
300	.724 410	1000	2.414 700
310	.748 557	1100	2.656 170

Cubic Feet Table.

The following table gives the horse-power of 1 cubic foot of water per minute under heads from 1 up to 1100 feet.

Heads, in feet.	Horse-power.	Heads, in feet.	Horse-power.
1	.001 6098	320	.515 136
20	.032 196	330	.531 234
30	.048 294	340	.547 332
40	.064 392	350	.563 430
50	.080 490	360	.579 528
60	.096 588	370	.595 626
70	.112 686	380	.611 724
80	.128 784	390	.627 822
90	.144 892	400	.643 920
100	.160 980	410	.660 018
110	.177 078	420	.676 116
120	.193 176	430	.692 214
130	.209 274	440	.708 312
140	.225 372	450	.724 410
150	.241 470	460	.740 508
160	.257 568	470	.756 606
170	.273 666	480	.772 704
180	.289 764	490	.788 802
190	.305 862	500	.804 900
200	.321 960	520	.837 096
210	.338 058	540	.869 292
220	.354 156	560	.901 488
230	.370 254	580	.933 684
240	.386 352	600	.965 880
250	.402 450	650	1.046 370
260	.418 548	700	1.126 860
270	.434 646	750	1.207 350
280	.450 744	800	1.287 840
290	.466 842	900	1.448 820
300	.482 940	1000	1.609 800
310	.499 038	1100	1.770 780

When the Exact Head is Found in Above Table:

Example. Have 100-foot head and 50 inches of water. How many horse-power?

By reference to above table the horse-power of 1 inch under 100-foot head is .241470. This amount, multiplied by the number of inches, 50, will give 12.07 horse-power.

When Exact Head is Not Found in Table:

Take the horse-power of 1 inch under 1-foot head and multiply by the number of inches, and then by number of feet head. The product will be the required horse-power.

The above formula will answer for the cubic feet table by substituting the equivalents therein for those of miner's inches.

NOTE.—The above tables are based upon an efficiency of 85 per cent.

Contents, in Cubic Feet and United States Gallons, of Pipes and Cylinders of Various Diameters and 1 Foot in Length.

1 gallon = 231 cubic inches. 1 cubic foot = 7.4805 gallons.

Diameter, in inches.	For 1 foot in length. Cubic feet; also, area in square feet.	For 1 foot in length. United States gallons 231 cubic inches.	Length, in inches, of cylinder of 1 cubic foot capacity.	Diameter, in inches.	For 1 foot in length. Cubic feet; also, area in square feet.	For 1 foot in length. United States gallons 231 cubic inches.	Length, in inches, of cylinder of 1 cubic foot capacity.
1/4	.0003	.0025	....	6	.1963	1.469	61.13
5/16	.0005	.0040	....	6 1/8	.2046	1.531	58.65
3/8	.0008	.0057	....	6 1/4	.2131	1.594	56.31
7/16	.0010	.0078	....	6 3/8	.2217	1.662	54.01
1/2	.0014	.0102	....	6 1/2	.2304	1.724	52.08
9/16	.0017	.0129	....	6 5/8	.2394	1.791	50.13
5/8	.0021	.0159	....	6 3/4	.2485	1.859	48.29
11/16	.0026	.0193	....	6 7/8	.2578	1.928	46.55
3/4	.0031	.0230	....	7	.2673	1.999	44.89
13/16	.0036	.0269	....	7 1/8	.2769	2.071	43.34
7/8	.0042	.0312	....	7 1/4	.2867	2.145	41.86
15/16	.0048	.0359	....	7 3/8	.2967	2.219	40.45
1	.0055	.0408	2181.81	7 1/2	.3068	2.295	39.11
1 1/8	.0069	.0516	1739.13	7 5/8	.3171	2.372	37.84
1 1/4	.0085	.0638	1411.76	7 3/4	.3276	2.450	36.63
1 3/8	.0103	.0770	1165.04	7 7/8	.3382	2.530	35.48
1 1/2	.0123	.0918	975.69	8	.3491	2.611	34.37
1 5/8	.0144	.1077	833.33	8 1/8	.3601	2.694	33.32
1 3/4	.0167	.1249	718.56	8 1/4	.3712	2.777	32.33
1 7/8	.0192	.1436	625.00	8 3/8	.3826	2.862	31.36
2	.0218	.1632	550.44	8 1/2	.3941	2.948	30.45
2 1/8	.0246	.1840	487.80	8 5/8	.4057	3.035	29.58
2 1/4	.0276	.2066	434.76	8 3/4	.4176	3.125	28.74
2 3/8	.0308	.2304	389.52	8 7/8	.4296	3.214	27.93
2 1/2	.0341	.2550	351.84	9	.4418	3.305	27.16
2 5/8	.0376	.2813	319.14	9 1/8	.4541	3.397	26.43
2 3/4	.0412	.3085	291.26	9 1/4	.4667	3.491	25.71
2 7/8	.0451	.3374	266.07	9 3/8	.4794	3.586	25.03
3	.0491	.3672	244.39	9 1/2	.4922	3.682	24.38
3 1/8	.0533	.3987	225.14	9 5/8	.5053	3.780	23.75
3 1/4	.0576	.4309	208.33	9 3/4	.5185	3.879	23.14
3 3/8	.0621	.4645	193.23	9 7/8	.5319	3.979	22.56
3 1/2	.0668	.4998	178.14	10	.5454	4.080	22.00
3 5/8	.0717	.5361	167.36	10 1/8	.5591	4.182	21.46
3 3/4	.0767	.5738	156.45	10 1/4	.5730	4.286	20.94
3 7/8	.0819	.6127	146.52	10 3/8	.5871	4.392	20.44
4	.0873	.6528	137.43	10 1/2	.6013	4.498	19.96
4 1/8	.0928	.6942	129.31	10 5/8	.6157	4.606	19.49
4 1/4	.0985	.7369	121.82	10 3/4	.6303	4.715	19.04
4 3/8	.1044	.7810	114.94	10 7/8	.6450	4.825	18.60
4 1/2	.1104	.8263	108.69	11	.6600	4.937	18.18
4 5/8	.1167	.8727	102.82	11 1/8	.6751	5.050	17.78
4 3/4	.1231	.9206	97.50	11 1/4	.6903	5.164	17.38
4 7/8	.1296	.9695	92.59	11 3/8	.7057	5.279	17.00
5	.1364	1.020	87.98	11 1/2	.7213	5.396	16.63
5 1/8	.1433	1.072	83.74	11 5/8	.7370	5.513	16.28
5 1/4	.1503	1.125	79.84	11 3/4	.7530	5.633	15.94
5 3/8	.1576	1.179	76.14	11 7/8	.7691	5.753	15.60
5 1/2	.1650	1.234	72.73	12	.7854	5.875	15.28
5 5/8	.1726	1.291	69.52	12 1/8	.8018	5.998	14.94
5 3/4	.1803	1.349	66.56	12 1/4	.8184	6.122	14.66
5 7/8	.1883	1.409	63.72	12 3/8	.8352	6.248	14.37

Contents, in Cubic Feet and United States Gallons, of Pipes and Cylinders of Various Diameters and 1 Foot in Length.—*Continued.*

1 gallon = 231 cubic inches. 1 cubic foot = 7.4805 gallons.

Diameter, in inches.	For 1 foot in length. Cubic feet; also, area in square feet.	United States gallons 231 cubic inches.	Length, in inches, of cylinder of 1 cubic foot capacity.	Diameter, in inches.	For 1 foot in length. Cubic feet; also, area in square feet.	United States gallons 231 cubic inches.	Length, in inches, of cylinder of 1 cubic foot capacity.
12½	.8522	6.375	14.080	21¼	2.463	18.42	4.872
12⅝	.8693	6.503	13.800	21½	2.521	18.86	4.760
12¾	.8866	6.632	13.530	21¾	2.580	19.30	4.651
12⅞	.9041	6.763	13.270	22	2.640	19.75	4.545
13	.9218	6.895	13.020	22¼	2.700	20.20	4.445
13⅛	.9395	7.028	12.780	22½	2.761	20.66	4.347
13¼	.9575	7.163	12.530	22¾	2.823	21.12	4.251
13⅜	.9757	7.299	12.300	23	2.885	21.58	4.160
13½	.994	7.436	12.070	23¼	2.948	22.05	4.070
13⅝	1.013	7.578	11.850	23½	3.012	22.53	3.990
13¾	1.031	7.712	11.640	23¾	3.076	23.01	3.901
13⅞	1.051	7.855	11.420	24	3.142	23.50	3.819
14	1.069	7.997	11.230	25	3.409	25.50	3.520
14⅛	1.088	8.139	11.030	26	3.678	27.58	3.263
14¼	1.107	8.281	10.840	27	3.976	29.74	3.018
14⅜	1.127	8.431	10.650	28	4.276	31.99	2.806
14½	1.147	8.578	10.460	29	4.587	34.31	2.616
14⅝	1.167	8.730	10.280	30	4.909	36.72	2.444
14¾	1.187	8.879	10.110	31	5.241	39.21	2.290
14⅞	1.207	9.029	9.940	32	5.585	41.78	2.149
15	1.227	9.180	9.780	33	5.940	44.43	2.020
15⅛	1.248	9.336	9.620	34	6.305	47.16	1.903
15¼	1.268	9.485	9.460	35	6.681	49.98	1.796
15⅜	1.289	9.642	9.310	36	7.069	52.88	1.698
15½	1.310	9.801	9.160	37	7.467	55.86	1.607
15⅝	1.332	9.964	9.010	38	7.876	58.92	1.527
15¾	1.353	10.121	8.870	39	8.296	62.06	1.446
15⅞	1.374	10.278	8.730	40	8.727	65.28	1.375
16	1.396	10.440	8.600	41	9.168	68.58	1.309
16¼	1.440	10.772	8.330	42	9.621	71.91	1.247
16½	1.485	11.11	8.081	43	10.085	75.44	1.190
16¾	1.530	11.45	7.843	44	10.559	78.99	1.136
17	1.576	11.79	7.511	45	11.045	82.62	1.087
17¼	1.623	12.14	7.394	46	11.541	86.33	1.040
17½	1.670	12.49	7.186	47	12.048	90.13	.996
17¾	1.718	12.85	6.985	48	12.566	94.00	.955
18	1.768	13.22	6.787	49	13.095	97.96	.916
18¼	1.817	13.59	6.604	50	13.635	102.00	.880
18½	1.867	13.96	6.427	51	14.186	106.12	.846
18¾	1.917	14.34	6.259	52	14.748	110.32	.814
19	1.969	14.73	6.094	53	15.320	114.60	.783
19¼	2.021	15.12	5.938	54	15.904	118.97	.755
19½	2.074	15.51	5.786	55	16.499	122.82	.727
19¾	2.128	15.92	5.639	56	17.104	127.95	.702
20	2.182	16.32	5.500	57	17.720	132.55	.677
20¼	2.237	16.73	5.365	58	18.347	137.24	.654
20½	2.292	17.15	5.236	59	18.985	142.02	.632
20¾	2.348	17.56	5.110	60	19.637	146.89	.611
21	2.405	17.99	4.989				

To find the capacity of pipes greater than the largest given in the table, look in the table for a pipe one-half the given size and multiply its capacity by 4, or one of one-third its size, and multiply its capacity by 9, etc.

Table for Tank Measurement.

Giving the Number of Cubic Feet of Water Discharged per Minute through an Orifice 1 inch square under any Head of Water from 3 to 72 inches.

Heads, in inches.	Cubic feet discharged per minute.	Heads, in inches.	Cubic feet discharged per minute.	Heads, in inches.	Cubic feet discharged per minute.	Heads, in inches.	Cubic feet discharged per minute.	Heads, in inches.	Cubic feet discharged per minute.
3	1.12	17	2.51	31	3.36	45	4.05	59	4.63
4	1.27	18	2.58	32	3.41	46	4.09	60	4.65
5	1.40	19	2.64	33	3.47	47	4.12	61	4.72
6	1.52	20	2.71	34	3.52	48	4.18	62	4.74
7	1.64	21	2.78	35	3.57	49	4.21	63	4.78
8	1.75	22	2.84	36	3.62	50	4.27	64	4.81
9	1.84	23	2.90	37	3.67	51	4.30	65	4.85
10	1.94	24	2.97	38	3.72	52	4.34	66	4.89
11	2.03	25	3.03	39	3.77	53	4.39	67	4.92
12	2.12	26	3.08	40	3.81	54	4.42	68	4.97
13	2.20	27	3.14	41	3.86	55	4.46	69	5.00
14	2.28	28	3.20	42	3.91	56	4.52	70	5.03
15	2.36	29	3.25	43	3.95	57	4.55	71	5.07
16	2.43	30	3.31	44	4.00	58	4.58	72	5.09

Example. Suppose the opening to be 36 inches long and 2 inches high, and the head of water above the opening 25 inches. Multiply the length, 36, by 2, the height of the opening, and it gives 72. Referring to the above table, opposite 25-inch head will be found 3.03. This, multiplied by 72, gives 218.16, the number of cubic feet of water passing through the opening per minute.

Water-wheels.

Water-wheels may be divided into two classes, vertical and horizontal, according to the position of the plane in which the revolving wheel is placed.

Vertical wheels include Overshot-wheels, Undershot-wheels, Breast-wheels, and Impact-wheels of the Pelton type. Horizontal wheels include practically all forms of turbines, although in some cases turbine wheels are placed on horizontal axes and revolve in vertical planes, without, however, suffering any material change in construction or action. In the pages immediately following the data for the various kinds of vertical wheels are:

Q = quantity of water, in cubic feet, per second;
h = head, in feet;
V = velocity of water, in feet, per second;
v = velocity of wheel buckets, in feet, per second;
u = angle of entrance;
a = area of float, in square feet.

Example. The vertical section of the immersed floats of an undershot-wheel in a mid-stream is $a = 27$ square feet; velocity of the stream, $V = 8.6$; and $v = 4$ feet per second. Required the horse-power of the wheel?

$$HP = \frac{av}{200}(V - v)^2 = \frac{27 \times 4}{200}(8.6 - 4)^2 = 11.4\ HP.$$

Example. On a breast-wheel is acting $Q = 88$ cubic feet of water per second; the head, $h = 8$ feet; velocity of the wheel at the centre of the buckets, $v = 5$ feet per second. The water strikes the buckets at an angle, $u = 8°$, and velocity, $V = 7$ feet per second. Required the horse-power of the wheel?

$$HP = \frac{88}{11.4}\left(8 + \frac{5}{25}(7 \times \cos 8° - 5)\right) = 65\ HP.$$

Example. Required the effect of a Poncelet wheel: the head, $h = 4$ feet; and the orifice, $a = 5$ square feet; the velocity of the wheel at the centre of the pressure of the floats is $v = 6.78$ feet per second?

$$V = 6.91\sqrt{4} = 13.82 \text{ feet per second;}$$

$$Q = 6.5 \times 5 \times \sqrt{4} = 65 \text{ cubic feet per second;}$$

$$HP = \frac{65 \times 6.78}{197}(13.82 - 6.78) = 15.8\ HP.$$

Example. A saw-mill wheel is to be built under a fall of $h = 18$ feet, and to make $n = 110$ revolutions per minute. Required the proper diameter of the wheel?

$$D = \frac{100}{110}\sqrt{18} = 3.857 \text{ feet}$$

at the centre of pressure of the buckets.

Velocity,

$$V = 8\sqrt{18} = 33.94 \text{ feet per second.}$$

Velocity,

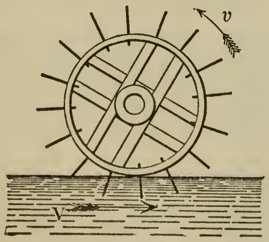
$$v = \frac{3.14 \times 3.857 \times 110}{60} = 22.2 \text{ feet per second.}$$

The fall discharged 30 cubic feet of water per second. Required the horse-power of the wheel?

$$HP = \frac{30 \times 22.2}{200}(33.94 - 22.2) = 39\ HP.$$

In general, the maximum efficiency of such wheels is obtained when $v = \frac{1}{2}V$.

Undershot Stream-wheel.

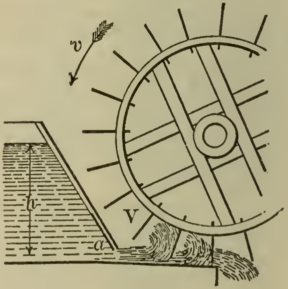


$$HP = \frac{av}{200}(V - v)^2.$$

When $V = 2v$, about, the effect will be

$$HP = \frac{aV^3}{1600}; \quad a = \text{area of float.}$$

Undershot-wheel.

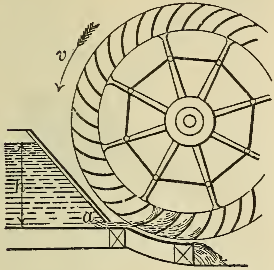


$$HP = \frac{Qv}{454}(V - v);$$

$$HP = \frac{mav}{56.6}(V - v)\sqrt{h};$$

When $V = 2v$, about, $HP = \dfrac{ah\sqrt{h}}{3.9}$.

Poncelet Wheel.



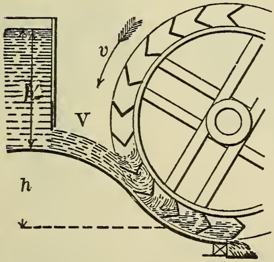
$$HP = \frac{Qv}{228}(V - v), \text{ when } h > 5 \text{ feet};$$

$$HP = \frac{Qv}{200}(V - v), \text{ when } h < 5 \text{ feet};$$

$$Q = 8ma\sqrt{h};$$

$$V = 6.91\sqrt{h}.$$

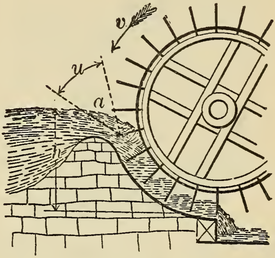
Breast-wheel with Parabolic Drain.



$$HP = \frac{Q}{12}\left[h + \frac{v}{28}(V - v)\right];$$

$$Q = 6.5a\sqrt{h'}.$$

Low Breast-wheel.



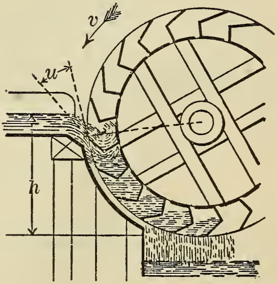
$$HP = \frac{Q}{11.2}\left[h + \frac{v}{32}(V \cos u - v)\right];$$

$$Q = kb;$$

$$V = \frac{Q}{a}.$$

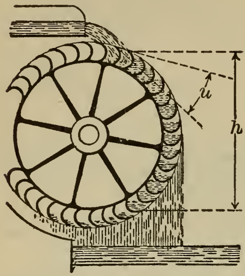
See table for weirs.

Breast-wheel.



$$HP = \frac{Q}{11.4}\left[h + \frac{v}{25}(V \cos u - v)\right].$$

Overshot-wheel.



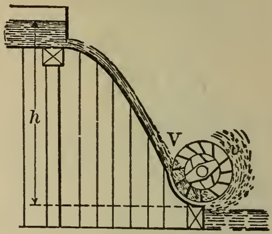
$$HP = \frac{Q}{13.7}\left[h + \frac{v}{21.5}(V\cos u - v)\right].$$

Proper velocity about

$$n = \frac{35D + 100}{D}$$

revolutions per minute.

Saw-mill Wheel.



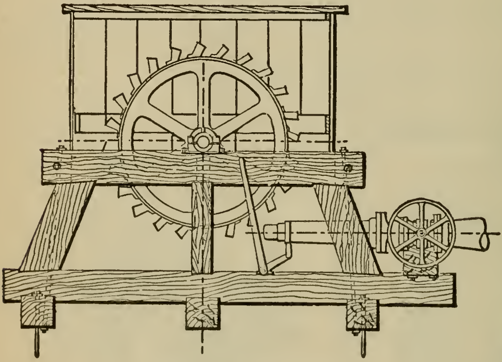
$$HP = \frac{Qv}{200}(V - v).$$

Proper diameter of the wheel:

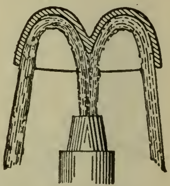
$$D = \frac{100}{n}\sqrt{h}\text{, in feet;}$$

n = revolutions per minute.

For high heads of water the most effective form of wheel is the tangential impact type, of which the well-known Pelton wheel is a good example.



Pelton Water-wheel.



Pelton Bucket.

The buckets of the Pelton wheel are made double, with centre fin, producing a side discharge, and permitting the maximum transfer of energy from the jet to the wheel, an efficiency of 85 per cent. being obtained.

Pelton Water-wheel Table.

The calculations for power in this table are based upon the application of one stream to the wheel, as also upon an 85 per cent. efficiency and effective heads, no allowance being made for loss of head, in pipe, by friction. The smaller figures under those denoting the various heads give the equivalent pressure, in pounds, and spouting velocity of the water, in feet, per minute. The cubic feet measurement is also based on the flow per minute.

Head, in feet.	Size of wheels.	6 in.	12 inch.	15 inch.	18 inch.	24 inch.	3 foot.	4 foot.	5 foot.	6 foot.
100	Horse-power	.60	1.40	2.32	4.21	7.49	16.84	29.93	46.85	67.36
43 lb.	Cubic feet...	3.74	8.74	14.81	26.22	46.58	104.88	186.32	291.51	419.52
4812.00	Miner's inc's	2.33	5.46	9.25	17.48	31.05	69.93	124.21	194.34	279.72
	Revolutions.	1530	765	612	510	382	255	191	152	127
120	Horse-power	.79	1.84	3.12	5.54	9.85	22.18	39.41	61.66	88.75
52 lb.	Cubic feet...	4.10	9.57	16.21	28.72	51.02	114.91	204.10	319.33	459.64
5271.30	Miner's inc's	2.56	5.98	10.13	19.15	34.01	76.60	136.06	212.89	306.43
	Revolutions.	1677	838	671	559	419	279	209	167	139
140	Horse-power	99	2.33	3.94	6.99	12.41	27.96	49.64	77.71	111.85
60 lb.	Cubic feet...	4.43	10.34	17.53	31.03	55.11	124.12	220.44	344.92	496.48
5693.65	Miner's inc's	2.76	6.46	10.95	20.68	36.74	82.72	146.96	229.94	330.88
	Revolutions.	1812	906	725	604	453	302	226	181	151
160	Horse-power	1.22	2.84	4.82	8.54	15.17	34.16	60.68	94.94	136.65
69 lb.	Cubic feet...	4.73	11.05	18.74	33.17	58.92	132.68	235.68	368.73	530.75
6086.74	Miner's inc's	2.95	6.90	11.71	22.11	39.28	88.46	157.12	245.82	353.84
	Revolutions.	1938	969	775	646	484	323	242	193	161
180	Horse-power	1.45	3.39	5.75	10.19	18.10	40.77	72.41	113.30	163.08
78 lb.	Cubic feet...	5.02	11.72	19.87	35.18	62.49	140.74	249.97	391.10	562.96
6455.97	Miner's inc's	3.13	7.32	12.41	23.45	41.66	93.82	166.64	260.73	375.29
	Revolutions.	2049	1024	820	683	513	342	256	206	171
200	Horse-power	1.70	3.97	6.74	11.93	21.20	47.75	84.81	132.70	191.00
87 lb.	Cubic feet...	5.29	12.36	20.94	37.08	65.87	148.35	263.49	412.25	593.40
6805.17	Miner's inc's	3.30	7.72	13.08	24.72	43.91	98.90	175.66	274.83	395.60
	Revolutions.	2160	1080	864	720	540	360	270	216	180
220	Horse-power	1.96	4.59	7.77	13.77	24.46	55.09	97.85	153.10	220.36
95 lb.	Cubic feet...	5.55	12.96	21.96	38.89	69.08	155.59	276.35	432.38	622.36
7137.35	Miner's inc's	3.46	8.10	13.72	25.93	46.05	103.72	184.23	288.25	414.91
	Revolutions.	2268	1134	906	756	567	378	283	226	189
240	Horse-power	2.24	5.23	8.86	15.69	27.87	62.77	111.50	174.45	251.10
105 lb.	Cubic feet...	5.80	13.54	22.93	40.62	72.16	162.50	288.64	451.60	650.03
7454.70	Miner's inc's	3.62	8.46	14.33	27.08	48.10	108.34	192.42	301.07	433.36
	Revolutions.	2370	1185	948	790	592	395	296	237	197
260	Horse-power	2.52	5.89	10.05	17.69	31.43	70.78	125.72	196.71	283.15
113 lb.	Cubic feet...	6.04	14.09	23.88	42.28	75.10	169.14	300.43	470.04	676.59
7759.10	Miner's inc's	3.77	8.80	14.92	28.19	50.07	112.76	200.28	313.36	451.05
	Revolutions.	2466	1233	986	822	617	411	308	247	206
280	Horse-power	2.82	6.59	11.16	19.77	35.12	79.11	140.51	219.84	316.44
121 lb.	Cubic feet...	6.26	14.62	24.79	43.88	77.94	175.53	311.77	487.79	702.12
8052.01	Miner's inc's	3.91	9.13	15.49	29.25	51.29	117.02	205.18	325.19	468.06
	Revolutions.	2562	1281	1025	854	639	427	319	255	213
300	Horse-power	3.13	7.31	12.38	21.93	38.95	87.73	155.83	243.82	350.94
130 lb.	Cubic feet...	6.48	15.13	25.66	45.42	80.67	181.69	322.71	504.91	726.76
8334.62	Miner's inc's	4.05	9.45	16.03	30.28	53.78	121.12	215.14	336.60	484.51
	Revolutions.	2652	1326	1060	884	663	442	331	265	221
320	Horse-power	3.45	8.05	13.64	24.16	42.91	96.65	171.68	268.60	386.62
139 lb.	Cubic feet...	6.70	15.63	26.50	46.91	83.32	187.65	333.29	521.46	750.60
8607.94	Miner's inc's	4.18	9.76	16.56	31.27	55.55	125.10	222.19	347.64	500.40
	Revolutions.	2739	1369	1095	913	685	456	342	274	228
340	Horse-power	3.78	8.82	14.94	26.46	47.00	105.86	188.02	294.18	423.44
147 lb.	Cubic feet...	6.90	16.12	27.31	48.35	85.88	193.42	343.55	537.51	773.71
8872.89	Miner's inc's	4.31	10.07	17.06	32.24	57.26	128.98	229.04	358.34	515.93
	Revolutions.	2823	1411	1130	941	706	470	353	282	235

Pelton Water-wheel Table.—*Continued.*

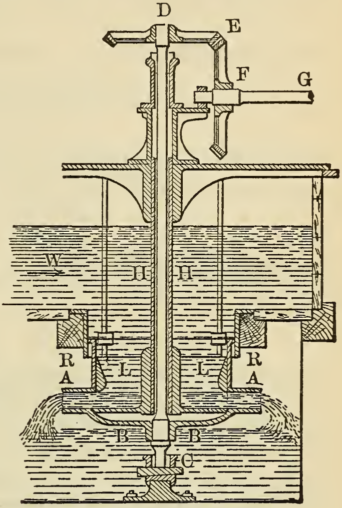
The calculations for power in this table are based upon the application of one stream to the wheel, as also upon an 85 per cent. efficiency and effective heads, no allowance being made for loss of head, in pipe, by friction. The smaller figures under those denoting the various heads give the equivalent pressure, in pounds, and spouting velocity of the water, in feet, per minute. The cubic feet measurement is also based on the flow per minute.

Head, in feet.	Size of wheels.	6 in.	12 inch.	15 inch.	18 inch.	24 inch.	3 foot.	4 foot.	5 foot.	6 foot.
360	Horse-power	4.10	9.61	16.28	28.83	51.21	115.34	204.86	320.52	461.36
156 lb.	Cubic feet...	7.10	16.58	28.10	49.75	88.37	199.03	353.51	553.10	796.14
9130.14	Miner's inc's	4.43	10.36	17.56	33.17	58.91	132.68	235.64	368.73	530.75
	Revolutions.	2907	1453	1161	969	726	484	363	290	242
380	Horse-power	4.46	10.42	17.66	31.27	55.54	125.08	222.16	347.60	500.33
165 lb.	Cubic feet...	7.30	17 04	28.88	51.12	90.80	204.48	363.20	568.25	817.95
9380.32	Miner's inc's	4.56	10.65	18.03	34.08	60.53	136.32	242.13	378.83	545.29
	Revolutions.	2985	1492	1194	995	746	497	373	298	248
400	Horse-power	4.82	11.25	19.07	33.77	59.98	135.08	239.94	375.40	540.35
173 lb.	Cubic feet...	7.49	17.48	29.63	52.45	93.16	209.80	372.64	583.02	839.20
9624.00	Miner's inc's	4.68	10.92	18.51	34.96	62.10	139.84	248.40	388.68	559.35
	Revolutions.	3063	1531	1225	1021	765	510	382	306	255
420	Horse-power	5.19	12.11	20.52	36.33	64.54	145.34	258.16	403.91	581.39
182 lb.	Cubic feet...	7.67	17.91	30.36	53.74	95.46	214.98	381.84	597.41	859.93
9861.66	Miner's inc's	4.79	11.19	18.93	35.83	63.64	143.32	254.56	398.28	573.28
	Revolutions.	3141	1570	1255	1047	785	523	392	313	261
440	Horse-power	5.56	12.98	22.01	38.96	69.20	155.85	276.82	433.11	623.40
191 lb.	Cubic feet...	7.85	18.33	31.07	55.01	97.70	220.04	390.82	611.47	880.16
10093.74	Miner's inc's	4.90	11.45	19.41	36.66	65.13	146.64	260.53	407.65	586.56
	Revolutions.	3213	1606	1285	1071	803	535	401	320	267
460	Horse-power	5.95	13.88	23.53	41.65	73.97	166.60	295.91	462.97	666.40
200 lb.	Cubic feet...	8.03	18.74	31.77	56.24	99.90	224.98	399.61	625.22	899.95
10320.58	Miner's inc's	5.01	11.71	19.79	37.50	66.60	150.00	266.40	416.80	600.00
	Revolutions.	3285	1642	1315	1095	821	547	410	327	273
480	Horse-power	6.34	14.79	25.07	44.39	78.85	177.58	315.42	493.49	710.33
208 lb.	Cubic feet...	8.20	19.15	32.45	57.45	102.05	229.82	408.20	638.66	919.29
10542.56	Miner's inc's	5.12	11.96	20.28	38.30	68.00	153.20	272.12	425.78	612.80
	Revolutions.	3357	1678	1343	1119	839	559	419	335	279
500	Horse-power	6.74	15.73	20.66	47.20	83.83	188.80	335.34	524.66	755.20
217 lb.	Cubic feet...	8.37	19.54	33.12	58.64	104.15	234.56	416.62	651.83	938.25
10759.96	Miner's inc's	5.23	12.21	20.72	39.09	69.41	156.36	277.64	434.56	625.44
	Revolutions.	3426	1713	1370	1142	856	571	428	342	285
600	Horse-power	....	.....	.....	.....	......	248.16	440.77	689.63	992.65
260 lb.	Cubic feet...	....	.....	.....	.....	......	256.95	456.38	714.05	1027.80
11786.94	Miner's inc's	....	.....	.....	.....	......	171.30	304.24	476.03	685.20
	Revolutions.	....	.....	.....	.....	......	625	469	375	312
700	Horse-power	....	.....	.....	.....	......	312.73	555.46	869.06	1250.92
304 lb.	Cubic feet...	....	.....	.....	.....	......	277.54	492.95	771.26	1110.16
12731.34	Miner's inc's	....	.....	.....	.....	......	185.02	328.63	514.18	740.09
	Revolutions.	....	.....	.....	.....	......	675	506	405	337
800	Horse-power	....	.....	.....	.....	......	382.09	678.66	1061.81	1528.36
348 lb.	Cubic feet...	....	.....	.....	.....	......	296.70	526.99	824.51	1186.81
13610.40	Miner's inc's	....	.....	.....	.....	......	197.80	351.32	549.68	791.21
	Revolutions.	....	.....	.....	.....	......	722	542	433	361
900	Horse-power	....	.....	.....	.....	......	455.94	809.82	1267.02	1823.76
391 lb.	Cubic feet...	....	.....	.....	.....	......	314.70	558.96	874.53	1258.81
14436.00	Miner's inc's	....	.....	.....	.....	......	209.80	372.64	583.02	839.20
	Revolutions.	....	.....	.....	.....	......	766	574	459	383
1000	Horse-power	....	.....	.....	.....	......	534.01	948.48	1483.97	2136.04
434 lb.	Cubic feet...	....	.....	.....	.....	......	331.72	589.19	921.83	1326.91
15216.89	Miner's inc's	....	.....	.....	.....	......	221.15	392.79	614.56	884.61
	Revolutions.	....	.....	.....	.....	......	807	605	484	403

Turbines.

For moderate and low heads and large volumes of water turbines are now generally used. Various forms of turbines are in use, but they may all be considered as variants of the two original types, the Fourneyron and the Jonval.

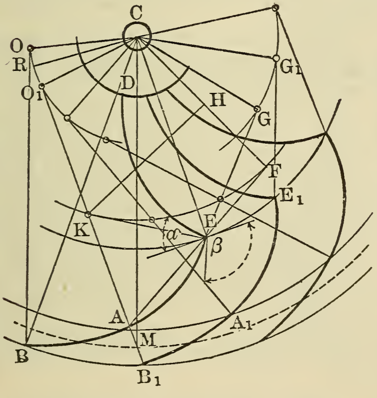
In the Fourneyron turbine the water flows down and out through the guides, *LL*, and is delivered into the curved buckets, *AA*, of the wheel, this latter being connected to the vertical shaft by the plate, *BB*. In the illustration the power is transmitted by the gear-wheels, *DE*, but the rotor of a dynamo may be mounted directly on the shaft.



Fourneyron Turbine.

The proportions of the Fourneyron turbine may be determined as follows, according to Weisbach:* The data are given in the accompanying diagram, the dimensions being in feet, and Q being the quantity of water delivered, in cubic feet, per second under a head, h, in feet. The inner radius, $r_1 = CE = 0.326\sqrt{Q}$. The outer radius, $r = CM = \nu r_1 = \frac{5}{4}r_1$ to $\frac{3}{2}r_1$. The angle of the guides at the entrance, E, may then be made $\alpha = 15°$ to $30°$, and the bucket angle at the same point $= \beta = 2\alpha + 20°$ to $2\alpha + 30°$. The inner velocity of the wheel will then be determined by the formula,

$$v_1 = \sqrt{\frac{2gh}{\frac{2\sin\beta\cos\alpha}{\sin(\beta-\alpha)} + 0.1\left[\left(\frac{\sin\beta}{\sin(\beta-\alpha)}\right)^2 + \nu^2\right]}}.$$



Curves for the Fourneyron Turbine.

The outer velocity will then be given $v = \nu v_1 = \frac{r}{r_1}v_1$, ν being the ratio of the inner and outer radii of the wheel. From this we obtain the number of revolutions:

$$u = \frac{30v}{\pi r} = 9.55\frac{v}{r} = 9.55\frac{v_1}{r_1}.$$

The velocity, c, with which the water issues through the guides will be

$$c = \frac{v_1 \sin\beta}{\sin(\alpha-\beta)},$$

and the cross-section, F, of the sum of all the openings will be

$$F = \frac{Q}{c} = \frac{Q\sin(\alpha-\beta)}{v_1\sin\beta} \text{ square}$$

feet. If e be the height of a

* "Der Ingenieur," *Seventh Edition*, 1896, pages 596–602.

bucket, and d = the width, as at AB_1, we have for their ratio $\lambda = \frac{e}{d} = 2$ to 5, according to the head of water, the larger value being for the lower head. The thickness of metal in the floats may be made

$$s = 0.015r.$$

We then have for the height, e, of the wheel,

$$e = \frac{F}{2\pi r_1 \sin \alpha}\left(1 + \frac{2\pi r \sin \alpha \,.\, \lambda s}{F}\right).$$

The number of guide buckets, $n_1 = \frac{\lambda F}{e^2}$, and the number of wheel buckets,

$$n = \frac{\sin \beta}{\sin \alpha} n_1 = \frac{\lambda F \sin \beta}{e^2 \sin \alpha}.$$

The exit angle, δ, at the middle point, M, of a bucket is found from

$$\sin \delta = \frac{F_2 + nse}{2\pi re},$$

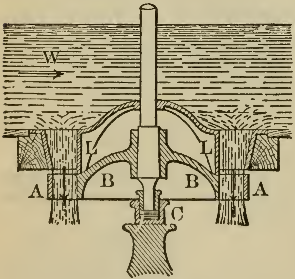
F_2 being the sum of all the discharge openings of the wheel buckets.

The curvature of the floats is determined as follows:

From $CM = r$ lay off the angle, $CMR = \delta$, and drop the perpendicular, CR, to MR. From M and from R lay off $MA = MB_1 = RO = RO_1 = r \sin \delta \tan \frac{\phi}{2}$, ϕ being $= \frac{360^\circ}{n}$. AB_1 will then be the width of a bucket mouth, neglecting the thickness of the metal of the floats, and O and O_1 will be the centres for the arcs, AB and A_1B_1, of the outer portions of the floats.

Lay off the line, $AF = CE = r_1$, making the angle, $RAF = 180^\circ - \beta$, join CF, and at the middle point, H, of the latter line erect the perpendicular, HK. The intersection, K, with RA will then be the centre from which the remainder of the bucket curve is struck.

Lay off the angle, α, from the ends, C and E, of the inner radius, CE, making the isosceles triangle, CEG, and G will be the centre from which to strike the curve, DE, of the guide. Having determined the centres for one pair of guides and bucket floats, the rest of the buckets can be readily drawn.



Jonval Turbine.

The inward discharge turbines, such as that of Francis, may be designed in the same manner, simply by changing r to r_1 and v to v_1, and vice versa.

The Jonval turbine is constructed as shown in the illustration, the guide blades, LL, being arranged in a ring above the turbine buckets, AA, the flow of water being parallel to the axis. This form of turbine is especially adapted for use with a draft tube, surrounding the wheel and adding the suction head of the discharge to the head above the wheel. When such a draft tube is used, the effective head, h, used in the computations is the sum of the heads above and below the wheel.

In this form of turbine $\alpha = 15^\circ$ to 25° and $\beta = 100^\circ$ to 120°, and the most efficient velocity for the wheel is given by the formula,

$$v = \sqrt{\frac{2gh}{\frac{2 \sin \beta \cos \alpha}{\sin (\beta - \alpha)} + 0.1 \,.\, \left[1 + \left(\frac{\sin \beta}{\sin (\beta - \alpha)}\right)^2\right]}}.$$

The velocity of entrance of the water is

$$c = \frac{v \sin \beta}{\sin (\beta - \alpha)}.$$

The total cross-section of the entrance spaces between the guides will then be

$$F = \frac{Q}{c},$$

and the total discharge section of the wheel buckets,

$$F_2 = \frac{Q}{v},$$

Q being the quantity of water, in cubic feet, per second.

If r_1 and r_2 be the inner and outer radii of the wheel, the mean radius will be

$$r = \frac{r_1 + r_2}{2},$$

and the width of the annular operative portion of the wheel, measured radially, will be

$$e = r_2 - r_1.$$

Usually, e is made equal to $\rho r = 0.4r$, whence

$$r_1 = r(1 - \tfrac{1}{2}\rho) = 0.8r,$$

and

$$r_2 = r(1 + \tfrac{1}{2}\rho) = 1.2r.$$

The ratio,

$$\lambda = \frac{e}{d},$$

of the length, e, of the floats to their width, d, may be made from 2 to 4. The radius may be determined from the formula,

$$r = \sqrt{\frac{F}{2\pi\rho \sin \alpha}}\left(1 + \lambda s \sqrt{\frac{\pi \sin \alpha}{2\rho F}}\right).$$

Approximately, we may take

$$r = \sqrt{\frac{F}{2\pi\rho \sin \alpha}},$$

and the thickness of the floats may be made

$$s = 0.02r.$$

The length of a float will then be

$$e = \rho r.$$

The number of guides,

$$n_1 = \frac{F}{de} + \frac{\lambda F}{e_2},$$

while the number of floats in the wheel,

$$n = \frac{\sin \beta}{\sin \alpha} \cdot n_1.$$

The angle of discharge, δ, is obtained from

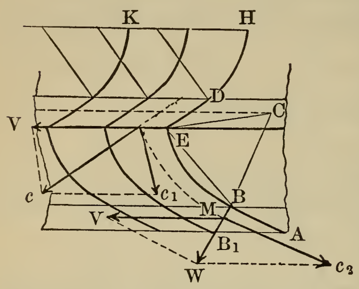
$$\sin \delta = \frac{F_2 + nse}{2\pi re},$$

and the number of revolutions,

$$u = \frac{30v}{\pi r} = 9.55\frac{v}{r}.$$

The height, a, of the wheel, as well as of the guides, may be made $0.5r$ to $0.6r$.

Both guides and floats are formed of warped surfaces, whose generating line is at right angles with the axis and follows the curves, HE and EA, of the illustration. The lower portions of the guides and buckets are straight lines, inclined at angles, α and δ, with the horizontal lines, as at DE and BA. The upper portions are arcs of circles, DH and BE. The centre, K, for DH, is at the intersection of a normal, DK, to the straight part of the guide with the upper line of the guides. To find the centre, C, of the curve, BE, of the wheel float, draw BC perpendicular to AB, make the angles, CBE and BEC, equal to $\frac{\beta + \delta}{2}$, when the intersection, C, of EC and BC will be the desired centre.



PUMPS.

Dimensions.

Let

D = diameter of plunger, in inches;
Q = quantity of water, in cubic feet, per minute;
v = plunger speed, in feet, per minute.

$$D = \sqrt{\frac{Q}{0.00545v}},$$

$$Q = 0.00545vD^2.$$

Approximately, the number of United States gallons delivered per minute, with a plunger speed of 100 feet per minute, will be

$$G = 4D^2.$$

The loss by leakage and slip varies from 10 to 40 per cent. For a new pump, well packed, the delivery should be 90 per cent. of the theoretical.

Ordinarily, the speed of pump plungers should not exceed 100 feet per minute. At higher speeds there are apt to be concussions and water-hammer produced, due mainly to the sudden stoppages in the movement of the column of water. The study of the movement of water in pumps has been greatly facilitated by the use of the indicator, the instrument being attached, not only with the recording drum operated from the pump plunger, but also by the life movement of the valves. The result of the latter method of investigation has shown the necessity for proper timing of the valves, especially when pumping against high heads at high speeds. In the designs of Professor Riedler a combination movement is used, the valve being closed mechanically, and opened by the action of the water.

Smooth running may also be improved by judicious use of air chambers to receive the impact of the water. Air chambers on the suction side of the plunger are especially important. The proper arrangement is to have the air chamber in the direct line of the suction; and recent designs of high-speed pumps provide a large air chamber directly beneath the cylinder, each suction valve having its own suction tube extending down nearly to the bottom of the air chamber. This construction is both simple and effective, and should be followed, when practicable.

General practice indicates that the air vessel on the delivery side should be from 3 to 6 times the capacity of the pump, while on the suction side it may be made from 2 to 3 times the capacity of the pump.

Delivery of Double-acting Pumps, in Cubic Feet.

Diameter of pump, in inches.	Piston speed, 50 feet per minute.		Piston speed, 60 feet per minute.		Piston speed, 70 feet per minute.		Piston speed, 80 feet per minute.		Piston speed, 90 feet per minute.		Piston speed, 100 feet per minute.	
	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.
1	.272	16.32	.326	19.58	.368	22.08	.435	26.11	.489	29.37	.544	32.64
1¼	.424	25.44	.508	30.52	.593	35.61	.680	40.80	.764	45.88	.849	50.97
1½	.611	36.67	.734	44.06	.856	51.36	.979	58.75	1.100	66.04	1.224	73.44
1¾	.832	49.92	.998	59.90	1.164	69.88	1.332	79.96	1.499	89.95	1.665	99.93
2	1.088	65.28	1.305	78.33	1.523	91.39	1.740	104.4	1.942	116.5	2.176	130.5
2¼	1.376	82.56	1.651	99.07	1.926	115.5	2.203	132.1	2.478	148.7	2.753	165.2
2½	1.699	101.9	2.040	122.4	2.379	142.7	2.720	163.2	3.059	183.5	3.400	204.0
2¾	2.056	123.3	2.467	148.0	2.879	172.7	3.291	197.4	3.702	222.1	4.113	246.8
3	2.448	146.8	2.937	176.2	3.427	205.6	3.916	235.0	4.406	264.3	4.896	293.7
3¼	2.872	172.3	3.446	206.7	4.020	241.2	4.596	275.8	5.171	310.2	5.745	344.7
3½	3.331	199.8	3.998	239.9	4.760	285.6	5.331	319.8	5.996	359.8	6.664	399.8
3¾	3.824	229.4	4.588	275.3	5.353	321.2	6.120	367.2	6.884	413.0	7.649	458.9
4	4.352	261.1	5.222	313.3	6.092	365.5	6.963	417.7	7.833	470.0	8.704	522.2
4½	5.347	320.8	6.609	396.5	7.710	462.6	8.812	528.7	9.913	594.8	11.01	660.9
5	6.800	408.0	8.160	489.6	9.520	571.2	10.88	652.8	12.24	734.4	13.60	816.0
5½	8.227	493.6	9.875	592.5	11.51	690.9	13.16	789.8	14.80	888.5	16.44	986.8
6	9.792	587.5	11.75	705.0	13.70	822.5	15.66	940.0	17.61	1056.0	19.58	1175.0
6½	11.49	689.4	13.79	827.4	16.08	964.8	18.38	1103.0	20.67	1240.0	22.97	1378.0
7	13.32	799.6	15.99	959.6	18.65	1119.0	21.31	1278.0	23.98	1439.0	26.65	1599.0

Delivery of Double-acting Pumps, in Cubic Feet.—*Continued.*

Diameter of pump, in inches.	Piston speed, 50 feet per minute.		Piston speed, 60 feet per minute.		Piston speed, 70 feet per minute.		Piston speed, 80 feet per minute.		Piston speed, 90 feet per minute.		Piston speed, 100 feet per minute.	
	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.	Cubic feet per minute.	Cubic feet per hour.
7½	15.29	917.9	18.35	1101	21.40	1284	24.48	1468	27.53	1652	30.59	1835
8	17.39	1043.0	20.88	1252	24.35	1461	27.84	1670	31.31	1878	34.81	2088
8½	19.64	1178.0	23.56	1414	27.50	1650	31.44	1886	35.36	2121	39.29	2357
9	22.03	1321.0	26.43	1585	30.83	1849	35.24	2114	39.64	2378	44.06	2643
9½	24.54	1472.0	29.45	1767	34.35	2061	39.26	2355	44.17	2650	49.08	2945
10	27.20	1632.0	32.64	1958	38.08	2284	43.52	2611	48.96	2937	54.40	3264
10½	29.98	1799.0	35.98	2159	41.96	2518	47.96	2878	53.96	3238	59.96	3598
11	32.91	1974.0	39.48	2369	46.06	2763	52.65	3159	59.23	3553	65.82	3949
11½	35.96	2158.0	43.15	2589	50.35	3021	57.55	3453	64.73	3884	71.93	4316
12	39.16	2350.0	46.99	2819	54.83	3289	62.65	3759	71.49	4229	78.33	4700
12½	42.49	2549.0	50.99	3059	59.48	3569	68.00	4080	76.49	4589	84.99	5099
13	45.96	2758.0	55.15	3309	64.35	3861	73.53	4412	82.73	4964	91.93	5516
13½	49.56	2974.0	59.47	3568	69.39	4163	79.31	4758	89.21	5352	99.13	5948
14	53.31	3198.0	63.96	3838	74.62	4477	85.29	5117	95.95	5757	106.6	6397
14½	57.18	3431.0	68.62	4117	80.04	4802	91.48	5489	102.9	6175	114.3	6862
15	61.20	3672.0	73.44	4406	85.68	5140	97.92	5875	110.1	6609	122.4	7344
15½	65.34	3920.0	78.41	4704	91.47	5488	104.5	6272	117.6	7056	130.6	7841
16	69.63	4177.0	83.55	5013	97.47	5848	111.4	6684	125.3	7519	139.2	8355
16½	74.04	4442.0	88.84	5330	103.6	6219	118.4	7108	133.2	7996	148.0	8885
17	78.60	4716.0	94.32	5659	110.0	6602	125.7	7545	141.4	8489	157.2	9432
17½	83.29	4997.0	99.95	5997	116.6	6996	133.2	7996	149.9	8996	166.5	9995

18	88.12	5287.0	105.7	6344	123.3	7402	140.9	8459	158.6	9517	176.2	10575
18½	93.07	5584.0	111.6	6701	130.3	7818	148.9	8935	167.5	10052	186.1	11170
19	98.19	5891.0	117.8	7069	137.4	8247	157.1	9426	176.7	10604	196.3	11783
19½	103.4	6205.0	124.1	7446	144.7	8687	165.4	9928	186.1	11168	206.8	12410
20	108.8	6528.0	130.5	7833	152.3	9139	174.0	10444	195.8	11750	217.6	13056
20½	114.3	6858.0	137.1	8230	160.0	9600	182.8	10972	205.7	12344	228.6	13716
21	119.9	7197.0	143.9	8636	167.9	10075	191.9	11515	215.9	12954	239.9	14394
21½	125.7	7543.0	150.8	9051	176.0	10560	201.1	12070	226.3	13578	251.8	14511
22	131.6	7898.0	157.9	9478	184.3	11058	210.6	12637	236.9	14217	263.2	15797
22½	137.6	8261.0	165.2	9913	192.7	11566	220.3	13219	247.8	14871	275.3	16523
23	143.8	8633.0	172.6	10359	201.4	12086	230.2	13812	258.9	15539	287.7	17266
23½	150.2	9012.0	180.2	10814	210.2	12617	240.3	14420	270.3	16222	300.4	18024
24	156.6	9400.0	188.1	11287	219.3	13159	250.6	15040	282.0	16920	313.3	18800
25	170.0	10200.0	204.0	12240	238.0	14280	272.0	16320	306.0	18360	340.0	20400
26	183.8	11032.0	220.6	13238	257.4	15444	294.1	17651	330.9	19857	367.7	22064
27	198.2	11897.0	237.9	14276	277.6	16656	317.2	19034	356.9	21414	396.5	23794
28	213.2	12794.0	255.8	15353	298.5	17912	341.1	20471	383.8	23030	426.4	25589
29	228.7	13725.0	274.4	16469	320.2	19214	366.0	21960	411.7	24704	457.5	27450
30	244.8	14688.0	293.7	17625	342.7	20563	391.6	23500	440.6	26438	489.6	29376
31	261.3	15683.0	313.6	18819	365.9	21956	418.2	25093	470.4	28229	522.7	31367
32	278.5	16711.0	334.2	20053	389.9	23396	445.6	26737	501.3	30080	557.0	33423
33	296.2	17772.0	355.4	21326	414.6	24881	473.9	28435	533.1	31990	592.4	35544
34	314.4	18865.0	377.3	22638	440.1	26411	503.0	30185	565.9	33958	628.8	37731
35	333.2	19992.0	399.8	23990	466.4	27988	533.1	31987	599.7	35985	666.4	39984
36	352.5	21150.0	423.0	25380	493.5	29610	564.0	33840	634.5	38070	705.0	42301
37	372.3	22342.0	446.8	26809	521.3	31278	595.7	35746	670.2	40215	744.7	44684
38	392.7	23566.0	471.3	28278	549.8	32992	628.4	37704	706.9	42418	785.5	47132
39	413.7	24822.0	496.4	29786	579.1	34751	661.9	39716	744.6	44680	827.4	49645
40	435.2	26112.0	522.2	31334	609.2	36556	696.3	41779	783.3	47001	870.4	52224

Standard Sizes of Deane Steam Pumps.

For Ordinary Service.

Diameter of steam cylinder, in inches.	Diameter of water cylinder, in inches.	Length of stroke, in inches.	United States gallons per stroke.	Strokes, per minute.	Capacity, per minute, at given speed.		Extreme length, in inches.	Extreme width, in inches.	Size of steam-supply pipe.	Size of steam-exhaust pipe.	Size of suction.	Size of discharge.
					Strokes.	United States gallons.						
4	3½	5	.14	1 to 300	130	18	33	9½	½	¾	2	1½
4	4	5	.27	1 to 300	130	35	33	9½	½	¾	2	1½
5	4	7	.39	1 to 300	125	49	45½	15	¾	1	3	2½
5½	5	7	.51	1 to 275	125	64	45½	15	¾	1	3	2½
5½	5½	7	.72	1 to 275	125	90	45½	15	¾	1	3	2½
7	7	10	1.64	1 to 250	110	180	58	17	1	1½	5	4
7½	7½	10	1.91	1 to 250	110	210	58	17	1	1½	5	4
7½	8	10	2.17	1 to 250	110	239	58	17	1	1½	5	4
8	6	12	1.47	1 to 250	100	147	67	20½	1	1½	4	4
8	7	12	2.00	1 to 250	100	200	67	20½	1	1½	5	4
8	8	12	2.61	1 to 250	100	261	68	30	1	1½	5	5
8	10	12	4.08	1 to 250	100	408	68	20½	1	1½	8	8
10	8	12	2.61	1 to 250	100	261	68½	30	1½	2	5	5
10	10	12	4.08	1 to 250	100	408	68½	30	1½	2	8	8
10	12	12	5.87	1 to 250	100	587	68½	30	1½	2	8	8
12	10	12	4.08	1 to 250	100	408	64	24	2	2½	8	8
12	10	18	6.12	1 to 200	70	428	68½	30	2	2½	8	8
12	12	12	5.87	1 to 250	100	587	64	28½	2	2½	8	8
12	12	18	8.80	1 to 175	70	616	88	28½	2	2½	8	8
12	14	18	12.00	1 to 175	70	840	88	28½	2	2½	8	8
14	10	12	4.08	1 to 250	100	408	69	30	2	2½	8	8
14	10	18	6.12	1 to 175	70	428	93	25	2	2½	8	8
14	10	24	8.16	1 to 150	50	408	112	26	2	2½	8	8
14	12	12	5.87	1 to 250	100	587	69	30	2	2½	8	8
14	12	18	8.80	1 to 175	70	616	88	28½	2	2½	8	8
14	12	24	11.75	1 to 150	50	587	112	26	2	2½	10	8
14	14	24	15.99	1 to 150	50	800	112	34	2	2½	12	10
14	16	16	13.92	1 to 175	80	1114	84	34	2	2½	12	10
14	16	24	20.88	1 to 150	50	1044	112	38	2	2½	12	10
16	14	18	12.00	1 to 175	70	840	89	27	2	2½	8	8
16	14	24	15.99	1 to 150	50	800	109	34	2	2½	12	10
16	16	16	13.92	1 to 175	80	1114	85	34	2	2½	12	10
16	16	24	20.88	1 to 150	50	1044	115	34	2	2½	12	10
16	18	24	26.43	1 to 125	50	1322	115	40	2	2½	14	12
18	16	24	20.88	1 to 125	50	1044	118	38	3	3½	12	10
18	18	24	26.43	1 to 125	50	1322	118	40	3	3½	14	12
18	20	24	32.64	1 to 125	50	1632	118	40	3	3½	16	14
20	18	24	26.43	1 to 125	50	1322	118	40	3	3½	14	12
20	20	24	32.64	1 to 125	50	1632	118	40	3	3½	16	14
20	22	24	39.50	1 to 125	50	1975	120	40	3	3½	18	14

Sizes of Worthington Standard Duplex Feed Pumps.

Size of pump.			Maximum gallons per hour recommended.	Floor-space required.	Size of pipes.			
Diameter of steam cylinders.	Diameter of water cylinders.	Length of stroke.			Steam.	Exhaust.	Suction.	Delivery.
Inch.	Inch.	Inch.		Inch.	Inch.	Inch.	Inch.	Inch.
2	1⅛	2¾	100	21 × 6	⅜	½	1	¾
3	1½	3	200	26 × 10	⅜	½	1¼	1
3	1¾	3	300	26 × 10	⅜	½	1¼	1
3	2	3	400	26 × 10	⅜	½	1¼	1
4½	2¾	4	1000	33 × 13	½	¾	2	2
5¼	3½	5	1800	38 × 15	¾	1¼	2½	2½
6	4	6	2500	44 × 16	1	1½	3	3
7½	4½	6	3300	48 × 24	1½	2	4	3
7½	5	6	4000	48 × 24	1½	2	4	3
7½	4½	10	4000	72 × 29	1½	2	4	3
7½	5	10	5000	72 × 30	1½	2	5	5
7½	5¼	10	5500	72 × 30	1½	2	5	5
9	5¼	10	5500	72 × 30	2	2½	5	5
9	6	10	7200	72 × 31	2	2½	6	5
10	6	10	7200	72 × 31	2	2½	6	5
10	7	10	10000	72 × 33	2	2½	6	6
12	8½	10	15000	80 × 42	2½	3	6	6
12	9¼	10	18000	80 × 42	2½	3	8	7
14	9¼	10	18000	80 × 42	2½	3	8	7

Sizes of Knowles Standard Duplex Feed Pumps.

Steam cylinders.	Water cylinders.	Stroke.	Gallons per stroke of each piston.	Strokes, per minute, of each piston, ordinary speed.	Capacity of both cylinders at speed stated, per minute.	Steam pipe.	Exhaust pipe.	Suction pipe.	Delivery pipe.	Floor-space required.
In.	In.	In.			Gallons.	Inch.	Inch.	Inch.	Inch.	Inch.
3	2	3	.046	75 to 200	9 to 18	⅜	½	1¼	1	36 × 12
4½	2¾	4	.10	75 to 150	15 to 30	½	¾	2	1½	38 × 13
5¼	3½	6	.24	75 to 150	36 to 72	¾	1¼	2½	2	52 × 20
6	4	7	.39	75 to 125	58 to 97	1	1½	3	2½	54 × 24
7½	4½	10	.69	60 to 120	82 to 164	1½	1½	4	3	60 × 24
8	5	12	1.02	50 to 100	102 to 204	1½	1½	5	4	80 × 31
8	6	12	1.47	50 to 100	147 to 294	1½	1½	5	4	80 × 31
10	6	12	1.47	50 to 100	147 to 294	2	2½	5	4	80 × 31
12	7	12	2.00	50 to 100	200 to 400	2	2½	6	5	83 × 35
12	8½	12	3.00	50 to 100	300 to 600	2	2½	6	5	83 × 37
14	8½	12	3.00	50 to 100	300 to 600	2½	3	6	5	83 × 37
14	10½	12	4.50	50 to 100	450 to 900	2½	3	8	7	83 × 44
16	10½	12	4.50	50 to 100	450 to 900	3	3	8	7	83 × 44
16	12	12	5.87	50 to 100	587 to 1174	3	3	10	8	83 × 50
18½	10½	12	4.50	50 to 100	450 to 900	3	3	8	7	83 × 44
18½	12	12	5.87	50 to 100	587 to 1174	3	3	10	8	83 × 50
18½	14	12	8.00	50 to 100	800 to 1600	3	3	12	10	83 × 57
20	14	18	12.00	40 to 70	960 to 1680	4	6	......	......	....

Standard Method of Conducting Duty Trials of Pumping Engines.

Report of Committee of the American Society of Mechanical Engineers, 1890.

Abstract.

The basis upon which the duty of a pumping engine is to be determined is 1,000,000 British thermal units. This is the equivalent of 100 pounds of coal when each pound of coal imparts 10,000 heat units to the water in the boiler, this corresponding to an evaporation of 10.355 pounds of water from and at 212° F. per pound of fuel.

The duty should be computed from the quantity of heat supplied to the complete plant, including all auxiliaries. The work done by the pump is to be determined by the plunger displacement, the loss by leakage to be subsequently determined.

The necessary data having been obtained, the duty of an engine may be computed by the use of the following formulas:

$$1.\ \text{Duty} = \frac{\text{Foot-pounds of work done}}{\text{Total number of heat units consumed}} \times 1\,000\,000,$$

$$= \frac{A\,(P \pm p + s) \times L \times N}{H} \times 1\,000\,000 \text{ (foot-pounds).}$$

$$2.\ \text{Percentage of leakage} = \frac{C \times 144}{A \times L \times N} \times 100 \text{ (per cent.).}$$

$$3.\ \text{Capacity} = \text{number of gallons of water discharged in 24 hours,}$$

$$= \frac{A \times L \times N \times 7.4805 \times 24}{D \times 144},$$

$$= \frac{A \times L \times N \times 1.24675}{D} \text{ (gallons).}$$

4. Percentage of total frictions

$$= \left[\frac{I.H.P. - \dfrac{A\,(P \pm p + s) \times L \times N}{D \times 60 \times 33000}}{I.H.P.} \right] \times 100,$$

$$= \left[1 - \frac{A\,(P \pm p + s) \times L \times N}{A_s \times M.E.P. \times L_s \times N_s} \right] \times 100 \text{ (per cent.);}$$

or, in the usual case, where the length of the stroke and number of strokes of the plunger are the same as that of the steam piston, this last formula becomes

$$\text{Percentage of total frictions} = \left[1 - \frac{A\,(P \pm p + s)}{A_s \times M.E.P.} \right] \times 100 \text{ (per cent.).}$$

In these formulas the letters refer to the following quantities:

A = Area, in square inches, of pump plunger or piston, corrected for area of piston-rod. (When one rod is used at one end only, the correction is one-half the area of the rod. If there is more than one rod, the correction is multiplied accordingly.)

P = Pressure, in pounds, per square inch, indicated by the gauge on the force main.

p = Pressure, in pounds, per square inch, corresponding to indication of the vacuum gauge on suction main (or pressure gauge, if the suction pipe is under a head). The indication of the vacuum gauge, in inches of mercury, may be converted into pounds by dividing it by 2.035.

s = Pressure, in pounds, per square inch, corresponding to distance between the centres of the two gauges. The computation for this pressure is made by multiplying the distance, expressed in feet, by the weight of one cubic foot of water at the temperature of the pump well, and dividing the product by 144.

L = Average length of stroke of pump plunger, in feet.

N = Total number of single strokes of pump plunger made during the trial.

A_s = Area of steam cylinder, in square inches, corrected for area of piston-rod. The quantity, $A_s \times M.E.P.$, in an engine having more than one cylinder is the sum of the various quantities relating to the respective cylinders.

L_s = Average length of stroke of steam piston, in feet.

N_s = Total number of single strokes of steam piston during trial.

$M.E.P.$ = Average mean effective pressure, in pounds, per square inch, measured from the indicator diagrams taken from the steam cylinder.

$I.H.P.$ = Indicated horse-power developed by the steam cylinder.

C = Total number of cubic feet of water which leaked by the pump plunger during the trial, estimated from the results of the leakage test.

D = Duration of trial, in hours.

H = Total number of heat units ($B.T.U.$) consumed by engine = weight of water supplied to boiler by main feed pump $\times$ total heat of steam of boiler pressure reckoned from temperature of main feed water + weight of water supplied by jacket pump $\times$ total heat of steam of boiler pressure reckoned from temperature of jacket water + weight of any other water supplied $\times$ total heat of steam reckoned from its temperature of supply. The total heat of the steam is corrected for the moisture or superheat which the steam may contain. For moisture the correction is subtracted, and is found by multiplying the latent heat of the steam by the percentage of moisture, and dividing the product by 100. For superheat the correction is added, and is found by multiplying the number of degrees of superheating—*i.e.*, the excess of the temperature of the steam above the normal temperature of saturated steam—by 0.48. No allowance is made for heat added to the feed water, which is derived from any source, except the engine or some accessory of the engine. Heat added to the water by the use of a flue heater at the boiler is not to be deducted. Should heat be abstracted from the flue by means of a steam reheater connected with the intermediate receiver of the engine, this heat must be included in the total quantity supplied by the boiler.

The leakage test of the pump plunger should be made as soon as possible after the completion of the main trial.

The leakage of an inside plunger (the only type which requires testing) is most satisfactorily determined by making the test with the cylinder head removed. A wide board or plank may be temporarily bolted to the lower part of the end of the cylinder, so as to hold back the water in the manner of a dam, and an opening made in the temporary head thus provided for the reception of an overflow pipe. The plunger is blocked at

some intermediate point in the stroke (or, if this position is not practicable, at the end of the stroke) and the water from the force main is admitted at full pressure behind it. The leakage escapes through the overflow pipe, and is collected in barrels and measured.

Should the escape of the water into the engine-room be objectionable, a spout may be constructed to carry it out of the building. Where the leakage is too great to be readily measured in barrels, or where other objections arise, resort may be had to weir or orifice measurement, the weir or orifice taking the place of the overflow pipe in the wooden head. The apparatus may be constructed, if desired, in a somewhat rude manner, and yet be sufficiently accurate for practical requirements. The test should be made, if possible, with the plunger in various positions.

In the case of a pump so planned that it is difficult to remove the cylinder head, it may be desirable to take the leakage from one of the openings which are provided for the inspection of the suction valves, the head being allowed to remain in place.

It is here assumed that there is a practical absence of valve leakage, a condition of things which ought to be attained in all well-constructed pumps. Examination for such leakage should be made first of all, and if it occurs, and it is found to be due to disordered valves, it should be remedied before making the plunger test. Leakage of the discharge valves will be shown by water passing down into the empty cylinder at either end when they are under pressure. Leakage of the suction valves will be shown by the disappearance of water which covers them.

If valve leakage is found which cannot be remedied, the quantity of water thus lost should also be tested. The determination of the quantity which leaks through the suction valves, where there is no gate in the suction pipe, must be made by indirect means. One method is to measure the amount of water required to maintain a certain pressure in the pump cylinder when this is introduced through a pipe temporarily erected, no water being allowed to enter through the discharge valves of the pump.

The exact methods to be followed in any particular case, in determining leakage, must be left to the judgment and ingenuity of the person conducting the test.

Table of Data and Results.

In order that uniformity may be secured, it is suggested that the data and results, worked out in accordance with the standard method, be tabulated in the manner indicated in the following scheme:

DUTY TRIAL OF ENGINE.

Dimensions.

1. Number of steam cylinders ....
2. Diameter of steam cylinders .... ins.
3. Diameter of piston-rods of steam cylinders .... ins.
4. Nominal stroke of steam pistons .... ft.
5. Number of water plungers ....
6. Diameter of plungers .... ins.
7. Diameter of piston-rods of water cylinders .... ins.
8. Nominal stroke of plungers .... ft.
9. Net area of plungers .... sq. ins.
10. Net area of steam pistons .... sq. ins.
11. Average length of stroke of steam pistons during trial .... ft.
12. Average length of stroke of plungers during trial .... ft.

(Give also complete description of plant.)

Temperatures.

13. Temperature of water in pump well .... degs.
14. Temperature of water supplied to boiler by main feed pump. degs.
15. Temperature of water supplied to boiler from various other sources .... degs.

Feed Water.

16. Weight of water supplied to boiler by main feed pump....... lbs.
17. Weight of water supplied to boiler from various other sources. lbs.
18. Total weight of feed water supplied from all sources... ..... lbs.

Pressures.

19. Boiler pressure indicated by gauge.......................... lbs.
20. Pressure indicated by gauge on force main.................. lbs.
21. Vacuum indicated by gauge on suction main.................. ins.
22. Pressure corresponding to vacuum given in preceding line... lbs.
23. Vertical distance between the centres of the two gauges..... ins.
24. Pressure equivalent to distance between the two gauges...... lbs.

Miscellaneous Data.

25. Duration of trial.. hrs.
26. Total number of single strokes during trial..................
27. Percentage of moisture in steam supplied to engine, or number of degrees of superheating.................... } per cent. or deg.
28. Total leakage of pump during trial, determined from results of leakage test... lbs.
29. Mean effective pressure, measured from diagrams taken from steam cylinders... M. E. P.

Principal Results.

30. Duty.. ft.-lbs.
31. Percentage of leakage....................................... per cent.
32. Capacity... gals.
33. Percentage of total frictions................................ per cent.

Additional Results.*

34. Number of double strokes of steam piston per minute........
35. Indicated horse-power developed by the various steam cylinders.. I. H. P.
36. Feed water consumed by the plant per hour.................. lbs.
37. Feed water consumed by the plant, per indicated horse-power, per hour, corrected for moisture in steam................. lbs.
38. Number of heat units consumed, per indicated horse-power, per hour.. B. T. U.
39. Number of heat units consumed, per indicated horse-power, per minute.. B. T. U.
40. Steam accounted for by indicator at cut-off and release in the various steam cylinders.................................. lbs.
41. Proportion which steam accounted for by indicator bears to the feed water consumption..................................

Sample Diagrams taken from Steam Cylinders.

(Also, if possible, full measurements of the diagrams, embracing pressures at the initial point, cut-off, release, and compression; also, back-pressure and the proportions of the stroke completed at the various points noted.)

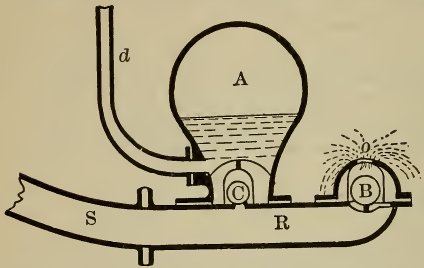
42. Number of double strokes of pump per minute...............
43. Mean effective pressure, measured from pump diagrams..... M. E. P.
44. Indicated horse-power exerted in pump cylinders............ I. H. P.
45. Work done (or duty) per 100 pounds of coal.................. ft.-lbs.

(Sample diagrams taken from pump cylinders.)

* These are not necessary to the main object, but it is desirable to give them.

The Hydraulic Ram.

The hydraulic ram is a device by means of which a large volume of water under a low head may be used to force a smaller quantity of water up against a higher head. Its chief advantages are its simplicity, moderate cost, and freedom from care or attendance.



The operation is as follows:

The water working the ram is supplied through the pipe, S, and escapes through an opening at o until it has gained a velocity sufficient to raise the valve or ball, B, which suddenly stops the current and causes an excessive pressure in the ram, R, which opens the valve or ball, C; the water is forced into the vessel and air chamber, A, and finally through the delivery pipe, d, to its destination. When equilibrium of pressure is restored between S and R the valve, B, falls and the operation is repeated. The ram can make as many as 200 strokes per minute, depending upon its size.

The length of the supply pipe, S, should not be less than 5 times the height of the fall, F, because it is the dynamic action of the water in the pipe which works the ram. The delivery pipe may be made 10 or more times the height of the fall.

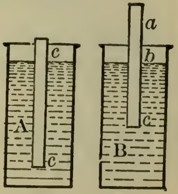
The useful effect of the ram, like that of water-wheels and turbines, depends much upon its construction. In ordinary cases it returns about 50 per cent. of the natural effect,—that is, the quantity of water, q, multiplied by the height, h, of the delivery above the ram will be about 50 per cent. of the quantity of water, Q, working the ram, multiplied by the head of fall, F, in the same unit of time.

$$qh = 0.5QF. \qquad q = \frac{0.5QF}{h}. \qquad Q = \frac{2qh}{F}.$$

Q and q can be expressed in any unit of volume or weight.
F and h can be expressed in any unit of length.
But let us assume Q and q to be cubic feet per minute;
F and h = fall and height, in feet;
L = length, in feet, and D = diameter, in inches, of the supply pipe, S;
l = length, and d = diameter of the delivery pipe, d;

then $$D = \sqrt[5]{\frac{2Q^2(L+5D)}{F}}, \text{ and } d = \sqrt[5]{\frac{4q^2(l+5d)}{h}}.$$

Hydrometer.

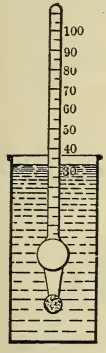


A body wholly immersed in a liquid will lose as much of its weight as the weight of the liquid it displaces.

A floating body will displace its own weight of the liquid in which it floats.

A cylindrical rod of wood or some light materials, being set down in two liquids, A and B, of different specific gravities, when in equilibrium will sink to the mark a in the liquid A, and to b in the liquid B; then the specific gravity of $A : B = b, c : a, c$, or inversely as the immersed parts of the rod. This is the principle upon which a hydrometer is constructed.

Table Showing the Comparative Scales of Gay Lussac and Baumé, with the Specific Gravity and Proof, at the Temperature of 60° Fahr.



	Gay Lussac's.	Baumé's.	Specific gravity.	Proof.	
Percentage of pure alcohol.	100	46	.796	100	Percentage over proof.
	95	40	.815	92	
	90	36	.833	82	
	85	33	.848	72	
	80	31	.863	62	
	75	28	.876	52	
	70	26	.889	42	
	65	24	.901	32	
	60	23	.912	22	
	55	21	.923	12	
	50	19	.933	0 Proof.	
	45	18	.942	8	Under proof.
	40	17	.951	18	
	35	16	.958	29	
	30	15	.964	35	
	25	14	.970	48	

Hydrostatics.

Notation.

A and **a** = areas of the pressed surfaces, in square feet;
I and p = hydrostatic pressure, in pounds;
d = depth of the centre of gravity of **A** or **a** under the surface of the liquids, in feet;
S = specific gravity of the liquid.

Example. Case I.—The plane **A** = 3.3 square feet at a depth of $d = 6$ feet under the surface of fresh water. Required the pressure, $P = ?$ Specific gravity of fresh water, $S = 1$.

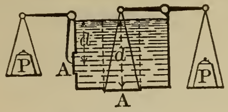
$$P = 62.3\mathbf{A}d = 62.3 \times 3.3 \times 6 = 1237.5 \text{ pounds.}$$

Example. Case IV.—The area of the pistons, **A** = 8.5 square feet, **a** = 0.02 square feet, $l = 4$ feet, $e = 9$ inches, and $F = 18$ pounds. Required the pressure, $P =$

$$P = \frac{Fl\mathbf{A}}{e\mathbf{a}} = \frac{18 \times 4 \times 8.5}{0.75 \times 0.02} = 40800 \text{ pounds.}$$

It must be distinguished that the centre of pressure and centre of gravity of the planes are two different points; the centre of pressure is below the centre of gravity when the plane is inclined or vertical.

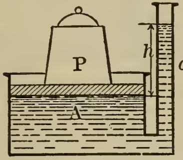
I.



$$P = 62.3SAd,$$

$$A = \frac{P}{62.3Sd}, \qquad d = \frac{P}{62.3SA}.$$

II.

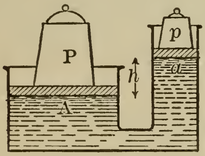


The Hydrostatic Paradox.

The pressure, P, is independent of the width of column, C.

$P = 62.3Sah$. (Same as above.)

III.

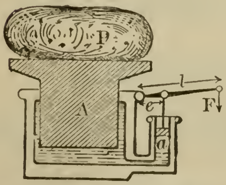


$$P = A\left(62.3Sh + \frac{p}{a}\right),$$

$$p = a\left(\frac{P}{A} - 62.3Sh\right),$$

$$h = \frac{Pa - pA}{62.3SAa}.$$

IV.

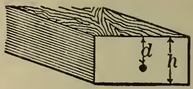


Bramah's Hydraulic Press.

$$P = \frac{FlA}{ea}, \qquad A = \frac{Pea}{Fl},$$

$$F = \frac{Pea}{A}, \qquad a = \frac{FAl}{Pe}.$$

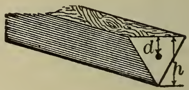
V.



Centre of Pressure of a Rectangle,

the upper edge at the surface of the liquid, $d = \frac{2}{3}h$.

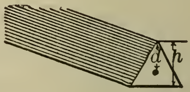
VI.



Centre of Pressure of a Triangle,

the base being at the surface of the liquid, $d = \frac{1}{2}h$.

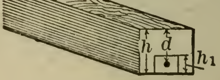
VII.



Centre of Pressure of a Triangle,

the vertex being at the surface of the liquid, $d = \frac{3}{4}h$.

VIII.



$$d = \frac{2}{3} \cdot \frac{3h^2 - 3hh_1 + h_1^2}{2h - h_1}.$$

Resistance Caused by Obstructions.

In nearly all hydraulic work numerous bends, valves, etc., must be inserted in the connections, and these produce frictional resistance to the flow. Such resistances are conveniently computed in terms of additional head, representing the number of feet-head to be added to that for a smooth pipe in order that the final discharge or pressure may be realized.

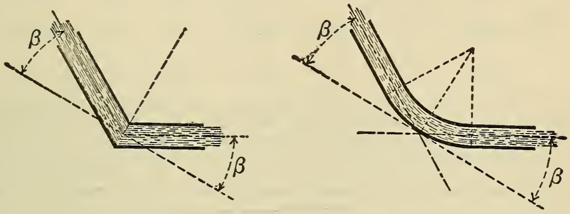
Resistance in Angles and Bends.—The resistance due to an angle is important, and is dependent upon what Weisbach calls the semi-angle of deviation, β, according to the following formula:

$$h_2 = \zeta_2 \frac{v^2}{2g} = (0.9457 \sin^2 \beta + 2.047 \sin^4 \beta)\frac{v^2}{2g},$$

from which we get:

$\beta =$	10	20	30	40	45	50	60	70
$\zeta_2 =$	0.046	0.139	0.364	0.74	0.985	1.26	1.861	2.431

Example. In a right-angle bend $\beta = 45°$, and the loss is practically equal to $\frac{v^2}{2g}$.



In the case of bends the resistance is not so great, but is too large to be neglected, since we have

$$h_2 = \zeta_2 \frac{\beta}{90} \cdot \frac{v^2}{2g}.$$

The ratio of the radius of the tube to the radius of the curvature of the bend affects the coefficient, as below:

$\frac{0.5D}{r} =$	0.1	0.2	0.3	0.4	0.5
$\zeta_2 =$	0.131	0.138	0.158	0.206	0.294
$\frac{0.5D}{r} =$	0.6	0.7	0.8	0.9	1.0
$\zeta_2 =$	0.440	0.661	0.977	1.408	1.978

Example. For a right-angle bend in which $r = D$, we have

$$h_2 = 0.294 \frac{45}{90} \cdot \frac{v^2}{2g} = 0.147 \frac{v^2}{2g},$$

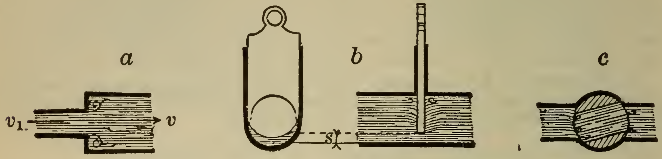
or only about $\frac{1}{7}$ the resistance of a sharp bend with any curvature.

Resistances due to Sudden Changes of Cross-section.—When water which is moving at a velocity, v_1, suddenly changes to another velocity, v, as at a, it experiences a loss of pressure which, according to Weisbach, is equivalent to a height:

$$h_3 = \frac{v_1^2 - v^2}{2g} = \left(\frac{F}{F_1} - 1\right)^2 \frac{v^2}{2g} = \zeta_3 \frac{v^2}{2g},$$

F and F_1 being the respective cross-sections; also, $Fv = F_1 v_1$. Doubling the cross-section causes a loss of head equal to $\frac{v^2}{2g}$.

For gate valves, as at b, or cocks, as at c, there is a loss due to the amount of contraction. For gate valves we have from Weisbach:



Openings =	⅛	¼	⅜	½	⅝	¾	⅞
$\frac{F_1}{F}$ =	0.159	0.315	0.466	0.609	0.74	0.856	0.948
ζ_3 =	97.8	17.0	5.52	2.06	0.81	0.26	0.07

and for cocks:

Angle =	10°	20°	30°	40°	50°	60°	65°	82⅛°
$\frac{F_1}{F}$ =	0.850	0.692	0.535	0.385	0.25	0.137	0.091	0
ζ =	0.29	1.56	5.47	17.3	52.6	206	486	∞

From the above tables it will be seen how important an influence is exerted by valve chests, mud traps, and the like upon the flow of water. In all such cases it is important to modify the suddenness of the change of velocity by rounding and curving all angles in the passages, and in this way a large part of the loss may be obviated. For gaseous fluids the resistance is less, but is at the same time sufficiently important to be carefully considered. For a fuller discussion of the resistances offered to water in canals and streams the reader must be referred to special treatises on the subject.

Centrifugal Pumps.

Let

v = velocity of rim of wheel, in feet, per second;
h = height of delivery, in feet, including suction;
D = diameter of wheel, in feet;
Q = cubic feet of water per minute;
d = diameter of discharge pipe, in feet.

Then

$$v = 10 + 8\sqrt{h},$$

$$d = 0.36\sqrt{\frac{Q}{\sqrt{29h}}},$$

$$D = \sqrt{\frac{Q}{\sqrt{h}}} \times 0.18.$$

The inlet opening in the side of the wheel is made equal to $0.5D$. The blades are sometimes made in the form of an Archimedean spiral, but a better efficiency is obtained with the reversed curve, designed according to the method of Rittinger, as follows:

Let

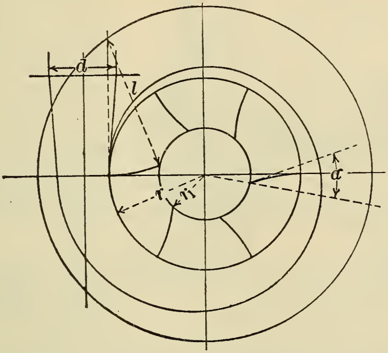
r = the radius of the propeller wheel;
r_1 = the radius of the inlet opening;
a = the angle between radius and initial line of blade;
l = radius of curvature of blade;
n = number of revolutions per minute;
c = velocity of inflowing water per minute;
$= \frac{Q}{\pi r_1^2}$.

We have

$$\tan \alpha = 0.1047\frac{nr_1}{c},$$

$$l = \frac{r^2 - r_1^2}{2r_1 \sin \alpha}.$$

The case is made in the form of an Archimedean spiral.



Centrifugal Pump.

Hydraulic Transmission of Power.

For many purposes where power has to be distributed over a limited area for working machinery, such as presses, lifts, cranes, riveting and flanging machinery, and the like, it has been found advantageous to use water under high pressure, the system being piped to accumulators and pumps, so that a supply of stored energy is available for the varied and irregular demands.

The accumulator is simply a vertical cylinder fitted with a weighted plunger, the area of the plunger and its load being proportioned to equal the pressure, in pounds, per square inch to be maintained in the system. The pumps deliver water under the plunger, and the pipe system is also connected to the cylinder. Unless the demand upon the pumps is equal to their full capacity, the plunger of the accumulator will be forced upward, the excess energy being thus stored in the lifted weights. When the plunger reaches its upper limit it shuts off the steam to the pumps and checks their action ; as it falls, the steam is turned on, and the pumps are again started. When any machine connected with the system, as a riveter or a press, is put in motion the accumulator plunger falls as the water is drawn from the pipes, but the pressure is maintained by the weights upon the plunger. The pumps promptly respond to the fall of the plunger, so that the latter is kept oscillating up and down in response to the demand from the machines and the supply from the pumps.

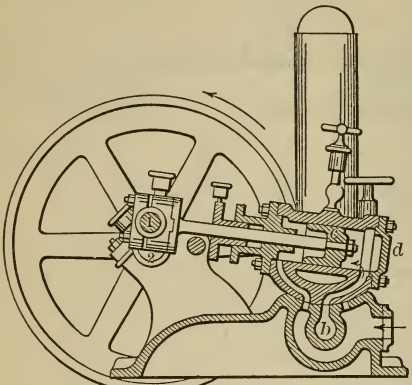
The amount of energy stored in an accumulator will be

$$\text{Foot-pounds} = 2240\,Ws = \frac{\pi d^2}{4}sp,$$

in which

W = weight on plunger, in tons;
d = diameter of plunger, in inches;
p = pressure, in pounds, per square inch;
s = vertical travel of plunger.

The efficiency of an accumulator may be as high as 98 per cent., 1 per cent. being lost in charging and as much in discharging. While the total amount of energy which can be stored is not great, it can be discharged at a high rate for a short time, and by care in proportioning the capacity of the pumps to the probable demand a very satisfactory service may be maintained.



Schmid Hydraulic Motor.

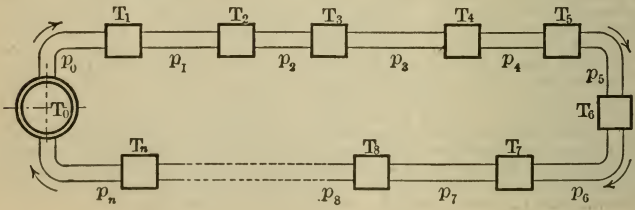
The pressures in such systems range from 600 to 800 pounds per square inch. At Hull, England, a pressure of 610 pounds per square inch is maintained. In London the pressure is 800 pounds, and at Birmingham it is 730 pounds. For lifts the water is used in cylinders, usually having a stroke equal to but a fraction of the entire hoist, the travel of the cage being multiplied by a reduplication of the hoisting cable over a system of sheaves. Plunger elevators are now coming into use, however, the valves used permitting speeds of 600 feet per minute, when required.

The high-pressure water is used in various forms of motors. On the Continent the Schmid oscillating engine is much used, while in England the 3-cylinder engine of Brotherhood and the 4-cylinder engine of Rigg are found.

The principal feature of such engines is the regulation. Throttling is unsatisfactory, and the rigidity of the water column must be contended with. The most satisfactory principle is that of a variable stroke, the pressure of the water being left unchanged.

An example of such a regulator is that of Helfenberger. This is made with a hydraulic ratchet mechanism arranged in the crank disk in such a manner as to move the crank pin to or from the centre, the ratchet being operated by tappets, which strike each time the crank passes the dead centres. The throw of the crank is thus varied to correct for variations of speed, the mechanism being controlled by a governor.

The Pelton water-wheel has also been employed with success as a small motor for use with high-pressure water, and is both simple and convenient.

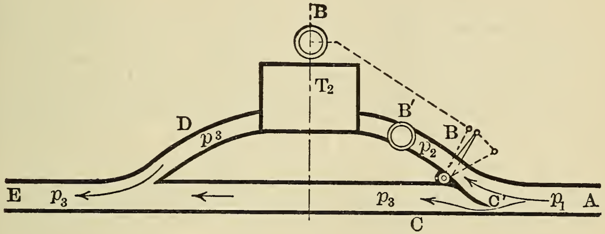


Professor Reuleaux has suggested a very effective method for power distribution by hydraulic pressure, using the water in a circuit or "ring," not unlike methods of electrical distribution.

Taking into consideration high-pressure hydraulic systems, we find two distinct kinds of "ring" systems which may be used.

In the first method, shown above, the flow of water under pressure starts from the power station, T_0, with a pressure, p_0, and proceeds to the

first station, T_1, where it operates a water-pressure engine, and passes on with a reduced pressure, p_1. It has, therefore, operated at the station, T_1, with a pressure, $p_0 - p_1$. With the pressure, p_1, it passes on to the second, third, fourth, — — nth station, T_n, each time losing pressure until it returns to the power station with a final pressure, p_n, where it is again raised to the initial pressure of p_0. It is apparent that the water-pressure engines (escapements) at T_1, T_2, T_3, — — — T_n, should all be of equal size, in order to utilize the entire flow without excessive resistance. Automatic regulation, such as Helfenberger's, is also desirable.



The second system is shown above. It will be seen that at each station there is a branch or shunt tube leading through the motor (or escapement), T_2, and then reuniting with the main pipe. The main pipe, A, forks at the station into the two branches, B and C, of which the first diverts any required fraction of the power of the main flow, as $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{3}$, as the case may be. At the fork is a swing valve, C', operated by a speed governor, R, driven by the motor. This governor requires the assistance of some form of power reinforcement. The discharge pipe, D, of the motor unites with the by-pass, C, to form again the main conductor, E. At the entrance in the main pipe, A, we have the pressure, p_1, of the original flow. The motor, T_2, is now supposed to be stationary, the stop valve at B' having been closed by hand. The flap valve, C', which has been disconnected from the regulator before stopping the motor, is also closed. The flow of water then passes through C to E with the pressure, p_1.

When the motor, T_2, is to be started, the valve, B', is opened and the flap valve, C', gradually opened until the motor begins to move, when it is connected to the governor, which regulates it thereafter so as to keep the motor at its normal speed. When a heavy load is thrown on the valve is opened so that the pressure, p_2, in B becomes a greater fraction of p_1, and when the work is less it is reduced. The pressure of discharge, p_3, acts as a back pressure, so that the motor works with an effective pressure, $p_2 - p_3$. The flow of water in the by-pass pipe, C, also passes the valve, C', with a pressure, p_3, and unites with the discharge at E, to be further utilized at subsequent stations until it returns to the power station, where, if it has reached the minimum pressure, it is permitted to flow into a tank, from which it is again drawn by the pressure pumps. If the return water is delivered under pressure it may be allowed to enter the suction pipe of the pressure pumps direct, and so form a closed ring system to start anew on the circuit.

The ring system of hydraulic power transmission is to be recommended when the various stations are distributed over a wide area and are readily connected by a continuous line of pipe. The pipe can be kept from freezing in winter by occasional gas flames, as has already been demonstrated by experience with Armstrong's hydraulic cranes. The ring system should be carefully distinguished from those forms in which the flow of water passes through the motor and is allowed to flow off at lowest pressure of discharge.

Full detailed descriptions of a variety of hydraulic machinery will be found in Professor Henry Robinson's treatise on "Hydraulic Power and Hydraulic Machinery," and reference should also be made to Reuleaux's "Constructor."

FUEL.

The fuels used in engineering consist of compounds of carbon and of hydrogen, which when uniting with oxygen produce heat.

Fuels may be classified as solid, liquid, and gaseous.

The solid fuels are coal in its various grades from anthracite, bituminous, lignite, and peat; charcoal, coke, and wood. Liquid fuels include the mineral, vegetable, and animal oils. The gaseous fuels are natural gas and the various artificial gases, as coal gas produces gas, etc.

The **calorific power** of solid and liquid fuels, or, as it is sometimes called, the **thermal value,** is measured by the number of thermal units or calories developed during the combustion of a unit weight. Usually, the calorific power is expressed in the number of British thermal units evolved by the combustion of a pound of the fuel, or the number of calories produced by the combustion of a kilogramme of fuel. To convert B. T. U. per pound to calories per kilogramme multiply by 0.555, to convert calories per kilogramme to B. T. U. per pound multiply by 1.8, this being the ratio of the Centigrade to the Fahrenheit degree. The calorie involves the raising of the temperature of a kilogramme of water, and the B. T. U. involves the raising of the temperature of only a pound of water, but this corresponds exactly to the different weights of fuels respectively consumed, so that the ratio is simply that due to the thermometer scales.

The calorific power of gaseous fuels is generally determined in B. T. U. per cubic foot or in calories per cubic metre. To convert calories per cubic metre to B. T. U. per cubic foot multiply by 0.11235, to convert B. T. U. per cubic foot to calories per cubic metre multiply by 8.9.

When the calorific powers of solids and gases are compared, the gas should be taken by weight, in order to have the data in comparable form, otherwise it is more convenient to consider gases separately by volume, as they are measured.

Since the fuels used in engineering are carbon, hydrogen, and their compounds, the calorific value of these elements form the foundations upon which other values are computed.

One pound of pure carbon, completely burned to carbonic acid, CO_2, evolves 14,500 B. T. U. One kilogramme of carbon, burned in like manner, evolves 8080 calories.

One pound of pure hydrogen, burned to water, evolves 62,100 B. T. U., and one kilogramme of hydrogen evolves 34,500 calories.

Having these facts we may determine the calorific value of any combination of carbon and hydrogen when we know the ultimate chemical composition,—*i.e.*, the percentage of carbon and hydrogen contained. When there is only carbon and hydrogen present, the calorific value of the combination is expressed by the sum of the calorific value of the constituents. Thus, if h be the number of heat units evolved by the complete combustion of a combination of carbon and hydrogen, we have

$$h = 8080C + 34500H \text{ calories},$$

or

$$h = 14500C + 62100H \text{ B. T. U.}$$

When, however, as is usually the case, there is oxygen present in the fuel it will unite with a portion of the hydrogen, and in such case a deduction should be made. We therefore have, for the computation of the calorific value of a fuel from its chemical composition,

$$h = 8080C + 34500\left(H - \frac{O}{8}\right), \text{ for calories per kilogramme,}$$

and

$$h = 14500C + 62100\left(H - \frac{O}{8}\right), \text{ for B. T. U. per pound.}$$

Whenever practicable, it is desirable that the calorific power of a fuel be determined directly by experiment. Various devices have been made for

this purpose, depending for their action upon the complete combustion of a determinate weight of the fuel in a closed chamber immersed in a known weight of water. The rise in the temperature of the water then gives the information from which the heat evolved may be determined. The most reliable apparatus of this kind is the so-called calorimetric "bomb" of Bethelot, Vielle, and Mahler, in which the fuel is enclosed in a steel vessel, lined with platinum or enamel, together with sufficient compressed oxygen to complete the combustion. The ignition is effected by means of an electric current, and the heat evolved is measured by the rise in temperature of the bath of water in which the bomb is immersed.

For full details of calorimetric apparatus and work reference may be made to Poole's work on the "Calorific Power of Fuels."

In important investigations the fuel used should be carefully sampled, and its calorific power determined, either by computation—using Dulong's formula—from a chemical analysis or by the use of the bomb, such work being performed in the testing laboratory. For general purposes, however, the calorific value of a fuel may well be taken by selecting from existing tests that of a fuel corresponding most nearly with the one under consideration.

Calorific Values of Fuels.

Substance.	Approximate total heat of combustion of 1 pound of fuel.	Equivalent evaporation from and at 212° Fahr. per pound of fuel.
	Thermal units.	Lb. water.
Hydrogen	62000	64.20
Petroleum oils (benzine, etc.)	27500	28.56
Petroleum, crude	20400	21.13
Petroleum refuse	20000	20.70
Coal gas	17800	18.43
Coal gas, per cubic foot, at 62° Fahr.	630	0.70
Coal, good average quality	14700	15.22
Carbon, pure	14500	15.07
Coke	13500	13.87
Wood charcoal, dessicated	13000	13.46
Wood, dessicated	11000	11.39
Peat, dessicated	10000	10.35
Wood, air dried	8000	8.28
Straw	8000	8.40
Peat, 25 per cent. moisture	7000	7.25
Sulphur	4000	7.14

Theoretical Heating Value of Coals.

(Babcock and Wilcox.)

Heating Power of Coals of Great Britain, United States, Germany, France, Belgium, and Austria-Hungary.

Coals. Locality of beds.	B. T. U.	Calories.	Nature.
Great Britain.			
Welsh Coal:			
Ebbw Vale, 1848	16214	8998	Almost pure anthracites, having 84 to 89 per cent. of carbon.
Powell Duffryn, 1848	15715	8710	
Llangennech, 1848	14998	8318	
Llangennech, 1871	14964	8305	
Graigole, 1848	14689	8152	
Nixon's Navigation	15000	8325	Pure hard anthracite.
Gwaun Cae Gurwen	15123	8402	Called smokeless.
Newcastle	14820	8225	Bituminous coal, having 77 to 82 per cent. of carbon.
Derbyshire and Yorkshire	13860	7692	
Lancashire	13918	7724	
Scotch	14164	7861	Bituminous coal, having 78 per cent. of carbon.
United States.			
Pennsylvania	14221	7892	Anthracite, having 88 per cent. of carbon.
Pennsylvania	13143	7293	Cannel coal.
Pennsylvania	13155	7301	Bituminous coking.
Kentucky	14391	7987	
Kentucky	15198	8434	Cannel coke.
Kentucky	9326	5175	Lignite, good.
Illinois	13123	7283	Bituminous coking.
Indiana	14146	7851	
Indiana	13097	7268	Cannel coal.
Virginia	13100	7270	Bituminous coking.
Arkansas	9215	5114	Lignite, good.
Germany.			
Rhenish Prussia.			
Ruhr Coal:			
Dortmund	14518	8066	Cannel coal.
Witten	15125	8403	
Bochum	13514	7508	
Bommern	13212	7340	Short-flame coal, semi-anthracite.
Essen	14985	8325	Cannel coal.
Saar coal	11511	6395	
Saxony.			
Zwickau	11964	6647	
Hohndorf	11343	6302	
Oelsnitz	10674	5930	
Lower Saxony, Anhalt, and Brunswig.			
Unseburg	5769	3205	Brown coal or lignite, low grade.
Atzendorf	6444	3580	
Neudorf	6093	3385	
Görzig	3852	2140	
Halle a S.	4165	2314	
Bitterfeld	3830	2128	
Naumburg	4563	2535	

Theoretical Heating Value of Coals.—*Continued.*

Coals. Locality of beds.	B. T. U.	Calories.	Nature.
Germany.—*Continued.*			
Hanover.			
Osnabrück	10789	5994	Semi-anthracite, low grade.
Obernkirchen	12718	7066	Bituminous.
Silesia (Prussia).			
Carlssegen	10422	5790	Long-flaming, semi-bituminous.
Myslowitz	10758	5977	
Waterloa	11412	6340	
Königshülle	12247	6804	
Paulusgrube	12425	6903	
Waldenburg	12637	7021	
Brandenburg	12193	6774	
Neurode	13393	7441	
Freienstein	9651	5362	
Maxgrube	10087	5604	
Bavaria.			
Hanshamer coal	9821	5456	Lignite or brown, low grade.
Peipenberg	8186	4548	
Penzberg	8921	4956	
France.			
Anthracite de la Mayenne	15566	8646	Anthracite.
Anthracite de Lamure (Isère)	13782	7657	
Bassin du Bas-de-Calais.			
Marles	14175	7875	Bituminous hard coal.
Bully	15120	8400	
Hessin	15352	8529	Bituminous coking.
Lens	15258	8477	Bituminous hard coal.
Naux	15256	8476	Bituminous coking.
l'Escarpelle	15400	8556	
Les Courrières	14265	7925	Semi-bituminous coal.
Bassin de la Saône.			
Blanzy	13127	7293	Semi-bituminous coal, long flame.
Epinac	14086	7826	Bituminous coal, long flame.
Bassin de la Loire.			
Rive-de-Gier, puits Henry	15481	8601	Bituminous hard coal.
Rive-de-Gier, No. 1	15472	8596	
Rive-de-Gier, Cimetière 1	14493	8052	Bituminous hard coal, long flame.
Rive-de-Gier, Cimetière 2	15309	8505	
Rive-de-Gier, Couson	14770	8206	
Bassin de l'Aveyron.			
Lavaysse	14630	8128	
Céral	13203	7335	Semi-bituminous coal.
Bassin d'Alais Rochbelle	15643	8691	Bituminous coking.

Theoretical Heating Value of Coals.—*Continued.*

Coals. Locality of beds.	B. T. U.	Calories.	Nature.
France.—*Continued.*			
Bassin de Valenciennes.			
Denain, Fosse Renard.......	15244	8469	
Denain, Fosse Lelvet 1......	15100	8389	Bituminous coal, long flame.
Denain, Fosse Lelvet 2......	15316	8509	
St. Wast, Fosse de la Réussite	15105	8392	
St. Wast, Grande Fosse......	15188	8438	Bituminous coal, short flame.
St. Wast, Fosse Tinchon.....	15082	8379	
Anzin, Fosse Chauffour.....	14353	7974	
Anzin, Fosse la Cave........	14549	8083	Bituminous coking.
Anzin, Fosse St. Louis.......	15397	8554	
Fresne, Fosse Bonnepart....	15228	8460	Semi-bituminous coal.
Vieux-Condé, Fosse Sarteau.	15409	8561	
Belgium.			
Bassin de Mons.			
Haut-flenu..................	14576	8098	
Belle et Bonne, Fosse No. 21.	14326	7959	
Levant de flenu.............	14508	8060	
Couchant...................	14446	8037	
Midi........................	14553	8085	Semi-bituminous hard coal.
Grand-Hornu................	14943	8302	
Nord du bois de Bossu......	14407	8004	
Grand-Buisson..............	14877	8265	
Escouffiaux.................	15217	8454	
St. Hortense, bonne veine...	15107	8393	
Bassin du Centre.			
Haine St. Pierre.............	14702	8168	
Bois du Luc.................	14358	7977	
La Louvière.................	15127	8404	Semi-bituminous coking.
Bracquegnies................	15363	8535	
Mariemont..................	15168	8427	
Bascoup.....................	14911	8284	
Sars-Longchamps...........	14895	8275	Bituminous hard coal.
Houssu.....................	14945	8303	
Bassin de Charleroi.			
St. Martin, Fosse No. 3......	14954	8308	
Trieukaisin..................	15069	8372	Semi-bituminous coking.
Poirier, Fosse St. Louie......	14421	8012	
Bayemont, Fosse St. Charles.	13806	7670	
Sacré-Madame..............	15204	8447	
Sars-les-Moulins, Fosse No. 7	15125	8403	Semi-bituminous hard coal.
Carabinier-française No. 7...	14911	8284	
Roton, veine Greffier........	14311	7951	
Pont-du-Loup..............	14947	8304	
Austria-Hungary.			
Lower Austria.			
Grünbach...................	11458	6366	Semi-bituminous coal.
Thallern....................	7057	3921	
Upper Austria.			Lignite or brown coal.
Wolfsegg-Trannthal..... ...	6006	3337	

Theoretical Heating Value of Coals.—*Continued.*

Coals. Locality of beds.	B. T. U.	Calories.	Nature.
Austria-Hungary.—*Continued.*			
Styria.			
Leoben	9666	5370	Lignite or brown coal.
Fohnsdorf	9187	5104	
Göriagh	6222	3457	
Wies	7997	4443	
Trifail	7556	4198	
Bohemia.			
Kladno	10675	5931	Semi-bituminous coal.
Buschtehrad	8865	4925	
Libuschin	9900	5500	
Schlan	7979	4433	
Rakonitz-Lubna	7257	4032	
Pilsen	9318	5177	
Schatzlar	6552	5307	
Aussig	6408	3560	Lignite or brown coal.
Dux	7808	4338	
Bilin	8182	4546	
Brüx	8274	4597	
Moravia.			
Rossitz	12553	6974	....
M. Ostran	12623	7013	....
Gaya	4858	2699	Lignite or brown coal.
Göding	5056	2809	
Silesia.			
P. Ostran	12564	6980	Bituminous coal.
Orlan-Lazy	12389	6883	
Poremba	11057	6143	
Karwin	13021	7234	
Taklowetz	11932	6632	
Hungary.			
Fünfkirchen	10276	5709	Cannel coal.
Anina	11356	6309	
Neufeld	5200	2889	Lignite or brown coal.
Brennberg	8325	4625	
Aika	6913	3841	
Salgo-Tarjan	7966	4426	
Dorog-Annathal	7709	4283	
Tokod	8069	4483	
Dalmatia.			
Siveric	8087	4493	
Istria.			
Arsa	10182	5657	
Transylvania.			
Petrozsény	11286	6270	
Egeres	8692	4829	
Bosnia.			
Zenica	7911	4359	

American Coals.

State. Kind of coal.	Per cent. of ash.	Theoretical value.	
		In heat units.	Pounds of water evaporated.
Pennsylvania anthracite	3.49	14199	14.70
	6.13	13535	14.01
	2.90	14221	14.72
Pennsylvania cannel	15.02	13143	13.60
Pennsylvania, Connellsville	6.50	13368	13.84
Pennsylvania semi-bituminous	10.70	13155	13.62
Pennsylvania, Stone's gas	5.00	14021	14.51
Pennsylvania, Youghiogheny	5.60	14265	14.76
Pennsylvania brown	9.50	12324	12.75
Kentucky caking	2.75	14391	14.89
Kentucky cannel	2.00	15198	16.76
	14.80	13360	13.84
Kentucky lignite	7.00	9326	9.65
Illinois, Bureau County	5.20	13025	13.48
Illinois, Mercer County	5.60	13123	13.58
Illinois, Montauk	5.50	12659	13.10
Indiana block	2.50	13588	14.38
Indiana caking	5.66	14146	14.64
Indiana cannel	6.00	13097	13.56
Maryland, Cumberland	13.88	12226	12.65
Arkansas lignite	5.00	9215	9.54
Colorado lignite	9.25	13562	14.04
	4.50	13866	14.35
Texas lignite	4.50	12962	13.41
Washington Territory lignite	3.40	11551	11.96

Wood

as fuel is estimated to have about 0.4 times the calorific value as the same weight of coal. The relative calorific values of various woods are therefore proportional to their weights. The following table gives the weight, in pounds, per cord.

Kind of wood.	Weight.	Kind of wood.	Weight.
Hickory, shell-bark	4469	Beech	3126
Hickory, red heart	3705	Hard maple	2878
White oak	3821	Southern pine	3375
Red oak	3254	Virginia pine	2680
Spruce	2325	Yellow pine	1904
New Jersey pine	2137	White pine	1868

Total Heat of Combustion of Fuels.

(Rankine.)

The following table shows the total heat of combustion with oxygen of 1 pound of each of the substances named in it, in British thermal units, and also in pounds of water evaporated from 212°. It also shows the weight of oxygen required to combine with each pound of the combustible and the weight of air necessary in order to supply that oxygen. The quantities of heat are given on the authority of MM. Favre and Silbermann.

Combustible.	Pounds of oxygen per pound of combustible.	Pounds of air (about).	Total British heat units.	Evaporative power from 212° Fahr.
				Lb.
Hydrogen gas..................	8	36	62032	64.20
Carbon, imperfectly burned, so as to make carbonic oxide	$1\frac{1}{3}$	6	4400	4.55
Carbon, perfectly burned, so as to make carbonic acid...	$2\frac{2}{3}$	12	14500	15.0
Olefiant gas, 1 pound	$3\frac{3}{7}$	$15\frac{3}{7}$	21344	22.1
Various liquid hydrocarbons, 1 pound....................	....	....	from 21700 to 19000	from $22\frac{1}{2}$ to 20
Carbonic oxide, as much as is made by the imperfect combustion of 1 pound of carbon,—viz., $2\frac{1}{3}$ pounds	$1\frac{1}{2}$	6	10000	10.45

Evaporative Power and Composition of Liquid Fuels.

Fuel.	Specific gravity at 32° Fahr. Water=1.	Chemical composition.			Heating power, in B. T. U.	Theoretical evaporation, in pounds, of water per pound of fuel, from and at 212° Fahr.
		C.	H.	O.		
		P. c.	P. c.	P. c.		Lb.
Pennsylvania heavy crude oil	.886	84.9	13.7	1.4	20736	21.48
Caucasian light crude oil .	.884	86.3	13.6	.1	22027	22.79
Caucasian heavy crude oil	.938	86.6	12.3	1.1	20138	20.85
Petroleum refuse..........	.928	87.1	11.7	1.2	19832	20.53
Good English coal, mean of 98 samples............	1.380	80.0	5.0	8.0	14112	14.61

Comparative Evaporation of Coal and Oil.

Taken from the United States Geological Report on Petroleum for 1900.

1 pound of combustible.	Pounds of water evaporated at 212° per pound of combustible.	Barrels of petroleum required to do same amount of evaporation as 1 ton of coal.
Petroleum, 18° to 40° Baumé	....	....
Pittsburg lump and nut, Pennsylvania	10.0	4.0
Pittsburg nut and slack, Pennsylvania	8.0	3.2
Anthracite, Pennsylvania	9.8	3.9
Indiana block	9.5	3.8
Georges Creek lump, Maryland	10.0	4.0
New River, West Virginia	9.7	3.8
Pocahontas lump, West Virginia	10.5	4.2
Cardiff lump, Wales	10.0	4.0
Cape Breton, Canada	9.2	3.7
Nanaimo, British Columbia	7.3	2.9
Co-operative, British Columbia	8.9	3.6
Greta, Washington	7.6	3.0
Carbon Hill, Washington	7.6	3.0

Under favorable conditions 1 pound of oil will evaporate from 14 to 16 pounds of water from and at 212°; 1 pound of coal will evaporate from 7 to 10 pounds of water from and at 212°; 1 pound of natural gas will evaporate from 18 to 20 pounds of water from and at 212°.

Relative Values in Coal and Oil.

Petroleum residuum	19500
Beaumont crude	18500
Anthracite coal—East Middle coal-field	13400
Semi-bituminous coal—Cumberland, Maryland	14400
Semi-bituminous coal—Pocahontas, Virginia	15070
Bituminous coal—Jackson County, Ohio	13090
Bituminous coal—Hocking Valley, Ohio	12130
Bituminous coal—Missouri	12230
Bituminous coal—Alabama	13500
Bituminous coal—McAllester, Indian Territory	12789
Bituminous coal—New Mexico	12000
Bituminous coal—Texas lignite	10000

Gaseous Fuels.

The most valuable gaseous fuel is the natural gas of Pennsylvania and Ohio; the calorific power being about 1100 B. T. U. per cubic foot, or 10,000 calories per cubic metre. In comparison, 57.25 pounds of coal or 63 pounds of coke are about equal to 1000 cubic feet of natural gas.

Producer gas, made by the partial combustion of coal to carbonic oxide, is a lean gas composed of about 25 per cent. of CO and about 60 per cent. of nitrogen, with small quantities of CO_2 and hydrogen. The calorific value is about 150 B. T. U. per cubic foot.

Blast-furnace gas is almost identical with producer gas in composition, except that there is usually more CO_2 present, the calorific power falling to about 120 B. T. U. per cubic foot.

These lean gases can be used to advantage in properly designed gas engines with a high thermal efficiency, and engines of 1000 horse-power and more are in successful operation, using the waste gases from blast furnaces.

Calorific Power of Gas Fuels.

Authority.	Gas.	B. T. U.
A. G. Glasgow, M.E.	Plain water gas.	327,268 per 1000 cubic feet.
A. C. Humphreys	Plain water gas (theoretical).	323,003 per 1000 cubic feet.
F. E. Taylor, M.E.	Plain water gas.	8,335 per pound.
Dr. Greene	Plain water gas.	6,223 per pound.
Newbigging	Plain water gas.	290,000 per 1000 cubic feet.
	Plain water gas.	6,649 per pound.
Dr. Gideon Moore	Carburetted water gas, 22 candle-power.	650,000 per 1000 cubic feet.
	Coal gas, 18 candle-power.	642,000 per 1000 cubic feet.
Newbigging	Coal gas, 17 candle-power.	673,224 per 1000 cubic feet.
	Coal gas, 17 candle-power.	21,696 per pound.
R. D. Wood & Co.	Coal gas.	735,000 per 1000 cubic feet.
	Water gas.	322,000 per 1000 cubic feet.
	Producer gas (anthracite).	137,000 per 1000 cubic feet.
	Producer gas (bituminous).	156,000 per 1000 cubic feet.

STEAM.

Steam is the common name for water which has been converted into the gaseous state by heat. When heat is applied to water in an open vessel at or near the level of the sea the temperature of the water will rise until it reaches 212° F. or 100° C., after which it will remain constant until all the water is vaporized.

If we consider one pound of water at atmospheric pressure, it will require the expenditure of 180.9 B. T. U. to raise the temperature from the freezing-point, 32° F., to the boiling-point, 212° F. If heat is further supplied until the pound of water at 212° is converted into a pound of steam at 212°, it will be found to require 965.7 additional thermal units, so that a pound of steam at amospheric pressure will have a sensible temperature of 212° F., and will contain energy equal to $180.9 + 965.7 = 1146.6$ B. T. U.

Furthermore, its volume will have increased to 1641.5 times that of the original pound of water at its greatest density (39° F.).

If the steam is confined, and heat further applied, its temperature will rise,—the temperature, pressure, volume, and heat absorbed bearing certain relations to each other. These relations, of continual importance in steam engineering, have been the subject of much study and investigation, and have been tabulated in various ways by numerous authors.

The data upon which all steam tables at present in use are founded are the result of experiments made by the French physicist, Regnault, in 1847. Regnault's observations covered only temperatures from 40° C. (104° F.) to 230° C. (446° F.), advancing by 10° C., there being thus 20 observations in all; and upon these 20 observations all the existing steam tables have been built by various computers who have devised formulas representing more

or less accurately the results of the experiments, and therefore available for interpolating the intermediate values.

Since modern steam engineering is beginning to demand the use of pressures higher than the maximum examined by Regnault, some of the tables have been extended to higher pressures; but it must be understood that such figures are based upon the assumption that the relations developed within the range of Regnault's experiments continue beyond the limit of his work. A study of this feature of the subject will be found in the important paper of Macfarlane Gray, presented before the British Institution of Mechanical Engineers in 1889.

The tables here given in British units are those computed from the experiments of Regnault by the late John W. Nystrom, and may be accepted as being as reliable as any. The figures for temperatures above 446° F. agree fairly well with those deduced by Macfarlane Gray, and, until experimental researches at these higher pressures and temperatures are made, they may be used.

The metric steam tables given have been compiled from those of Zeuner and Fliegner.

Introduction to Steam Tables.

(Nystrom.)

Properties of Steam.

Column P contains the total steam pressure, in pounds, per square inch, including the pressure of the atmosphere.

Column I is the same pressure, in inches, of mercury. The specific gravity of mercury at 32° F. is 13.5959, compared with water of maximum density at 39°. 1 cubic inch of mercury weighs 0.49086 pound, of which a column of 29.9218 inches is a mean balance of the atmosphere, or 14.68757 pounds per square inch.

Column T contains the temperature of the steam on Fahrenheit's scale, deduced from Regnault's experiments.

Column V contains the volume of steam of the corresponding temperature, T, compared with that of water of maximum density at 39° F. This column is calculated from the formula of Fairbairn and Tate, namely,—

$$V = 25.62 + \frac{49513}{I + 0.72}.$$

Column W contains the weight per cubic foot in fractions of a pound; and

Column C the cubic feet per pound of saturated steam under the pressure, P, and temperature, T.

Column H contains the heat units per pound of steam from 32° to temperature, T, and pressure, P, calculated from the formula,

$$H = 1081.91 + 0.305\,T.$$

Column H' contains the heat units per cubic foot of steam from 32° to temperature, T.

The columns H and H' give the heat units required to heat the water from 32° to the boiling-point, and evaporate the same to steam under the pressure, P, and of temperature, T.

Column L contains the latent units of heat per pound in steam of temperature, T, and pressure, P. The latent heat expresses the work done in the evaporation, or the difference between the number of heat units per pound in the steam and in the water of the same temperature.

Column L' contains the latent heat per cubic foot of steam.

Latent heat, $L = H - h$, the heat units required to evaporate each pound of water from the boiling-point into steam.

In the metric tables the pressures are given in kilogrammes per square centimetre, or so-called metric atmospheres (1 kilogramme per square centimetre = 14.22 pounds per square inch), the temperatures in degrees Centigrade, and the total and latent heats in calories per kilogramme and calories per cubic metre.

Steam Table. British System.

Absolute press.		Temp. Fahr. scale.	Volume water = 1 at 39°.	Wt. lb. per cubic foot.	Bulk, cubic feet per pound.	Units of heat, from 32° to T°.				Press. Ab. at. lb. per sq. in.
Lb. per sq. in.	Inch. of mer.					Total per lb.	Total per cubic ft.	Lat'nt per lb.	Latent per cubic ft.	
P	*I*	*T*	*V*	*W*	*C*	*H*	*H'*	*L*	*L'*	*p*
1	2.037	101.36	17983.00	.00347	288.2400	1112.8	3.8614	1043.40	3.6337	—14
2	4.074	126.21	10353.00	.00602	165.9400	1120.4	6.7449	1026.00	6.1165	—13
3	6.111	141.67	7283.80	.00856	116.7500	1125.1	9.6308	1015.20	8.6901	—12
4	8.149	153.27	5608.40	.01112	89.8950	1128.7	12.551	1007.10	11.199	—11
5	10.18	162.51	4565.60	.01366	73.1800	1131.5	15.456	1000.60	13.714	—10
6	12.22	170.25	3851.00	.01619	61.7420	1133.8	18.156	995.17	16.113	— 9
7	14.26	176.97	3330.80	.01872	53.3880	1135.9	20.846	990.44	18.194	— 8
8	16.29	182.96	2935.10	.02125	47.0460	1137.7	24.176	986.22	20.957	— 7
9	18.33	188.36	2624.00	.02377	42.0590	1139.4	27.083	982.41	23.352	— 6
10	20.37	193.20	2373.00	.02628	38.0370	1140.8	29.980	978.99	25.728	— 5
11	22.41	197.60	2166.30	.02880	34.7230	1142.2	32.895	975.88	28.099	— 4
12	24.44	201.90	1993.00	.03130	31.9450	1143.5	35.791	972.84	30.450	— 3
13	26.48	205.77	1845.70	.03380	29.5840	1144.7	38.691	970.11	32.789	— 2
14	28.52	209.55	1718.90	.03629	27.5510	1145.8	41.581	967.43	35.435	— 1
14.7	29.92	212.00	1641.50	.03800	26.3110	1146.6	43.571	965.70	36.706	0
15	30.55	213.04	1608.60	.03878	25.7840	1146.9	44.476	964.96	37.421	.3125
16	32.59	216.33	1511.70	.04123	24.2300	1147.9	47.328	962.63	39.690	1
17	34.63	219.45	1426.20	.04374	22.8590	1148.8	50.248	960.49	42.012	2
18	36.67	222.40	1349.80	.04622	21.6360	1149.7	53.138	958.32	44.393	3
19	38.71	225.25	1281.10	.04868	20.5390	1150.6	56.011	958.30	46.698	4
20	40.74	227.95	1219.70	.05119	19.5500	1151.4	58.894	954.38	48.655	5
21	42.78	230.60	1163.80	.05360	18.6540	1152.2	61.758	952.50	51.924	6
22	44.82	233.10	1112.90	.05605	17.8380	1153.0	64.637	950.62	53.282	7
23	46.85	235.49	1066.30	.05851	17.0920	1153.7	67.503	949.03	55.529	8
24	48.89	237.81	1023.60	.06095	16.4070	1154.5	70.367	947.37	57.743	9
25	50.93	240.07	984.23	.06338	15.7760	1155.1	73.410	945.76	59.942	10
26	52.97	242.24	947.86	.06582	15.1930	1155.8	76.074	944.25	62.161	11
27	55.00	244.32	914.14	.06824	14.6520	1156.4	78.913	942.74	64.423	12
28	57.04	246.35	882.80	.07067	14.1500	1157.1	81.772	941.29	66.521	13
29	59.08	248.33	853.60	.07308	13.6820	1157.7	84.604	939.88	68.686	14
30	61.11	250.26	826.32	.07550	13.2450	1158.2	87.444	938.50	70.857	15
31	63.15	252.13	800.79	.07791	12.8350	1158.8	90.166	937.17	73.015	16
32	65.19	253.98	766.83	.08031	12.4510	1159.4	93.121	935.45	75.126	17
33	67.23	255.77	754.31	.08271	12.0900	1159.9	95.861	934.57	77.298	18
34	69.26	257.52	733.09	.08510	11.7500	1160.5	98.782	933.32	79.425	19
35	71.30	259.22	713.08	.08749	11.4290	1161.0	101.48	932.10	81.549	20
36	73.34	260.88	694.17	.08987	11.1270	1161.5	104.38	930.92	83.662	21
37	75.38	262.50	676.27	.09225	10.8400	1162.0	107.19	929.76	85.770	22
38	77.41	264.09	659.31	.09462	10.5680	1162.5	109.98	928.62	87.866	23
39	79.45	265.65	643.21	.09700	10.3100	1162.9	112.79	927.51	89.968	24
40	81.49	267.17	627.91	.09936	10.0640	1163.4	115.59	926.42	92.059	25
41	83.52	268.66	613.34	.10172	9.8310	1163.9	118.39	925.35	94.126	26
42	85.56	270.12	599.46	.10407	9.6086	1164.3	121.17	924.30	96.192	27
43	87.60	271.55	586.23	.10642	9.3963	1164.7	123.95	923.28	98.255	28
44	89.64	272.96	573.58	.10877	9.1938	1165.2	126.74	922.27	100.32	29
45	91.67	274.33	561.50	.11111	9.0002	1165.6	129.51	921.29	102.36	30
46	93.71	275.68	549.94	.11344	8.8149	1166.0	132.29	920.32	104.40	31
47	95.75	277.01	538.87	.11577	8.6374	1166.4	135.07	919.36	106.43	32
48	97.78	278.32	528.25	.11810	8.4673	1166.8	137.83	918.43	108.46	33
49	99.82	279.62	518.07	.12042	8.3040	1167.2	140.69	917.49	110.48	34
50	101.86	280.89	508.29	.12273	8.1472	1167.6	143.30	916.58	112.49	35
51	103.90	282.14	498.89	.12505	7.9966	1167.9	146.08	915.68	114.50	36
52	105.93	283.39	489.85	.12736	7.8517	1168.4	148.85	914.79	116.51	37
53	107.97	284.58	481.15	.12966	7.7122	1168.7	151.63	913.93	118.50	38
54	110.01	285.76	472.77	.13196	7.5779	1169.0	154.48	913.08	120.49	39
55	112.04	286.96	464.69	.13428	7.4468	1169.4	157.02	912.22	122.47	40
56	114.08	288.09	456.90	.13652	7.3236	1169.8	159.74	911.42	124.43	41
57	116.12	289.24	449.38	.13883	7.2030	1170.1	162.45	910.48	126.40	42

Steam Table. British System.

Absolute press. Lb. per sq. in.	Absolute press. Inch. of mer.	Temp. Fahr. scale.	Volume water = 1 at 39°.	Wt. lb. per cubic foot.	Bulk, cubic feet per pound.	Units of heat, from 32° to T°. Total per lb.	Total per cubic ft.	Lat'nt per lb.	Latent per cubic ft.	Press. Ab. at. lb. per sq. in.
P	*I*	*T*	*V*	*W*	*C*	*H*	*H'*	*L*	*L'*	*p*
58	118.16	290.37	442.12	.14111	7.0866	1170.5	165.15	909.78	128.38	43
59	120.19	291.48	435.10	.14338	6.9741	1170.8	167.84	908.97	130.33	44
60	122.23	292.58	428.32	.14566	6.8654	1171.2	170.58	908.18	132.28	45
61	124.27	293.66	421.75	.14792	6.7601	1171.5	173.27	907.40	134.22	46
62	126.30	294.73	415.40	.15018	6.6583	1171.8	175.96	906.63	136.16	47
63	128.34	295.78	409.25	.15244	6.5597	1172.1	178.65	905.87	138.09	48
64	130.38	296.82	403.29	.15469	6.4642	1172.5	181.34	905.13	140.01	49
65	132.42	297.84	397.51	.15694	6.3715	1172.8	184.03	904.39	141.93	50
66	134.45	298.85	391.90	.15919	6.2817	1173.1	186.72	903.66	143.85	51
67	136.49	299.85	386.47	.16130	6.1994	1173.4	189.40	902.94	145.64	52
68	138.53	300.84	381.18	.16366	6.1099	1173.7	192.07	902.23	147.66	53
69	140.56	301.81	376.06	.16590	6.0277	1174.0	194.74	901.53	149.56	54
70	142.60	302.77	371.07	.16812	5.9478	1174.3	197.42	900.84	151.45	55
71	144.64	303.72	366.34	.17035	5.8702	1174.6	200.08	900.15	153.34	56
72	146.68	304.69	361.53	.17256	5.7948	1174.9	202.74	899.46	155.21	57
73	148.72	305.60	356.95	.17478	5.7214	1175.1	205.40	898.79	157.09	58
74	150.75	306.52	352.49	.17690	5.6500	1175.4	208.04	898.13	158.88	59
75	152.79	307.42	348.15	.17919	5.5805	1175.8	210.67	897.57	160.83	60
76	154.83	308.32	343.93	.18139	5.5129	1176.0	213.30	896.83	162.67	61
77	156.86	309.22	339.81	.18359	5.4468	1176.2	215.93	896.18	164.56	62
78	158.90	310.11	335.80	.18578	5.3825	1176.5	218.56	895.54	166.37	63
79	160.94	310.99	331.89	.18797	5.3190	1176.8	221.19	894.92	168.22	64
80	162.98	311.86	328.08	.19015	5.2588	1177.0	223.82	894.27	170.04	65
81	165.01	312.72	324.37	.19233	5.1992	1177.3	226.44	893.65	171.87	66
82	167.05	313.57	320.74	.19451	5.1410	1177.6	229.06	893.03	173.70	67
83	169.09	314.42	317.20	.19668	5.0843	1177.9	231.68	892.51	175.52	68
84	171.12	315.25	313.74	.19885	5.0289	1178.1	234.28	891.82	177.33	69
85	173.16	316.08	310.36	.20101	4.9748	1178.3	236.89	891.22	179.14	70
86	175.20	316.90	307.07	.20317	4.9219	1178.6	239.50	890.63	180.95	71
87	177.24	317.71	303.85	.20532	4.8703	1178.8	242.10	890.04	182.75	72
88	179.27	318.51	300.70	.20747	4.8198	1179.1	244.69	889.46	184.53	73
89	181.31	319.31	297.62	.20962	4.7704	1179.3	247.29	888.88	186.33	74
90	183.35	320.10	294.61	.21185	4.7222	1179.6	249.88	888.31	188.12	75
91	185.38	320.88	291.66	.21390	4.6750	1179.8	252.45	887.74	189.88	76
92	187.42	321.66	288.78	.21603	4.6288	1180.0	255.02	887.19	191.66	77
93	189.46	322.42	285.96	.21816	4.5836	1180.3	257.58	886.63	193.43	78
94	191.50	323.18	283.21	.22029	4.5394	1180.5	260.14	886.08	195.19	79
95	193.53	323.94	280.50	.22241	4.4961	1180.7	262.69	885.53	196.94	80
96	195.57	324.67	277.86	.22453	4.4537	1180.9	265.23	885.00	198.71	81
97	197.61	325.43	275.27	.22672	4.4106	1181.2	267.77	884.45	200.49	82
98	199.65	326.17	272.73	.22875	4.3715	1181.4	270.30	883.91	202.18	83
99	201.68	326.90	270.24	.23085	4.3316	1181.6	273.10	883.38	203.92	84
100	203.72	327.63	267.80	.23296	4.2926	1181.9	275.52	882.85	205.67	85
101	205.76	328.35	265.41	.23505	4.2543	1182.1	277.85	882.33	207.39	86
102	207.79	329.07	263.07	.23715	4.2167	1182.3	280.38	881.81	209.12	87
103	209.83	329.78	260.77	.23924	4.1799	1182.5	282.90	881.29	210.84	88
104	211.87	330.48	258.52	.24132	4.1438	1182.7	285.42	880.78	212.55	89
105	213.91	331.18	256.31	.24340	4.1083	1182.9	287.93	880.27	214.26	90
106	215.94	331.87	254.14	.24548	4.0736	1183.2	290.45	879.77	215.96	91
107	217.98	332.56	252.01	.24750	4.0394	1183.4	292.94	879.27	217.66	92
108	220.02	333.24	249.92	.24963	4.0058	1183.6	295.41	878.79	219.36	93
109	222.05	333.92	247.87	.25169	3.9731	1183.8	297.91	878.28	221.05	94
110	224.10	334.59	245.86	.25375	3.9408	1183.9	300.44	877.80	222.74	95
111	226.13	335.26	243.88	.25581	3.9091	1184.2	302.93	877.31	224.42	96
113	230.20	336.58	240.03	.25991	3.8474	1184.6	307.90	876.25	227.74	98
114	232.24	337.23	238.15	.26204	3.8100	1184.8	310.36	875.88	229.51	99
115	234.28	337.89	236.31	.26400	3.7878	1185.0	312.86	875.40	231.10	100
120	244.4	341.0	227.56	.27421	3.6475	1185.9	325.20	873.09	239.41	105

Steam Table. British System.

Absolute press. Lb. per sq. in.	Absolute press. Inch. of mer.	Temp. Fahr. scale.	Volume water = 1 at 39°.	Wt. lb. per cubic foot.	Bulk, cubic feet per pound.	Units of heat, from 32° to T°. Total per lb.	Total per cubic ft.	Lat'nt per lb.	Latent per cubic ft.	Press. Ab. at. lb. per sq. in.
P	*I*	*T*	*V*	*W*	*C*	*H*	*H'*	*L*	*L'*	*p*
125	254.6	344.1	219.50	.28422	3.5184	1186.9	337.39	870.85	247.51	110
130	264.8	347.1	212.07	.29419	3.3991	1187.8	349.44	868.68	255.55	115
135	275.0	350.0	205.18	.30406	3.2880	1188.7	361.42	866.56	263.48	120
140	285.2	352.8	198.78	.31385	3.1862	1189.5	373.34	864.49	271.32	125
145	295.4	355.6	192.83	.32354	3.0908	1190.4	385.20	862.48	278.97	130
150	305.6	358.4	187.26	.33315	3.0001	1191.2	396.86	860.45	286.66	135
155	315.8	361.6	180.00	.3466	2.8958	1191.8	413.20	858.4	297.5	140
160	325.9	364.5	174.20	.3601	2.7916	1192.5	429.54	856.5	308.3	145
165	336.0	367.3	167.90	.3736	2.6873	1193.6	445.88	854.0	319.1	150
170	346.3	369.8	161.10	.3871	2.5831	1194.7	462.22	852.5	329.9	155
175	356.5	372.0	157.00	.3973	2.5171	1195.4	475.80	851.0	338.7	160
180	366.7	374.2	152.80	.4075	2.4541	1196.1	488.96	849.4	347.1	165
185	376.9	376.4	148.80	.4182	2.3916	1196.8	502.10	847.8	355.5	170
190	387.1	378.5	145.00	.4292	2.3299	1197.4	515.20	846.2	363.9	175
195	397.3	380.6	141.50	.4409	2.2684	1198.1	528.27	844.8	372.4	180
200	407.4	382.6	138.10	.4517	2.2137	1198.7	542.07	843.3	381.0	185
210	427.8	386.6	132.00	.4719	2.1192	1199.8	568.40	840.3	398.0	195
220	448.2	390.4	126.30	.4935	2.0265	1201.0	574.70	837.5	414.8	205
230	468.5	394.0	120.80	.5165	1.9360	1202.2	620.96	835.0	431.3	215
240	488.9	397.6	116.10	.5364	1.8646	1203.2	647.41	832.3	447.9	225
250	509.3	401.0	111.70	.5595	1.7874	1204.2	673.85	829.8	464.4	235
260	529.7	404.3	107.50	.5803	1.7230	1205.2	700.28	827.4	480.8	245
270	550.0	407.5	103.70	.6016	1.6621	1206.2	726.66	825.0	497.1	255
280	570.4	410.6	100.20	.6238	1.6031	1207.2	753.04	822.8	513.3	265
290	590 8	413.5	97.01	.6459	1.5481	1208.1	779.40	820.7	529.4	275
300	611.1	416.5	94.22	.6681	1.4967	1209.0	805.74	818.6	545.4	285
310	631.5	419.2	91.13	.6896	1.4499	1209.8	832.96	816.5	561.4	295
320	651.9	422.1	88.21	.7107	1.4071	1210.6	858.36	814.4	577.3	305
330	672.3	424.8	85.44	.7302	1.3695	1211.5	884.63	812.4	593.2	315
340	692.6	427.4	83.19	.7547	1.3250	1212.3	910.89	810.5	608.9	325
350	713.0	430.0	80.99	.7745	1.2915	1213.1	937.13	808.6	624.5	335
360	733.4	432.4	78.84	.7943	1.2590	1213.9	963.34	806.9	640.2	345
370	753.8	434.9	76.74	.8146	1.2275	1214.7	989.51	805.1	655.8	355
380	774.1	437.3	74.66	.8353	1.1968	1215.5	1015.7	803.4	671.3	365
390	794.5	439.6	72.90	.8626	1.1597	1216.2	1041.8	801.7	686.7	375
400	814.9	441.9	71.19	.8745	1.1434	1216.8	1067.9	800.0	702.0	385
410	835.2	444.1	69.52	.8952	1.1170	1217.4	1094.0	799.4	717.2	395
420	855.6	446.4	67.90	.9142	1.0938	1218.0	1120.2	797.7	732.4	405
430	876.0	448.5	66.34	.9400	1.0634	1218.7	1146.3	795.0	747.6	415
440	896.4	450.6	64.91	.9599	1.0417	1219.4	1172.3	793.5	762.8	425
450	916.7	452.6	63.55	.9804	1.0201	1220.1	1198.3	792.0	777.9	435
460	937.1	454.6	62.22	1.0007	.9993	1220.7	1224.3	790.5	792.9	445
470	957.5	456.7	60.94	1.0211	.9793	1221.3	1250.4	789.0	807.8	455
480	977.8	458.7	59.72	1.0446	.9573	1221.9	1276.5	787.5	822.7	465
490	998.2	460.6	58.54	1.0652	.9388	1222.5	1302.3	786.1	837.4	475
500	1018.6	462.5	57.45	1.0859	.9209	1223.0	1328.1	784.7	852.1	485
525	1069.5	466.1	54.81	1.1381	.8786	1224.5	1392.6	782.3	881.8	510
550	1120.4	471.5	52.47	1.1890	.8410	1225.8	1456.9	778.0	921.3	535
575	1171.4	475.7	50.32	1.2397	.8066	1227.2	1521.0	775.0	960.4	560
600	1222.3	479.8	48.35	1.2901	.7751	1228.3	1584.8	771.8	1000.0	585
650	1324.2	487.6	44.75	1.3943	.7172	1230.6	1709.5	766.0	1082.0	635
700	1426.0	494.9	41.70	1.4961	.6684	1232.7	1933.8	760.4	1157.0	685
750	1527.9	501.8	39.05	1.5977	.6259	1234.9	2057.7	755.4	1234.0	735
800	1629.8	508.4	36.73	1.6986	.5887	1237.0	2101.2	750.6	1307.0	785
850	1731.6	514.6	34.68	1.7989	.5554	1238.9	2228.3	745.9	1374.0	835
900	1833.5	521.4	32.87	1.8979	.5269	1241.0	2355.4	740.0	1435.0	885
950	1935.5	526.0	31.21	1.9992	.5002	1242.4	2482.5	737.4	1490.0	935
1000	2037.2	531.6	29.73	2.0986	.4765	1243.5	2609.6	732.3	1538.0	985

Steam Table. *Metric System.*

Absolute pressure.		Temp. Centigrade scale.	Volume cubic metres per kilogram.	Weight kilograms. per cubic metre.	Volume water = 1 at 4° C.	Calories, from 0° C. to T°.			
Kilograms. per sq. cm.	Millimetres of mercury.					Total per kilogram.	Total per cubic metre.	Latent per kilogram.	Latent per cubic metre.
.10	73.6	45.6	15.0376	.0665	15038	620.40	41.25	574.75	38.50
.20	147.1	59.8	7.8064	.1281	7806	624.73	80.00	564.84	72.30
.30	220.7	68.7	5.3305	.1876	5330	627.46	116.50	558.53	104.40
.40	294.2	75.5	4.0667	.2459	4067	629.52	154.70	553.81	136.20
.50	367.8	80.9	3.2971	.3033	3297	631.18	191.80	549.99	167.00
.60	441.3	85.5	2.7777	.3600	2778	632.58	228.20	546.76	197.00
.70	514.9	89.5	2.4033	.4161	2403	633.95	264.10	544.11	227.00
.80	588.4	93.0	2.1191	.4719	2119	634.87	299.50	541.44	255.50
.90	662.0	96.2	1.8964	.5273	1896	635.64	335.20	539.20	284.70
1.00	735.5	99.1	1.7173	.5823	1717	636.72	370.76	537.15	312.79
1.10	809.1	101.8	1.5711	.6365	1571	637.54	405.5	535.26	341.0
1.20	882.6	104.2	1.4478	.6907	1449	638.29	447.5	533.50	368.2
1.30	956.2	106.5	1.3430	.7446	1343	639.00	486.2	531.86	396.0
1.40	1029.7	108.7	1.2527	.7983	1253	639.66	515.0	530.32	423.5
1.50	1103.3	110.8	1.1740	.8518	1174	640.29	540.0	528.27	450.0
1.60	1176.8	112.7	1.1050	.9050	1105	640.87	587.5	527.49	477.5
1.70	1250.4	114 5	1.0438	.9580	1044	641.43	614.1	526.18	504.3
1.80	1323.9	116.3	.9891	1.0109	989	641.96	649.0	524.93	531.0
1.90	1397.5	118.0	.9398	1.0637	940	642.48	683.0	523.64	556.0
2.00	1471.0	119.6	.8960	1.1161	896	642.97	718.0	522.60	583.3
2.10	1544.6	121.1	.8562	1.1684	856	643.44	752.0	521.51	609.0
2.20	1618.1	122.6	.8190	1.2206	819	643.88	785.5	520.44	635.0
2.30	1691.7	124.0	.7855	1.2726	785	644.32	821.0	519.42	661.5
2.40	1765.2	125.4	.7553	1.3245	755	644.74	854.1	518.44	686.5
2.50	1838.8	126.7	.7267	1.3763	727	645.15	888.0	517.49	712.0
2.60	1912.3	128.0	.7003	1.4280	700	645.54	922.0	516.57	738.0
2.70	1985.9	129.3	.6761	1.4793	676	645.94	956.0	515.68	763.0
2.80	2059.4	130.5	.6531	1.5307	653	646.29	990.0	514.81	788.0
2.90	2133.0	131.6	.6321	1.5820	632	646.65	1024.0	513.97	813.0
3.00	2206.5	132.8	.6124	1.6332	612	647.00	1057.0	513.15	838.0
3.10	2280.1	133.9	.5938	1.6843	594	647.34	1091.0	512.35	864.0
3.20	2353.6	135.0	.5763	1.7352	576	647.67	1123.0	511.57	888.0
3.30	2427.2	136.1	.5599	1.7864	560	648.00	1158.0	510.82	914.0
3.40	2500.7	137.1	.5444	1.8369	544	648.31	1195.0	510.07	938.0
3.50	2574.3	138.1	.5296	1.8879	530	648.62	1227.0	509.35	962.0
3.60	2647.8	139.1	.5160	1.9384	516	648.92	1258.0	508.64	987.0
3.70	2721.4	140.0	.5027	1.9889	503	649.21	1292.0	507.95	1011.0
3.80	2794.9	141.0	.4904	2.0392	490	649.50	1325.0	507.27	1034.0
3.90	2868.5	141.9	.4787	2 0894	479	649.78	1357.0	506.61	1058.0
4.00	2942.0	142.8	.4673	2.1400	467	650.06	1372.0	505.96	1088.0
4.10	3015.6	143.7	.4566	2.1901	457	650.33	1425.0	505.32	1107.0
4.20	3089.1	144.6	.4464	2.2401	446	650.60	1457.0	504.70	1131.0
4.30	3162.7	145.4	.4367	2.2904	437	650.86	1492.0	504.09	1156.0
4.40	3236.2	146.3	.4273	2.3403	427	651.10	1525.0	503.47	1177.0
4.50	3309.8	147.1	.4184	2.3901	418	651.35	1558.0	502.88	1203.0
4.60	3383.3	147.9	.4098	2.4402	410	651.60	1591.0	502.30	1227.0
4.70	3459.9	148.7	.4016	2.4900	402	651.85	1624.0	501.73	1250.0
4.80	3530.4	149.5	.3938	2.5394	394	652.09	1658.0	501.17	1274.0
4.90	3604.0	150.2	.3862	2.5893	386	652.31	1691.0	500.61	1297.0

Steam Table. Metric System.

Absolute pressure. Kilograms. per sq. cm.	Absolute pressure. Millimetres of mercury.	Temp. Centigrade scale.	Volume cubic metres per kilogram.	Weight kilograms. per cubic metre.	Volume water = 1 at 4° C.	Calories, from 0° C. to T°. Total per kilogram.	Total per cubic metre.	Latent per kilogram.	Latent per cubic metre.
5.00	3677.6	151.0	.3786	2.6412	379	652.45	1723	500.07	1321
5.10	3751.1	151.7	3720	2.6882	372	652.78	1756	499.54	1344
5.20	3824.7	152.5	.3654	2.7375	365	653.00	1789	499.01	1367
5.30	3898.2	153.2	.3588	2.7871	359	653.21	1821	498.48	1390
5.40	3971.8	153.9	.3524	2.8369	352	653.43	1854	497.97	1413
5.50	4045.3	154.6	.3465	2.8860	346	653.55	1887	497.47	1436
5.60	4118.9	155.3	.3407	2.9351	341	653.87	1920	496.98	1459
5.70	4192.4	156.0	.3351	2.9842	335	654.06	1943	496.48	1482
5.80	4266.0	156.6	.3297	3.0331	330	654.27	1985	496.00	1505
5.90	4339.5	157.3	.3243	3.0826	324	654.46	2018	495.52	1528
6.00	4413.1	157.9	.3193	3.1319	319	654.66	2051	495.04	1551
6.10	4486.6	158.6	.3144	3.1807	314	654.85	2083	494.60	1573
6.20	4560.2	159.2	.3096	3.2300	310	655.03	2116	494.16	1596
6.30	4633.7	159.8	.3049	3.2787	305	655.21	2148	493.72	1618
6.40	4707.3	160 5	.3004	3.3278	300	655.40	2181	493.28	1641
6.50	4780.8	161.1	.2962	3.3761	296	655.59	2213	492.83	1663
6.60	4854.4	161.7	.2919	3.4247	292	655.78	2246	492.39	1686
6.70	4927.9	162.3	.2879	3.4734	288	655.96	2278	491.95	1708
6.80	5001.5	162.9	.2839	3.5224	284	656.15	2311	491.52	1731
6.90	5075.0	163.4	.2800	3.5714	280	656.33	2343	491.07	1753
7.00	5148.6	164.0	.2763	3.6193	276	656.52	2376	490.63	1776
7.25	5332.4	165.4	.2673	3.7411	267	656.93	2443	489.64	1822
7.50	5516.3	166.8	.2590	3.8610	259	657.35	2540	488.66	1890
7.75	5700.2	168.1	.2511	3.9825	251	657.76	2620	487.67	1942
8.00	5884.1	169.5	.2437	4.1034	244	658.18	2700	486.69	1998
8.25	6068.0	170.7	.2368	4.2230	237	658.55	2782	485.79	2052
8.50	6251.8	172.0	.2302	4.3440	230	658.93	2867	484.89	2109
8.75	6435.7	173.2	.2241	4.4623	224	659.30	2942	483.99	2161
9.00	6619.6	174.4	.2182	4.5830	218	659.68	3022	483.10	2216
9.25	6803.5	175.5	.2127	4.7015	213	660.02	3105	482.28	2270
9.50	6987.4	176.7	.2074	4.8216	207	660.37	3185	481.46	2321
9.75	7171.2	177.8	.2024	4.9407	202	660.71	3265	480.64	2375
10.00	7355.1	178.9	.1975	5.0607	197	661.06	3345	479.82	2432
10.25	7539.0	180.0	.1931	5.1787	193	661.38	3425	479.06	2483
10.50	7722.9	181.0	.1888	5.2966	189	661.68	3505	478.29	2535
10.75	7906.7	182.0	.1847	5.4142	185	662.00	3585	477.53	2586
11.00	8090.6	183.0	.1807	5.5340	181	662.33	3665	476.77	2638
11.25	8274.5	184.0	.1769	5.6497	177	662.62	3745	476.04	2690
11.50	8458.4	185.0	.1733	5.7703	173	662.92	3825	475.32	2742
11.75	8642.2	186.0	.1698	5.8858	170	663.21	3905	474.61	2794
12.00	8826.1	186.9	.1665	6.0060	166	663.51	3985	473.92	2846
12.25	9010.0	187.9	.1634	6.1200	163	663.75	4064	473.24	2897
12.50	9193.9	188.8	.1603	6.2383	160	664.08	4143	472.57	2948
12.75	9377.8	189.7	.1573	6.3573	157	664.35	4222	471.90	2998
13.00	9561.6	190.6	.1545	6.4725	154	664.63	4301	471.25	3049
13.50	9929.4	192.3	.1491	6.7069	149	665.15	4467	469.97	3155
14.00	10297.1	194.0	.1441	6.9396	144	665.67	4620	468.73	3254
14.50	10664.9	195.6	.1394	7.1737	139	666.17	4780	467.51	3353
15.00	11032.7	197.2	.1351	7.4019	135	666.67	4935	466.35	3452

In the preceding tables the temperatures are given which correspond to the respective pressures, it being understood that these are the temperatures at which the steam is formed from the water under those pressures. Such steam is said to be **saturated**; it contains no moisture; neither is it superheated. If, now, the steam be further supplied with heat, its temperature will rise and it will become superheated. The effect of the additional heat upon the steam is similar to that upon a gas, and the more highly it is superheated the more nearly it resembles a perfect gas. For any given pressure saturated steam can have but one temperature, as given in the tables. Superheated steam may have any higher temperature.

Flow of Steam.

The flow of steam from one pressure to another increases as the increase in difference in pressure, until the lower pressure becomes 58 per cent. of the higher pressure. If the lower pressure be diminished, or even is made a perfect vacuum, the flow will not be affected. Steam will expand in a nozzle until it reaches the external pressure, provided the latter is not less than 58 per cent. of the internal pressure. The ratio of expansion for all external pressures below 58 per cent. of the internal pressure is 1 to 1.624. The discharge will then have a constant velocity of 890 feet per second, and the amount discharged will be proportional to the density of the steam, which latter value can be obtained from the steam tables.

The following formulas, by Rankine, may be used in computing the discharge of steam:

Let

W = weight discharged, in pounds, per minute;
a = area of opening, in square inches;
p = absolute pressure, in pounds, per square inch;
d = difference in pressure, when more than 58 per cent.;
k = coefficient = 0.93 for short nozzle = 0.63 for hole in thin plate.

$$W = 0.85ap,$$

when discharging into atmosphere.

$$W = 1.9ak\sqrt{(p-d)d},$$

when the difference between the two pressures is more than 58 per cent.

The following table, compiled by D. K. Clark from experiments by Brownlee, will be useful in this connection.

Outflow of Steam from a given Initial Pressure into Various Lower Pressures.

Absolute initial pressure in boiler 75 pounds per square inch.

(D. K. Clark.)

Absolute pressure in boiler, in pounds, per square inch.	External pressure, in pounds, per square inch.	Ratio of expansion in nozzle.	Velocity of outflow at constant density, in feet, per second.	Actual velocity of outflow expanded, in feet, per second.	Discharge per square inch of orifice, in pounds, per minute.
75	74.00	1.012	227.5	230.0	16.68
75	72.00	1.037	386.7	401.0	28.35
75	70.00	1.063	490.0	521.0	35.93
75	65.00	1.136	660.0	749.0	48.38
75	61.62	1.198	736.0	876.0	53.97
75	60.00	1.219	765.0	933.0	56.12
75	50.00	1.434	873.0	1252.0	64 00
75	45.00	1.575	890.0	1401.0	65.24
75	43.46 (58%)	1.624	890.6	1446.5	65.3
75	15	1.624	890.6	1446.5	65.3
75	0	1.624	890.6	1446.5	65.3

Napier's rule, which is a close approximation, is that the absolute pressure, in pounds, per square inch, multiplied by the area in square inches, divided by 70, equals the discharge, in pounds, per second.

Brownlee's formula for the discharge of steam of varying pressures of the atmosphere is

$$v = 3.5953\sqrt{h},$$

in which $v =$ the velocity of outflow, in feet, per second as for steam of the initial density, and $h =$ the height, in feet, of a column of steam of the given absolute initial pressure of uniform density, the weight of which is equal to the pressure on the unit of base.

Example. Boiler pressure, 80 pounds per square inch above the atmosphere. With what velocity will steam flow out of an orifice in the shell,—for example, a safety valve?

Here the absolute pressure $= 80 + 14.7 = 94.7$ pounds per square inch. The volume of *one pound* of steam at this pressure $=$ 4.56 cubic feet; consequently, the height of a column of this steam 1 inch square, and weighing 94.7 pounds, will be

$$4.56 \times 144 \times 94.7 = 62183.81 \text{ feet} = h.$$

Then by the formula the velocity of outflow will be

$$v = 3.5953\sqrt{h} = 3.5953\sqrt{62183.81} = 3.5953 \times 249.37 = 896 \text{ feet per second.}$$

To find the amount of steam discharged from an orifice of any given size in a given time, we have merely to multiply the area of the orifice by the above velocity, and this product by the time in seconds, to obtain the volume of steam discharged, from which it is easy to calculate its weight by reference to a steam table.

Velocity of Efflux of Steam into the Atmosphere.

Pressure per gauge.	Velocity of discharge, in feet, per second.	Pounds of steam discharged, per minute, per square inch of opening.	Pressure per gauge.	Velocity of discharge, in feet, per second.	Pounds of steam discharged, per minute, per square inch of opening.
10	861	22.2	70	894	73.5
15	867	26.6	75	895	77.6
20	871	30.9	80	896	81.9
25	874	35.3	85	898	86.0
30	877	39.5	90	899	90.3
35	880	43.8	95	900	94.4
40	882	48.0	100	902	98.6
45	884	52.3	110	904	106.9
50	886	56.5	120	906	115.2
55	888	60.7	130	908	123.5
60	890	65.0	140	910	131.9
65	892	69.3	150	912	140.2

Flow of Steam in Pipes.

The quantity of steam flowing through a pipe under a given head increases directly as the square root of the density of the loss of pressure, and inversely as the square root of the length. A formula used for flow of steam in pipes is $V = 50\sqrt{\frac{H}{L}D}$, in which $V =$ velocity, in feet, per

second, L = length, and D = diameter of pipe, in feet, H = height, in feet, of a column of steam of the pressure of the steam at the entrance, which would produce a pressure equal to the difference of pressures at the two ends of the pipe.

If Q = quantity, in cubic feet, per minute, d = diameter, in inches, L and H being in feet, formula reduces to

$$Q = 4.7233\sqrt{\frac{H}{L}d^5},\ H = .0448\,\frac{Q^2L}{d^5},\ d = .5374\sqrt[5]{\frac{Q^2L}{H}}.$$

A pipe 1 inch in diameter, 100 feet long, carrying steam of 100 pounds gauge-pressure at 6000 feet velocity per minute, would have a loss of pressure of 8.8 pounds per square inch, while steam travelling at the same velocity in a pipe 8.8 inches in diameter would lose only 1 pound pressure.

The following generally-accepted formula gives the weight of steam which, with a given vertical pressure, will flow through a given pipe:

W = weight, in pounds avoirdupois;
D = density or weight per cubic foot;
d = diameter, in inches;
p_1 = initial pressure;
p_2 = pressure at end of pipe;
L = length, in feet.

$$W = 87\sqrt{\frac{D(p_1 - p_2)d^5}{L\left(1 + \frac{3.6}{d}\right)}}.$$

Flow of Steam through Pipes.

Initial pressure by gauge, in pounds, per square inch.	Diameter of pipe, in inches. Length of each = 240 diameters.													
	¾	1	1½	2	2½	3	4	5	6	8	10	12	15	18
	Weight of steam per minute, in pounds, with 1 pound loss of pressure.													
1	1.16	2.07	5.7	10.27	15.45	25.38	46.85	77.3	115.9	211.4	341.1	502.4	804	1177
10	1.44	2.57	7.1	12.72	19.15	31.45	58.05	95.8	143.6	262.0	422.7	622.5	996	1458
20	1.70	3.02	8.3	14.94	22.49	36.94	68.20	112.6	168.7	307.8	496.5	731.3	1170	1713
30	1.91	3.40	9.4	16.84	25.35	41.63	76.84	126.9	190.1	346.8	559.5	824.1	1318	1930
40	2.10	3.74	10.3	18.51	27.87	45.77	84.49	139.5	209.0	381.3	615.3	906.0	1450	2122
50	2.27	4.04	11.2	20.01	30.13	49.48	91.34	150.8	226.0	412.2	665.0	979.5	1567	2294
60	2.43	4.32	11.9	21.38	32.19	52.87	97.60	161.1	241.5	440.5	710.6	1046.7	1675	2451
70	2.57	4.58	12.6	22.65	34.10	56.00	103.37	170.7	255.8	466.5	752.7	1108.5	1774	2596
80	2.71	4.82	13.3	23.82	35.87	58.91	108.74	179.5	269.0	490.7	791.7	1166.1	1866	2731
90	2.83	5.04	13.9	24.92	37.52	61.62	113.74	187.8	281.4	513.3	828.1	1219.8	1951	2856
100	2.95	5.25	14.5	25.96	39.07	64.18	118.47	195.6	293.1	534.6	862.6	1270.1	2032	2975
120	3.16	5.63	15.5	27.85	41.93	68.87	127.12	209.9	314.5	573.7	925.6	1363.3	2181	3193
150	3.45	6.14	17.0	30.37	45.72	75.09	138.61	228.8	343.0	625.5	1009.2	1486.5	2378	3481

For any loss of pressure, multiply by the square root of the proposed loss.

For any other length of pipe, divide 240 by the given length expressed in diameters, and multiply the table figures by the square root of this quotient to get the flow for 1 pound loss of pressure.

The resistance due to steam entering pipe = 60 diameters additional length; to a globe valve = 60; to an elbow = 40, or ⅔ of a globe valve. All these equivalents must be added in getting out total length of pipe, with corresponding losses.

Moisture in Steam.

Various methods have been devised for determining the percentage of moisture in steam, but the principal difficulty involved in their use lies in the impossibility of obtaining an average sample of the steam.

Professor J. E. Denton has shown that the appearance of an escaping jet of steam will reveal to the eye the presence or absence of moisture up to about 2 per cent. of moisture. If the jet be transparent close to the orifice, the steam may be assumed to be so nearly dry that no portable condensing calorimeter will be capable of measuring the small amount of moisture present. If the jet be strongly white, the amount of water may be roughly judged up to about 2 per cent., but beyond this only a calorimeter can determine the amount of moisture present.

In the appendix to the report of the committee of the American Society of Mechanical Engineers on steam boiler trials, Mr. Kent says: "For scientific research and in all cases in which there is reason to suspect that the moisture may exceed 2 per cent., a steam separator should be placed in the steam pipe as near to the steam outlet of the boiler as convenient, well covered with felting, all the steam made by the boiler passing through it, and all the moisture caught by it carefully weighed after being cooled. A convenient method of obtaining the weight of the drip from the separator is to discharge it through a trap into a barrel of cold water standing on a platform scale. A throttling or a separating calorimeter should be placed in the steam pipe, just beyond the steam separator, for the purpose of determining, by the sampling method, the small percentage of moisture which may still be in the steam after passing through the separator."

The formula for calculating the percentage of moisture when the throttling calorimeter is used is the following:

$$w = 100 \times \frac{H - h - k(T - t)}{L},$$

in which w = percentage of moisture in the steam, H = total heat and L = latent heat per pound of steam at the pressure in the steam pipe, h = total heat per pound of steam at the pressure in the discharge side of the calorimeter, k = specific heat of superheated steam, T = temperature of the throttled and superheated steam in the calorimeter, and t = temperature due to the pressure in the discharge side of the calorimeter, = 212° F. at atmospheric pressure. Taking $k = 0.48$ and $t = 212$, the formula reduces to

$$w = 100 \times \frac{H - 1146.6 - 0.48(T - 212)}{L}.$$

For descriptions of the throttling calorimeter of Peabody, see "Transactions of the American Society of Mechanical Engineers," Vol. X., p. 327; for the Barrus calorimeter, Vol. XI., p. 790, and Vol. XVII., p. 617; and for the Carpenter calorimeter, Vol. XII., p. 640, and Vol. XVII., p. 608.

In treating of superheated steam it is customary to give the number of degrees of superheat,—that is, the excess of temperature over that due to the pressure, as shown in the steam tables. It is sometimes desirable to give the so-called "quality" of the steam, this being the percentage of excess heat.

The quality of the superheated steam is determined from the number of degrees of superheating by using the following formula:

$$Q = \frac{L + 0.48(T - t)}{L},$$

in which L is the latent heat, in British thermal units, in 1 pound of steam of the observed pressure; T, the observed temperature; and t, the normal

temperature due to the pressure. This normal temperature should be determined by obtaining a reading of the thermometer when the fires are in a dead condition and the superheat has disappeared, this temperature being observed when the pressure as shown by the gauge is the average of the readings taken during the trial.

STEAM BOILERS.

A steam boiler is essentially a device for the conversion of water from the liquid to the gaseous state by the means of heat. Its performance should therefore be based entirely upon thermal considerations: the conversion of the energy in the fuel into energy in the steam, regardless of the use to which the steam is to be put. To speak of the horse-power of a boiler is distinctly unscientific, and is to be as strongly discouraged as the expressions horse-power of a feed-water heater, of a condenser, of a chimney, or any similar device. The capacity of a boiler is fully indicated by a statement of the quantity of water it is capable of evaporating in a given time, and its economy by the proportion of combustible required to the quantity of water evaporated.

The fact that the number of pounds of water evaporated to equal a boiler horse-power has varied from time to time shows the unsuitability of the application of the term to a steam boiler. At the same time, the commercial requirements of the business demand some definition of a boiler horse-power, and at the present time the evaporation of 30 pounds of water from feed water at a temperature of 100° F., as established by the judges of the Centennial Exhibition of 1876, may be used. It is always desirable, however, that the capacity of a boiler should be stated in terms of the number of pounds of water it will evaporate, from and at the boiling-point.

According to the steam tables, it will be seen that 965.7 B. T. U. are required to convert a pound of water at 212° to a pound of steam at the same temperature. If we assume a pound of combustible in the fuel to be capable of supplying 14,500 B. T. U., a perfectly efficient steam boiler would be capable of evaporating

$$\frac{14500}{965.7} = 15.015 \text{ pounds}$$

of water for every pound of combustible burned. The actual efficiency of a boiler, therefore, is found by dividing the actual evaporation by 15.015. Thus, if a boiler evaporates 10 pounds of water per pound of combustible, its efficiency is

$$\frac{10}{15.015} = 0.66,$$

or 66 per cent.

In order to compute beforehand the proportions which will give the best efficiency, the formula of Rankine may be used.

Let

E = theoretical evaporative power of fuel used, pounds of water;
E' = actual evaporative power, pounds of water;
S = square feet of heating surface in boiler;
F = pounds of fuel burned per square foot of grate per hour;
$\left.\begin{array}{l} A = \text{a constant} \\ B = \text{a constant} \end{array}\right\}$ tabulated below.

Then we have

$$\text{Efficiency} = \frac{E'}{E} = \frac{BS}{S + AF},$$

or

$$E' = E\frac{BS}{S + AF}.$$

The value of E varies with the composition of the coal, and may be computed by Dulong's formula or determined by a calorimeter.

The constants A and B may be taken as follows:

I. Chimney draft, hottest gases meeting hottest water, economizer in flue, $B = 1$, $A = 0.5$.
II. Ordinary flow of gases, chimney draft, $B = 0.916$, $A = 0.5$.
III. Forced draft, hottest gases meeting hottest water, $B = 1$, $A = 0.3$.
IV. Forced draft, ordinary flow of gases, $B = 0.95$, $A = 0.3$.

From the above it will be seen that a high efficiency may be obtained by causing the gases to flow in such a manner as to bring the hottest portion into contact with that portion of the boiler containing the hottest water, the flow of water and gases being in the opposite direction; also, that a moderate rate of combustion is conducive to efficiency.

When the feed water is supplied to a boiler at a temperature of 212°, the only heat required to be supplied is that necessary to furnish the latent heat of evaporation and the heat to raise the steam to the working pressure. When, however, the feed water is not at the boiling-point, it is necessary to supply additional heat to raise it to 212° F. For this reason it is necessary to know the temperature of the feed water in order to correct the observed evaporation to the equivalent evaporation from and at 212°.

The factors for making this correction may be computed from the formula,

$$F = \frac{H - h}{965.7},$$

H being the total heat of the steam at the given pressure, and h being the total heat of the feed water.

The table on page 594 gives factors for various temperatures.

The evaporative performance of steam boilers is such an important matter, both from a commercial and technical point of view, that it is desirable for all tests to be conducted in such a manner as to be comparable. The standard method of testing steam boilers, according to the report of the Committee of the American Society of Mechanical Engineers, enables such uniform methods of testing possible, and an abridgement of this code is here given. The complete code will be found in Volume XXI. of the "Transactions" of the Society, and may be obtained in pamphlet form.

The Committee recommends that, as far as possible, the capacity of a boiler be expressed in terms of the "number of pounds of water evaporated per hour from and at 212 degrees." It does not seem expedient, however, to abandon the widely-recognized measure of capacity of stationary or land boilers expressed in terms of "boiler horse-power."

The unit of commercial boiler horse-power adopted by the Committee of 1885 was the same as that used in the reports of the boiler tests made at the Centennial Exhibition in 1876. The Committee of 1885 reported in favor of this standard in language of which the following is an extract:

"The Committee, after due consideration, has determined to accept the Centennial standard, and to recommend that in all standard trials the commercial horse-power be taken as an evaporation of 30 pounds of water per hour from a feed-water temperature of 100° F. into steam at 70 pounds gauge pressure, which shall be considered to be equal to 34½ units of evaporation,—that is, to 34½ pounds of water evaporated from a feed-water temperature of 212° F. into steam at the same temperature. This standard is equal to 33,305 thermal units per hour."

The present Committee accepts the same standard, but reverses the order of two clauses in the statement, and slightly modifies them to read as follows:

"The unit of commercial horse-power developed by a boiler shall be taken as 34½ units of evaporation per hour,—that is, 34½ pounds of water evaporated per hour from a feed-water temperature of 212° F. into dry steam of the same temperature. This standard is equivalent to 33,317 British thermal units per hour. It is also practically equivalent to an evaporation of 30 pounds of water from a feed-water temperature of 100° F. into steam at 70 pounds gauge pressure.

Factors of Equivalent Evaporation from and at 212° F.

Temp. feed.	Gauge pressure, in pounds, per square inch.																
	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
32	1.211	1.214	1.217	1.219	1.222	1.224	1.227	1.229	1.231	1.233	1.234	1.236	1.237	1.238	1.239	1.240	1.241
35	1.208	1.211	1.214	1.216	1.219	1.221	1.224	1.226	1.228	1.230	1.231	1.233	1.234	1.235	1.236	1.237	1.238
40	1.203	1.206	1.209	1.211	1.214	1.216	1.219	1.221	1.223	1.224	1.226	1.228	1.229	1.230	1.231	1.232	1.233
45	1.197	1.200	1.203	1.205	1.208	1.210	1.213	1.216	1.217	1.219	1.220	1.222	1.223	1.224	1.225	1.226	1.227
50	1.192	1.195	1.198	1.200	1.203	1.205	1.208	1.210	1.212	1.214	1.215	1.217	1.218	1.219	1.220	1.221	1.222
55	1.187	1.190	1.193	1.195	1.198	1.200	1.203	1.205	1.207	1.209	1.210	1.212	1.213	1.214	1.215	1.216	1.217
60	1.182	1.185	1.188	1.190	1.193	1.195	1.198	1.200	1.202	1.204	1.205	1.207	1.208	1.209	1.210	1.211	1.212
65	1.177	1.180	1.183	1.185	1.188	1.190	1.193	1.195	1.197	1.198	1.200	1.202	1.203	1.204	1.205	1.206	1.207
70	1.172	1.175	1.178	1.180	1.183	1.185	1.188	1.190	1.192	1.193	1.195	1.196	1.198	1.199	1.200	1.201	1.202
75	1.167	1.170	1.173	1.175	1.178	1.180	1.183	1.184	1.187	1.188	1.190	1.191	1.193	1.194	1.195	1.196	1.197
80	1.161	1.164	1.167	1.169	1.172	1.174	1.177	1.179	1.181	1.183	1.184	1.186	1.187	1.188	1.189	1.190	1.191
85	1.156	1.159	1.162	1.164	1.167	1.169	1.172	1.174	1.176	1.178	1.179	1.181	1.182	1.183	1.184	1.185	1.186
90	1.151	1.154	1.157	1 159	1.162	1.164	1.167	1.169	1.171	1.173	1.174	1.176	1.177	1.178	1.179	1.180	1.181
95	1.146	1.149	1.152	1.154	1.157	1.159	1.162	1.164	1.166	1.167	1.169	1.171	1.172	1.173	1.174	1.175	1.176
100	1.141	1.144	1.147	1.149	1.152	1.154	1.157	1.158	1.161	1.162	1.164	1.165	1.167	1.168	1.169	1.170	1.171
105	1.135	1.138	1.141	1 143	1.146	1.148	1.151	1.153	1.155	1.157	1.158	1.160	1.161	1.162	1.163	1.164	1.165
110	1.130	1.133	1.136	1.138	1.141	1.143	1.146	1.148	1.150	1.152	1.153	1.155	1.156	1.157	1.158	1.159	1.160
115	1.125	1.128	1.131	1.133	1.136	1.138	1.141	1.143	1.145	1.147	1.148	1.150	1.151	1.152	1.153	1.154	1.155
120	1.120	1.123	1.126	1.128	1.131	1.133	1.136	1.138	1.140	1.141	1.143	1.145	1.146	1.147	1.148	1.149	1 150
125	1.115	1.118	1 121	1.123	1.126	1 128	1.131	1.133	1.135	1.136	1.138	1.139	1.141	1.142	1.143	1.144	1.145
130	1.109	1.112	1.115	1.117	1.120	1.122	1.125	1.127	1.129	1.131	1.132	1.134	1.135	1.136	1.137	1.138	1.139
135	1.104	1.107	1.110	1 112	1.115	1.117	1.120	1.122	1.124	1.126	1.127	1.129	1.130	1.131	1.132	1.133	1.134
140	1.099	1.102	1.105	1.107	1.110	1.112	1.115	1.117	1.119	1.121	1.122	1.124	1.125	1.126	1.127	1.128	1.129
145	1.094	1.097	1.100	1.102	1.105	1.107	1.110	1.112	1.114	1.115	1.117	1.119	1.120	1.121	1.122	1.123	1.124
150	1.089	1.092	1.095	1.097	1.100	1.102	1.105	1.107	1.109	1.110	1.112	1.113	1.115	1.116	1.117	1.118	1.119
160	1 078	1.081	1.084	1.086	1.089	1.091	1.094	1.096	1.098	1.100	1.101	1.103	1.104	1.105	1.106	1.107	1.108
170	1.068	1.071	1.074	1.076	1.079	1.081	1.084	1.086	1.088	1.089	1.091	1.092	1.094	1.095	1.096	1.097	1.098
180	1.057	1.060	1.063	1.065	1.068	1.070	1.073	1.075	1.077	1.079	1.080	1.082	1.083	1.084	1.085	1.086	1.087
190	1.047	1.050	1.053	1.055	1.058	1.060	1.063	1.065	1.067	1.068	1.070	1.072	1.073	1.074	1.075	1.076	1.077
200	1.037	1.040	1.043	1.045	1.048	1.050	1.053	1.055	1.057	1.058	1.060	1.061	1.063	1.064	1.065	1.066	1.067
210	1.026	1.029	1.032	1.035	1.037	1.040	1.042	1.044	1.046	1.047	1.049	1.051	1.052	1.054	1.056	1.057	1.058
212	1.024	1 027	1.030	1.033	1.035	1.038	1.040	1.042	1.044	1.045	1.047	1.049	1.050	1.052	1.054	1.055	1.056

Rules for Conducting Boiler Trials.

Code of 1899.

I. **Determine at the outset** the specific object of the proposed trial, whether it be to ascertain the capacity of the boiler, its efficiency as a steam generator, its efficiency and its defects under usual working conditions, the economy of some particular kind of fuel, or the effect of changes of design, proportion, or operation; and prepare for the trial accordingly.

II. **Examine the boiler,** both outside and inside; ascertain the dimensions of grates, heating surfaces, and all important parts; and make a full record, describing the same, and illustrating special features by sketches. The area of heating surface is to be computed from the surfaces of shells, tubes, furnaces, and fire-boxes in contact with the fire or hot gases. The outside diameter of water-tubes and the inside diameter of fire-tubes are to be used in the computation. All surfaces below the mean water-level which have water on one side and products of combustion on the other are to be considered as water-heating surface, and all surfaces above the mean water-level which have steam on one side and products of combustion on the other are to be considered as superheating surface.

III. **Notice the general condition** of the boiler and its equipment, and record such facts in relation thereto as bear upon the objects in view.

If the object of the trial is to ascertain the maximum economy or capacity of the boiler as a steam generator, the boiler and all its appurtenances should be put in first-class condition. Clean the heating surface inside and outside, remove clinkers from the grates and from the sides of the furnace. Remove all dust, soot, and ashes from the chambers, smoke connections, and flues. Close air-leaks in the masonry and poorly-fitted cleaning doors. See that the damper will open wide and close tight. Test for air-leaks by firing a few shovels of smoky fuel and immediately closing the damper, observing the escape of smoke through the crevices, or by passing the flame of a candle over cracks in the brickwork.

IV. **Determine the character of the coal** to be used. For tests of the efficiency or capacity of the boiler for comparison with other boilers the coal should, if possible, be of some kind which is commercially regarded as a standard. For New England and that portion of the country east of the Allegheny Mountains, good anthracite egg coal, containing not over 10 per cent. of ash, and semi-bituminous Clearfield (Pennsylvania), Cumberland (Maryland), and Pocahontas (Virginia) coals are thus regarded. West of the Allegheny Mountains, Pocahontas (Virginia) and New River (West Virginia) semi-bituminous and Youghiogheny or Pittsburg bituminous coals are recognized as standards.* There is no special grade of coal mined in the Western States which is widely recognized as of superior quality or considered as a standard coal for boiler testing. Big Muddy lump, an Illinois coal mined in Jackson County, Illinois, is suggested as being of sufficiently high grade to answer these requirements in districts where it is more conveniently obtainable than the other coals mentioned above.

For tests made to determine the performance of a boiler with a particular kind of coal, such as may be specified in a contract for the sale of a boiler, the coal used should not be higher in ash and in moisture than that specified, since increase in ash and moisture above a stated amount is apt to cause a falling off of both capacity and economy in greater proportion than the proportion of such increase.

V. **Establish the correctness of all apparatus** used in the test for weighing and measuring. These are:

1. Scales for weighing coal, ashes, and water.
2. Tanks or water-meters for measuring water. Water-meters, as a rule,

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* These coals are selected because they are about the only coals which possess the essentials of excellence of quality, adaptability to various kinds of furnaces, grates, boilers, and methods of firing, and wide distribution and general accessibility in the markets.

should only be used as a check on other measurements. For accurate work, the water should be weighed or measured in a tank.

3. Thermometers and pyrometers for taking temperatures of air, steam, feed water, waste gases, etc.

4. Pressure gauges, draught gauges, etc.

The kind and location of the various pieces of testing apparatus must be left to the judgment of the person conducting the test, always keeping in mind the main object,—*i.e.*, to obtain authentic data.

VI. **See that the boiler is thoroughly heated** before the trial to its usual working temperature. If the boiler is new and of a form provided with a brick setting, it should be in regular use at least a week before the trial, so as to dry and heat the walls. If it has been laid off and become cold, it should be worked before the trial until the walls are well heated.

VII. **The boiler and connections** should be proved to be free from leaks before beginning a test, and all water connections, including blow and extra feed pipes, should be disconnected, stopped with blank flanges, or bled through special openings beyond the valves, except the particular pipe through which water is to be fed to the boiler during the trial. During the test the blow-off and feed pipes should remain exposed to view.

If an injector is used, it should receive steam directly through a felted pipe from the boiler being tested.*

If the water is metered after it passes the injector, its temperature should be taken at the point where it leaves the injector. If the quantity is determined before it goes to the injector, the temperature should be determined on the suction side of the injector, and if no change of temperature occurs other than that due to the injector, the temperature thus determined is properly that of the feed water. When the temperature changes between the injector and the boiler, as by the use of a heater or by radiation, the temperature at which the water enters and leaves the injector and that at which it enters the boiler should all be taken. In that case the weight to be used is that of the water leaving the injector, computed from the heat units if not directly measured, and the temperature, that of the water entering the boiler.

Let

w = weight of water entering the injector;
x = weight of steam entering the injector;
h_1 = heat units per pound of water entering injector:
h_2 = heat units per pound of steam entering injector;
h_3 = heat units per pound of water leaving injector.

Then $w + x$ = weight of water leaving injector,

$$x = w\frac{h_3 - h_1}{h_2 - h_3}.$$

See that the steam main is so arranged that water of condensation cannot run back into the boiler.

VIII. **Duration of the Test.**—For tests made to ascertain either the maximum economy or the maximum capacity of a boiler, irrespective of the particular class of service for which it is regularly used, the duration should be at least 10 hours of continuous running. If the rate of combustion exceeds 25 pounds of coal per square foot of grate surface per hour, it may be stopped when a total of 250 pounds of coal has been burned per square foot of grate.

* In feeding a boiler undergoing test with an injector taking steam from another boiler, or from the main steam pipe from several boilers, the evaporative results may be modified by a difference in the quality of the steam from such source compared with that supplied by the boiler being tested, and in some cases the connection to the injector may act as a drip for the main steam pipe. If it is known that the steam from the main pipe is of the same pressure and quality as that furnished by the boiler undergoing the test, the steam may be taken from such main pipe.

In cases where the service requires continuous running for the whole 24 hours of the day, with shifts of fireman a number of times during that period, it is well to continue the test for at least 24 hours.

When it is desired to ascertain the performance under the working conditions of practical running, whether the boiler be regularly in use 24 hours a day or only a certain number of hours out of each 24, the fires being banked the balance of the time, the duration should not be less than 24 hours.

IX. **Starting and Stopping a Test.**—The conditions of the boiler and furnace in all respects should be, as nearly as possible, the same at the end as at the beginning of the test. The steam pressure should be the same, the water-level the same, the fire upon the grates should be the same in quantity and condition, and the walls, flues, etc., should be of the same temperature. Two methods of obtaining the desired equality of conditions of the fire may be used,—viz., those which were called in the Code of 1885 "the standard method" and "the alternate method," the latter being employed where it is inconvenient to make use of the standard method.*

X. **Standard Method of Starting and Stopping a Test.**—Steam being raised to the working pressure, remove rapidly all the fire from the grate, close the damper, clean the ash-pit, and as quickly as possible start a new fire with weighed wood and coal, noting the time and the water-level,† while the water is in a quiescent state, just before lighting the fire.

At the end of the test remove the whole fire, which has been burned low, clean the grates and ash-pit, and note the water-level when the water is in a quiescent state, and record the time of hauling the fire. The water-level should be as nearly as possible the same as at the beginning of the test. If it is not the same, a correction should be made by computation, and not by operating the pump after the test is completed.

XI. **Alternate Method of Starting and Stopping a Test.**—The boiler being thoroughly heated by a preliminary run, the fires are to be burned low and well cleaned. Note the amount of coal left on the grate as nearly as it can be estimated; note the pressure of steam and the water-level; note the time, and record it as the starting time. Fresh coal, which has been weighed, should now be fired. The ash-pits should be thoroughly cleaned at once after starting. Before the end of the test the fires should be burned low, just as before the start, and the fires cleaned in such a manner as to leave a bed of coal on the grates of the same depth, and in the same condition, as at the start. When this stage is reached, note the time, and record it as the stopping time. The water-level and steam pressures should previously be brought as nearly as possible to the same point as at the start. If the water-level is not the same as at the start, a correction should be made by computation, and not by operating the pump after the test is completed.

XII. **Uniformity of Conditions.**—In all trials made to ascertain maximum economy or capacity the conditions should be maintained uniformly constant. Arrangements should be made to dispose of the steam so that the rate of evaporation may be kept the same from beginning to end. This may be accomplished in a single boiler by carrying the steam through a waste steam pipe, the discharge from which can be regulated as desired. In a battery of boilers, in which only one is tested, the draught may be regulated on the remaining boilers, leaving the test boiler to work under a constant rate of production.

Uniformity of conditions should prevail as to the pressure of steam,

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* The Committee concludes that it is best to retain the designations "standard" and "alternate," since they have become widely known and established in the minds of engineers and in the reprints of the Code of 1885. Many engineers prefer the "alternate" to the "standard" method, on account of its being less liable to error due to cooling of the boiler at the beginning and end of a test.

† The gauge glass should not be blown out within an hour before the water-level is taken at the beginning and end of a test, otherwise an error in the reading of the water-level may be caused by a change in the temperature and density of the water in the pipe leading from the bottom of the glass into the boiler.

the height of water, the rate of evaporation, the thickness of fire, the times of firing and quantity of coal fired at one time, and as to the intervals between the times of cleaning the fires.

The method of firing to be carried on in such tests should be dictated by the expert or person in responsible charge of the test, and the method adopted should be adhered to by the fireman throughout the test.

XIII. **Keeping the Records.**—Take note of every event connected with the progress of the trial, however unimportant it may appear. Record the time of every occurrence and the time of taking every weight and every observation.

The coal should be weighed and delivered to the fireman in equal proportions, each sufficient for not more than one hour's run, and a fresh portion should not be delivered until the previous one has all been fired. The time required to consume each portion should be noted, the time being recorded at the instant of firing the last of each portion. It is desirable that at the same time the amount of water fed into the boiler should be accurately noted and recorded, including the height of the water in the boiler and the average pressure of steam and temperature of feed during the time. By thus recording the amount of water evaporated by successive portions of coal, the test may be divided into several periods, if desired, and the degree of uniformity of combustion, evaporation, and economy analyzed for each period. In addition to these records of the coal and the feed water, half-hourly observations should be made of the temperature of the feed water, of the flue gases, of the external air in the boiler-room, of the temperature of the furnace when a furnace pyrometer is used; also, of the pressure of steam and of the readings of the instruments for determining the moisture in the steam. A log should be kept on properly-prepared blanks containing columns for record of the various observations.

When the "standard method" of starting and stopping the test is used the hourly rate of combustion and of evaporation and the horse-power should be computed from the records taken during the time when the fires are in active condition. This time is somewhat less than the actual time which elapses between the beginning and end of the run. The loss of time due to kindling the fire at the beginning and burning it out at the end makes this course necessary.

XIV. **Quality of Steam.**—The percentage of moisture in the steam should be determined by the use of either a throttling or a separating steam calorimeter The sampling nozzle should be placed in the vertical steam pipe rising from the boiler. It should be made of ½-inch pipe, and should extend across the diameter of the steam pipe to within half an inch of the opposite side, being closed at the end and perforated with not less than twenty ⅛-inch holes equally distributed along and around its cylindrical surface, but none of these holes should be nearer than ½ inch to the inner side of the steam pipe. The calorimeter and the pipe leading to it should be well covered with felting. Whenever the indications of the throttling or separating calorimeter show that the percentage of moisture is irregular, or occasionally in excess of 3 per cent., the results should be checked by a steam separator placed in the steam pipe as close to the boiler as convenient, with a calorimeter in the steam pipe just beyond the outlet from the separator. The drip from the separator should be caught and weighed, and the percentage of moisture computed therefrom added to that shown by the calorimeter.

Superheating should be determined by means of a thermometer placed in a mercury-well inserted in the steam pipe. The degree of superheating should be taken as the difference between the reading of the thermometer for superheated steam and the readings of the same thermometer for saturated steam at the same pressure, as determined by a special experiment, and not by reference to steam tables.

XV. **Sampling the Coal and Determining its Moisture.**—As each barrow-load or fresh portion of coal is taken from the coal-pile a representative shovelful is selected from it and placed in a barrel or box in a cool place and kept until the end of the trial. The samples are then mixed and broken into pieces not exceeding 1 inch in diameter, and reduced by the process of repeated quartering and crushing until a final sample weighing about 5 pounds is obtained and the size of the larger pieces is such that they will pass through a sieve with ¼-inch meshes. From this

sample two 1-quart, air-tight glass preserving jars, or other air-tight vessels which will prevent the escape of moisture from the sample, are to be promptly filled, and these samples are to be kept for subsequent determinations of moisture and of heating value and for chemical analyses. During the process of quartering, when the sample has been reduced to about 100 pounds, a quarter to a half of it may be taken for an approximate determination of moisture. This may be made by placing it in a shallow iron pan not over 3 inches deep, carefully weighing it, and setting the pan in the hottest place that can be found on the brickwork of the boiler setting or flues, keeping it there for at least 12 hours, and then weighing it. The determination of moisture thus made is believed to be approximately accurate for anthracite and semi-bituminous coals, and also for Pittsburg or Youghiogheny coal; but it cannot be relied upon for coals mined west of Pittsburg, or for other coals containing inherent moisture. For these latter coals it is important that a more accurate method be adopted. The method recommended by the Committee for all accurate tests, whatever the character of the coal, is described as follows:

Take one of the samples contained in the glass jars and subject it to a thorough air-drying by spreading it in a thin layer and exposing it for several hours to the atmosphere of a warm room, weighing it before and after, thereby determining the quantity of surface moisture it contains. Then crush the whole of it by running it through an ordinary coffee-mill, adjusted so as to produce somewhat coarse grains (less than $\frac{1}{16}$ inch), thoroughly mix the crushed sample, select from it a portion of from 10 to 50 grams, weigh it in a balance which will easily show a variation as small as 1 part in 1000, and dry it in an air- or sand-bath at a temperature between 240° and 280° F. for one hour. Weigh it and record the loss, then heat and weigh it again repeatedly, at intervals of an hour or less, until the minimum weight has been reached and the weight begins to increase by oxidation of a portion of the coal. The difference between the original and the minimum weight is taken as the moisture in the air-dried coal. This moisture test should preferably be made on duplicate samples, and the results should agree within 0.3 to 0.4 of one per cent., the mean of the two determinations being taken as the correct result. The sum of the percentage of moisture thus found and the percentage of surface moisture previously determined is the total moisture.

XVI. **Treatment of Ashes and Refuse.**—The ashes and refuse are to be weighed in a dry state. If it is found desirable to show the principal characteristics of the ash, a sample should be subjected to a proximate analysis and the actual amount of incombustible material determined. For elaborate trials a complete analysis of the ash and refuse should be made.

XVII. **Calorific Tests and Analysis of Coal.**—The quality of the fuel should be determined either by heat test or by analysis, or by both.

The rational method of determining the total heat of combustion is to burn the sample of coal in an atmosphere of oxygen gas, the coal to be sampled as directed in Article XV. of this code.

The chemical analysis of the coal should be made only by an expert chemist. The total heat of combustion computed from the results of the ultimate analysis may be obtained by the use of Dulong's formula (with constants modified by recent determinations),—viz., $14600C + 62000\left(H - \frac{O}{8}\right) + 4000S$, in which C, H, O, and S refer to the proportions of carbon, hydrogen, oxygen, and sulphur, respectively, as determined by the ultimate analysis.*

It is desirable that a proximate analysis should be made, thereby determining the relative proportions of volatile matter and fixed carbon. These proportions furnish an indication of the leading characteristics of the fuel, and serve to fix the class to which it belongs. As an additional indication of the characteristics of the fuel the specific gravity should be determined.

* Favre and Silberman give 14,544 B. T. U. per pound carbon ; Berthelot, 14,647 B. T. U. Favre and Silberman give 62,032 B. T. U. per pound hydrogen ; Thomsen, 61,816 B. T. U.

XVIII. **Analysis of Flue Gases.**—The analysis of the flue gases is an especially valuable method of determining the relative value of different methods of firing or of different kinds of furnaces. In making these analyses great care should be taken to procure average samples, since the composition is apt to vary at different points of the flue. The composition is also apt to vary from minute to minute, and for this reason the drawings of gas should last a considerable period of time. Where complete determinations are desired, the analyses should be intrusted to an expert chemist. For approximate determinations the Orsat* or the Hempel† apparatus may be used by the engineer.

For the continuous indication of the amount of carbonic acid present in the flue gases an instrument may be employed which shows the weight of the sample of gas passing through it.

XIX. **Smoke Observations.**—It is desirable to have a uniform system of determining and recording the quantity of smoke produced where bituminous coal is used. The system commonly employed is to express the degree of smokiness by means of percentages dependent upon the judgment of the observer. The Committee does not place much value upon a percentage method, because it depends so largely upon the personal element, but if this method is used it is desirable that, so far as possible, a definition be given in explicit terms as to the basis and method employed in arriving at the percentage. The actual measurement of a sample of soot and smoke by some form of meter is to be preferred.

XX. **Miscellaneous.**—In tests for purposes of scientific research, in which the determination of all the variables entering into the test is desired, certain observations should be made which are in general unnecessary for ordinary tests. These are the measurement of the air-supply, the determination of its contained moisture, the determination of the amount of heat lost by radiation, of the amount of infiltration of air through the setting, and (by condensation of all the steam made by the boiler) of the total heat imparted to the water.

As these determinations are rarely undertaken, it is not deemed advisable to give directions for making them.

XXI. **Calculations of Efficiency.**—Two methods of defining and calculating the efficiency of a boiler are recommended. They are,—

$$1.\ \text{Efficiency of the boiler} = \frac{\text{Heat absorbed per pound of combustible}}{\text{Calorific value of 1 pound of combustible}};$$

$$2.\ \text{Efficiency of the boiler and grate} = \frac{\text{Heat absorbed per pound of coal}}{\text{Calorific value of 1 pound of coal}}.$$

The first of these is sometimes called the efficiency based on combustible, and the second the efficiency based on coal. The first is recommended as a standard of comparison for all tests, and this is the one which is understood to be referred to when the word "efficiency" alone is used without qualification. The second, however, should be included in a report of a test, together with the first, whenever the object of the test is to determine the efficiency of the boiler and furnace together with the grate (or mechanical stoker), or to compare different furnaces, grates, fuels, or methods of firing.

The heat absorbed per pound of combustible (or per pound of coal) is to be calculated by multiplying the equivalent evaporation from and at 212° per pound of combustible (or of coal) by 965.7.

XXII. **The Heat Balance.**—An approximate "heat balance," or statement of the distribution of the heating value of the coal among the several items of heat utilized and heat lost, may be included in the report of a test when analyses of the fuel and of the chimney gases have been made. It should be reported in the following form :

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* See R. S. Hale's paper on "Flue Gas Analysis," "Transactions of the American Society of Mechanical Engineers," Vol. XVIII., p. 109.

† See Hempel's "Methods of Gas Analysis" (Macmillan & Co.).

Heat Balance, or Distribution of the Heating Value of the Combustible.

Total heat value of 1 pound of combustible................ B.T.U.

	B.T.U.	Per cent.
1. Heat absorbed by the boiler = evaporation from and at 212° per pound of combustible × 965.7.		
2. Loss due to moisture in coal = per cent. of moisture referred to combustible ÷ 100 × $[(212-t)+966+0.48(T-212)]$ (t = temperature of air in the boiler-room, T = that of the flue gases).		
3. Loss due to moisture formed by the burning of hydrogen = per cent. of hydrogen to combustible ÷ 100 × 9 × $[(212-t)+966+0.48(T-212)]$.		
4.* Loss due to heat carried away in the dry chimney gases = weight of gas per pound of combustible × $0.24 \times (T-t)$.		
5.† Loss due to incomplete combustion of carbon = $\frac{CO}{CO_2+CO} \times \frac{\text{per cent. C in combustible}}{100} \times 10150$.		
6. Loss due to unconsumed hydrogen and hydrocarbons, to heating the moisture in the air, to radiation, and unaccounted for. (Some of these losses may be separately itemized if data are obtained from which they may be calculated.)		
Totals..		100

XXIII. **Report of the Trial.**—The data and results should be reported in the manner given in either one of the two following tables, omitting lines where the tests have not been made as elaborately as provided for in such tables. Additional lines may be added for data relating to the specific object of the test. The extra lines should be classified under the headings provided in the tables, and numbered as per preceding line, with sub-letters *a*, *b*, etc. The short form of report, Table No. 2, is recommended for commercial tests and as a convenient form of abridging the longer form for publication when saving of space is desirable. For elaborate trials it is recommended that the full log of the trial be shown graphically, by means of a chart.

* The weight of gas per pound of carbon burned may be calculated from the gas analyses, as follows:

Dry gas per pound carbon $= \frac{11CO_2 + 8O + 7(CO + N)}{3(CO_2 + CO)}$, in which CO_2, CO, O, and N are the percentages by volume of the several gases. As the sampling and analyses of the gases in the present state of the art are liable to considerable errors, the result of this calculation is usually only an approximate one. The heat balance itself is also only approximate for this reason, as well as for the fact that it is not possible to determine accurately the percentage of unburned hydrogen or hydrocarbons in the flue gases.

The weight of dry gas per pound of combustible is found by multiplying the dry gas per pound of carbon by the percentage of carbon in the combustible, and dividing by 100.

† CO_2 and CO are respectively the percentage by volume of carbonic acid and carbonic oxide in the flue gases. The quantity 10150 = number of heat units generated by burning to carbonic acid 1 pound of carbon contained in carbonic oxide.

TABLE No. 2.

Data and Results of Evaporative Test.

Arranged in accordance with the Short Form advised by the Boiler Test Committee of the American Society of Mechanical Engineers. Code of 1899.

Made by........................on............boiler, at................to determine ....
Kind of fuel.................. ....
Kind of furnace.......................................
Method of starting and stopping the test ("standard" or "alternate," Art. X. and XI., Code) ....
Grate surface... sq. ft.
Water-heating surface.................................... sq. ft.
Superheating surface...................................... sq. ft.

Total Quantities.

1. Date of trial ....
2. Duration of trial.. hours.
3. Weight of coal as fired .... lbs.
4. Percentage of moisture in coal .... per cent.
5. Total weight of dry coal consumed.................. lbs.
6. Total ash and refuse .... lbs.
7. Percentage of ash and refuse in dry coal............. per cent.
8. Total weight of water fed to the boiler............... lbs.
9. Water actually evaporated, corrected for moisture or superheat in steam.................................. lbs.
10. Equivalent water evaporated into dry steam from and at 212°.. lbs.

Hourly Quantities.

11. Dry coal consumed per hour.......................... lbs.
12. Dry coal per square foot of grate surface per hour .... lbs.
13. Water evaporated per hour, corrected for quality of steam.. lbs.
14. Equivalent evaporation per hour from and at 212°.... lbs.
15. Equivalent evaporation per hour from and at 212° per square foot of water-heating surface............... lbs.

Average Pressures, Temperatures, etc.

16. Steam pressure by gauge............................... lbs. per sq. in.
17. Temperature of feed water entering boiler............ deg.
18. Temperature of escaping gases from boiler............ deg.
19. Force of draft between damper and boiler............ ins. of water.
20. Percentage of moisture in steam, or number of degrees of superheating .... per cent. or deg.

Horse-power.

21. Horse-power developed (Item 14 ÷ 34½) .... H. P.
22. Builders' rated horse-power.......................... H. P.
23. Percentage of builders' rated horse-power developed . per cent.

Economic Results.

24. Water apparently evaporated under actual conditions per pound of coal as fired (Item 8 ÷ Item 3)....... lbs.
25. Equivalent evaporation from and at 212° per pound of coal as fired (Item 10 ÷ Item 3)................... lbs.
26. Equivalent evaporation from and at 212° per pound of dry coal (Item 10 ÷ Item 5)........................ lbs.
27. Equivalent evaporation from and at 212° per pound of combustible [Item 10 ÷ (Item 5 — Item 6)]........ lbs.

(If Items 25, 26, and 27 are not corrected for quality of steam, the fact should be stated.)

Efficiency.

28. Calorific value of the dry coal per pound.............. B. T. U.
29. Calorific value of the combustible per pound .... B. T. U.
30. Efficiency of boiler (based on combustible)........... per cent.
31. Efficiency of boiler, including grate (based on dry coal) per cent.

Cost of Evaporation.

32. Cost of coal per ton of —— pounds, delivered in boiler-room.. $
33. Cost of coal required for evaporating 1000 pounds of water from and at 212° .... $

Although the capacity of a boiler should be specified by the quantity of water it is capable of evaporating per hour, it is often necessary to state the dimensions and proportions to be furnished. Certain general relations of heating and grate surface have come to be recognized as representing evaporative capacity, and although this practice is to be discouraged, it cannot be ignored.

The area of heating surface allowed for 1 horse-power, or the evaporation of 30 pounds of water from 100° F. per hour, is usually about 12 square feet for return tubular or water-tube boilers, with about ⅓ of a square foot of grate surface per horse-power. The proportion of heating surface varies, however, for various kinds of boilers, and the following formula may be used for the determination of the heating surface in designing boilers:

Let

S = heating surface, in square feet;
Q = quantity of water evaporated per hour;
t = total heat of steam at the working pressure in the boiler;
C = constant, as per table below.

Then

$$S = C\frac{Q}{t}.$$

Values of constant C:

Locomotive boilers	$C = 90$
Marine Scotch boilers	$C = 180$
Cornish boilers	$C = 220$
Plain cylinder boilers	$C = 280$
Return tubular boilers	$C = 400$
Water-tube boilers....................................	$C = 400$

Thus, for a return tubular boiler to evaporate 5000 pounds of water per hour into steam at 160 pounds pressure, we have $Q = 5000$; t, by steam table, $= 1195.4$; $C = 400$; hence,

$$S = 400\frac{5000}{1195.4} = 1677 \text{ square feet.}$$

Since 5000 pounds of water, at 30 pounds to the horse-power, is 166 horse-power, this corresponds to about 10 square feet per horse-power.

In estimating the heating surface, all parts of a boiler are not equally efficient. Rankine says that, on an average, from ¾ to ⅚ of the total heating surface may be taken as effective heating surface. In computing the heating surface of tubes the side next to the heated gases should be taken.

The relative value of different forms of heating surface, compared with flat horizontal surface above the fire, is as follows:

1 square foot of flat horizontal surface above the fire, such as the crown-plate of the fire-box of the boiler of a locomotive engine	1.00
1 square foot of circular surface above and concave to the fire, such as the crown-plates of the circular furnace of an internally-fired boiler	.95

1 square foot of circular surface above and convex to the fire, such as the surface plates of an externally-fired plain cylindrical boiler	.90
1 square foot of flat surface at right angles to the current of gases, exposed to direct impingement of flame, such as the fire-box tube-plate of a locomotive boiler	.80
1 square foot of water-tube surface at right angles to the current of hot gases	.70
1 square foot of sloping surface at the side of and inclined towards the fire, such as the sides of a fire-box when inclined sufficiently to facilitate evaporation	.65
1 square foot of vertical surface at the side of the fire, such as the sides of a fire-box when vertical	.50
1 square foot of the surface of the tubes of a locomotive boiler, contained in a length not exceeding 3 feet from the fire-box tube-plate	.30

Horizontal surfaces below the fire and the under portions of internally-heated tubes have practically no evaporative value, and cannot be considered as effective heating surface, therefore the lower half of a furnace tube below the grate bars should not be included in calculating the heating surface of a steam boiler.

The draught area through the tubes of a boiler should be proportional to the area of the grate, and also depends upon the intensity of the draught. For natural chimney draught the area is usually made about 0.2 of the grate area ; it may reach 0.25, or fall as low as 0.125, but these are extremes.

The ratio of grate surface to heating surface varies according to the type of boilers. Accepted proportions are as follows :

Ratio of Grate to Heating Surface.

Type of boiler.	Ratio.
Scotch marine boiler	25 to 38
Lancashire	26 to 33
Cornish	25 to 40
Horizontal return tubular	30 to 50
Water tube	35 to 65
Locomotive	60 to 90
Plain cylinder	10 to 15

The quantity of water evaporated for a given combustion of fuel, when the proportions of heating and grate surface are given, may be determined by the formulas of D. K. Clark.

Let

w = weight of water, in pounds, per square foot of grate per hour ;
c = pounds of fuel per square foot of grate per hour ;
r = ratio of heating to grate surface.

Then we have for

Stationary boilers	$w = 0.0222r^2 + 9.56c$
Marine boilers	$w = 0.016r^2 + 10.25c$
Portable engine boilers	$w = 0.008r^2 + 8.6c$
Locomotive boilers	$w = 0.009r^2 + 9.7c$

CHIMNEYS.

The proportions of chimneys to furnish proper draught for steam boilers depend upon so many variables that it is impracticable to give absolute rational formulas, and hence empirical rules are used.

It is generally assumed that the area should bear a direct proportion to the quantity of fuel burned, and an inverse proportion to the square root of the height. The force of draught, however, has not only to draw the air in to maintain combustion, but must enable it to overcome the resistance of the fuel bed upon the grate, and this is always an indeterminate resistance. Moreover, the force of the draught depends upon the temperature of the discharge gases; but this latter should not be too high, or heat will be lost which should have been absorbed by the boiler. The entire subject will be found very fully discussed in the "Transactions of the American Society of Mechanical Engineers," Vol. XI., pp. 451, 974, and 984.

A common, ready rule for chimney area is to make it equal to one-tenth of the area of the grate. Mr. A. F. Nagle gives the rule to allow 2 square inches of chimney area for every pound of coal burned per hour.

The following formulas are given by their respective authors, as based upon the results of experience, taking into account the investigations of Péclet, Rankine, and others.

Let

A = area of chimney, in square feet;
h = height, in feet;
F = total number of pounds of coal burned per hour;
t = temperature of discharge gases;
G = grate area, in square feet.

Then

$$\left.\begin{aligned} A &= \frac{0.0825F}{\sqrt{h}}, \\ h &= \left(\frac{0.0825F}{A}\right)^2, \end{aligned}\right\} \text{Smith;}$$

or

$$\left.\begin{aligned} A &= \frac{0.06F}{\sqrt{h}}, \\ h &= \left(\frac{0.06F}{A}\right)^2, \end{aligned}\right\} \text{Kent;}$$

or

$$\left.\begin{aligned} A &= 0.07F^{\frac{2}{3}}, \\ h &= \frac{180}{t}\left(\frac{F}{G}\right)^2, \end{aligned}\right\} \text{Gale.}$$

The last formulas, it will be observed, do not make the height and area interchangeable, and for that reason they are to be preferred. Colonel E. D. Meier suggests the use of Gale's formula for heights, so modified as to read:

$$h = \frac{120}{t}\left(\frac{F}{G}\right)^2,$$

after which any other formula, such as Kent's, may be used to find the area. The following table has been computed by Colonel Meier for heights and areas of boiler chimneys, based on an assumed evaporation of 7 pounds of water per pound of coal, which is equivalent to the combustion of 5 pounds of coal per horse-power per hour. If the coal burned per hour is given, divide by 5, and take the chimney dimensions for the corresponding horse-power.

Table of Chimney Dimensions.

Area, in square feet.	Diameter, in inches.	Heights, in feet.												
		75	80	85	90	95	100	110	120	130	140	150	175	200
		Commercial horse-power.												
3.14	24	75	78	81	....	....	....	....	....	....	....	....	....	....
3.69	26	90	92	95	98	....	....	....	....	....	....	....	....	....
4.28	28	....	106	110	114	117	120	....	....	....	....	....	....	....
4.91	30	....	122	127	130	133	137	....	....	....	....	....	....	....
5.59	32	....	....	144	149	152	156	164	....	....	....	....	....	....
6.31	34	....	....	162	168	171	176	185	....	....	....	....	....	....
7.07	36	....	....	....	188	192	198	208	215	....	....	....	....	....
8.73	40	....	....	....	....	237	244	257	267	279	....	....	....	....
10.56	44	....	....	....	....	287	296	310	322	337	....	....	....	....
12.57	48	....	....	....	....	....	352	370	384	400	413	....	....	....
15.90	54	....	....	....	....	....	445	468	484	507	526	....	....	....
19.63	60	....	....	....	....	....	....	577	600	627	650	672	....	....
23.76	66	....	....	....	....	....	....	697	725	758	784	815	....	....
28.27	72	....	....	....	....	....	....	....	862	902	932	969	1044	....
38.48	84	....	....	....	....	....	....	....	1173	1229	1270	1319	1422	....
50.27	96	....	....	....	....	....	....	....	....	1584	1660	1725	1859	1983
63.62	108	....	....	....	....	....	....	....	....	2058	2102	2181	2352	2511
78.54	120	....	....	....	....	....	....	....	....	....	2596	2693	2904	3100

The following formulas for chimney dimensions, for use in the metric system, are given in the "Ingenieurs Taschenbuch:"

Let

d = internal diameter, in metres;
h = height, in metres;
R = grate area, in square metres;
B = coal burned per hour, in kilogrammes.

Then

$$d = 0.1B^{0.4} \text{ metres,}$$

$$h = 0.00277\left(\frac{B}{R}\right) + 6d.$$

For use in British measures we have

d = internal diameter, in feet;
h = height, in feet;
R = grate area, in square feet;
B = coal burned per hour, in pounds.

Then

$$d = 0.242B^{0.4} \text{ feet,}$$

$$h = 0.216\left(\frac{B}{R}\right)^2 + 6d.$$

These appear to be the most satisfactory formulas of all. The diameter, and hence the area, is dependent solely upon the quantity of coal burned per hour, and the height is determined mainly by the rate of combustion

per square foot of grate, plus 6 diameters; the latter member providing for the relation of height to diameter. With these formulas no absurd relations of height to diameter are possible, and the range of heights for various rates of combustion accord well with practice.

When the rate of combustion is not known it may be taken according to the character of the boiler and furnace. Taking the grate surface at 0.01 square foot per pound of water evaporated per hour, or about ⅓ square foot per horse-power, and the quantity of water evaporated per pound of coal from 5 to 10 pounds,—that is, 0.20 to 0.1 pound of coal per pound of water,—we have corresponding ratios of 10 to 20 pounds of coal per square foot of grate. It is advisable to make the chimney capable of maintaining a rate of 20 pounds per square foot of grate, so that $\left(\frac{B}{R}\right)^2 = 20^2 = 400$, and this gives a minimum height of chimney for that rate as

$$h = 0.216 \times 400 = 86.4 \text{ feet,}$$

plus 6 diameters. The diameter is then determined by the total quantity of coal burned per hour; and taking this at 0.2 pound of coal per pound of water, or 6 pounds per horse-power, we have all the data necessary to determine the size of a chimney for any given evaporation of water.

Thus, for 3000 pounds of water per hour, or 100 horse-power, we have

$$B = 3000 \times 0.2 = 600,$$

and

$$d = 0.242 \times 600^{0.4} = 3.13,$$

or, say, 3 feet diameter, and the height will be

$$86.4 + 18 = 104.4 \text{ feet.}$$

The areas given by these formulas are somewhat larger than many rules, and may serve for boilers of at least 25 per cent. greater capacity than close computation will indicate.

Theoretically, about 12 pounds of air are required for the combustion of 1 pound of coal; but, in practice, from 18 to 24 pounds actually pass through the furnace. This excess is found necessary to insure combustion, owing to the imperfect mixture of the air and the gases.

Draught Pressure Required for Combustion of Different Fuels.

Kind of fuel.	Total draught, in inches, of water.	Kind of fuel.	Total draught, in inches, of water.
Straw	.20	Slack, very small	.7 to 1.1
Wood	.30	Coal-dust	.8 to 1.1
Sawdust	.35	Semi-anthracite coal	.9 to 1.2
Peat, light	.4	Mixture of breeze and slack	1.0 to 1.3
Peat, heavy	.5	Anthracite, round	1.2 to 1.4
Sawdust mixed with small coal	.6	Mixture of breeze and coal-dust	1.2 to 1.5
Steam coal, round	.4 to .7	Anthracite slack	1.3 to 1.8
Slack, ordinary	.6 to .9		

Flue Area, in Square Inches, Required for the Passage of a Given Volume of Air at a Given Velocity.

(B. F. Sturtevant.)

Volume, in cubic feet, per minute.	Velocity, in feet, per minute.													
	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
100	48	36	29	24	21	18	16	14	13	12	11	10	9.6	9.0
125	60	45	36	30	26	23	20	18	16	15	14	13	12.0	11.3
150	72	54	43	36	31	27	24	22	20	18	16	15	14.4	13.5
175	84	63	50	42	36	32	28	25	23	21	19	18	16.8	15.8
200	96	72	58	48	41	36	32	29	26	24	22	21	19.2	18.0
225	108	81	65	54	46	41	36	32	29	27	25	23	21.6	20.3
250	120	90	72	60	51	45	40	36	33	30	28	26	24.0	22.5
275	132	99	79	66	57	50	44	40	36	33	30	28	26.4	24.8
300	144	108	86	72	62	54	48	43	39	36	33	31	28.8	27.0
325	156	117	94	78	67	59	52	47	43	39	36	33	31.2	29.3
350	168	126	101	84	72	63	56	50	46	42	39	36	33.6	31.5
375	180	135	108	90	77	68	60	54	49	45	42	39	36.0	33.8
400	192	144	115	96	82	72	64	58	52	48	44	41	38.4	36.0
425	204	153	122	102	87	77	68	61	56	51	47	44	40.8	38.3
450	216	162	130	108	93	81	72	65	59	54	50	46	43.2	40.5
475	228	171	137	114	98	86	76	68	62	57	53	49	45.6	42.8
500	240	180	144	120	103	90	80	72	65	60	55	51	48.0	45.0
525	252	189	151	126	108	95	84	76	69	63	58	54	50.4	47.3
550	264	198	158	132	113	99	88	79	72	66	61	57	52.8	49.5
575	276	207	166	138	118	104	92	83	75	69	64	59	55.2	51.8
600	288	216	173	144	123	108	96	86	79	72	66	62	57.6	54.0
625	300	225	180	150	129	113	100	90	82	75	69	64	60.0	56.3
650	312	234	187	156	134	117	104	94	85	78	72	67	62.4	58.5
675	324	243	194	162	139	122	108	97	88	81	75	69	64.8	60.8
700	336	252	202	168	144	126	112	101	92	84	78	72	67.2	63.0
725	348	261	209	174	149	131	116	104	95	87	80	75	69.6	65.3
750	360	270	216	180	154	135	120	108	98	90	83	77	72.0	67.5
775	372	279	223	186	159	140	124	112	101	93	86	80	74.4	69.8
800	384	288	230	192	165	144	128	115	105	96	89	82	76.8	72.0
825	396	297	238	198	170	149	132	119	108	99	91	85	79.2	74.3
850	408	306	245	204	175	153	136	122	111	102	94	87	81.6	76.5
875	420	315	252	210	180	158	140	126	115	105	97	90	84.0	78.8
900	432	324	259	216	185	162	144	130	118	108	100	93	86.4	81.0
925	444	333	266	222	190	167	148	133	121	111	103	95	88.8	83.3
950	456	342	274	228	195	171	152	137	124	114	105	98	91.2	85.5
975	468	351	281	234	201	176	156	140	128	117	108	100	93.6	87.8
1000	480	360	288	240	206	180	160	144	131	120	111	103	96.0	90.0

Flue Area, in Square Inches, Required for the Passage of a Given Volume of Air at a Given Velocity.

(B. F. Sturtevant.)

Velocity, in feet, per minute.														Volume, in cubic feet, per minute.
1700	1800	1900	2000	2100	2200	2300	2400	2600	2700	2800	2900	3000	3100	
8.5	8	7.6	7.2	6.9	6.6	6.3	6.0	5.5	5.3	5.1	5.0	4.8	4.6	100
10.6	10	9.5	9.0	8.6	8.2	7.8	7.5	6.9	6.7	6.4	6.2	6.0	5.8	125
12.7	12	11.4	10.8	10.3	9.8	9.4	9.0	8.0	8.0	7.7	7.5	7.2	7.0	150
14.8	14	13.3	12.6	12.0	11.5	11.0	10.5	9.7	9.3	9.0	8.7	8.4	8.1	175
16.9	16	15.2	14.4	13.7	13.1	12.5	12.0	11.1	10.7	10.3	9.9	9.6	9.3	200
19.1	18	17.1	16.2	15.6	14.7	14.1	13.5	12.5	12.0	11.6	11.2	10.8	10.4	225
21.2	20	19.0	18.0	17.1	16.4	15.7	15.0	13.9	13.3	12.9	12.4	12.0	11.6	250
23.3	22	21.8	19.8	18.9	18.0	17.2	16.5	15.2	14.7	14.1	13.7	13.2	12.8	275
25.4	24	22.7	21.6	20.6	19.6	18.8	18.0	16.6	16.0	15.4	14.9	14.4	13.9	300
27.5	26	24.6	23.4	22.3	21.3	20.6	19.5	18.0	17.3	16.7	16.1	15.6	15.1	325
29.6	28	26.5	25.2	24.0	22.9	21.9	21.0	19.4	18.7	18.0	17.4	16.8	16.3	350
31.8	30	28.4	27.0	25.7	24.5	23.5	22.5	20.8	20.0	19.3	18.6	18.0	17.4	375
33.9	32	30.3	28.8	27.4	26.2	25.0	24.0	22.2	21.3	20.6	19.8	19.2	18.6	400
36.0	34	32.2	30.6	29.1	27.8	26.6	25.5	23.5	22.7	21.9	21.1	20.4	19.7	425
38.1	36	34.1	32.4	30.9	29.5	28.2	27.0	24.9	24.0	23.1	22.3	21.6	20.9	450
40.2	38	36.0	34.2	32.6	31.1	29.7	28.5	26.3	25.3	24.4	23.6	22.8	22.1	475
42.4	40	37.9	36.0	34.3	32.7	31.3	30.0	27.7	26.7	25.7	24.8	24.0	23.2	500
44.5	42	39.8	37.8	36.0	34.4	32.9	31.5	29.1	28.0	26.9	25.0	25.2	24.4	525
46.6	44	41.7	38.6	37.7	36.0	34.4	33.0	30.5	29.3	28.3	27.3	26.4	25.5	550
48.7	46	43.6	41.4	39.4	37.6	36.0	34.5	31.9	30.7	29.6	28.5	27.6	26.7	575
50.8	48	45.5	43.2	41.1	39.3	37.6	36.0	33.2	32.0	30.8	29.8	28.8	27.8	600
52.9	50	47.4	45.0	42.9	40.9	39.1	37.5	34.6	33.3	32.1	31.0	30.0	29.0	625
55.1	52	49.3	46.8	44.6	42.5	40.7	39.0	36.0	34.7	33.4	32.2	31.2	30.2	650
57.2	54	51.2	48.6	46.3	44.1	42.3	40.5	37.5	36.0	34.7	33.5	32.4	31.3	675
59.3	56	53.1	50.4	48.0	45.8	43.8	42.0	38.8	37.3	36.0	34.7	33.6	32.5	700
61.4	58	55.0	52.2	49.7	47.4	45.4	43.5	40.2	38.7	37.3	36.0	34.8	33.6	725
63.5	60	56.9	54.0	51.4	49.1	47.0	45.0	41.5	40.0	38.6	37.2	36.0	34.8	750
65.6	62	58.8	56.3	53.1	50.7	48.5	46.5	42.9	41.3	39.9	38.5	37.2	36.0	775
67.8	64	60.6	57.6	54.9	52.4	50.1	48.0	44.3	42.7	41.2	39.7	38.4	37.1	800
69.9	66	62.5	59.4	56.6	54.0	51.7	49.5	45.7	44.0	42.4	40.9	39.6	38.3	825
72.0	68	64.4	61.2	58.4	55.6	53.2	51.0	47.1	45.3	43.7	42.2	40.8	39.4	850
74.0	70	67.3	63.0	60.0	57.3	54.8	52.5	48.5	46.7	45.0	43.4	42.0	40.6	875
76.2	72	68.2	64.8	61.7	58.9	56.3	54.0	49.9	48.0	46.3	44.6	43.2	41.8	900
78.4	74	70.1	66.6	63.4	60.5	57.9	55.5	51.3	49.3	47.6	46.0	44.4	42.9	925
80.5	76	72.0	68.4	65.1	62.2	59.5	57.0	52.6	50.7	48.8	47.1	45.6	44.1	950
82.6	78	73.9	70.2	66.8	63.8	61.0	58.5	54.0	52.0	50.2	48.4	46.8	45.3	975
84.7	80	75.8	72.0	68.7	66.0	62.6	60.0	55.4	53.3	51.4	49.6	48.0	46.4	1000

Pressure, in Ounces, per Square Inch.

Corresponding to Various Heads of Water, in Inches.

Head, in inches.	Decimal parts of an inch.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	......	.06	.12	.17	.23	.29	.35	.40	.46	.52
1	.58	.63	.69	.75	.81	.87	.93	.98	1.04	1.09
2	1.16	1.21	1.27	1.33	1.39	1.44	1.50	1.56	1.62	1.67
3	1.73	1.79	1.85	1.91	1.96	2.02	2.08	2.14	2.19	2.25
4	2.31	2.37	2.42	2.48	2.54	2.60	2.66	2.72	2.77	2.83
5	2.89	2.94	3.00	3.06	3.12	3.18	3.24	3.29	3.35	3.41
6	3.47	3.52	3.58	3.64	3.70	3.75	3.81	3.87	3.92	3.98
7	4.04	4.10	4.16	4.22	4.28	4.33	4.39	4.45	4.50	4.56
8	4.62	4.67	4.73	4.79	4.85	4.91	4.97	5.03	5.08	5.14
9	5.20	5.26	5.31	5.37	5.42	5.48	5.54	5.60	5.66	5.72

Height of Water Column, in Inches.

Corresponding to Pressures, in Ounces, per Square Inch.

Pressure, in ounces, per square inch.	Decimal parts of an ounce.									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	......	.17	.35	.52	.69	.87	1.04	1.21	1.38	1.56
1	1.73	1.90	2.08	2.25	2.42	2.60	2.77	2.94	3.11	3.29
2	3.46	3.63	3.81	3.98	4.15	4.33	4.50	4.67	4.84	5.01
3	5.19	5.36	5.54	5.71	5.88	6.06	6.23	6.40	6.57	6.75
4	6.92	7.09	7.27	7.44	7.61	7.79	7.96	8.13	8.30	8.48
5	8.65	8.82	9.00	9.17	9.34	9.52	9.69	9.86	10.03	10.21
6	10.38	10.55	10.73	10.90	11.07	11.26	11.43	11.60	11.77	11.95
7	12.11	12.28	12.46	12.63	12.80	12.97	13.15	13.32	13.49	13.67
8	13.84	14.01	14.19	14.36	14.53	14.71	14.88	15.05	15.22	15.40
9	15.57	15.74	15.92	16.09	16.26	16.45	16.62	16.76	16.96	17.14

STEAM-BOILER DETAILS.

Material for Riveting.

Board of Trade.—Tensile strength of rivet bars between 26 and 30 tons; elongation in 10 inches not less than 25 per cent., and contraction of area not less than 50 per cent.

Lloyd's.—Tensile strength, 26 to 30 tons; elongation not less than 20 per cent. in 8 inches. The material must stand bending to a curve, the inner radius of which is not greater than 1½ times the thickness of the plate, after having been uniformly heated to a low cherry-red, and quenched in water at 82° F.

United States Statutes.—No special provision.

Bureau Veritas.—Tensile strength, 53,000 pounds.

German Lloyd's.—Tensile strength, 45,000 to 51,000 pounds; elongation, 23.5 per cent. to 26 per cent., depending on thickness of plate.

Rules Connected with Riveting.

Board of Trade.—The shearing resistance of the rivet steel to be taken at 23 tons per square inch, 5 to be used for the factor of safety independently of any addition to this factor for the plating. Rivets in double shear to have only 1.75 times the single section taken in the calculation, instead of 2. The diameter must not be less than the thickness of the plate, and the pitch never greater than 8½ inches. The thickness of double butt straps (each) not to be less than ⅝ the thickness of the plate; single butt straps not less than $\frac{9}{8}$.

Distance from centre of rivet to edge of hole = diameter of rivet × 1½.

Distance between rows of rivets

$$= 2 \times \text{diameter of rivet or} = [(\text{diameter} \times 4) + 1] \div 2, \text{ if chain, and}$$

$$= \frac{\sqrt{[(\text{pitch} \times 11) + (\text{diameter} \times 4)] \times (\text{pitch} \div \text{diameter} \times 4)}}{10}, \text{ if zigzag.}$$

Diagonal pitch = (pitch × 6 + diameter × 4) ÷ 10.

Lloyd's.—Rivets in double shear to have only 1.75 times the single section taken in the calculation, instead of 2. The shearing strength of rivet steel to be taken at 85 per cent. of the tensile strength of the material of shell plates. In any case where the strength of the longitudinal joint is satisfactorily shown by experiment to be greater than given by the formula, the actual strength may be taken in the calculation.

United States Statutes.—No rules.

Bureau Veritas.—Shearing strength assumed = 0.8 tensile strength; at working pressure shearing strength to be $\frac{1}{4.4}$ part of full shearing strength. Double shear twice single section. Circular seams to be double-riveted if plates exceed ½ inch.

German Lloyd's.—Shearing assumed = 0.8 tensile strength of plates,—factor of safety = 5 for lap joints and 1.15 × 5 for double butt joints,—total rivet area to be taken. Butt straps at least 0.75 of plate diameter of rivets not over twice, or less than thickness of plate for thin and thick plates, respectively. Pitch of rivets not over 8 times thickness of plate strap.

Proportions of Rivets.

(Thurston.)

Thickness of plate..............	¼″	$\frac{5}{16}$″	⅜″	$\frac{7}{16}$″	½″
Diameter of rivet...............	⅝″	$\frac{11}{16}$″	¾″	$\frac{13}{16}$″	⅞″
Diameter of rivet-hole..........	$\frac{11}{16}$″	¾″	$\frac{13}{16}$″	⅞″	$\frac{15}{16}$″
Pitch—single riveting	2″	$2\frac{1}{16}$″	2⅛″	$2\frac{3}{16}$″	2¼″
Pitch—double riveting	3″	3⅛″	3¼″	3⅜″	3½″
Strength of single-riveted joint..	.66%	.64%	.62%	.60%	.58%
Strength of double-riveted joint.	.77%	.76%	.75%	.74%	.73%

Lloyd's Proportions for Riveted Joints.

Single-riveted Joints.

Thickness of iron plate.	Diameter of iron rivet.	Pitch.	Lap.	Percentage.	
				Plate.	Rivet.
Inch.	Inch.	Inch.	Inch.		
$\frac{5}{16}$	$\frac{11}{16}$	$1\frac{3}{4}$	$2\frac{1}{16}$	67.9	60.7
$\frac{3}{8}$	$\frac{13}{16}$	$2\frac{1}{16}$	$2\frac{7}{16}$	67.0	60.0
$\frac{7}{16}$	$\frac{15}{16}$	$2\frac{1}{4}$ and $\frac{3}{32}$	$2\frac{13}{16}$	67.4	60.1
$\frac{1}{2}$	1	$2\frac{7}{16}$	3	64.9	58.9
$\frac{9}{16}$	$1\frac{1}{8}$	$2\frac{5}{8}$ and $\frac{3}{32}$	$3\frac{3}{8}$	65.1	58.6
$\frac{5}{8}$	$1\frac{3}{16}$	$2\frac{3}{4}$ and $\frac{1}{32}$	$3\frac{9}{16}$	63.7	57.3

Double-riveted Joints.

Thickness of iron plate.	Diameter of iron rivet.	Pitch.	Lap.	Percentage.	
				Rivet.	Plate.
Inch.	Inch.	Inch.	Inch.		
$\frac{3}{8}$	$\frac{11}{16}$	$2\frac{3}{8}$ and $\frac{3}{32}$	$3\frac{7}{16}$	80.2	72.1
$\frac{7}{16}$	$\frac{3}{4}$	$2\frac{1}{2}$ and $\frac{3}{32}$	$3\frac{3}{4}$	77.7	71.1
$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{5}{8}$ and $\frac{1}{16}$	$4\frac{1}{16}$	77.2	69.4
$\frac{9}{16}$	$\frac{15}{16}$	$3\frac{1}{8}$	$4\frac{11}{16}$	78.5	70.0
$\frac{5}{8}$	1	$3\frac{1}{4}$	5	77.3	69.2
$\frac{11}{16}$	$1\frac{1}{16}$	$3\frac{3}{8}$	$5\frac{5}{16}$	76.4	68.5
$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{2}$ and $\frac{1}{32}$	$5\frac{5}{8}$	75.0	68.1

Treble-riveted Joints.

Thickness of iron plate.	Diameter of iron rivet.	Pitch.	Lap.	Percentage.	
				Rivet.	Plate.
Inch.	Inch.	Inch.	Inch.		
$\frac{1}{2}$	$\frac{3}{4}$	$3\frac{1}{8}$ and $\frac{1}{32}$	$4\frac{1}{2}$	83.9	76.2
$\frac{9}{16}$	$\frac{13}{16}$	$3\frac{1}{4}$ and $\frac{1}{32}$	$4\frac{7}{8}$	84.3	75.2
$\frac{5}{8}$	$\frac{7}{8}$	$3\frac{7}{16}$	$5\frac{1}{4}$	83.9	75.4
$\frac{11}{16}$	1	$4\frac{1}{16}$	6	84.4	75.4
$\frac{3}{4}$	$1\frac{1}{16}$	$4\frac{1}{4}$	$6\frac{3}{8}$	83.4	75.0
$\frac{13}{16}$	$1\frac{1}{8}$	$4\frac{3}{8}$ and $\frac{1}{32}$	$6\frac{3}{4}$	83.3	74.5
$\frac{7}{8}$	$1\frac{3}{16}$	$4\frac{1}{2}$ and $\frac{3}{32}$	$7\frac{1}{8}$	82.5	74.1
$\frac{15}{16}$	$1\frac{1}{4}$	$4\frac{3}{4}$ and $\frac{1}{32}$	$7\frac{1}{2}$	82.1	73.8
1	$1\frac{5}{16}$	$4\frac{15}{16}$	$7\frac{7}{8}$	82.2	73.4

Materials for Boiler Shells.

Board of Trade.—Tensile strength between 27 and 32 tons. In the normal condition, elongation not less than 18 per cent. in 10 inches, but should be about 25 per cent.; if annealed, not less than 20 per cent. Strips 2 inches wide should stand bending until the sides are parallel at a distance from each other of not more than 3 times the plate's thickness.

Lloyd's.—Tensile strength between the limits of 26 and 30 tons per square inch. Elongation not less than 20 per cent. in 8 inches. Test strips heated to a low cherry-red and plunged into water at 82° F. must stand bending to a curve, the inner radius of which is not greater than $1\frac{1}{2}$ times the plate's thickness.

United States Statutes.—Plates of $\frac{1}{2}$ inch thickness and under shall show a contraction of not less than 50 per cent.; when over $\frac{1}{2}$ inch, and up to $\frac{3}{4}$ inch, not less than 45 per cent.; when over $\frac{3}{4}$ inch, not less than 40 per cent.

Bureau Veritas.—Tensile strength not over 61,000 pounds. Elongation, 20 to 31 per cent. for various tensile strengths. Quench strips must bend 180° around diameter $= 3t$.

German Lloyd's.—Tensile strength not over 61,000 pounds. Elongation, 20 to 26 per cent. for various tensile strengths. Quench strips must bend 180° around diameter $= 4t$.

Proportions of Boiler Shells.

Board of Trade.—$P = \dfrac{T \times B \times t \times 2}{D \times F}$.

D = diameter of boiler, in inches;
P = working pressure, in pounds, per square inch;
t = thickness, in inches;
B = percentage of strength of joint compared to solid plate;
T = tensile strength allowed for the material, in pounds, per square inch;
F = a factor of safety, being 4.5, with certain additions depending on method of construction.

Lloyd's.—$P = \dfrac{C \times (t - 2) \times B}{D}$.

t = thickness of plate, in sixteenths of an inch;
B and D as before;
C = a constant depending on the kind of joint.

When longitudinal seams have double butt straps, $C = 20$. When longitudinal seams have double butt straps of unequal width, only covering on one side, the reduced section of plate at the outer line of rivets, $C = 19.5$.
When the longitudinal seams are lap-jointed, $C = 18.5$.

United States Statutes.—Using same notation as for Board of Trade. $P = \dfrac{t \times 2 \times T}{D \times 6}$ for single riveting; add 20 per cent. for double riveting where T is the lowest tensile strength stamped on any plate.

Bureau Veritas.—$P = \dfrac{T \times B \times (t - 0.042)^2}{D \times 4.4 \times 100}$.

B = per cent. of plate section at joint.
P also depends on rivet section.

German Lloyd's.—$P = \dfrac{t \times 2 \times B \times T}{D \times F \times 100}$.

F varies from 4.65 to 5, depending on thickness of plate.

Proportions for Flat Plates.

Board of Trade.—$P = \dfrac{C(t+1)^2}{S-6}$.

P = working pressure, in pounds, per square inch;
S = surface supported, in square inches;
t = thickness, in sixteenths of an inch;
C = a constant, as per following table.

$C = 125$ for plates not exposed to heat or flame, the stays fitted with nuts and washers, the latter at least 3 times the diameter of the stay and ⅔ the thickness of the plate.
$C = 187.5$ for the same condition, but the washers ⅔ the pitch of stays in diameter, and thickness not less than plate.
$C = 200$ for the same condition, but doubling plates in place of washers, the width of which is ⅔ the pitch and thickness the same as the plate.
$C = 112.5$ for the same condition, but stays fitted with nuts only.
$C = 75$ when exposed to impact of heat or flame and steam in contact with the plates, and the stays fitted with nuts and washers 3 times the diameter of the stay and ⅔ the plate's thickness.
$C = 67.5$ for the same condition, but stays fitted with nuts only.
$C = 100$ when exposed to heat or flame and water in contact with the plates, and stays screwed into the plates and fitted with nuts.
$C = 66$ for the same condition, but stays with riveted heads.

United States Statutes.—Using same notations as for Board of Trade.
$P = \dfrac{C \times t}{p^2}$, where p = greatest pitch, in inches, P and t as above.

$C = 112$ for plates $\frac{7}{16}$ of an inch thick and under, fitted with screw stay-bolts and nuts, or plain bolt fitted with single nut and socket or riveted head and socket.
$C = 120$ for plates above $\frac{7}{16}$ of an inch, under the same conditions.
$C = 140$ for flat surfaces where the stays are fitted with nuts inside and outside.
$C = 200$ for flat surfaces under the same condition, but with the addition of a washer riveted to the plate at least half the plate's thickness and of a diameter equal to $\frac{2}{5}$ pitch.

N.B.—Plates fitted with double angle-irons and riveted to plate, with leaf at least ⅔ the thickness of plate and depth at least ¼ of pitch, would be allowed the same pressure as determined by formula for plate with washer riveted on.

N.B.—No brace or stay-bolt used in marine boilers to have a greater pitch than 10½ inches on fire-boxes and back connections.

Certain experiments were carried out by the Board of Trade which showed that the resistance to bulging does not vary as the square of the plate's thickness. There seems, also, good reason to believe that it is not inversely as the square of the greatest pitch.

Bureau Veritas.—$P = \dfrac{(t-1)^2}{a^2+b^2} \times \dfrac{T}{C}$.

T = tensile strength, in tons, per square inch;
a = pitch in one row, in inches;
b = distance between rows;
C = factor depending on method of supporting, and varies from 0.055 to 0.084.

German Lloyd's.—$P = \dfrac{t^2}{C^2 \times p^2}$.

C varies from 0.00425 to 0.00639, depending on exposure and method of supporting.

Plates for Flanging.

The Board of Trade gives the following rule for the strength of furnaces stiffened with flanged seams, provided the pitch of the flanges does not exceed $120T - 12$, and the flanging is of suitable design and effected at one heat:

$$P = \frac{9900 \times T}{3 \times D}\left(5 - \frac{L + 12}{60 \times T}\right).$$

P = working pressure per square inch;
T = thickness of plate, in inches;
L = pitch of flanges, in inches;
D = outside diameter of tubes, in inches.

Bureau Veritas.—Tensile strength not over 61,000 pounds. Elongation, 20 to 31 per cent. for various tensile strengths. Quench strips must bend 180° around diameter $= 3t$.

German Lloyd's.—Tensile strength not over 53,000 pounds. Elongation not under 22½ per cent. Quench strips must bend 180° around diameter $= 4t$.

Furnace Flues.

Board of Trade. Long Furnaces.—$P = \dfrac{C \times t^2}{(L+1) \times D}$, but not where L is shorter than $(11.5t - 1)$, at which length the rule for short furnaces comes into use.

P = working pressure, in pounds, per square inch;
t = thickness, in inches;
D = outside diameter, in feet;
L = length of furnaces, in feet, up to 10 feet;
C = a constant, as below, for drilled holes.

C = 99,000 for welded or butt-jointed, with single straps, double-riveted.
C = 88,000 for butts with single straps, single-riveted.
C = 99,000 for butts with double straps, single-riveted.

Provided, always, that the pressure so found does not exceed that given by the following formulas, which apply also to short furnaces:

$P = \dfrac{C \times t}{D}$ for all the patent furnaces named.
C = 8800 for plain furnaces.
C = 14,000 for Fox. Minimum thickness, $\frac{5}{16}$ inch; greatest, ⅝ inch; plain part not to exceed 6 inches in length.
C = 13,500 for Morison. Minimum thickness, $\frac{5}{16}$ inch; greatest, ⅝ inch; plain part not to exceed 6 inches in length.
C = 14,000 for Purves-Brown. Limits of thickness, $\frac{7}{16}$ and ⅝ inch; plain part 9 inches in length.

United States Statutes. Long Furnaces.—Same notation.

$$P = \frac{89{,}600 \times t^2}{L \times D}, \text{ but } L \text{ not to exceed 8 feet.}$$

Short Furnaces, Plain and Patent.—P as before, when not 8 feet long $= \dfrac{89{,}600 \times t^2}{L \times D}$.

$$P = \frac{t \times C}{D}, \text{ when}$$

C = 14,000 for Fox corrugations, where D = mean diameter.
C = 14,000 for Purves-Brown, where D = diameter of flue.
C = 5677 for plain flues over 16 inches diameter and less than 40 inches, when not over 3-foot lengths.

Lloyd's and Bureau Veritas for Morison Suspension Furnaces.— $WP = \frac{1259(T-2)}{D}$.

T = thickness, in sixteenths of an inch;
D = greatest diameter, in inches;
WP = working pressure.

Stays.

MATERIAL.

Board of Trade.—The tensile strength to lie between the limits of 27 and 32 tons per square inch, and to have an elongation of not less than 20 per cent. in 10 inches. Steel stays which have been welded or worked in the fire should not be used.

Lloyd's.—26 to 30 ton steel, with elongation not less than 20 per cent. in 8 inches.

United States Statutes.—Reduction of area must not be less than 40 per cent. if the test bar is more than ¾ of an inch in diameter.

Bureau Veritas.—Same as for shell plates.

German Lloyd's.—Large stays, tensile strength 45,800 to 61,200 pounds. Elongation same as shell plates. Screwed stays, tensile strength 44,600 to 53,400 pounds, and corresponding elongation.

Loads on Stays.

Board of Trade.—9000 pounds per square inch is allowed on the net section, provided the tensile strength ranges from 27 to 32 tons. Steel stays are not to be welded or worked in the fire.

Lloyd's.—For screwed and other stays not exceeding 1½ inches in diameter effective, 8000 pounds per square inch is allowed; for stays above 1½ inches, 9000 pounds. No stays are to be welded.

United States Statutes.—Braces and stays shall not be subjected to a greater stress than 6000 pounds per square inch.

Bureau Veritas.—$\frac{1}{5.75}$ of lower test limit on net section. Then add ⅛ inch to diameter of stay.

German Lloyd's.—Not to exceed $\frac{1}{7}$ of tensile strength, or about 8500 pounds per square inch.

Stay Girders.

Board of Trade.—$P = \frac{C \times d^2 \times t}{(W-p)D \times L}$.

P = working pressure, in pounds, per square inch;
W = width of flame-box, in inches;
L = length of girder, in inches;
p = pitch of bolts, in inches;
D = distance between girders from centre to centre, in inches;
d = depth of girder, in inches;
t = thickness of sum of same, in inches;
C = a constant = 6600 for 1 bolt, 9900 for 2 or 3 bolts, and 11,220 for 4 bolts.

Lloyd's.—The same formula and constants, except that C = 11,000 for 4 or 5 bolts, 11,550 for 6 or 7 bolts, and 11,880 for 8 or more.

Tube Plates.

Board of Trade.—$P = \dfrac{t(D-d) \times 20000}{W \times D}$.

D = least horizontal distance between centres of tubes, in inches;
d = inside diameter of ordinary tubes;
t = thickness of tube plate, in inches;
W = extreme width of combustion-box, in inches, from front of tube plate to back of fire-box, or distance between combustion-box tube plates, when the boiler is double-ended, and the box common to both ends.

The crushing stress on tube plates caused by the pressure on the flame-box top is to be limited to 10,000 pounds per square inch.

Fox and Purves Furnace Tubes.

Working Pressures allowed by Board of Trade and Lloyd's.

Diameter of furnace inside corrugations.		Working pressure, in pounds, per square inch.																	
		$\frac{3}{8}$ inch thick.		$\frac{13}{32}$ inch thick.		$\frac{7}{16}$ inch thick.		$\frac{15}{32}$ inch thick.		$\frac{1}{2}$ inch thick.		$\frac{17}{32}$ inch thick.		$\frac{9}{16}$ inch thick.		$\frac{19}{32}$ inch thick.		$\frac{5}{8}$ inch thick.	
Ft.	In.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.	B. of T.	Lloyd's.
2	6	164	145	177	163	191	181	205	199	218	217	232	235	246	254	259	272	273	290
2	7	159	141	172	158	185	176	198	193	212	211	225	229	238	246	251	264	265	282
2	8	154	137	167	154	180	171	193	188	205	205	218	222	231	239	244	257	257	274
2	9	150	133	162	150	175	166	187	183	200	200	212	216	225	233	237	250	250	266
2	10	145	129	157	146	170	162	182	178	194	194	206	211	218	227	230	243	243	259
2	11	141	126	153	142	165	158	177	174	189	189	201	205	212	221	224	237	236	253
3	0	138	123	149	138	161	154	172	169	184	185	195	200	207	215	218	231	230	246
3	1	134	120	145	135	157	150	168	165	179	180	190	195	201	210	213	225	224	240
3	2	131	117	142	132	153	146	164	161	175	176	185	190	196	205	207	220	218	235
3	3	128	114	138	129	149	143	160	157	170	172	181	186	192	200	202	215	213	229
3	4	125	112	135	126	145	140	156	154	166	168	177	182	187	196	197	210	208	224
3	5	122	109	132	123	142	137	152	150	162	164	172	178	183	191	193	205	203	219
3	6	119	107	129	120	139	134	149	147	159	160	169	174	178	187	188	201	198	214
3	7	116	105	126	118	136	131	145	144	155	157	165	170	175	183	184	196	194	210
3	8	114	102	123	115	133	128	142	141	152	154	161	167	171	179	180	192	190	205
3	9	111	100	121	113	130	125	139	138	148	151	158	163	167	176	176	188	186	201
3	10	109	98	118	111	127	123	136	135	145	148	154	160	164	172	173	185	182	197
3	11	107	96	116	108	125	120	133	133	142	145	151	157	160	169	169	181	178	193
4	0	105	94	113	106	122	118	131	130	140	142	148	154	157	166	166	177	175	189
4	1	102	93	111	104	120	116	128	128	137	139	145	151	154	162	162	174	171	186
4	2	100	91	109	102	117	114	126	125	134	137	143	148	151	159	159	171	168	182
4	3	99	89	107	100	115	112	123	123	132	134	140	145	148	157	156	168	165	179
4	4	97	88	105	99	113	110	121	121	129	132	137	143	145	154	153	165	162	176
4	5	95	86	103	97	111	108	119	119	127	129	135	140	143	151	151	162	159	173
4	6	93	85	101	95	109	106	117	117	125	127	132	138	140	148	148	159	156	170

Internal flues should be so constructed as to allow for expansion.

Table Showing Working Pressure and Thickness of Morison Suspension Furnaces.

Inside diameter of furnace.		Working pressure, in pounds, per square inch. Thickness of furnace.														
		$\frac{5}{16}$ inch.	$\frac{11}{32}$ inch.	$3/8$ inch.	$\frac{13}{32}$ inch.	$\frac{7}{16}$ inch.	$\frac{15}{32}$ inch.	$1/2$ inch.	$\frac{17}{32}$ inch.	$\frac{9}{16}$ inch.	$\frac{19}{32}$ inch.	$5/8$ inch.	$\frac{21}{32}$ inch.	$\frac{11}{16}$ inch.	$\frac{23}{32}$ inch.	$3/4$ inch.
Ft.	In.															
2	4	146	160	175	189	204	219	233	247	262	276	290	304	318	332	347
2	5	142	156	170	183	197	212	225	239	253	267	281	294	308	322	336
2	6	137	151	164	178	191	205	218	232	245	259	272	285	299	312	325
2	7	133	146	159	172	186	199	212	225	238	251	264	277	290	302	315
2	8	129	142	154	166	180	193	205	218	231	243	256	268	281	294	306
2	9	125	138	150	162	175	187	200	212	224	236	249	261	273	285	297
2	10	122	134	146	158	170	182	194	206	218	230	242	254	265	277	289
2	11	119	130	142	154	165	177	189	200	212	224	235	247	258	270	281
3	0	115	127	138	149	161	172	184	195	207	218	229	240	252	263	274
3	1	112	123	135	146	157	168	179	190	201	212	223	234	245	256	267
3	2	110	120	131	142	153	164	175	185	196	207	218	228	239	250	260
3	3	107	117	128	138	149	160	170	181	191	202	212	223	233	244	254
3	4	104	115	125	135	146	156	166	176	187	197	207	217	228	238	248
3	5	102	112	122	132	142	152	162	172	183	192	202	212	222	232	242
3	6	100	109	119	129	139	149	159	168	178	188	198	207	217	227	237
3	7	97	107	116	126	136	146	155	165	174	184	193	203	213	222	232
3	8	95	105	114	123	133	142	152	161	171	180	189	199	208	217	227
3	9	93	102	112	121	130	139	148	158	167	176	185	194	203	213	222
3	10	91	100	109	118	127	137	145	154	163	172	181	190	199	209	217
3	11	89	98	107	116	125	134	142	151	160	169	178	186	195	204	213
4	0	87	96	105	113	122	131	140	148	157	166	174	183	191	200	208
4	1	86	94	103	111	120	128	137	145	154	162	170	179	188	196	204
4	2	84	92	101	109	118	126	134	142	151	159	167	176	184	192	200
4	3	82	91	99	107	115	123	132	140	148	156	164	172	180	189	197
4	4	81	88	97	105	113	121	129	137	145	153	161	169	177	185	193
4	5	79	87	95	103	111	119	127	135	143	150	158	166	174	182	190
4	6	78	86	93	101	109	117	125	132	140	148	155	163	171	178	186
4	7	77	84	92	99	107	115	122	130	138	145	153	160	168	175	183
4	8	75	83	90	98	105	113	120	128	135	143	150	157	165	172	180
4	9	74	81	89	96	103	111	118	125	133	140	147	155	162	169	177
4	10	73	80	87	94	102	109	116	123	131	138	145	152	159	167	174
4	11	71	78	86	93	100	107	114	121	129	136	143	150	157	164	171

Dimensions of Standard Boiler Tubes, Lap-welded, Wrought-iron.

Outside.		Thickness, in inches.	Weight per foot, in pounds.	Heating surface 1 foot in length.		Area of opening.	
Diameter, in inches.	Circumference, in inches.			Outside, square feet.	Inside, square feet.	Square feet.	Square inches.
1½	4.71	.08	1.25	.393	.349	.0097	1.40
1¾	5.50	.10	1.67	.458	.408	.0133	1.91
2	6.28	.10	1.98	.524	.472	.0177	2.56
2¼	7.07	.10	2.34	.589	.540	.0230	3.31
2½	7.85	.11	2.76	.655	.598	.0284	4.09
2¾	8.64	.11	3.05	.720	.663	.0350	5.04
3	9.43	.11	3.33	.785	.729	.0422	6.08
3¼	10.21	.12	3.96	.851	.789	.0495	7.12
3½	11.00	.12	4.27	.916	.854	.0580	8.36
3¾	11.78	.12	4.59	.982	.919	.0673	9.69
4	12.57	.13	5.32	1.047	.979	.0763	10.99
4½	14.14	.13	6.01	1.178	1.110	.0981	14.13
5	15.71	.14	7.23	1.309	1.234	.1215	17.50
6	18.85	.15	9.35	1.571	1.492	.1771	25.51
7	21.99	.17	12.44	1.833	1.743	.2417	34.81
8	25.13	.18	15.11	2.094	1.998	.3180	45.80
9	28.27	.19	18.00	2.356	2.254	.4048	58.29
10	31.42	.21	22.19	2.618	2.506	.4998	71.98
11	34.56	.22	25.49	2.880	2.764	.6075	87.48
12	37.70	.23	28.52	3.142	3.022	.7205	103.75
13	40.84	.24	32.21	3.403	3.279	.8554	123.19
14	43.98	.25	36.27	3.665	3.534	.9943	143.19
15	47.12	.26	40.61	3.927	3.791	1.1438	164.72
16	50.27	.27	45.20	4.189	4.047	1.3032	187.67
17	53.41	.28	49.90	4.451	4.305	1.4738	212.23
18	56.55	.29	54.82	4.712	4.560	1.6543	238.22
19	59.69	.30	59.48	4.974	4.817	1.8465	265.90
20	62.83	.32	66.77	5.219	5.068	2.0443	294.37
21	65.97	.34	73.40	5.498	5.320	2.2522	324.31

Proportions for Stay Bolts for Flat Surfaces.

(Barr.)

Pressure per square inch.	Centre to centre of stay bolts, in inches.				
	¼-inch plate. ¾-inch stay.	5/16-inch plate. ¾-inch stay.	⅜-inch plate. ⅞-inch stay.	7/16-inch plate. 1-inch stay.	½-inch plate. 1¼-inch stay.
50	6	7	8	9	10
60	5⅜	6⅜	7¼	8⅛	9
70	5	5¾	6⅝	7½	8⅜
80	4⅝	5½	6¼	7⅛	7⅞
90	4½	5⅛	5⅞	6⅝	7⅜
100	4¼	4¾	5½	6¼	7
110	4	4⅝	5¼	5⅞	6⅝
120	3⅞	4¼	5	5¾	6⅜
130	3¾	4¼	4⅞	5½	6⅛
140	3⅝	4⅛	4⅝	5¼	6
150	3½	4	4½	5	5⅝

Working Pressures for Flat Stayed Surfaces.

Pounds per Square Inch.

Diameter of circle.	Thickness of plates, in inches.								
	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾
In.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
10	62½	90	122	160	202	250	302	360	422
11	50	73	99	129	163	202	244	290	341
12	42	60	82	107	135	167	202	240	282
13	35	50	69	90	113	140	170	201	237
14	30	43	59	76	97	120	145	172	202
15	26	37	51	66	84	103	125	148	174
16	23	32	44	58	73	90	109	130	152
17	20	29	39	51	64	80	96	114	134
18	18	25	35	45	57	71	85	101	119
19	16	23	31	40	51	63	76	91	107
20	14	20	28	36	46	57	69	82	96
21	13	18	25	33	41	51	62	74	87
22	12	17	23	30	38	47	56	67	79
23	11	15	21	27	35	43	51	61	72
24	10	14	19	25	32	39	47	56	66
25	9	13	18	23	29	36	43	52	61
26	8	12	16½	21	27	33	40	48	56
27	7½	11	15	19½	25	31	37	44	52
28	7	10	14	18	23	29	34½	41	48
29	6½	9½	13	17	21½	27	32	38	45
30	6	9	12	16	20	25	30	35½	42
31	5½	8½	11	15	19	23	28	33	39
32	5¼	8	10½	14	18	21	26	31	36
33	5	7½	10	13	17	20	25	29½	34
34	4¾	7	9½	12	16	19	23	28	32
35	4½	6½	9	11	15	18	22	26	30
36	4¼	6	8½	11	14	17	21	25	29
37	4	6	8	10	13	16	20	23	27
38	3¾	5½	7½	10	12½	15	19	22	26
39	3½	5	7	9½	12	14½	18	21	25
40	3½	5	6½	9	11	14	17	20	23½

The rules of the Boiler Inspection Department of the city of Philadelphia have been extensively used, and are as follows:

Philadelphia City Rules for Boiler Dimensions.

In estimating the strength of the longitudinal seams for rating maximum working pressure on cylindrical boiler shells, two rules should be applied:

Rule A.—From the pitch of the rivets, in inches, subtract the diameter of holes punched to receive the rivets; divide the remainder by the pitch of the rivets. The quotient represents the percentage of strength of the solid part of the sheet.

Rule B.—Multiply the area of the hole filled by the rivet by the number of rows of rivets in the seam; divide the product by the pitch of the rivets multiplied by the thickness of the sheet. This product, multiplied by the shearing strength of the rivet, divided by the tensile strength of the sheet, will give the percentage of the strength of the rivets in the seam as compared with the strength of the solid part of the sheet.

The shearing strength of a rivet in a composite joint made of iron rivets and steel plates shall not be considered in excess of 40,000 pounds. Take the lowest of the percentages as found by Rules A and B and apply that percentage as the value of the seam in the following rule (C), which determines the strength of the longitudinal seams.

Rule C.—Multiply the thickness of the boiler plate, in parts of an inch, by the value of the seam as obtained by Rules A or B and by the ultimate tensile strength of the metal used in the plates; divide this product by the internal radius of the boiler, in inches, multiplied by the factor of safety. The quotient will be the pressure per square inch at which the safety valve may be set.

Working Pressures for Cylindrical Shells of Steam Boilers, *Lap Joints*, Double-riveted.

(Barr.)

Factor of Safety, 5.

Diameter.	Thickness.	Iron shell, iron rivets.	Steel shell, iron rivets.	Steel shell, steel rivets.
Inch.	Inch.	Lb.	Lb.	Lb.
36	1/4 5/16	91 112	111 128	111 137
38	1/4 5/16	86 106	105 121	105 129
40	1/4 5/16	82 101	100 115	100 123
42	1/4 5/16	78 96	95 110	95 117
44	1/4 5/16	74 91	91 105	91 112
46	1/4 5/16	71 87	87 100	87 107
48	5/16 3/8	84 99	96 107	102 121
50	5/16 3/8	81 95	92 103	98 116
52	5/16 3/8	77 92	89 99	95 112
54	5/16 3/8	75 88	85 96	91 108
56	5/16 3/8	72 85	82 92	88 104
58	5/16 3/8	69 82	79 89	85 100
60	5/16 3/8	67 79	77 85	82 97
62	3/8 7/16	77 88	83 92	94 108
64	3/8 7/16	74 86	81 89	91 105
66	3/8 7/16	72 83	78 87	88 102
68	3/8 7/16	70 81	76 80	86 99
70	3/8 7/16	68 78	74 82	83 96
72	3/8 7/16 1/2	66 76 85	72 79 89	81 93 104

Working Pressures for Cylindrical Shells of Steam Boilers, *Lap Joints,* Triple-riveted.

(Barr.)

Factor of Safety, 5.

Diameter.	Thickness.	Iron shell, iron rivets.	Steel shell, iron rivets.	Steel shell, steel rivets.
Inch.	Inch.	Lb.	Lb.	Lb.
36	$\frac{1}{4}$ $\frac{5}{16}$	100 124	121 139	123 151
38	$\frac{1}{4}$ $\frac{5}{16}$	95 117	115 132	116 144
40	$\frac{1}{4}$ $\frac{5}{16}$	90 112	109 125	110 136
42	$\frac{1}{4}$ $\frac{5}{16}$	86 106	104 119	105 130
44	$\frac{1}{4}$ $\frac{5}{16}$	83 101	99 114	100 124
46	$\frac{1}{4}$ $\frac{5}{16}$	79 97	95 109	96 119
48	$\frac{5}{16}$ $\frac{3}{8}$	93 110	104 118	114 135
50	$\frac{5}{16}$ $\frac{3}{8}$	89 106	100 113	109 129
52	$\frac{5}{16}$ $\frac{3}{8}$	86 102	96 109	105 124
54	$\frac{5}{16}$ $\frac{3}{8}$	83 98	93 105	101 120
56	$\frac{5}{16}$ $\frac{3}{8}$	80 95	89 101	97 116
58	$\frac{5}{16}$ $\frac{3}{8}$	77 91	86 98	94 112
60	$\frac{5}{16}$ $\frac{3}{8}$	74 88	83 95	91 108
62	$\frac{3}{8}$ $\frac{7}{16}$	85 98	92 103	104 120
64	$\frac{3}{8}$ $\frac{7}{16}$	83 95	89 100	101 117
66	$\frac{3}{8}$ $\frac{7}{16}$	80 93	86 97	98 113
68	$\frac{3}{8}$ $\frac{7}{16}$	78 90	84 94	95 110
70	$\frac{3}{8}$ $\frac{7}{16}$	76 87	81 91	92 107
72	$\frac{3}{8}$ $\frac{7}{16}$ $\frac{1}{2}$	74 85 97	79 89 98	90 104 117

Working Pressures for Cylindrical Shells of Steam Boilers, *Butt Joints,* Triple-riveted.

(Barr.)

Factor of Safety, 5.

Diameter.	Thickness.	Iron shell, iron rivets.	Steel shell, iron rivets.	Steel shell, steel rivets.
Inch.	Inch.	Lb.	Lb.	Lb.
36	1/4	108	134	134
	5/16	135	165	165
	3/8	161	197	197
38	1/4	102	127	127
	5/16	128	156	156
	3/8	152	187	187
40	1/4	97	120	120
	5/16	121	148	148
	3/8	145	178	178
42	1/4	93	115	115
	5/16	116	141	141
	3/8	138	169	169
44	1/4	89	109	109
	5/16	110	135	135
	3/8	132	161	161
46	1/4	85	105	105
	5/16	106	129	129
	3/8	126	154	154
48	5/16	101	124	124
	3/8	121	148	148
	7/16	141	172	172
50	5/16	97	119	119
	3/8	116	142	142
	7/16	135	165	165
52	5/16	93	114	114
	3/8	111	137	137
	7/16	130	159	159
54	5/16	90	110	110
	3/8	107	132	132
	7/16	125	153	153
56	5/16	87	106	106
	3/8	103	127	127
	7/16	121	148	148
58	5/16	84	102	102
	3/8	100	123	123
	7/16	117	142	142
60	3/8	97	118	118
	7/16	111	138	138
	1/2	128	157	157
62	3/8	93	115	115
	7/16	109	133	133
	1/2	124	152	152

Working Pressures for Cylindrical Shells of Steam Boilers, *Butt Joints*, Triple-riveted.

(Barr.)

Factor of Safety, 5.

Diameter.	Thickness.	Iron shell, iron rivets.	Steel shell, iron rivets.	Steel shell, steel rivets.
Inch.	Inch.	Lb.	Lb.	Lb.
64	3/8	90	111	111
	7/16	106	129	129
	1/2	120	147	147
	9/16	135	165	165
66	3/8	88	108	108
	7/16	102	125	125
	1/2	117	143	143
	9/16	131	160	160
68	3/8	85	105	105
	7/16	99	121	121
	1/2	113	138	138
	9/16	127	155	155
70	3/8	83	102	102
	7/16	97	118	118
	1/2	110	134	134
	9/16	123	151	151
72	3/8	80	99	99
	7/16	94	115	115
	1/2	107	131	131
	9/16	120	147	147
	5/8	134	163	163
75	7/16	90	110	110
	1/2	102	125	125
	9/16	115	141	141
	5/8	128	157	157
78	7/16	87	106	106
	1/2	99	121	121
	9/16	111	135	135
	5/8	123	151	151
81	7/16	83	102	102
	1/2	95	116	116
	9/16	107	130	130
	5/8	119	145	145
84	1/2	92	112	112
	9/16	103	126	126
	5/8	115	140	140
	11/16	126	158	158
	3/4	137	167	167
87	1/2	89	108	108
	9/16	99	121	121
	5/8	111	135	135
	11/16	121	148	148
	3/4	132	162	162
90	1/2	86	105	105
	9/16	96	117	117
	5/8	107	131	131
	11/16	117	143	143
	3/4	128	156	156

Working Pressures for Cylindrical Shells of Steam Boilers, *Butt Joints,* Triple-riveted.

(Barr.)

Factor of Safety, 5.

Diameter.	Thickness.	Iron shell, iron rivets.	Steel shell, iron rivets.	Steel shell, steel rivets.
Inch.	Inch.	Lb.	Lb.	Lb.
93	9/16 5/8 11/16	93 103 114	114 126 139	114 126 139
96	5/8 11/16 3/4	100 110 120	123 134 146	123 134 146
99	5/8 11/16 3/4	97 107 116	119 130 142	119 130 142
102	5/8 11/16 3/4	94 104 113	115 127 138	115 127 138
105	5/8 11/16 3/4	92 101 110	112 123 134	112 123 134
108	5/8 11/16 3/4	89 98 107	109 120 130	109 120 130
111	5/8 11/16 3/4	87 95 104	106 116 127	106 116 127
114	5/8 11/16 3/4	84 93 101	103 113 123	103 113 123
117	5/8 11/16 3/4	82 90 99	100 110 120	100 110 120
120	5/8 11/16 3/4	80 88 96	98 108 117	98 108 117

The formulas for boilers given by Reuleaux in the "Constructor" are as follows:

Let

D = diameter, in metres;
a = pressure, in atmospheres;
δ = thickness of shell, in millimetres;
S = fibre stress on material, in kilogrammes, per square millimetre.

$$\delta = 1.54aD + 2.6.$$

The stress in the longitudinal seams will be

$$S = \frac{a}{200} \cdot \frac{D}{\delta},$$

D being taken in millimetres.

From these we have

$a =$	4 atmospheres.		7 atmospheres.		10 atmospheres.		13 atmospheres.	
D metres.	δ mm.	S kg. per sq. mm.	δ mm.	S kg. per sq. mm.	δ mm.	S kg. per sq. mm.	δ mm.	S kg. per sq. mm.
.6	6.3	1.90	9.1	2.31	11.8	2.54	14.6	2.67
.8	7.5	2.13	11.2	2.50	14.9	2.70	18.6	2.80
1.0	8.8	2.27	13.4	2.61	18.0	2.78	22.6	2.87
1.5	11.8	2.54	18.8	2.79	25.7	2.92	32.6	2.99
2.0	14.9	2.68	24.2	2.89	33.4	2.99	42.6	3.06

The stresses in the circumferential seams are one-half those in the longitudinal seams; hence, single riveting may be used when the longitudinal seams are double-riveted.

For spherical ends, or boiler heads which are formed in the shape of a segment of a sphere, if R is the radius of curvature, we have for the thickness, δ_1.

$$\delta_1 = R_1 \frac{a}{200S}.$$

The above formulas, when adapted for English measures, are as follows:

Let

D = diameter, in inches;
a = pressure, in atmospheres;
p = pressure, in pounds, per square inch;
δ = thickness of shell, in inches;
S = fibre stress, in pounds, per square inch.

We then have

$$\delta = 0.0015aD + 0.1,$$

$$S = \frac{p}{2} \cdot \frac{D}{2},$$

$$\delta_1 = \frac{R_1}{2} \cdot \frac{p}{S}.$$

For the usual diameters we have the following results:

$a =$	4 = 60 pounds.		7 = 105 pounds.		10 = 150 pounds.		13 = 175 pounds.	
D	δ	S	δ	S	δ	S	δ	S
24	.24	3000	.35	3600	.48	3750	.58	3700
36	.31	3500	.48	3900	.64	4000	.80	4000
42	.35	3600	.54	4000	.73	4300	.92	4000
72	.43	5000	.85	4400	1.18	4600	1.50	4200

The general character of the material entering into boiler work is well described in the specifications of the American Boiler Manufacturers' Association, given herewith:

Uniform American Boiler Specifications

Adopted by the American Boiler Manufacturers' Association.

(See Proceedings 1889, pages 49, 50, 66–81, 84–88.)
(See Proceedings 1897, pages 42–54, 61–77, 207–208.)
(See Proceedings 1898, pages 49–100.)

MATERIALS.

1. **Cast-iron.**—Should be of soft, gray texture and high degree of ductility. To be used only for hand-hole plates, crabs, yokes, etc., and man-heads. It is a dangerous metal to be used in mud drums, legs, necks, headers, man-hole rings, or any part of a boiler subject to tensile strains; its use is prohibited for such parts.

2. **Steel.**—Homogeneous steel made by the open-hearth or crucible processes, and having the following qualities, is to be used in all boilers.

Tensile Strength, Elongation, Chemical Tests.—Shell plates *not* exposed to the direct heat of the fire or gases of combustion, as in the external shells of internally-fired boilers, may have from 65,000 to 70,000 pounds tensile strength; elongation not less than 24 per cent. in 8 inches; phosphorus not over 0.035 per cent.; sulphur not over 0.035 per cent.

Shell plates in any way exposed to the direct heat of the fire or the gases of combustion, as in the external shells or heads of externally-fired boilers, or plates on which any flanging is to be done, to have from 60,000 to 65,000 pounds tensile strength; elongation not less than 27 per cent. in 8 inches; phosphorus not over 0.03 per cent.; sulphur not over 0.025 per cent.

Fire-box plates, or such as are exposed to the direct heat of the fire or flanged on the greater portion of their periphery, to have 55,000 to 62,000 pounds tensile strength; elongation, 30 per cent. in 8 inches; phosphorus not over 0.03 per cent.; sulphur not over 0.025 cent.

For all plates the elastic limit to be at least one-half the ultimate strength; percentage of manganese and carbon left to the judgment of the steel maker.

Test Section to be 8 inches long, planed or milled edges; its cross-sectional area not less than one-half of 1 square inch, nor width less than the thickness of the plate.

Bending Test.—Steel up to ½-inch thickness must stand bending double and being hammered down on itself; above that thickness it must bend round a mandrel of diameter of 1½ times the thickness of plate down to 180 degrees. All without showing signs of distress.

Bending test piece to be in length not less than 16 times the thickness of plate, and rough, shear edges milled or filed off. Such pieces to be cut both lengthwise and crosswise of the plate.

All tests to be made at the steel mill. Three pulling tests and three bending tests to be made from each heat. If one fails the manufacturer may furnish and test a fourth piece, but if two fail the entire heat to be rejected.

Certified copies of tests to be furnished each member of A. B. M. A. from heats from which his plates are made.

3. **Rivets** to be of good charcoal iron or a soft, mild steel, having the same physical and chemical properties as the fire-box plates, and must test hot and cold by driving down on an anvil with the head in a die, by nicking and bending, by bending back on themselves cold, without developing cracks or flaws.

4. **Boiler Tubes** of charcoal iron or mild steel specially made for the purpose, and lap-welded or drawn. They should be round, straight, free from scales, blisters, and mechanical defects, each tested to 500 pounds internal hydrostatic pressure.

This fact and manufacturer's name to be plainly stencilled on each tube.

Standard Thicknesses by Birmingham wire gauge to be

No. 13 for tubes 1 inch, 1¼ inches, 1½ inches, and 1¾ inches diameter;
No. 12 for tubes 2 inches, 2¼ inches, and 2½ inches diameter;
No. 11 for tubes 2¾ inches, 3 inches, 3¼ inches, and 3½ inches diameter;
No. 10 for tubes 3¾ inches and 4 inches diameter;
No. 9 for tubes 4½ inches and 5 inches diameter.

Tests.—A section cut from 1 tube taken at random from a lot of 150 or less must stand hammering down cold vertically without cracking or splitting when down solid.

Length of test pieces:

¾ inch for tubes from 1 inch to 1¾ inches diameter;
1 inch for tubes from 2 inches to 2½ inches diameter;
1¼ inches for tubes from 2¾ inches to 3¼ inches diameter;
1½ inches for tubes from 3½ inches to 4 inches diameter;
1¾ inches for tubes from 4½ inches to 5 inches diameter.

All tubes must stand expanding flange over on tube plate and bending without flaw, crack, or opening of the weld.

5. **Stay Bolts** to be made of iron or mild steel specially manufactured for the purpose, and must show on

Test Section 8 inches long, net:

For Iron, tensile strength not less than 46,000 pounds; elastic limit not less than 26,000 pounds; elongation not less than 22 per cent. for bolts of less than one (1) square inch area, nor less than 20 per cent. for bolts one (1) square inch and more in net area.

For Steel, tensile strength not less than 55,000 pounds; elastic limit not less than 33,000 pounds; elongation not less than 25 per cent. for bolts of less than one (1) square inch area, nor less than 22 per cent. for bolts one (1) square inch and more in net area.

Tests.—A bar taken from a lot of 1000 pounds or less at random, threaded with a sharp die "V" thread with rounded edges, must bend cold 180° around a bar of same diameter without showing any crack or flaws.

Another piece, similarly chosen and threaded, to be screwed into well-fitting nuts formed of pieces of the plates to be stayed, and riveted over so as to form an exact counterpart of the bolt in the finished structure; to be pulled in testing machine and breaking stress noted; if it fails by pulling apart the tensile stress per square inch of net section is its measure of strength; if it fails by shearing the shear stress per square inch of mean section in shear is this measure. The mean section in shear is the product of half the thickness of the plate by the circumference at half height of thread.

6. **Braces and Stays.**—Material to be fully equal to stay-bolt stock, and tensile strength to be determined by testing a bar not less than ten (10) inches long from each lot of 1000 pounds or less.

II. WORKMANSHIP AND DIMENSIONS.

7. **Flanging, Bending, and Forming** to be done at a heat suited to the material, but no bending must be done or blow struck on any plate

which no longer shows red by daylight at the working point and at least 4 inches beyond it.

8. **Rolling** must be done cold by gradual and regular increments from the straight plate to the exact circle required, and the whole circumference, including the lap, rolled to a true circle.

9. **Bumped Head** uniformly dished to a segment of a sphere should have a thickness equal to that of a cylindrical shell of solid plate of same material, whose diameter is equal to the radius of curvature of the dished head.

Rivet-holes, man-holes, etc., to be allowed for by proportionate increase in the thickness.

10. **Riveting.**—Holes made perfectly true and fair by clean-cutting punches or drills. Sharp edges and burrs removed by slight countersinking and burr-reaming before and after sheets are joined together.

Under side of original rivet head must be flat, square, and smooth. For rivets $\frac{5}{8}$ inch to $\frac{13}{16}$ inch diameter allow $1\frac{1}{2}$ diameters for length of stock to form the head, and less for larger rivets. Allow 5 per cent. more stock for driven head for button set or snap rivets. Use light regulation riveting hammers until rivet is well upset in the hole; after that, snap and heavy mauls. For machine riveting more stock to be left for driven head to make it equal to original head, as fixed by experiment.

Total pressure on the die about 80 tons for $1\frac{1}{8}$-inch to $1\frac{1}{4}$-inch rivets; 65 tons for 1-inch rivets; 57 tons for $\frac{15}{16}$-inch rivets; 35 tons for $\frac{3}{4}$-inch rivets.

Make heads of rivets equal in strength to shanks by making head at periphery of shank of a height equal to one-third the diameter of shank and giving a slight fillet at this point.

Approximately, make rivet-holes double thickness of thinnest plate; pitch, 3 times rivet-hole; pitch lines of staggered rows $\frac{1}{2}$ pitch apart; lap for single riveting equal to pitch, for double riveting $1\frac{1}{3}$ pitch, and $\frac{1}{2}$ pitch more for each additional row of rivets; *exact dimensions determined by* making resistance to shear of aggregate rivet section at least 10 per cent. greater than tensile strength of net or standing metal.

11. **Rivet-holes** punched with good, sharp punches and well-fitting dies in A. B. M. A. steel up to $\frac{5}{8}$-inch thickness; in thicker plates punch and ream with a fluted reamer or drill the holes.

12. **Drift Pin** to be used only with light hammers to pull plates into place and round up the hole, but never to enlarge or gouge holes with heavy hammers.

13. **Calking** to be done by hand or pneumatic hammer and Conery or round-nosed tool. Avoid excessive calking; the fit must be made in the laying of the plates. The square-nosed tool may be used for finishing, with great care to avoid nicking lower plate. Calking edges must be prepared by bevel planing, shearing, or chipping.

14. **Flat Surfaces.**—State the thickness of the plate, t, in sixteenths of an inch; the pitch, p, in inches, and use a constant:

$C = 112$ for plates $\frac{7}{16}$ inch and under, with screw stays with riveted ends.
$C = 120$ for plates over $\frac{7}{16}$ inch, with screw stays with riveted ends.
$C = 140$ for all plates when, in addition to screw threads in the plates, a nut is used inside and outside of each plate.

When salt, acids, or alkali are contained in the feed water, this latter construction is imperative.

Rule.—Multiply this constant, C, by the square of the thickness of the plate expressed in sixteenths of an inch, and divide by the square of the pitch expressed in inches; the quotient is the safe working pressure, P.

$$P = \frac{CXt^2}{p^2}.$$

15. **Tube-holes,** either punched $\frac{1}{8}$ inch less than required diameter and reamed to full size, or drilled, then slightly countersunk on both sides, should be $\frac{1}{64}$ inch to $\frac{1}{16}$ inch larger than diameter of tube, according to

size of tube; if copper ferrules are used, the hole to be a neat fit for the ferrule. Tube sheet to be annealed after punching and before reaming.

16. **Tube Setting.**—Ends of tubes to be annealed (in the tube mill) before setting. The tube to extend through the sheet $\frac{1}{16}$ inch for every inch of diameter. Expand until tight in hole and no more. On end exposed to direct flame, flange the tube partly over on sheet, finishing by beading tool, which must not come in contact with the plate; expand slightly after beading.

Copper ferrules, No. 18 to 14 wire gauge, should be used in fire-tube boilers on ends subject to direct heat.

17. **Riveted and Lap-welded Flues,** as prescribed in Rule II., Sections 8, 9, 10, 11, 12, and 13 of Regulations of Board of Supervising Inspectors of Steam Vessels, approved February, 1895.

18. **Corrugated Furnace Flues,** as prescribed in Sections 14 and 15 of the same Rule.

19. **Stay Bolts** to be carefully threaded with sharp, clean dies, "V" thread, with rounded edges; threading machine equipped with a lead screw; holes tapped with tap extending through both sheets to neat, smooth fit, so that bolts can be put in by hand-lever or wrench with a steady pull; $\frac{1}{4}$ diameter to project for riveting over; with hollow stay bolts use slender drift pin in the bore while riveting, and drive it home to expand the bolt after riveting.

Height of nuts used on screw stays to be at least 50 per cent. of diameter of stay. Largest permissible pitch for screw stays is 10 inches.

20. **Braces and Stays** shall be subjected to careful inspection and tests, as per Sections 6 and 2. Welding to be avoided where possible, but good, clean welds to be allowed a value of 80 per cent. of the solid bar. Rivets by which braces are attached, when the pull on them is other than at right angles, to be allowed only half the stress permitted for rivets in the seams.

21. **Man-holes** should be flanged in, out of the solid plate, on a radius not less than 3 times the metal thickness to a straight flange; when the plate is $\frac{1}{2}$ inch or less in thickness a reinforce ring to be shrunk around it. Cast-iron reinforce flanges never to be used.

22. **Domes** to be avoided when possible; cylindrical portion to be flanged down to the shell of the boiler, and this shell flanged up inside the dome or reinforced by a collar flanged at the joint, the flanges double-riveted.

23. **Drums** should be put on with collar flanges of A. B. M. A. steel not less than $\frac{3}{8}$ inch thick, double-riveted to shell and drum and single-riveted to the neck or leg, or the flanges may be formed on these legs.

24. **Saddles or Nozzles** to be of flanged steel plate or of soft cast-steel, never of cast-iron.

III. FACTORS OF SAFETY.

25. **Rivet Seams,** when proportioned as prescribed in Section 10 with materials tested as per Sections 2 and 3, shall have $4\frac{1}{2}$ as factor of safety; when not so tested, but inspection of materials indicates good quality, a factor of safety of 5 is to be taken, and at most 55,000 pounds tensile strength assumed for the steel plate and 40,000 pounds shear strength for the rivets, all figured on the actual net standing metal.

26. **Flat Surfaces,** proportioned as per Section 14, have, in the constants there given, a factor of safety of 5 or a little over.

27. **Bumped Heads,** proportioned as per Section 9, to be subject to a factor of safety of 5.

28. **Stay Bolts,** proportioned and tested as per Sections 19 and 5, to have a factor of safety of 5 applied to the lowest stress found.

29. **Braces and Stays,** when tested as per Sections 6 and 2, to be allowed a factor of safety of 5; when not so tested, but careful inspection

shows good stock, they may be used up to 6500 pounds actual direct pull for wrought-iron, and 8000 pounds for mild steel, all per square inch of actual net metal.

IV. HYDROSTATIC PRESSURE.

30. **The hydrostatic test** to be made on completed boilers built strictly to these specifications is never to exceed working pressure by more than one-third of itself, and this excess limited to 100 pounds per square inch. The water used for testing to have a temperature of at least 125° F.

V. HANGING OR SUPPORTING THE BOILER.

31. **The boiler should be supported** on points where there is the greatest excess of stress. Excessive local stresses from weight of boiler and contents must be avoided, and distortion of parts prevented, by using long lugs or brackets; and only half the stress which they may carry in the seams to be allowed on rivets.

The supports must permit rebuilding the furnace without disturbing the proper suspension of the boiler. The boiler should be slightly inclined, so that a little less water shows at the gauge cocks than at the opposite end.

SAFETY VALVES.

Weighted Valves.

Let

A = area of valve, in square inches;
F = distance from centre of valve to fulcrum, in inches;
L = length of lever, in inches, from fulcrum to weight;
W = weight of ball, in pounds;
P = blowing-off pressure, in pounds, per square inch.

Then we have

$$P = \frac{WL}{AF},$$

$$L = \frac{AFP}{W},$$

$$W = \frac{AFP}{L}.$$

If lever is not balanced, its effect, and the effect of valve and spindle, must be added to pressure and be taken into account in calculating L and W. If w = weight of lever and v = weight of valve and spindle, in pounds; c = distance of centre of gravity of lever from fulcrum; then, if p = pressure per square inch on valve due to weight of lever and valve alone,

$$p = \frac{w \times c}{AF} + \frac{v}{A}.$$

In most cases effect of valve and spindle may be neglected. With long, heavy levers p will require adding to P to ascertain the blowing-off pressure.

Various rules are given for the area of safety valves, these usually being based on a certain number of square inches of valve area per square foot of grate surface, although sometimes the area of the valve is based on the heating surface of the boiler.

The United States Treasury Department, through its Board of Supervising Inspectors of Steam Vessels, has established the following rules:

"Lever safety valves to be attached to marine boilers shall have an area of not less than *one square inch to two square feet* of grate surface in

the boiler, and the seats of all such safety valves shall have an angle of inclination of 45° to the centre line of their axes.

"The valves shall be so arranged that each boiler shall have one separate safety valve, unless the arrangement is such as to preclude the possibility of shutting off the communication of any boiler with the safety valve or valves employed. This arrangement shall also apply to lock-up safety valves when they are employed.

"Any spring-loaded safety valves constructed so as to give an increased lift by the operation of steam after being raised from their seats, or any spring-loaded safety valve constructed in any other manner, or so as to give an effective area equal to that of the afore-mentioned spring-loaded safety valve, may be used in lieu of the common lever-weighted valves on all boilers on steam vessels, and all such spring-loaded safety valves shall be required to have an area of not less than 1 square inch to 3 square feet of grate surface of the boiler, and each spring-loaded safety valve shall be supplied with a lever that will raise the valve from its seat a distance of not less than that equal to one-eighth the diameter of the valve opening, and the seats of all such safety valves shall have an angle of inclination to the centre line of their axis of 45°. But in no case shall any spring-loaded safety valve be used in lieu of the lever-weighted safety valve without having first been approved by the Board of Supervising Inspectors."

The Boiler Inspection Department of the city of Philadelphia gives the following formula for boilers with natural draft:

$$A = \frac{22.5G}{P + 8.62},$$

in which A is the area of combined safety valves, in inches; G is area of grate, in square feet; P is pressure of steam, in pounds, per square inch to be carried in the boiler above the atmosphere.

The following table gives the results of the formula for 1 square foot of grate, as applied to boilers used at different pressures.

Pressure per Square Inch.

10	20	30	40	50	60	70	80	90	100	110	120	150	175
1.21	0.79	0.58	0.46	0.38	0.33	0.29	0.25	0.23	0.21	0.19	0.17	0.142	0.123

Valve area in square inches, corresponding to 1 square foot of grate.

When forced draft is used, the area of grate for purposes of safety valve computation is to be estimated at 1 square foot for each 16 pounds of fuel burned per hour.

Hutton's rule is

$$A = \frac{4G}{\sqrt{P}}.$$

A = area of valve, in square inches;
G = area of grate, in square feet;
P = pressure, in pounds, per square inch.

The area of a safety valve may be determined from the evaporative power of the boiler.

Let

A = area of safety valve, in square inches;
P = steam pressure, in pounds, per square inch;
E = evaporative capacity of the boiler, in pounds of water, per hour.

Then we have

$$A = \frac{E}{40\sqrt{P}}.$$

Minimum Size of Safety Valve Areas Allowed by Board of Trade.

Boiler pressure, in pounds.	Area of valve per square foot of fire-grate, in square inches.	Boiler pressure, in pounds.	Area of valve per square foot of fire-grate, in square inches.	Boiler pressure, in pounds.	Area of valve per square foot of fire-grate, in square inches.	Boiler pressure, in pounds.	Area of valve per square foot of fire-grate, in square inches.	Boiler pressure, in pounds.	Area of valve per square foot of fire-grate, in square inches.
15	1.250	52	.559	89	.360	126	.265	163	.210
16	1.209	53	.551	90	.357	127	.264	164	.209
17	1.171	54	.543	91	.353	128	.262	165	.208
18	1.136	55	.535	92	.350	129	.260	166	.207
19	1.102	56	.528	93	.347	130	.258	167	.206
20	1.071	57	.520	94	.344	131	.256	168	.204
21	1.041	58	.513	95	.340	132	.255	169	.203
22	1.013	59	.506	96	.337	133	.253	170	.202
23	.986	60	.500	97	.334	134	.251	171	.201
24	.961	61	.493	98	.331	135	.250	172	.200
25	.937	62	.487	99	.328	136	.248	173	.199
26	.914	63	.480	100	.326	137	.246	174	.198
27	.892	64	.474	101	.323	138	.245	175	.197
28	.872	65	.468	102	.320	139	.243	176	.196
29	.852	66	.462	103	.317	140	.241	177	.195
30	.833	67	.457	104	.315	141	.240	178	.194
31	.815	68	.451	105	.312	142	.238	179	.193
32	.797	69	.446	106	.309	143	.237	180	.192
33	.781	70	.441	107	.307	144	.235	181	.191
34	.765	71	.436	108	.304	145	.234	182	.190
35	.750	72	.431	109	.302	146	.232	183	.189
36	.735	73	.426	110	.300	147	.231	184	.188
37	.721	74	.421	111	.297	148	.230	185	.187
38	.707	75	.416	112	.295	149	.228	186	.186
39	.694	76	.412	113	.292	150	.227	187	.185
40	.681	77	.407	114	.290	151	.225	188	.184
41	.669	78	.403	115	.288	152	.224	189	.183
42	.657	79	.398	116	.286	153	.223	190	.182
43	.646	80	.394	117	.284	154	.221	191	.181
44	.635	81	.390	118	.281	155	.220	192	.181
45	.625	82	.386	119	.279	156	.219	193	.180
46	.614	83	.382	120	.277	157	.218	194	.179
47	.604	84	.378	121	.275	158	.216	195	.178
48	.595	85	.375	122	.273	159	.215	196	.177
49	.585	86	.371	123	.271	160	.214	197	.176
50	.576	87	.367	124	.269	161	.213	198	.176
51	.568	88	.364	125	.267	162	.211	200	.174

Lloyd's Rules for Safety Valves.

Two safety valves to be fitted to each boiler and loaded to the working pressure in the presence of the surveyor. In the case of boilers of greater working pressure than 60 pounds per square inch, the safety valves may be loaded to 5 pounds above the working pressure. If common valves are used, their combined areas to be at least half a square inch to each square foot of grate surface. If improved valves are used, they are to be tested under steam in the presence of the surveyor; the accumulation in no case to exceed 10 per cent. of the working pressure.

An approved safety valve also to be fitted to the superheater.

In winch boilers one safety valve will be allowed, provided its area be not less than half a square inch per square foot of grate surface.

Each valve to be arranged so that no extra load can be added when steam is up, and to be fitted with easing gear, which must lift the valve itself. All safety valve spindles to extend through the covers and to be fitted with sockets and cross handles, allowing them to be lifted and turned round in their seats, and their efficiency tested at any time.

The German rule for safety valves, as given in the "Ingenieurs Taschenbuch Hütte," is

$$f = 15\sqrt{\frac{v}{p}},$$

in which f is the area of valve, in square millimetres, per square metre of heating surface in the boiler; p is the maximum boiler pressure, in atmospheres; and v is the volume of steam, in litres, per kilogramme at the pressure, p, as given in the steam tables.

We have from this formula

Areas of Safety Valves,

in Square Millimetres, per Square Metre of Heating Surface.

p = press. atm. .	1	2	3	4	5	6	7	8	9	10	11	12	13	14
v = sp. volume.	896	612	467	378	313	276	244	218	198	181	167	154	144	135
f = sq. mm. per sq. metre..	449	263	188	147	119	102	89	78	71	63	59	54	50	48

INCRUSTATION IN BOILERS.

Whenever possible, pure water should be used for feeding boilers. When the water is impure the result is the formation of scale, producing diminished efficiency and possible injury to the boiler from overheating.

The principal impurities in water are calcium carbonate and calcium sulphate, together with suspended earth and organic matter. Water of condensation from steam engines contains more or less oil from the lubricant used in the steam, and this may produce a very injurious coating in the boiler.

By far the best plan is to remove or neutralize the impurities of the water before it is fed into the boiler, since the scale, when it is once formed, is difficult to remove.

No general rules can be given for the purification of water, since different waters require different treatment. The best plan is to have the water analyzed and adopt the course indicated by the nature of the salts found in it.

The following extracts from a paper by Messrs. Hunt and Clapp, in the "Transactions of the American Institute of Mining Engineers for 1888," is an authoritative statement of the subject:

"By far the most common commercial analysis of water is made to determine its fitness for making steam. Water containing more than 5 parts per 100,000 of free sulphuric or nitric acid is liable to cause serious corrosion, not only of the metal of the boiler itself, but of the pipes, cylinders, pistons, and valves with which the steam comes in contact. Sulphuric acid is the only one of these acids liable to be present in the water

from natural sources, it being often produced in the water of the coal and iron districts by the oxidation of iron pyrites to sulphate of iron, which, being soluble, is lixiviated from the earth strata and carried into the stream, the presence of organic matter taken up by the water in its after-course reducing the iron and lining the bottom of the stream with red oxide of iron, leaving a considerable proportion of the sulphuric acid free in the water. This is a troublesome feature with the water necessarily used in many of the iron districts of this country. The sulphuric acid may come from other natural chemical reactions than the one described above. Muriatic and nitric acids, as well as sulphuric acid, may be conveyed into water through the refuse of various kinds of manufacturing establishments being discharged into it.

"The large total residue in water used for making steam causes the interior linings of the boilers to become coated, clogs their action, and often produces a dangerous hard scale, which prevents the cooling action of the water from protecting the metal against burning.

"Lime and magnesia bicarbonates in water lose their excess of carbonic acid on boiling, and often, especially when the water contains sulphuric acid, produce, with the other solid residues constantly being formed by the evaporation, a very hard and insoluble scale.

"A larger amount than 100 parts per 100,000 of total solid residue will ordinarily cause troublesome scale, and should condemn the water for use in steam boilers, unless a better supply cannot be obtained.

"The following is a tabulated form of the causes of trouble with water for steam purposes, and the proposed remedies, given by Professor L. M. Norton in his lecture on 'Industrial Chemistry.'

"Causes of Incrustation.

"1. Deposition of suspended matter.

"2. Deposition of dissolved salts from concentration.

"3. Deposition of carbonates of lime and magnesia by boiling off carbonic acid, which holds them in solution.

"4. Deposition of sulphates of lime, because sulphate of lime is but slightly soluble in cold water, less soluble in hot water, insoluble above 140° C. (284° F.).

"5. Deposition of magnesia, because magnesium salts decompose at high temperature.

"6. Deposition of lime soap, iron soap, etc., formed by saponification of grease.

"Various Means of Preventing Incrustation.

"1. Filtration.

"2. Blowing off.

"3. Use of internal collecting apparatus or devices for directing the circulation.

"4. Heating feed water.

"5. Chemical or other treatment of water in boiler.

"6. Introduction of zinc into boiler.

"7. Chemical treatment of water outside of boiler."

Prevention and Cure of Boiler Troubles Due to Water.

Incrustation.	Sediment, mud, clay, etc.	Filtration. Blowing off.
	Readily soluble salts	Blowing off.
	Bicarbonate of magnesia, lime, and iron	Heating feed and precipitating. Caustic soda. Lime. Magnesia.
	Sulphate of lime	Carbonate of soda. Barium chloride.

Corrosion	Organic matter	Precipitate with alum and filter. Precipitate with ferric chloride and filter.
	Grease	Slaked lime and filter. Carbonate of soda and filter.
	Chloride or sulphate of magnesia	Carbonate of soda.
	Acid	Alkali.
	Dissolved carbonic acid and oxygen	Slaked lime. Caustic soda. Heating.
Priming	Sewage	Precipitate with alum or ferric chloride and filter.
	Carbonate of soda in large quantities	Barium chloride.

The following table shows the solubility of various scale-making materials in steam boilers, showing in the last column the temperatures at which they become insoluble. Although sulphate of lime does not become entirely insoluble until a temperature of nearly 400° F., corresponding to a pressure of about 225 pounds, a large proportion of it is precipitated at about 310° F., or about 65 pounds pressure. It will be seen, therefore, that most of these impurities may be precipitated by using a feed-water heater of sufficient size to permit the precipitated impurities to settle and be blown off before passing into the boiler.

Solubilities of Scale-making Minerals.

Substance.	Soluble in parts of pure water at 30° F.	Soluble in parts of carbonic acid, water cold.	Soluble in parts of pure water at 212° F.	Insoluble in water at
Carbonate of lime	62500	150	62500	302° F.
Sulphate of lime	500	....	460	392° F.
Carbonate of magnesia	5500	150	9600	....
Phosphate of lime	....	1333	....	212° F.
Oxide of iron	....	....	....	212° F.
Silica	....	Undetermined.	....	212° F.

Analyses of Boiler Scale.

(Chandler.)

Sulphate of lime.	Magnesia.	Silica.	Peroxide of iron.	Water.	Carbonate of lime.
74.07	9.19	.65	.08	1.14	14.78
71.37	....	1.76	....	....	....
62.86	18.95	2.60	.92	1.28	12.62
53.05	....	4.79	....	....	....
46.83	....	5.32	....	....	....
30.80	31.17	7.75	1.08	2.44	26.93
4.95	2.61	2.07	1.03	.63	86.25
.88	2.84	.65	.36	.15	93.19
4.81	....	2.92	....	....	....
30.07	....	8.24	....	....	....

Analysis, in Parts per 100,000, of Water Giving Bad Results in Steam Boilers.

(A. E. Hunt.)

Waters.	Bicarbonate of lime deposited on boiling.	Bicarbonate of magnesia deposited on boiling.	Total lime.	Total magnesia.	Sulphuric acid.	Chlorine.	Iron.	Organic matter.	Alumina.	Chloride of sodium.
Coal-mine water......	110	25	119.0	39.00	890	590.0	780	30	640	....
Salt well..............	151	38	1.9	48.00	360	990.0	38	21	30	13.1
Spring	75	89	95.0	120.00	310	21.0	75	10	80	36.0
Monongahela River ..	130	21	161.0	33.00	210	38.0	70	....	....	....
Monongahela River ..	80	70	94.0	81.00	219	210.0	90	....	....	....
Monongahela River ..	32	82	61.0	1.04	28	1.9	38	....	....	....
Allegheny River, near oil-works......	30	50	41.0	68.00	890	42.0	23	....	....	....

THE STEAM ENGINE.

Horse-power.

The measure of the power of steam engines is the **Horse-power,** originally selected by Watt as a basis on which to sell his engines. Tests of a number of powerful draught horses showed an effort corresponding to 22,000 foot-pounds per minute, and Watt increased this by 50 per cent., in order to assure his customers that he was furnishing ample power; this being the origin of the well-known value of 33,000 foot-pounds per minute, or 550 foot-pounds per second, as a commercial horse-power.

In the metric system the *cheval-vapeur* is taken as 75 kilogrammetres per second, this corresponding to 32,548 foot-pounds per minute, the metric horse-power thus being 0.9863 times the British horse-power. The latter will always be understood, unless otherwise stated.

In France it has been suggested to use a new unit, equal to 100 kilogrammetres per second, this being called the *Poncelet*, and being practically equivalent to the kilowatt.

Since 1 B. T. U. = 778 foot-pounds, it requires the expenditure of 42.416 B. T. U. per minute to produce 1 horse-power, if all the heat is converted into mechanical energy.

In the steam engine the power is usually developed by the pressure of the expansive force of the steam upon the piston in the cylinder. Since the speed of the piston is not uniform, varying from zero to a maximum twice for every revolution of the crank, it is necessary to take the total distance travelled in one minute as the average or mean speed.

The pressure of the steam upon the piston is also variable, and hence it is necessary to determine the mean effective pressure, in order that the horse-power may be computed. For a completed engine the mean effective pressure may be determined by use of the indicator, but for a proposed design it is computed in accordance with the laws of the expansion of steam.

According to the law of Mariotte, considering steam as a gas, the product of the pressure and the volume is constant, or

$$pv = C.$$

When the steam in a cylinder be permitted to follow a portion of the stroke at full boiler pressure, and is then cut off and allowed to expand for the remainder of the stroke, the expansion curve may be considered as an equilateral hyperbola, the pressure at any point being inversely as the volume. When the volume has been doubled, the pressure will fall to one-half the initial; when it becomes three times what it was at the point of cut-off, the pressure will be one-third the initial pressure, and so on. In this way it is quite possible to construct a theoretical diagram for any degree of cut-off or any expansion ratio, and measure the mean pressure throughout the stroke.

Instead of performing this work, however, the mean effective pressure may be computed immediately by means of a table of hyperbolic logarithms.

Let P = initial pressure, absolute,—*i.e.*, above vacuum;
p = mean effective pressure, including vacuum;
r = expansion ratio = total stroke divided by length up to point of cut-off.

Then

$$p = P \cdot \frac{1 + \text{hyp. log. } r}{r}.$$

Hence, by taking the hyperbolic logarithm of the expansion ratio and adding 1, and dividing by the expansion ratio, we have a number which, multiplied by the initial pressure, will give the mean effective pressure.

Thus, if the steam is admitted at 100 pounds gauge pressure, or 114.7 pounds absolute pressure, and cut off at ¼ the stroke, we have

$$r = 4,$$

and

$$p = 114.7 \frac{1 + \text{hyp. log. } 4}{4}.$$

The hyperbolic logarithm of 4 is 1.3863, and hence we have

$$\begin{aligned} p &= 114.7 \frac{1 + 1.3863}{4} \\ &= 114.7 \times 0.5966 \\ &= 68.43 \text{ pounds absolute} \\ &= 53.73 \text{ pounds above atmosphere.} \end{aligned}$$

There is always a loss of pressure in practice due to cylinder consideration, etc., and in practice about 70 per cent. of the theoretical mean effective pressure is attained.

In the above computations care must be taken always to use the *absolute* pressure,—*i.e.*, the pressure above vacuum,—the resulting mean effective pressure being that existing above vacuum. For a high-pressure engine, therefore, atmospheric pressure must be deducted.

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
1.01	.009 9503	1.65	.500 7752	2.29	.828 5518	2.93	1.075 0024
1.02	.019 8026	1.66	.506 8175	2.30	.832 9091	2.94	1.078 4095
1.03	.029 5588	1.67	.512 8236	2.31	.837 2475	2.95	1.081 8051
1.04	.039 2207	1.68	.518 7937	2.32	.841 5671	2.96	1.085 1892
1.05	.048 7902	1.69	.524 7285	2.33	.845 8682	2.97	1.088 5619
1.06	.058 2689	1.70	.530 6282	2.34	.850 1509	2.98	1.091 9233
1.07	.067 6586	1.71	.536 4933	2.35	.854 4153	2.99	1.095 2733
1.08	.076 9610	1.72	.542 3242	2.36	.858 6616	3.00	1.098 6123
1.09	.086 1777	1.73	.548 1214	2.37	.862 8899	3.01	1.101 9400
1.10	.095 3102	1.74	.553 8851	2.38	.867 1004	3.02	1.105 2568
1.11	.104 3600	1.75	.559 6157	2.39	.871 2933	3.03	1.108 5626
1.12	.113 3287	1.76	.565 3138	2.40	.875 4687	3.04	1.111 8575
1.13	.122 2176	1.77	.570 9795	2.41	.879 6267	3.05	1.115 1415
1.14	.131 0283	1.78	.576 6133	2.42	.883 7675	3.06	1.118 4149
1.15	.139 7619	1.79	.582 2156	2.43	.887 8912	3.07	1.121 6775
1.16	.148 4200	1.80	.587 7866	2.44	.891 9980	3.08	1.124 9295
1.17	.157 0037	1.81	.593 3268	2.45	.896 0880	3.09	1.128 1710
1.18	.165 5144	1.82	.598 8365	2.46	.900 1613	3.10	1.131 4021
1.19	.173 9533	1.83	.604 3159	2.47	.904 2181	3.11	1.134 6227
1.20	.182 3215	1.84	.609 7655	2.48	.908 2585	3.12	1.137 8330
1.21	.190 6203	1.85	.615 1856	2.49	.912 2826	3.13	1.141 0330
1.22	.198 8508	1.86	.620 5764	2.50	.916 2907	3.14	1.144 2227
1.23	.207 0141	1.87	.625 9384	2.51	.920 2827	3.15	1.147 4024
1.24	.215 1113	1.88	.631 2717	2.52	.924 2589	3.16	1.150 5720
1.25	.223 1435	1.89	.636 5768	2.53	.928 2193	3.17	1.153 7315
1.26	.231 1117	1.90	.641 8538	2.54	.932 1640	3.18	1.156 8811
1.27	.239 0169	1.91	.647 1032	2.55	.936 0933	3.19	1.160 0209
1.28	.246 8600	1.92	.652 3251	2.56	.940 0072	3.20	1.163 1508
1.29	.254 6422	1.93	.657 5200	2.57	.943 9058	3.21	1.166 2709
1.30	.262 3642	1.94	.662 6879	2.58	.947 7893	3.22	1.169 3813
1.31	.270 0271	1.95	.667 8293	2.59	.951 6578	3.23	1.172 4821
1.32	.277 6317	1.96	.672 9444	2.60	.955 5114	3.24	1.175 5733
1.33	.285 1789	1.97	.678 0335	2.61	.959 3502	3.25	1.178 6549
1.34	.292 6696	1.98	.683 0968	2.62	.963 1743	3.26	1.181 7271
1.35	.300 1045	1.99	.688 1346	2.63	.966 9838	3.27	1.184 7899
1.36	.307 4846	2.00	.693 1472	2.64	.970 7789	3.28	1.187 8434
1.37	.314 8107	2.01	.698 1347	2.65	.974 5596	3.29	1.190 8875
1.38	.322 0834	2.02	.703 0974	2.66	.978 3261	3.30	1.193 9224
1.39	.329 3037	2.03	.708 0357	2.67	.982 0784	3.31	1.196 9481
1.40	.336 4722	2.04	.712 9497	2.68	.985 8167	3.32	1.199 9647
1.41	.343 5897	2.05	.717 8397	2.69	.989 5411	3.33	1.202 9722
1.42	.350 6568	2.06	.722 7059	2.70	.993 2517	3.34	1.205 9707
1.43	.357 6744	2.07	.727 5485	2.71	.996 9486	3.35	1.208 9603
1.44	.364 6431	2.08	.732 3678	2.72	1.000 6318	3.36	1.211 9409
1.45	.371 5635	2.09	.737 1640	2.73	1.004 3015	3.37	1.214 9127
1.46	.378 4364	2.10	.741 9373	2.74	1.007 9579	3.38	1.217 8757
1.47	.385 2624	2.11	.746 6879	2.75	1.011 6008	3.39	1.220 8299
1.48	.392 0420	2.12	.751 4160	2.76	1.015 2306	3.40	1.223 7754
1.49	.398 7761	2.13	.756 1219	2.77	1.018 8473	3.41	1.226 7122
1.50	.405 4651	2.14	.760 8058	2.78	1.022 4509	3.42	1.229 6405
1.51	.412 1096	2.15	.765 4678	2.79	1.026 0415	3.43	1.232 5605
1.52	.418 7103	2.16	.770 1082	2.80	1.029 6194	3.44	1.235 4714
1.53	.425 2677	2.17	.774 7271	2.81	1.033 1844	3.45	1.238 3742
1.54	.431 7824	2.18	.779 3248	2.82	1.036 7368	3.46	1.241 2685
1.55	.438 2549	2.19	.783 9015	2.83	1.040 2766	3.47	1.244 1545
1.56	.444 6858	2.20	.788 4573	2 84	1.043 8040	3.48	1.247 0322
1.57	.451 0756	2.21	.792 9925	2.85	1.047 3189	3.49	1.249 9017
1.58	.457 4248	2.22	.797 5071	2.86	1.050 8216	3.50	1.252 7629
1.59	.463 7340	2.23	.802 0015	2.87	1.054 3120	3.51	1.255 6160
1.60	.470 0036	2.24	.806 4758	2.88	1.057 7902	3.52	1.258 4609
1.61	.476 2341	2.25	.810 9302	2.89	1.061 2564	3.53	1.261 2978
1.62	.482 4261	2.26	.815 3648	2.90	1.064 7107	3.54	1.264 1266
1.63	.488 5800	2.27	.819 7798	2.91	1.068 1530	3.55	1.266 9475
1.64	.494 6962	2.28	.824 1754	2.92	1.071 5836	3.56	1.269 7605

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
3.57	1.272 5655	4.21	1.437 4626	4.85	1.578 9787	5.49	1.702 9282
3.58	1.275 3627	4.22	1.439 8351	4.86	1.581 0384	5.50	1.704 7481
3.59	1.278 1521	4.23	1.442 2020	4.87	1.583 0939	5.51	1.706 5646
3.60	1.280 9338	4.24	1.444 5632	4.88	1.585 1452	5.52	1.708 3778
3.61	1.283 7077	4.25	1.446 9189	4.89	1.587 1923	5.53	1.710 1878
3.62	1.286 4740	4.26	1.449 2691	4.90	1.589 2352	5.54	1.711 9944
3.63	1.289 2326	4.27	1.451 6138	4.91	1.591 2739	5.55	1.713 7979
3.64	1.291 9836	4.28	1.453 9530	4.92	1.593 3085	5.56	1.715 5981
3.65	1.294 7271	4.29	1.456 2867	4.93	1.595 3389	5.57	1.717 3950
3.66	1.297 4631	4.30	1.458 6149	4.94	1.597 3653	5.58	1.719 1887
3.67	1.300 1916	4.31	1.460 9379	4.95	1.599 3875	5.59	1.720 9792
3.68	1.302 9127	4.32	1.463 2553	4.96	1.601 4057	5.60	1.722 7666
3.69	1.305 6264	4.33	1.465 5675	4.97	1.603 4198	5.61	1.724 5507
3.70	1.308 3328	4.34	1.467 8743	4.98	1.605 4298	5.62	1.726 3316
3.71	1.311 0318	4.35	1.470 1758	4.99	1.607 4358	5.63	1.728 1094
3.72	1.313 7236	4.36	1.472 4720	5.00	1.609 4379	5.64	1.729 8840
3.73	1.316 4082	4.37	1.474 7630	5.01	1.611 4359	5.65	1.731 6555
3.74	1.319 0856	4.38	1.477 0487	5.02	1.613 4300	5.66	1.733 4238
3.75	1.321 7558	4.39	1.479 3292	5.03	1.615 4200	5.67	1.735 1891
3.76	1.324 4189	4.40	1.481 6045	5.04	1.617 4060	5.68	1.736 9512
3.77	1.327 0749	4.41	1.483 8746	5.05	1.619 3882	5.69	1.738 7102
3.78	1.329 7240	4.42	1.486 1396	5.06	1.621 3664	5.70	1.740 4661
3.79	1.332 3660	4.43	1.488 3995	5.07	1.623 3408	5.71	1.742 2189
3.80	1.335 0010	4.44	1.490 6543	5.08	1.625 3112	5.72	1.743 9687
3.81	1.337 6291	4.45	1.492 9040	5.09	1.627 2778	5.73	1.745 7155
3.82	1.340 2504	4.46	1.495 1487	5.10	1.629 2405	5.74	1.747 4591
3.83	1.342 8648	4.47	1.497 3883	5.11	1.631 1994	5.75	1.749 1998
3.84	1.345 4723	4.48	1.499 6230	5.12	1.633 1544	5.76	1.750 9374
3.85	1.348 0731	4.49	1.501 8527	5.13	1.635 1056	5.77	1.752 6720
3.86	1.350 6671	4.50	1.504 0774	5.14	1.637 0530	5.78	1.754 4036
3.87	1.353 2544	4.51	1.506 2971	5.15	1.638 9967	5.79	1.756 1323
3.88	1.355 8351	4.52	1.508 5119	5.16	1.640 9365	5.80	1.757 8579
3.89	1.358 4091	4.53	1.510 7219	5.17	1.642 8726	5.81	1.759 5805
3.90	1.360 9765	4.54	1.512 9269	5.18	1.644 8050	5.82	1.761 3002
3.91	1.363 5373	4.55	1.515 1272	5.19	1.646 7336	5.83	1.763 0170
3.92	1.366 0916	4.56	1.517 3226	5.20	1.648 6586	5.84	1.764 7308
3.93	1.368 6394	4.57	1.519 5132	5.21	1.650 5798	5.85	1.766 4416
3.94	1.371 1807	4.58	1.521 6990	5.22	1.652 4974	5.86	1.768 1496
3.95	1.373 7156	4.59	1.523 8800	5.23	1.654 4112	5.87	1.769 8546
3.96	1.376 2440	4.60	1.526 0563	5.24	1.656 3214	5.88	1.771 5567
3.97	1.378 7661	4.61	1.528 2278	5.25	1.658 2280	5.89	1.773 2559
3.98	1.381 2818	4.62	1.530 3947	5.26	1.660 1310	5.90	1.774 9523
3.99	1.383 7912	4.63	1.532 5568	5.27	1.662 0303	5.91	1.776 6458
4.00	1.386 2943	4.64	1.534 7143	5.28	1.663 9260	5.92	1.778 3364
4.01	1.388 7912	4.65	1.536 8672	5.29	1.665 8182	5.93	1.780 0242
4.02	1.391 2818	4.66	1.539 0154	5.30	1.667 7068	5.94	1.781 7091
4.03	1.393 7663	4.67	1.541 1590	5.31	1.669 5918	5.95	1.783 3912
4.04	1.396 2446	4.68	1.543 2981	5.32	1.671 4733	5.96	1.785 0704
4.05	1.398 7168	4.69	1.545 4325	5.33	1.673 3512	5.97	1.786 7469
4.06	1.401 1829	4.70	1.547 5625	5.34	1.675 2256	5.98	1.788 4205
4.07	1.403 6429	4.71	1.549 6879	5.35	1.677 0965	5.99	1.790 0914
4.08	1.406 0969	4.72	1.551 8087	5.36	1.678 9639	6.00	1.791 7594
4.09	1.408 5449	4.73	1.553 9252	5.37	1.680 8278	6.01	1.793 4247
4.10	1.410 9869	4.74	1.556 0371	5.38	1.682 6882	6.02	1.795 0872
4.11	1.413 4230	4.75	1.558 1446	5.39	1.684 5453	6.03	1.796 7470
4.12	1.415 8531	4.76	1.560 2476	5.40	1.686 3989	6.04	1.798 4040
4.13	1.418 2774	4.77	1.562 3462	5.41	1.688 2491	6.05	1.800 0582
4.14	1.420 6957	4.78	1.564 4405	5.42	1.690 0958	6.06	1.801 7098
4.15	1.423 1083	4.79	1.566 5304	5.43	1.691 9391	6.07	1.803 3586
4.16	1.425 5150	4.80	1.568 6159	5.44	1.693 7790	6.08	1.805 0047
4.17	1.427 9160	4.81	1.570 6971	5.45	1.695 6155	6.09	1.806 6481
4.18	1.430 3112	4.82	1.572 7739	5.46	1.697 4487	6.10	1.808 2887
4.19	1.432 7007	4.83	1.574 8464	5.47	1.699 2786	6.11	1.809 9267
4.20	1.435 0845	4.84	1.576 9147	5.48	1.701 1051	6.12	1.811 5621

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
6.13	1.813 1947	6.77	1.912 5011	7.41	2.002 8305	8.05	2.085 6720
6.14	1.814 8247	6.78	1.913 9771	7.42	2.004 1790	8.06	2.086 9135
6.15	1.816 4520	6.79	1.915 4509	7.43	2.005 5258	8.07	2.088 1534
6.16	1.818 0767	6.80	1.916 9226	7.44	2.006 8708	8.08	2.089 3918
6.17	1.819 6988	6.81	1.918 3921	7.45	2.008 2140	8.09	2.090 6287
6.18	1.821 3182	6.82	1.919 8594	7.46	2.009 5553	8.10	2.091 8640
6.19	1.822 9351	6.83	1.921 3247	7.47	2.010 8949	8.11	2.093 0984
6.20	1.824 5493	6.84	1.922 7877	7.48	2.012 2327	8.12	2.094 3306
6.21	1.826 1608	6.85	1.924 2486	7.49	2.013 5687	8.13	2.095 5613
6.22	1.827 7699	6.86	1.925 7074	7.50	2.014 9030	8.14	2.096 7905
6.23	1.829 3763	6.87	1.927 1641	7.51	2.016 2354	8.15	2.098 0182
6.24	1.830 9801	6.88	1.928 6186	7.52	2.017 5661	8.16	2.099 2444
6.25	1.832 5814	6.89	1.930 0710	7.53	2.018 8950	8.17	2.100 4691
6.26	1.834 1801	6.90	1.931 5214	7.54	2.020 2221	8.18	2.101 6923
6.27	1.835 7763	6.91	1.932 9696	7.55	2.021 5475	8.19	2.102 9140
6.28	1.837 3699	6.92	1.934 4157	7.56	2.022 8711	8.20	2.104 1341
6.29	1.838 9610	6.93	1.935 8598	7.57	2.024 1929	8.21	2.105 3529
6.30	1.840 5496	6.94	1.937 3017	7.58	2.025 5131	8.22	2.106 5702
6.31	1.842 1356	6.95	1.938 7416	7.59	2.026 8315	8.23	2.107 7861
6.32	1.843 7191	6.96	1.940 1794	7.60	2.028 1482	8.24	2.108 9998
6.33	1.845 3002	6.97	1.941 6152	7.61	2.029 4631	8.25	2.110 2128
6.34	1.846 8787	6.98	1.943 0489	7.62	2.030 7763	8.26	2.111 4243
6.35	1.848 4547	6.99	1.944 4805	7.63	2.032 0878	8.27	2.112 6343
6.36	1.850 0283	7.00	1.945 9101	7.64	2.033 3976	8.28	2.113 8428
6.37	1.851 5994	7.01	1.947 3376	7.65	2.034 7056	8.29	2.115 0499
6.38	1.853 1680	7.02	1.948 7632	7.66	2.036 0119	8.30	2.116 2555
6.39	1.854 7342	7.03	1.950 1866	7.67	2.037 3166	8.31	2.117 4596
6.40	1.856 2979	7.04	1.951 6080	7.68	2.038 6195	8.32	2.118 6622
6.41	1.857 8592	7.05	1.953 0275	7.69	2.039 9207	8.33	2.119 8634
6.42	1.859 4181	7.06	1.954 4449	7.70	2.041 2203	8.34	2.121 0632
6.43	1.860 9745	7.07	1.955 8604	7.71	2.042 5181	8.35	2.122 2615
6.44	1.862 5285	7.08	1.957 2739	7.72	2.043 8143	8.36	2.123 4584
6.45	1.864 0801	7.09	1.958 6853	7.73	2.045 1088	8.37	2.124 6539
6.46	1.865 6293	7.10	1.960 0947	7.74	2.046 4016	8.38	2.125 8479
6.47	1.867 1761	7.11	1.961 5022	7.75	2.047 6928	8.39	2.127 0405
6.48	1.868 7205	7.12	1.962 9077	7.76	2.048 9823	8.40	2.128 2317
6.49	1.870 2625	7.13	1.964 3112	7.77	2.050 2701	8.41	2.129 4214
6.50	1.871 8021	7.14	1.965 7127	7.78	2.051 5563	8.42	2.130 6098
6.51	1.873 3394	7.15	1.967 1123	7.79	2.052 8408	8.43	2.131 7967
6.52	1.874 8743	7.16	1.968 5099	7.80	2.054 1237	8.44	2.132 9822
6.53	1.876 4069	7.17	1.969 9056	7.81	2.055 4049	8.45	2.134 1664
6.54	1.877 9371	7.18	1.971 2993	7.82	2.056 6845	8.46	2.135 3491
6.55	1.879 4650	7.19	1.972 6911	7.83	2.057 9624	8.47	2.136 5304
6.56	1.880 9906	7.20	1.974 0810	7.84	2.059 2388	8.48	2.137 7104
6.57	1.882 5138	7.21	1.975 4689	7.85	2.060 5135	8.49	2.138 8889
6.58	1.884 0347	7.22	1.976 8549	7.86	2.061 7866	8.50	2.140 0661
6.59	1.885 5533	7.23	1.978 2390	7.87	2.063 0580	8.51	2.141 2419
6.60	1.887 0696	7.24	1.979 6212	7.88	2.064 3278	8.52	2.142 4163
6.61	1.888 5837	7.25	1.981 0014	7.89	2.065 5961	8.53	2.143 5893
6.62	1.890 0954	7.26	1.982 3798	7.90	2.066 8627	8.54	2.144 7609
6.63	1.891 6048	7.27	1.983 7562	7.91	2.068 1277	8.55	2.145 9312
6.64	1.893 1119	7.28	1.985 1308	7.92	2.069 3911	8.56	2.147 1001
6.65	1.894 6168	7.29	1.986 5035	7.93	2.070 6530	8.57	2.148 2676
6.66	1.896 1194	7.30	1.987 8743	7.94	2.071 9132	8.58	2.149 4339
6.67	1.897 6198	7.31	1.989 2432	7.95	2.073 1719	8.59	2.150 5987
6.68	1.899 1179	7.32	1.990 6103	7.96	2.074 4290	8.60	2.151 7622
6.69	1.900 6138	7.33	1.991 9754	7.97	2.075 6845	8.61	2.152 9243
6.70	1.902 1075	7.34	1.993 3387	7.98	2.076 9384	8.62	2.154 0851
6.71	1.903 5989	7.35	1.994 7002	7.99	2.078 1907	8.63	2.155 2445
6.72	1.905 0881	7.36	1.996 0599	8.00	2.079 4415	8.64	2.156 4026
6.73	1.906 5751	7.37	1.997 4177	8.01	2.080 6907	8.65	2.157 5593
6.74	1.908 0600	7.38	1.998 7736	8.02	2.081 9384	8.66	2.158 7147
6.75	1.909 5425	7.39	2.000 1278	8.03	2.083 1845	8.67	2.159 8687
6.76	1.911 0228	7.40	2.001 4800	8.04	2.084 4290	8.68	2.161 0215

N.	Logarithm.	N.	Logarithm.	N.	Logarithm.	N.	Logarithm.
8.69	2.162 1729	9.33	2.233 2350	9.97	2.299 5806	71	4.262 6799
8.70	2.163 3230	9.34	2.234 3062	9.98	2.300 5831	72	4.276 6661
8.71	2.164 4718	9.35	2.235 3763	9.99	2.301 5846	73	4.290 4594
8.72	2.165 6192	9.36	2.236 4452	10.0	2.302 5851	74	4.304 0651
8.73	2.166 7653	9.37	2.237 5130	11.0	2.397 8953	75	4.317 4881
8.74	2.167 9101	9.38	2.238 5797	12.0	2.484 9067	76	4.330 7333
8.75	2.169 0536	9.39	2.239 6452	13.0	2.564 9494	77	4.343 8054
8.76	2.170 1959	9.40	2.240 7096	14.0	2.639 0573	78	4.356 7088
8.77	2.171 3367	9.41	2.241 7729	15.0	2.708 0502	79	4.369 4479
8.78	2.172 4763	9.42	2.242 8350	16.0	2.772 5887	80	4.382 0266
8.79	2.173 6146	9.43	2.243 8960	17.0	2.833 2133	81	4.394 4492
8.80	2.174 7517	9.44	2.244 9559	18 0	2.890 3718	82	4.406 7193
8.81	2.175 8874	9.45	2.246 0147	19.0	2.944 4390	83	4.418 8406
8.82	2.177 0218	9.46	2.247 0723	20.0	2.995 7323	84	4.430 8168
8.83	2.178 1550	9.47	2.248 1288	21.0	3.044 5224	85	4.442 6513
8.84	2.179 2868	9.48	2.249 1843	22.0	3.091 0425	86	4.454 3473
8.85	2.180 4174	9.49	2.250 2386	23.0	3.135 4942	87	4.465 9081
8.86	2.181 5467	9.50	2.251 2917	24.0	3.178 0538	88	4.477 3368
8.87	2.182 6747	9.51	2.252 3438	25.0	3.218 8758	89	4.488 6364
8.88	2.183 8015	9.52	2.253 3948	26.0	3.258 0965	90	4.499 8097
8.89	2.184 9270	9.53	2.254 4446	27.0	3.295 8369	91	4.510 8595
8.90	2.186 0512	9.54	2.255 4934	28.0	3.332 2045	92	4.521 7886
8.91	2.187 1742	9.55	2.256 5411	29.0	3.367 2958	93	4.532 5995
8.92	2.188 2959	9.56	2.257 5877	30.0	3.401 1974	94	4.543 2948
8.93	2.189 4163	9.57	2.258 6332	31.0	3.433 9872	95	4.553 8769
8.94	2.190 5355	9.58	2.259 6776	32.0	3.465 7359	96	4.564 3482
8.95	2.191 6535	9.59	2.260 7209	33.0	3.496 5076	97	4.574 7110
8.96	2.192 7702	9.60	2.261 7631	34.0	3.526 3605	98	4.584 9675
8.97	2.193 8856	9.61	2.262 8042	35.0	3.555 3481	99	4.595 1199
8.98	2.194 9998	9.62	2.263 8442	36.0	3.583 5189	100	4.605 1702
8.99	2.196 1128	9.63	2.264 8832	37.0	3.610 9179	101	4.615 1205
9.00	2.197 2245	9.64	2.265 9211	38.0	3.637 5862	102	4.624 9728
9.01	2.198 3350	9.65	2.266 9579	39.0	3.663 5617	103	4.634 7290
9.02	2.199 4443	9.66	2.267 9936	40.0	3.688 8795	104	4.644 3909
9.03	2.200 5523	9.67	2.269 0282	41.0	3.713 5721	105	4.653 9604
9.04	2.201 6591	9.68	2.270 0618	42.0	3.737 6696	106	4.663 4391
9.05	2.202 7647	9.69	2.271 0944	43.0	3.761 2001	107	4.672 8288
9.06	2.203 8691	9.70	2.272 1258	44.0	3.784 1896	108	4.682 1312
9.07	2.204 9722	9.71	2.273 1562	45.0	3.806 6525	109	4.691 3479
9.08	2.206 0741	9.72	2.274 1856	46.0	3.828 6414	110	4.700 4804
9.09	2.207 1748	9.73	2.275 2138	47.0	3.850 1476	111	4.709 5302
9.10	2.208 2744	9.74	2.276 2411	48.0	3.871 2010	112	4.718 4989
9.11	2.209 3727	9.75	2.277 2673	49.0	3.891 8203	113	4.727 3878
9.12	2.210 4697	9.76	2.278 2924	50.0	3.912 0230	114	4.736 1985
9.13	2.211 5656	9.77	2.279 3165	51.0	3.931 8256	115	4.744 9321
9.14	2.212 6603	9.78	2.280 3395	52.0	3.951 2437	116	4.753 5902
9.15	2.213 7538	9.79	2.281 3614	53.0	3.970 2919	117	4.762 1739
9.16	2.214 8461	9.80	2.282 3823	54.0	3.988 9841	118	4.770 6846
9.17	2.215 9372	9.81	2.283 4022	55.0	4.007 3332	119	4.779 1235
9.18	2.217 0272	9.82	2.284 4211	56.0	4.025 3517	120	4.787 4917
9.19	2.218 1160	9.83	2.285 4389	57.0	4.043 0513	121	4.795 7906
9.20	2.219 2034	9.84	2.286 4556	58.0	4.060 4430	122	4.804 0210
9.21	2.220 2898	9.85	2.287 4714	59.0	4.077 5374	123	4.812 1844
9.22	2.221 3750	9.86	2.288 4861	60.0	4.094 3446	124	4.820 2816
9.23	2.222 4590	9.87	2.289 4998	61.0	4.110 8739	125	4.828 3137
9.24	2.223 5418	9.88	2.290 5124	62.0	4.127 1344	126	4.836 2819
9.25	2.224 6235	9.89	2.291 5241	63.0	4.143 1347	127	4.844 1871
9.26	2.225 7040	9.90	2.292 5347	64.0	4.158 8839	128	4.852 0303
9.27	2.226 7833	9.91	2.293 5443	65.0	4.174 3873	129	4.859 8124
9.28	2.227 8615	9.92	2.294 5529	66.0	4.189 6547	130	4.867 5345
9.29	2.228 9385	9.93	2.295 5604	67.0	4.204 6926	131	4.875 1973
9.30	2.230 0144	9.94	2.296 5670	68.0	4.219 5077	132	4.882 8019
9.31	2.231 0890	9.95	2.297 5725	69.0	4.234 1065	133	4.890 3491
9.32	2.232 1626	9.96	2.298 5770	70.0	4.248 4952	134	4.897 8398

Mean Pressure Above Vacuum of Expanding Steam.

Absolute steam pressure, *P*.	Expansion ratio.							
	1.333	1.5	1.6	2	2.666	3	4	8
	Steam cut-off, fraction of stroke.							
	¾	⅔	⅝	½	⅜	⅓	¼	⅛
25	24.130	23.481	22.938	21.164	18.567	17.488	19.913	9.6232
30	28.956	28.100	27.524	25.396	22.280	20.986	17.897	11.548
35	33.782	32.874	32.110	29.630	25.992	24.484	20.880	13.472
40	38.608	37.468	36.700	33.862	28.964	27.982	23.862	15.396
45	43.474	42.151	41.287	38.095	32.677	31.479	26.845	17.320
50	48.262	46.835	45.875	42.328	37.133	34.977	29.828	19.246
55	53.088	51.518	50.462	46.561	40.846	38.474	32.811	21.170
60	57.914	56.202	55.050	50.794	44.559	41.972	35.794	23.095
65	62.740	60.885	59.637	55.027	48.273	45.470	38.777	25.020
70	67.566	65.569	64.225	59.260	51.986	48.967	41.760	26.944
75	72.393	70.252	68.812	63.493	55.700	52.465	44.743	28.869
80	77.216	74.936	73.400	67.726	59.413	55.963	47.726	30.794
85	82.042	79.619	77.987	71.959	63.126	59.461	50.709	32.718
90	86.866	85.303	82.574	76.192	66.840	62.958	53.692	34.643
95	91.699	89.986	87.163	80.425	70.553	66.456	56.675	36.568
100	96.524	93.670	91.750	84.657	74.267	69.954	59.657	38.493
105	101.35	98.353	96.337	88.890	77.981	73.451	62.640	40.417
110	106.17	103.04	100.92	93.123	81.694	76.949	65.622	42.342
115	111.00	107.72	105.51	97.356	85.407	80.447	68.606	44.267
120	115.83	112.40	110.10	101.59	89.121	83.944	71.589	46.191
125	120.65	117.08	114.68	105.82	92.834	87.442	74.572	48.116
130	125.48	121.77	119.27	110.05	96.548	90.940	77.555	50.041
135	130.30	126.45	123.86	114.28	100.26	94.437	80.538	51.966
140	135.13	131.13	128.45	118.52	103.97	97.935	83.520	53.890
145	139.96	135.82	133.03	122.75	107.68	101.43	86.502	55.815
150	144.78	140.50	137.62	126.98	111.40	104.93	89.485	57.739
155	149.60	145.18	142.20	131.22	115.11	108.42	92.468	59.663
160	154.43	149.87	146.79	135.45	118.82	111.92	95.451	61.588
165	159.26	154.55	151.38	139.68	122.54	115.42	98.434	63.513
170	164.08	159.23	155.97	143.92	126.25	118.92	101.41	65.437
175	168.91	163.92	160.55	148.15	129.96	122.42	104.40	67.362
180	173.73	168.60	165.14	152.38	133.68	125.91	107.38	69.287
185	178.56	173.28	169.73	156.61	137.39	129.41	110.36	71.212
190	183.39	177.97	174.32	160.85	141.10	132 91	113.35	73.136
195	188.21	182.65	178.90	165.08	144.82	136.41	116.33	75.061
200	193.04	187.34	183.50	169.31	148.53	139.91	119.31	76.986
210	202.69	196.71	192.68	177.78	155.96	146.90	125.27	80.835
220	212.34	205.08	201.85	186.25	163.39	153.90	131.24	84.684
230	221.99	215.45	211.03	194.71	170.82	160.89	137.20	88.534
240	231.65	224.81	220.20	203.18	178.23	167.89	143.17	92.383
250	241.30	234.18	229.38	211.64	185.67	174.88	149.13	96.232
260	250.96	243.55	238.55	220.11	193.18	181.88	155.11	100.08
270	260.61	252.91	247.73	228.57	200.52	188.87	161.07	103.93
280	270.26	262.28	256.90	237.04	207.95	195.87	167.04	107.78
300	289.56	281.00	275.24	253.96	222.80	209.86	178.97	115.48

Mean Pressure for High-pressure Engines Above Atmosphere.

Pressure above atmosphere, P.	Expansion ratio.							
	1.333	1.5	1.6	2	2.666	3	4	8
	Steam cut-off, fraction of stroke.							
	¾	⅔	⅝	½	⅜	⅓	¼	⅛
25	23.908	22.768	22.000	19.162	14.264	13.282	9.162	.696
30	28.774	27.451	26.587	23.395	17.977	16.779	12.145	2.620
35	33.562	32.135	31.175	27.628	22.433	20.277	15.128	4.546
40	38.388	36.818	35.762	31.861	26.146	23.774	18.111	6.470
45	43.214	41.502	40.350	36.094	29.859	27.272	21.094	8.395
50	48.040	46.185	44.937	40.327	33.573	30.770	24.077	10.320
55	52.866	50.869	49.625	44.560	37.286	34.267	27.060	12.244
60	57.693	55.552	54.112	48.793	41.000	37.765	30.043	14.169
65	62.516	60.236	58.700	53.026	44.713	41.263	33.026	16.094
70	67.342	64.919	63.287	57.259	48.426	44.761	36.009	18.018
75	72.166	70.603	67.874	61.492	52.140	48.258	38.992	19.943
80	76.999	75.286	72.463	65.725	55.853	51.756	41.975	21.868
85	81.824	78.970	77.050	69.957	59.567	55.254	44.957	23.793
90	86.65	83.653	81.637	74.190	63.281	58.751	47.940	25.717
95	91.47	88.34	86.22	78.423	66.994	62.249	50.922	27.642
100	96.30	93.02	90.81	82.656	70.707	65.747	53.906	29.567
105	101.13	97.70	95.40	86.89	74.421	69.244	56.889	31.491
110	105.95	102.38	99.98	91.12	78.134	72.742	59.872	33.416
115	110.78	107.07	104.57	95.35	81.848	76.240	62.855	35.341
120	115.60	111.75	109.16	99.58	85.56	79.737	65.838	37.266
125	120.43	116.43	113.75	103.82	89.27	83.235	68.820	39.190
130	125.26	121.12	118.33	108.05	92.98	86.73	71.802	41.115
135	130.08	125.80	122.92	112.28	96.70	90.23	74.785	43.039
140	134.90	130.48	127.50	116.52	100.41	93.72	77.768	44.963
145	139.73	135.17	132.09	120.75	104.12	97.22	80.751	46.888
150	144.56	139.85	136.68	124.98	107.84	100.72	83.734	48.813
155	149.38	144.83	141.27	129.22	111.85	104.22	86.71	50.737
160	154.21	149.22	145.85	133.45	115.26	107.72	89.70	52.662
165	159.03	153.90	150.44	137.68	118.98	111.21	92.68	54.587
170	163.86	158.58	155.03	141.91	122.69	114.71	95.66	56.812
175	168.69	163.27	159.62	146.15	126.40	118.21	98.65	58.436
180	173.51	167.95	164.20	150.38	130.12	121.71	101.63	60.361
185	178.34	172.64	168.80	154.81	133.83	125.21	104.61	62.286
190	183.16	177.32	173.39	158.81	137.54	128.71	107.59	64.210
195	187.99	182.01	177.98	163.08	141.26	132.20	110.57	66.135
200	192.81	186.69	182.58	167.31	144.97	135.70	113.55	68.060
210	202 46	195.06	191.74	175.78	152.40	142.70	119.52	71.908
220	212.11	205.43	200.93	184.24	159.83	149.69	125.48	75.758
230	221.77	214.79	210.10	192.71	167.24	156.69	131.39	79.603
240	231.42	224.16	219.27	201.17	174.68	163.68	137.41	83.456
250	241.08	233.57	228.45	209.64	182.19	170.68	143.39	87.30
260	250.73	242.89	237.62	218.10	189.53	177.67	149.35	91.15
270	260.38	252.26	246.79	226.57	196.96	184.67	155.32	95.00
280	270.04	261.62	255.94	235.03	204.39	191.66	161.29	98.86
300	289.34	280.35	264.30	251.95	219.24	205.56	173.22	106.55

In the preceding computations and tables it has been assumed that there was no clearance or waste space between the piston and the cylinder head at the end of the stroke. In practice, the clearance amounts to from 2 to 7 per cent. of the cylinder volume. This may be taken into account by adding the clearance to both the length of the stroke and the length of the admission portion in determining the expansion ratio, r. Thus, if the stroke is 24 inches and the steam is cut off at 6 inches, the expansion ratio will be $\frac{24}{6} = 4$, if clearance is neglected. If, however, there is a space of ½ inch between the piston and the cylinder head at the end of the stroke, we have

$$r = \frac{24.5}{6.5} = 3.77;$$

and this is the ratio to be used in computation.

Most Economical Point of Cut-off.

(W. D. Marks.)

To find the most economical point of cut-off,—that is, its inverse, that number of expansions which will result in the greatest economy of steam from the boiler, per horse-power, per hour.

Notation.

e = the true point of cut-off = the reciprocal of the true number of expansions;
B = the absolute back pressure during exhaust, in pounds, per square inch;
P_b = the absolute pressure at cut-off;
s = the stroke of piston, in feet;
d = the diameter of cylinder, in feet;
$A = \frac{62.5}{S}$;
S = the specific volume of steam at cut-off;
$D = 2\frac{T_b - T_e}{N}C$;
T_b = the temperature of the steam at cut-off (Fahr.);
T_e = the temperature of the steam during exhaust;
N = the number of strokes per minute = twice the revolutions of crank;
C = the constant of condensation = 0.018 pounds of steam for about 82 pounds gauge pressure.

$$e = \frac{B}{P_b} + \left(\frac{1}{s} + \frac{0.194}{d}\right)\frac{Dd}{Ad + D} \text{ nat. log. } \frac{1}{e}.$$

Example. Let

P_b = 100 pounds absolute;
B = 15 pounds absolute;
s = 4 feet;
d = 1.5 feet;
N = 150 per minute.

We have

$A = 0.233$,
$D = 0.0274$,

$$e = 0.15 + \left(\frac{1}{4} + \frac{0.194}{1.5}\right)\frac{0.0274 \times 1.5 \times 2.3026}{0.233 \times 1.5 + 0.0274} \text{ com. log. } \frac{1}{e},$$

$$e = 0.15 + 0.3793\frac{0.0944}{0.3764} \log. \frac{1}{e},$$

$$e = 0.15 + 0.0952 \log. \frac{1}{e}.$$

We must solve this transcendental equation tentatively, trying values until the two members balance.

Assume $e = \frac{1}{5}$ of stroke plus clearance. We have

$$0.20 = 0.15 + 0.066 = 0.216.$$

This error of 0.016 is closer work than can be realized in practice, and we can take 5 expansions as the best number.

Between $\frac{1}{5}$ and $\frac{1}{4}$ would have been near enough for all practical purposes.

To find the proper ratio of stroke to diameter under the given conditions, assuming 5 *expansions and diameter* $= 1\frac{1}{2}$ *feet.*

Inverting the above equation, we have

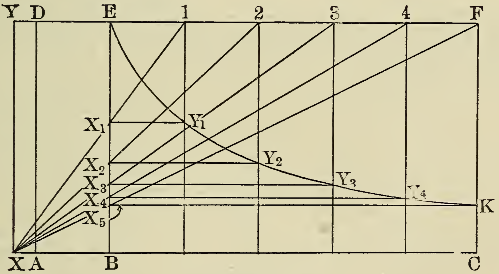
$$s = \frac{d}{\left(\frac{A}{D}d + 1\right)\left(\frac{e - \frac{B}{P_b}}{\text{nat. log.} \frac{1}{e}}\right) - 0.194},$$

$$\frac{A}{D} = 8.56,$$

$$s = \frac{1.5}{\left(8.56 \times 1.5 + 1\right)\left(\frac{0.20 - 0.15}{1.61}\right) - 0.194} = 6.4 \text{ feet, nearly.}$$

With slow-moving engines it will be found that long stroke is most economical, while on the other hand high-speed engines require short stroke for greatest economy. If we double the speed of this engine, making $N = 300$, the stroke $s = 2.4$ feet, for greatest economy.

In order to construct the curve representing the expansion of steam in a cylinder, under the assumption that the expansion is isothermal,—*i.e.*, that $pv =$ constant,—the following method may be used:



Isothermal Curve.

Draw the line, AC, to represent the position of zero pressure, or perfect vacuum, making the length, AC, represent the stroke of the piston. Make AX equal to the clearance, expressed in terms of the stroke,—that is,

$$AX = AC \frac{\text{clearance volume}}{\text{volume swept through by piston}}.$$

Erect the perpendicular, DF, to represent the admission pressure on any convenient scale, and draw the horizontal line, YDF. Mark the point, E, so that DE represents the length of the stroke during which steam is admitted,—*i.e.*, if the expansion ratio is 6, DE will be one-sixth of AC,—and draw BE. Take any points, 1, 2, 3, 4, and join them to X, and also drop

perpendiculars from 1, 2, 3, 4. Draw horizontal lines from the intersections, X_1, X_2, X_3, X_4, on EB, and where these horizontal lines intersect the corresponding verticals will be points on the curve, as at Y_1, Y_2, Y_3, Y_4.

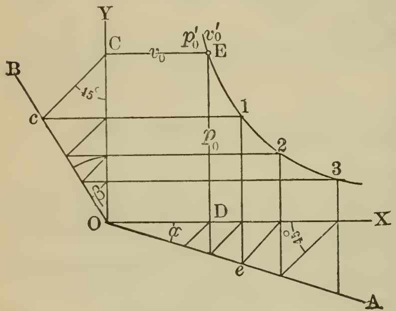
The hyperbolic curve represents isothermal expansion, it being assumed that the temperature is kept constant. Other curves have been considered in connection with the expansion of steam, and according to the investigations of Rankine, Zeuner, and others, these may be expressed by the general equation:

$$pv^m = \text{constant},$$

the exponent, m, being varied according to the curve under consideration.

For dry saturated steam, according to Rankine, $m = \frac{17}{16} = 1.0625$; while, according to Zeuner, $m = 1.0646$. For adiabatic expansion, in which the expanding steam neither receives nor gives out heat, Rankine gives $m = \frac{10}{9} = 1.111$.

Any of these curves may be constructed by the computation of any desired number of ordinates, using logarithms, or more conveniently by the so-called "polytropic" diagram.



Polytropic Curve.

Draw the rectangular axes, YOX, and make v_0 equal the portion of the stroke during which the initial pressure, p_0, is maintained. Draw OA, making the angle, α, any convenient value, and also draw OB, making the angle, β, so that

$$1 + \tan\beta = (1 + \tan\alpha)^m,$$

choosing m according to the curve to be drawn, as above. Then, starting from C, draw Cc at 45° from OC and back at right angles to CO, zigzagging back and forth, as shown; also drop the vertical, ED, prolonged to e, and construct a similar zigzag between OA and OX, making alternate angles of 45° and 90° with OX. The intersections of the normals to OY and OX, when prolonged, will then give points in the curve, as at 1, 2, 3, etc.

In practice, it has been found that the isothermal curve represents practical working conditions as closely as any which can be drawn.

The principal source of loss in steam engines is the initial condensation of the steam, which takes place when it first enters the cylinder. The difference in temperature between steam at 100 pounds and steam at atmospheric pressure is about 125° F., and as the cylinder walls absorb and part with heat readily the incoming steam meets the walls which have just been cooled to the temperature of the previous exhaust. For this reason it has been found wasteful to attempt to realize the high economy due to large expansion ratios in a single cylinder. For high-pressure engines the best results are obtained with 4 or 5 expansions,—*i.e.*, cutting off the steam at ¼ to ⅕ of the stroke,—while for condensing engines from 8 to 10 expansions is about the highest that can be used to advantage.

In order to use higher expansion ratios successfully, the expansion is performed in two or more cylinders, giving compound or multiple-expansion engines. A number of rules and methods have been given for determining the best relative areas of cylinders for compound and multiple-expansion engines, depending upon the desire of the designer. In many cases, it is wished to make the work performed in the various cylinders approximately equal; in others, it is desired to equalize the initial stresses; and in others, to equalize the drop in temperature.

For compound engines various empirical rules have been given, generally based upon the initial pressure or upon the expansion ratio.

Thus, if r be the expansion ratio, the cylinder ratio is often made equal to $\sqrt{r}$. In marine practice, the cylinder ratio usually ranges from 1 to 4 for 100 pounds pressure to 1 to 5 for 120 pounds pressure.

For triple-expansion engines the ratios found in practice, according to Whitham, are about as follows:

Cylinder Ratios for Triple-expansion Engines.

Initial pressure.	High pressure.	Intermediate.	Low pressure.
130	1	2.25	5.00
140	1	2.40	5.85
150	1	2.55	6.90
160	1	2.70	7.25

For quadruple-expansion engines, operating at pressures of 160 pounds and over, the following proportions are found:

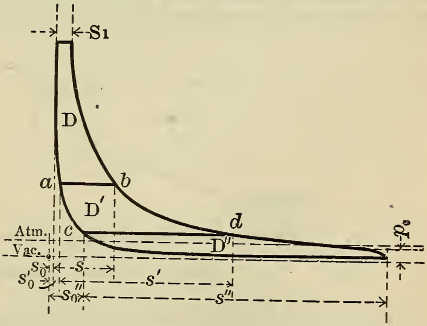
Cylinder Ratios for Quadruple-expansion Engines.

Initial pressure.	High pressure.	First intermediate.	Second intermediate.	Low pressure.
160	1	2.00	4.00	8
180	1	2.10	4.20	9
200	1	2.15	4.60	10
220	1	2.20	4.80	11

The subject is best studied by drawing a single diagram for the initial pressure and expansion ratio given, this being then divided up according to the distribution of power desired among two, three, or four cylinders, as the proposed design may be for a compound, triple, or quadruple engine.

For this purpose the isothermal curve will be sufficiently accurate.

Thus, the diagram may be drawn as for a single engine, as shown herewith, and divided into three portions of equal area, D, D', and D''; this being best done tentatively, the areas being measured by the planimeter. If the total area is first measured and divided by three, the portion D can be laid off very closely after one or two trials, and the same for D' and D''. The areas of the several parts will then be proportional to the volumes of the various cylinders, and, since they are all made of the same stroke in practice, the cylinder ratios will be proportional to the lengths, s, s', and s''. Any other subdivision of the total expansion may be considered in the same manner.



Triple-expansion Diagram.

The thermal efficiency of any heat motor is limited by the range of temperature through which the impelling fluid acts. This efficiency is the

ratio obtained by dividing the heat converted into work by the total heat taken in. This ratio must always be less than unity, and its maximum value for any range of temperature is found from the ratio

$$\frac{T_1 - T_2}{T_1},$$

in which T_1 is the absolute temperature of reception, = temperature F. + 461, = temperature C. + 273; while T_2 is the absolute temperature of rejection. Considering all temperatures as absolute,—that is, as measured from the absolute zero, we have

$$\text{Maximum efficiency} = \frac{\text{temperature of reception} - \text{temperature of rejection}}{\text{temperature of reception}}.$$

Thus, in the case of an engine in which the steam enters at a temperature of 341° F., or 802° absolute, corresponding to an absolute pressure of 120 pounds per square inch, and is rejected in the condenser at a temperature of 60° F., or 521° absolute, we have

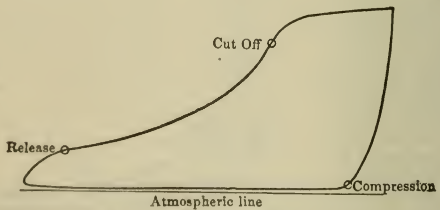
$$\frac{T_1 - T_2}{T_2} = \frac{802^\circ - 521^\circ}{802^\circ} = 0.35,$$

so that, if all the heat in the steam were converted into mechanical energy, the efficiency could not exceed 35 per cent.

In actual practice the thermal efficiency rarely attains 12 per cent., the highest recorded efficiency being that of the Reynolds pumping engine at Boston, Massachusetts. This engine has the record of a performance of 187.8 B. T. U. per indicated horse-power, corresponding to a thermal efficiency of 22⅝ per cent.

Indicator Diagrams.

The steam-engine indicator is a form of recording pressure gauge, arranged to be attached to the cylinder of a steam engine so as to draw a curve representing the pressure within the cylinder at every point in the stroke. Originally invented by Watt, and greatly improved by McNaught, Richards, Thompson, and others, it is now a standard instrument of the engineer. The details of construction of the various styles of instruments on the market are fully given in the hand-books issued by the manufacturer, and hence the diagrams themselves will only be discussed here.



Typical Indicator Diagram.

In the typical diagram, given herewith, the general form obtained from a single-cylinder engine in good condition is shown. If the area of the diagram (best measured by a planimeter) is divided by the length and this multiplied by the scale of the spring, the *mean effective pressure* in the cylinder is obtained. This mean effective pressure multiplied by the area of the piston, in square inches, gives the total force acting upon the piston, in pounds, and by multiplying this force by the number of feet of piston

travel per minute, the power, in foot-pounds, per minute is obtained. From this the horse-power is found by dividing by 33,000.

Thus, if

p = mean effective pressure, in pounds, per square inch;
a = area of piston, in square inches;
s = piston speed, in feet, per minute.

$$HP = \frac{a \times p \times s}{33000}.$$

If a number of computations are to be made upon a given engine, the area of the piston may conveniently be divided by 33,000 to obtain a constant factor, corresponding to the horse-power developed by 1 pound mean effective pressure at 1 foot piston speed. This constant need then only be multiplied by the actual speed and pressure to give the power in each case.

It must be remembered that the indicator is only a recording pressure gauge, and that it merely shows the pressure at every point in the stroke. The interpretation given to the record is a matter in which the judgment of the observer must in great measure supply.

In general, the indicator diagram shows the action of the valve gear, including the points of cut-off, release, and compression; also, the freedom of the exhaust and the equality of action in both the forward and backward strokes. To this extent the indicator is of great assistance in adjusting the valves and in maintaining a correct adjustment.

The indicator diagram may also be used to determine the steam consumption of the engine,—at least the theoretical consumption may thus be determined, and by comparison with actual measurements the proportion of steam accounted for by the indicator may be computed.

The steam consumption is usually stated in terms of the equivalent weight of water. Several methods may be used in computing the rate of water consumption. The following, due to Mr. Jesse Warrington, is convenient in that it does not require any data concerning the dimensions or speed of the engine, being determined solely from the indicator diagram.

Divide the constant number 859,375 by the *volume* of steam at the terminal pressure and by the mean effective pressure. The quotient will be the desired rate.

This constant is the number of pounds of water that would be used in 1 hour by an engine developing 1 horse-power, if run by water (instead of steam), at 1 pound pressure per square inch. Then, with pressure of more than 1 pound, the amount required would be as many times less as the pressure was greater than 1 pound, and when steam is used the amount would be as much less as the volume of the steam at the pressure at which it is released is greater than that of an equal weight of water; hence, the above rule. The constant is found as follows: The standard horse-power being 33,000 foot-pounds, or 33,000 pounds lifted 1 foot per minute, would be equivalent to $33{,}000 \times 12 = 396{,}000$ pounds lifted 1 inch per minute; hence, an engine whose piston displacement was 396,000 cubic inches per minute would develop 1 horse-power with 1 pound mean effective pressure on the piston. This for 1 hour would be $396{,}000 \times 60$ minutes $= 23{,}760{,}000$ cubic inches per hour. Then suppose the engine to be run by water at 1 pound pressure per square inch, instead of steam, and taking 27.648 as the number of cubic inches of water per pound, $23{,}760{,}000 \div 27.648 = 859{,}375$, which is the desired constant.

The water consumption thus determined is not corrected for clearance or for compression, but this may be done from the diagram, as follows: Prolong the expansion curve beyond the point of release until it reaches the end of the diagram, this giving the terminal point of the curve as it would have been had the exhaust valve not been opened. Draw a horizontal line from this terminal point through the compression curve to the other end of the diagram. The ratio of the length from terminal to compression curve, divided by the total length of the diagram, will give a factor which, when multiplied by the previously-computed water consumption, will give the result corrected for clearance and compression. These methods are naturally dependent upon the tightness of the valves for their accuracy.

In order to simplify the work of computation, the following table has been made.

Water Consumption Table.

P	W	P	W	P	W	P	W	P	W	P	W	P	W
3	39.10	20	34.99	37	33.72	54	32.98	71	32.46	88	32.07	105	31.73
4	38.47	21	34.89	38	33.67	55	32.94	72	32.43	89	32.05	106	31.71
5	37.95	22	34.79	39	33.62	56	32.91	73	32.40	90	32.03	107	31.69
6	37.54	23	34.70	40	33.57	57	32.88	74	32.38	91	32.00	108	31.67
7	37.22	24	34.61	41	33.52	58	32.85	75	32.36	92	31.98	109	31.65
8	36.93	25	34.53	42	33.47	59	32.82	76	32.34	93	31.96	110	31.63
9	36.67	26	34.45	43	33.42	60	32.79	77	32.32	94	31.94	111	31.61
10	36.44	27	34.37	44	33.38	61	32.76	78	32.30	95	31.92	112	31.59
11	36.24	28	34.29	45	33.34	62	32.73	79	32.27	96	31.90	113	31.57
12	36.06	29	34.22	46	33.30	63	32.70	80	32.25	97	31.88	114	31.55
13	35.89	30	34.15	47	33.26	64	32.67	81	32.23	98	31.86	115	31.54
14	35.73	31	34.08	48	33.22	65	32.64	82	32.20	99	31.84	116	31.53
15	35.59	32	34.01	49	33.18	66	32.61	83	32.18	100	31.82	117	31.52
16	35.46	33	33.95	50	33.14	67	32.58	84	32.16	101	31.80	118	31.51
17	35.34	34	33.89	51	33.10	68	32.55	85	32.14	102	31.78	119	31.50
18	35.22	35	33.83	52	33.06	69	32.52	86	32.12	103	31.77	120	31.49
19	35.10	36	33.77	53	33.02	70	32.49	87	32.09	104	31.75	121	31.48

Under P is found the absolute terminal pressure. Under W, opposite the terminal pressure, is found a factor which, when multiplied by the absolute terminal pressure and divided by the mean effective pressure, will give the theoretical water consumption. From this it will be seen that the best economy is attained by a low terminal pressure combined with a high mean effective pressure, conditions which are incompatible either with underloading or overloading.

The relation between the actual and the computed water consumption of simple engines, both condensing and non-condensing, for various points of cut-off is given in the following table from the practice of the Buckeye Engine Company.

Table of Standard Engine Performance.

Initial pressure.	Mean effective pressure, in pounds. Non-condensing.	Mean effective pressure, in pounds. Condensing.	Terminals.	Rates, in pounds of water per indicated horse-power per hour. Actual. Non-condensing.	Actual. Condensing.	Theoretical. Non-condensing.	Theoretical. Condensing.	Throt.
				$\frac{1}{10}$ cut-off.				
40	3.65	13.65	6.41	72.0	38.0	51.4	16.4	146
45	5.42	15.42	7.00	58.5	35.0	38.5	16.0	120
50	7.19	17.19	7.59	49.0	33.0	31.9	15.6	93
55	8.96	18.96	8.17	43.5	31.5	28.1	15.2	80
60	10.73	20.73	8.76	39.0	30.0	25.3	14.9	70
65	12.50	22.50	9.35	35.7	28.6	23.3	14.6	62
70	14.27	24.27	9.93	33.0	27.7	21.8	14.4	55
75	16.04	26.04	10.52	31.0	26.7	20.6	14.2	50
80	17.81	27.81	11.11	29.0	26.0	19.7	14.0	46
85	19.58	29.58	11.70	27.5	25.3	19.0	13.8	43
90	21.36	31.36	12.28	26.0	24.5	18.4	13.6	40
95	23.13	33.13	12.87	25.0	23.7	17.9	13.5	37
100	24.9	34.9	13.46	24.0	23.0	17.5	13.4	35
				$\frac{15}{100}$ cut-off.				
40	9.05	19.05	9.07	54.0	30.0	31.3	16.8	64
45	11.32	21.32	9.87	47.0	28.5	27.7	16.4	56
50	13.59	23.59	10.72	42.0	27.0	25.3	16.1	51
55	15.86	25.86	11.55	38.0	26.0	23.4	15.8	47
60	18.12	28.12	12.38	34.5	25.0	22.1	15.6	43
65	20.39	30.39	13.20	32.0	24.0	21.1	15.4	40
70	22.66	32.66	14.03	30.0	23.0	20.3	15.2	38
75	24.92	34.92	14.86	28.0	22.2	19.5	15.0	36
80	27.19	37.19	15.69	26.0	21.3	18.8	14.8	35
85	29.46	39.46	16.51	24.5	20.4	18.4	14.6	34
90	31.72	41.72	17.34	23.0	19.5	18.0	14.5	33
95	33.93	43.93	18.17	22.0	18.7	17.6	14.4	32
100	36.26	46.26	19.0	21.0	18.0	17.3	14.3	32
				$\frac{1}{5}$ cut-off.				
40	13.46	23.46	11.79	45.0	24.0	27.9	17.7	51
45	16.15	26.15	12.87	41.5	23.3	25.7	17.3	45
50	18.85	28.85	13.94	37.0	22.5	24.0	16.9	40
55	21.54	31.54	15.00	33.6	21.7	22.7	16.6	38
60	24.24	34.24	16.08	31.0	21.0	21.7	16.4	36
65	26.93	36.93	17.15	29.0	20.3	20.9	16.2	35
70	29.63	39.63	18.23	27.5	19.6	20.2	16.0	34
75	32.32	42.32	19.31	26.0	19.0	19.6	15.8	33
80	35.02	45.02	20.39	24.5	18.4	19.1	15.7	33
85	37.71	47.71	21.46	23.3	18.0	18.7	15.6	32
90	40.41	50.41	22.54	22.0	17.4	18.4	15.5	32
95	43.1	53.1	23.62	21.0	16.9	18.1	15.4	31
100	45.8	55.8	24.7	20.0	16.4	17.8	15.3	31

Table of Standard Engine Performance.—*Continued.*

Initial pressure.	Mean effective pressure, in pounds.		Terminals.	Rates, in pounds of water per indicated horse-power per hour.				
				Actual.		Theoretical.		Throt.
	Non-condensing.	Condensing.		Non-condensing.	Condensing.	Non-condensing.	Condensing.	
	$\frac{25}{100}$ or $\frac{1}{4}$ cut-off.							
40	17.34	27.34	14.49	39.0	22.0	27.2	18.5	46
45	20.39	30.39	15.81	36.0	21.5	25.3	18.2	44
50	23.45	33.45	17.13	33.5	21.0	24.0	17.9	42
55	26.50	36.50	18.45	31.2	20.5	22.9	17.6	40
60	29.56	39.56	19.77	29.0	20.0	22.0	17.4	39
65	32.61	42.61	21.09	27.6	19.5	21.3	17.2	38
70	35.67	45.67	22.41	26.4	19.0	20.8	17.0	37
75	38.72	48.72	23.73	25.3	18.5	20.4	16.8	36
80	41.78	51.78	25.05	24.0	18.0	20.0	16.6	35
85	44.83	54.83	26.37	23.0	17.7	19.6	16.5	34
90	47.89	57.89	27.69	22.0	17.4	19.3	16.4	33
95	50.94	60.94	29.01	21.2	17.2	19.0	16.3	32
100	54.0	64.0	30.33	20.4	17.0	18.7	16.2	31
	$\frac{3}{10}$ cut-off.							
40	20.75	30.75	17.11	38.0	22.5	27.0	19.4	43
45	24.13	34.13	18.67	35.0	22.0	25.5	19.1	41
50	27.50	37.50	20.24	33.0	21.6	24.3	18.8	40
55	30.87	40.87	21.80	31.2	21.2	23.3	18.5	39
60	34.24	44.24	23.37	29.5	20.7	22.5	18.3	38
65	37.61	47.61	24.94	28.2	20.3	21.9	18.1	37
70	40.98	50.98	26.51	27.0	19.9	21.4	17.9	36
75	44.35	54.35	28.07	26.0	19.5	21.0	17.7	35
80	47.72	57.72	29.64	25.0	19.0	20.6	17.5	34
85	51.09	61.09	31.20	24.0	18.8	20.2	17.3	33
90	54.46	64.46	32.77	23.0	18.5	19.9	17.2	32
95	57.83	67.83	34.33	22.2	18.3	19.6	17.1	31
100	61.2	71.2	35.9	21.5	18.0	19.4	17.0	30
	$\frac{35}{100}$ cut-off.							
40	23.70	33.70	19.80	37.0	24.0	27.5	20.4	41
45	27.32	37.32	21.61	35.2	23.5	26.3	20.0	40
50	30.94	40.94	23.42	33.7	23.0	25.3	19.7	39
55	34.56	44.56	25.23	32.0	22.5	24.4	19.5	38
60	38.18	48.18	27.04	30.4	22.0	23.6	19.3	37
65	41.80	51.80	28.85	29.3	21.7	22.9	19.1	36
70	45.42	55.42	30.66	28.0	21.2	22.3	18.9	35
75	49.05	59.05	32.47	27.0	20.8	21.8	18.7	34
80	52.68	62.68	34.28	26.0	20.5	21.4	18.5	33
85	56.31	66.31	36.09	25.4	20.2	21.1	18.4	32
90	59.94	69.94	37.90	24.0	20.0	20.8	18.3	31
95	63.57	73.57	39.71	23.2	19.7	20.6	18.2	30
100	67.20	77.20	41.52	22.3	19.4	20.4	18.1	30

Table of Standard Engine Performance.—*Continued.*

Initial pressure.	Mean effective pressure, in pounds. Non-condensing.	Mean effective pressure, in pounds. Condensing.	Terminals.	Rates, in pounds of water per indicated horse-power per hour. Actual. Non-condensing.	Actual. Condensing.	Theoretical. Non-condensing.	Theoretical. Condensing.	Throt.
				$\frac{4}{10}$ cut-off.				
40	26.22	36.22	22.44	38.0	25.0	28.3	21.4	40
45	30.08	40.08	24.49	36.3	24.6	26.9	21.1	39
50	33.95	43.95	26.55	34.5	24.3	25.8	20.8	38
55	37.81	47.81	28.60	33.0	23.8	25.0	20.5	37
60	41.68	51.68	30.66	31.5	23.5	24.4	20.2	36
65	45.54	55.54	32.71	30.0	23.0	23.9	20.0	35
70	49.41	59.41	34.77	29.0	22.7	23.4	19.8	34
75	53.27	63.27	36.82	28.0	22.3	23.0	19.6	33
80	57.14	67.14	38.88	27.0	22.0	22.6	19.4	32
85	61.00	71.00	40.93	26.0	21.7	22.2	19.3	31
90	64.87	74.87	42.99	25.0	21.5	21.9	19.2	30
95	68.73	78.73	45.04	24.2	21.2	21.6	19.1	29
100	72.6	82.6	47.1	23.4	21.0	21.4	19.0	29
				$\frac{1}{2}$ cut-off.				
40	30.50	40.50	27.78	41.0	29.5	28.5	23.4	39
45	34.75	44.75	30.33	39.0	28.8	27.6	23.1	38
50	39.00	49.00	32.88	37.0	28.3	26.9	22.8	37
55	43.25	53.25	35.43	35.5	27.9	26.3	22.5	36
60	47.50	57.50	37.98	34.0	27.5	25.8	22.2	35
65	51.75	61.75	40.52	32.5	27.1	25.3	22.0	34
70	56.00	66.00	43.07	31.0	26.7	24.9	21.8	33
75	60.25	70.25	45.61	30.0	26.3	24.5	21.6	32
80	64.50	74.50	48.16	29.0	25.8	24.2	21.5	31
85	68.75	78.75	50.70	28.0	25.4	23.9	21.4	30
90	73.00	83.00	53.25	27.0	24.9	23.7	21.3	30
95	77.25	87.25	55.79	26.0	24.5	23.5	21.2	29
100	81.5	91.5	58.34	25.0	24.0	23.3	21.1	29

The water rates given under the heading "Throt." in the tables show the number of pounds of water per indicated horse-power per hour used by throttling engines, at same (non-condensing) mean effective pressure and initial pressure as on same line.

Standard Engine Tests.

The final report of the Committee of the American Society of Mechanical Engineers upon the standardizing of steam-engine tests (1902) contains a large amount of valuable information upon the whole subject of engine performance, and an abridgement of it is here given.

The Committee recommends the heat-unit basis, believing it to be the only fundamental basis for the determination of engine performance.

The expressions of engine economy which meet all the requirements noted are the number of heat units consumed per hour, both per indicated and per brake horse-power, and these are recommended as the desired

standards of comparison. The heat-unit standard does not interfere in any way with the common terms of expressing economy of engines. The hourly weights of coal, gas, oil, or other fuel, or weight of steam consumed per horse-power, heretofore commonly employed, are additional forms of stating economy, and are none the less useful within their limitations. They should by no means be abandoned. In the scheme now presented these additional or subsidiary forms of stating economy, as applied to particular classes of engines, are suitably provided for.

The heat consumption of a steam-engine plant required for the standard test is ascertained by measuring the quantity of steam consumed by the plant, calculating the total heat of the entire quantity, and crediting this total with that portion of the heat rejected by the plant, which is utilized and returned to the boiler. The term "engine plant," as here used, should include the entire equipment of the steam plant which is concerned in the production of the power, embracing the main cylinder or cylinders; the jackets and reheaters; the air, circulating, and boiler feed pumps, if steam driven; and any other steam-driven mechanism or auxiliaries necessary to the working of the engine.

The indicated horse-power for the proposed standard is that determined by the use of steam-engine indicators. It should be confined to the power developed in the main cylinder or cylinders, and should not include that developed in the cylinders of auxiliaries.

One of the important subsidiary forms of expressing efficiency is that based on a so-called "standard coal" unit. The assumption is made that the heat consumed by the engine is generated from coal of a fixed heat value, as implied by the term "standard coal."

The term "standard coal" refers to a coal which imparts to the steam 10,000 B. T. U. for each pound of the dry coal consumed. It is coal having a calorific value of 12,500 B. T. U., used in what may be termed a "standard boiler," which gives an efficiency of 80 per cent. (referred to the coal). Although chosen arbitrarily, these figures, as a matter of fact, apply closely to the average coals of the United States.

In treating of the subject of engine testing as relating primarily to the determination of matters of economy, it must not be forgotten that capacity is often of even greater importance than economy. In that large class of steam engines which are required to run at a certain limited and constant speed there should be a considerable reserve of capacity beyond the rated power. It is recommended that when a steam engine is operating at its rated power at a given pressure there should be a sufficient reserve to allow a drop of at least 15 per cent. in the gauge pressure without sensible reduction in the working speed of the engine, and allow an overload at the stated pressure amounting to at least 25 per cent.

Rules for Conducting Steam-engine Tests.

Code of 1902.

American Society of Mechanical Engineers.

I. **Object of Test.**—Ascertain at the outset the specific object of the test, whether it be to determine the fulfilment of a contract guarantee, to ascertain the highest economy obtainable, to find the working economy and defects under conditions as they exist, to ascertain the performance under special conditions, to determine the effect of changes in the conditions, or to find the performance of the entire boiler and engine plant, and prepare for the test accordingly.

II. **General Condition of the Plant.**—Examine the engine and the entire plant concerned in the test; note its general condition and any points of design, construction, or operation which bear on the objects in view. Make a special examination of the valves and pistons for leakage by applying the working pressures with the engine at rest, and observe the quantity of steam, if any, blowing through per hour.

If the trial has for an object the determination of the highest efficiency obtainable, the valves and pistons must first be made tight, and all parts of the engine and its auxiliaries, and all other parts of the plant concerned, should be put in the best possible working condition.

III. **Dimensions, etc.**—Measure or check the dimensions of the cylinders in any case, this being done when they are hot. If they are much worn the average diameter should be determined. Measure also the clearance, which should be done, if possible, by filling the spaces with water previously measured, the piston being placed at the end of the stroke. If the clearance cannot be measured directly, it can be determined approximately from the working drawings of the cylinder.

Measure also the dimensions of auxiliaries and accessories; also those of the boilers, so far as concerned in attaining the objects. It is well to supplement these determinations with a sketch or sketches showing the general features and arrangement of the different parts of the plant.

IV. **Coal.**—When the trial involves the complete plant, embracing boilers as well as engine, determine the character of coal to be used. The class, name of the mine, size, moisture, and quality of the coal should be stated in the report. It is desirable, for purposes of comparison, that the coal should be of some recognized standard quality for the locality where the plant is situated.

V. **Calibration of Instruments.**—All instruments and apparatus should be calibrated and their reliability and accuracy verified by comparison with recognized standards. Such apparatus as is liable to change or become broken during a test, as gauges, indicator springs, and thermometers, should be calibrated before and after the test. The accuracy of scales should be verified by standard weights. When a water-meter is used, special attention should be given to its calibration, verifying it both before and after the trial, and, if possible, during its progress, the conditions in regard to water pressure and rate of flow being made the same in the calibrations as exist throughout the trial.

VI. **Leakages of Steam, Water, etc.**—In all tests except those of a complete plant made under conditions as they exist, the boiler and its connections, both steam and feed, as also the steam piping leading to the engine and its connections, should, so far as possible, be made tight. If absolute tightness cannot be obtained (in point of fact it rarely can be), proper allowance should be made for such leakage in determining the steam actually consumed by the engine. This, however, is not required where a surface condenser is used and the water consumption is determined by measuring the discharge of the air pump. In such cases it is necessary to make sure that the condenser is tight, both before and after the test, against the entrance of circulating water, or, if such occurs, to make proper correction for it, determining it under the working difference of pressure. When the steam consumption is determined by measuring the discharge of the air pump, any leakage about the valve or piston-rods of the engine should be carefully guarded against.

Make sure that there is no leakage at any of the connections with the apparatus provided for measuring and supplying the feed water which could affect the results. All connections should, so far as possible, be visible and be blanked off, and where this cannot be done satisfactory assurance should be obtained that there is no leakage either in or out.

VII. **Duration of Test.**—The duration of a test should depend largely upon its character and the objects in view. The standard heat test of an engine, and, likewise, a test for the simple determination of the feed-water consumption, should be continued for at least five hours, unless the class of service precludes a continuous run of so long duration. It is desirable to prolong the test the number of hours stated to obtain a number of consecutive hourly records as a guide in analyzing the reliability of the whole.

Where the water discharged from a surface condenser is measured for successive short intervals of time, and the rate is found to be uniform, the test may be of a much shorter duration than where the feed water is measured to the boiler. The longer the test with a given set of conditions the more accurate the work, and no test should be so short that it cannot be divided into several intervals which will give results agreeing substantially with each other.

The commercial test of a complete plant, embracing boilers as well as engine, should continue at least one full day of twenty-four hours, whether the engine is in motion during the entire time or not. A continuous coal

test of a boiler and engine should be of at least ten hours' duration, or the nearest multiple of the interval between times of cleaning fires.

VIII. **Starting and Stopping a Test.**—(*a*) *Standard Heat Test and Feed-water Test of Engine:* The engine having been brought to the normal condition of running, and operated a sufficient length of time to be thoroughly heated in all its parts, and the measuring apparatus having been adjusted and set to work, the height of water in the gauge glasses of the boilers is observed, the depth of water in the reservoir from which the feed water is supplied is noted, the exact time of day is observed, and the test held to commence. Thereafter the measurements determined upon for the test are begun and carried forward until its close. If practicable, the test may be commenced at some even hour or minute, but it is of the first importance to begin at such time as reliable observations of the water heights are obtained, whatever the exact time happens to be when these are satisfactorily determined. When the time for the close of the test arrives, the water should, if possible, be brought to the same height in the glasses and to the same depth in the feed-water reservoir as at the beginning, delaying the conclusion of the test, if necessary, to bring about this similarity of conditions. If differences occur, the proper corrections must be made.

(*b*) *Complete Engine and Boiler Test:* For a continuous running test of combined engine or engines, and boiler or boilers, the same directions apply for beginning and ending the feed-water measurements as that just referred to under Section *a*. The time of beginning and ending such a test should be the regular time of cleaning the fires and the exact time of beginning and ending should be the time when the fires are fully cleaned, just preparatory to putting on fresh coal. In cases where there are a number of boilers, and it is inconvenient or undesirable to clean all fires at once, the time of beginning the test should be deferred until they are all cleaned and in a satisfactory state, all the fires being then burned down to a uniformly thin condition, the thickness and condition being estimated and the test begun just before firing the new coal previously weighed. The ending of the test is likewise deferred until the fires are all satisfactorily cleaned, being again burned down to the same uniformly thin condition as before, and the time of closing being taken just before replenishing the fires with new coal.

For a commercial test of a combined engine and boiler, whether the engine runs continuously for the full twenty-four hours of the day or only a portion of the time, the fires in the boilers being banked during the time when the engine is not in motion, the beginning and ending of the test should occur at the regular time of cleaning the fires, the method followed being that already given. In cases where the engine is not in continuous motion, as, for example, in textile mills, where the working time is ten or eleven hours out of the twenty-four, and the fires are cleaned and banked at the close of the day's work, the best time for starting and stopping a test is the time just before banking, when the fires are well burned down and the thickness and condition can be most satisfactorily judged. In these, as in all other cases noted, the test should be begun by observing the exact time, the thickness and condition of the fires on the grates, the height of water in the gauge glasses of the boilers, the depth of the water in the reservoir from which the feed water is supplied, and other conditions relating to the trial, the same observations being again taken at the end of the test, and the conditions in all respects being made as nearly as possible the same as at the beginning.

IX. **Measurement of Heat Units Consumed by the Engine.**—The measurement of the heat consumption requires the measurement of each supply of feed water to the boiler,—that is, the water supplied by the main feed pump, that supplied by auxiliary pumps, such as jacket water, water from separators, drips, etc., and water supplied by gravity or other means; also, the determination of the temperature of the water supplied from each source, together with the pressure and quality of the steam.

The temperatures at the various points should be those applying to the working conditions. The temperature of the feed water should be taken near the boiler. This causes the engine to suffer a disadvantage from the heat lost by radiation from the pipes which carry the water to the boiler, but it is, nevertheless, advisable on the score of simplicity. Such pipes

would, therefore, be considered a portion of the engine plant. This conforms with the rule already recommended for the tests of pumping engines where the duty per million heat units is computed from the temperature of the feed water taken near the boiler. It frequently happens that the measurement of the water requires a change in the usual temperature of supply. For example, where the main supply is ordinarily drawn from a hot-well in which the temperature is, say, 100° F., it may be necessary, owing to the low level of the well, to take the supply from some source under a pressure or head sufficient to fill the weighing tanks used, and this supply may have a temperature much below that of the hot-well; possibly as low as 40° F. The temperature to be used is not the temperature of the water as weighed in this case, but that of the working temperature of the hot-well. The working temperature in cases like this must be determined by a special test and included in the log sheets.

The heat to be determined is that used by the entire engine equipment, embracing the main cylinders and all auxiliary cylinders and mechanism concerned in the operation of the engine, including the air pump, circulating pump, and feed pumps, also the jacket and reheater, when these are used. No deduction is to be made for steam used by auxiliaries, unless these are shown by test to be unduly wasteful. In this matter an exception should be made in cases of guarantee tests where the engine contractor furnishes all the auxiliaries referred to. He should, in that case, be responsible for the whole, and no allowance should be made for inferior economy, if such exists. Should a deduction be made on account of the auxiliaries being unduly wasteful, the method of waste and its extent, as compared with the wastes of the main engine or other standard of known value, shall be reported definitely.

The steam pressure and the quality of the steam are to be taken at some point conveniently near the throttle valve. The quantity of steam used by the calorimeter must be determined and properly allowed for. (See Article XVI., on "Quality of Steam.")

X. **Measurement of Feed Water or Steam Consumption of Engine, etc.**—The method of determining the steam consumption applicable to all plants is to measure all the feed water supplied to the boilers, and deduct therefrom the water discharged by separators and drips, as also the water and steam which escapes on account of leakage of the boiler and its pipe connections and leakage of the steam main and branches connecting the boiler and the engine. In plants where the engine exhausts into a surface condenser, the steam consumption can be measured by determining the quantity of water discharged by the air pump, corrected for any leakage of the condenser, and adding thereto the steam used by jackets, reheaters, and auxiliaries, as determined independently. If the leakage of the condenser is too large to satisfactorily allow for it, the condenser should, of course, be repaired and the leakage again determined before making the test.

In measuring the water it is best to carry it through a tank or tanks resting on platform weighing scales suitably arranged for the purpose, the water being afterwards emptied into a reservoir beneath, from which the pump is supplied.

Where extremely large quantities of water must be measured, or in some places relatively small quantities, the orifice method of measuring is one that can be applied with satisfactory results. In this case the average head of water on the orifice must be determined, and, furthermore, it is important that means should be at hand for calibrating the discharge of the orifice under the conditions of use.

The corrections or deductions to be made for leakage above referred to should be applied only to the standard heat-unit test and tests for determining simply the steam or feed-water consumption, and not to coal tests of combined engine and boiler equipment. In the latter, no correction should be made except for leakage of valves connecting to other engines and boilers, or for steam used for purposes other than the operation of the plant under test. Losses of heat due to imperfections of the plant should be charged to the plant, and only such losses as are concerned in the working of the engine alone should be charged to the engine.

In measuring jacket water or any supply under pressure which has a temperature exceeding 212° F., the water should first be cooled, as may be done by discharging it into a tank of cold water previously weighed, or by

passing it through a coil of pipe submerged in running and colder water, preventing thereby the loss of evaporation which occurs when such hot water is discharged into the open air.

XI. **Measurement of Steam Used by Auxiliaries.**—Although the steam used by the auxiliaries—embracing the air pump, circulating pump, feed pump, and any other apparatus of this nature, supposing them to be steam-driven, also the steam jackets, reheaters, etc., which consume steam required for the operation of the engine—is all included in the measurement of the steam consumption, as pointed out in Article X., yet it is highly desirable that the quantity of steam used by the auxiliaries, and in many cases that used by each auxiliary, should be determined exactly, so that the net consumption of the main engine cylinders may be ascertained and a complete analysis made of the entire work of the engine plant. Where the auxiliary cylinders are non-condensing, the steam consumption can often be measured by carrying the exhaust for the purpose into a tank of cold water resting on scales or through a coil of pipe surrounded by cold running water. Another method is to run the auxiliaries as a whole, or one by one, from a spare boiler (preferably a small vertical one), and measure the feed water supplied to this boiler. The steam used by the air and circulating pumps may be measured by running them under, as near as possible, the working conditions and speed, the main engine and other auxiliaries being stopped, and testing the consumption by the measuring apparatus used on the main trial. For a short trial, to obtain approximate results, measurement can be made by the water gauge-glass method, the feed supply being shut off. When the engine has a surface condenser, the quantity of steam used by the auxiliaries may be ascertained by allowing the engine alone to exhaust into the condenser, measuring the feed water supplied to the boiler and the water discharged by the air pump, and subtracting one from the other, after allowing for losses by leakage.

XII. **Coal Measurement.**—(*a*) *Commercial Tests:* In commercial tests of the combined engine and boiler equipment, or those made under ordinary conditions of commercial service, the test should, as pointed out in Article VII., extend over the entire period of the day,—that is, twenty-four hours,—or a number of days of that duration. Consequently, the coal consumption should be determined for the entire time. If the engine runs but a part of the time, and during the remaining portion the fires are banked, the measurement of coal should include that used for banking. It is well, however, in such cases, to determine separately the amount consumed during the time the engine is in operation and that consumed during the period while the fires are banked, so as to have complete data for purposes of analysis and comparison, using suitable precautions to obtain reliable measurements. The measurement of coal begins with the first firing, after cleaning the furnaces and burning down at the beginning of the test, as pointed out in Article VIII., and ends with the last firing, at the expiration of the allotted time.

(*b*) *Continuous Running Tests:* In continuous running tests which, as pointed out in Article VII., cover one or more periods which elapse between the cleaning of the fires, the same principle applies as that mentioned under the above heading (*a*),—viz., the coal measurement begins with the first firing, after cleaning and burning down, and the measurement ends with the last firing, before cleaning and burning down at the close of the trial.

(*c*) *Coal Tests in General:* When not otherwise specially understood, a coal test of a combined engine and boiler plant is held to refer to the commercial test above noted, and the measurement of coal should conform thereto.

In connection with coal measurements, whatever the class of tests, it is important to ascertain the percentage of moisture in the coal, the weight of ashes and refuse, and, where possible, the approximate and ultimate analysis of the coal, following all the methods and details advocated in the latest report of the Boiler Test Committee of the Society. (See "Transactions of the American Society of Mechanical Engineers," Volume XXI., page 34.)

(*d*) *Other Fuels than Coal:* For all other solid fuels than coal the same directions in regard to measurement should be followed as those given for coal. If the boilers are run with oil or gas, the measurements relating to

stopping and starting are much simplified, because the fuel is burned as fast as supplied and there is no body of fuel constantly in the furnace, as in the case of using solid fuel. When oil is used it should be weighed, and when gas is used it should be measured in a calibrated gas-meter or a gasometer.

XIII. **Indicated Horse-power.**—The indicated horse-power should be determined from the average mean effective pressure of diagrams taken at intervals of twenty minutes, and at more frequent intervals if the nature of the test makes this necessary, for each end of each cylinder. With variable loads, such as those of engines driving generators for electric railroad work, and of rubber-grinding and rolling-mill engines, the diagrams cannot be taken too often. In cases like the latter, one method of obtaining suitable averages is to take a series of diagrams on the same blank card without unhooking the driving cord, and apply the pencil at successive intervals of ten seconds until two minutes' time or more has elapsed, thereby obtaining a dozen or more indications in the time covered. This tends to insure the determination of a fair average for that period. In taking diagrams for variable loads, as, indeed, for any load, the pencil should be applied long enough to cover several successive revolutions, so that the variations produced by the action of the governor may be properly recorded. To determine whether the governor is subject to what is called "racing" or "hunting," a "variation diagram" should be obtained, —that is, one in which the pencil is applied a sufficient time to cover a complete cycle of variations. When the governor is found to be working in this manner the defect should be remedied before proceeding with the test.

It is seldom necessary, as far as average power measurements are concerned, to obtain diagrams at precisely the same instant at the two ends of the cylinder, or at the same instant on all the cylinders, when there are more than one. All that is required is to take the diagrams at regular intervals. Should the diagrams vary so much among themselves that the average may not be a fair one, it signifies that they should be taken more frequently, and not that special care should be employed to obtain the diagrams of each set at precisely the same time. When diagrams are taken during the time when the engine is working up to speed at the start, or when a study of valve setting and steam distribution is being made, they should be taken at as nearly the same time as practicable. In cases where the diagrams are to be taken simultaneously, the best plan is to have an operator stationed at each indicator. This is desirable, even where an electric or other device is employed to operate all the instruments at once, for, unless there are enough operators, it is necessary to open the indicator-cocks some time before taking the diagrams and run the risk of clogging the pistons and heating the high-pressure springs above the ordinary working temperature.

The most satisfactory driving rig for indicating seems to be some form of well-made pantagraph, with driving cord of fine annealed wire leading to the indicator. The reducing motion, whatever it may be, and the connections to the indicator should be so perfect as to produce diagrams of equal lengths when the same indicator is attached to either end of the cylinder, and produce a proportionate reduction of the motion of the piston at every point of the stroke, as proved by test.

The use of a three-way cock and a single indicator connected to the two ends of the cylinder is not advised, except in cases where it is impracticable to use an indicator close to each end. If a three-way cock is used the error produced should be determined and allowed for.

To determine the average power developed in cases where the engine starts from rest during the progress of the trial, as in a commercial test of a plant where the engine runs only a portion of the twenty-four hours, a number of diagrams should be taken during the period of getting up speed and applying the working load, the corresponding speed for each set of diagrams being counted. The power shown by these diagrams for the proportionate time should be included in the average for the whole run, and the duration should be the time the throttle valve is open.

XIV. **Testing Indicator Springs.**—To make a perfectly satisfactory comparison of indicator springs with standards, the calibration should be made, if this were practical, under the same conditions as those pertaining

to their ordinary use. Owing to the fact that the pressure of the steam in the indicator cylinder and the corresponding temperature are undergoing continual changes, it becomes almost impossible to compare the springs with any standard under such conditions. There must be a constant pressure during the time that the comparison is being made. Although the best that can be done is not altogether satisfactory, it seems that we must be content with it. To bring the conditions as nearly as possible to those of the working indicator, the steam should be admitted to the indicator as short a time as practicable for each of the pressures tried, and then the indicator cock should be closed and the steam exhausted therefrom before another pressure is tried. By this means the parts are heated and cooled somewhat the same as under the working conditions. We recommend, therefore, that for each required pressure the first step be to open and close the indicator cock a number of times in quick succession, then to quickly draw the line on the paper for the desired record, observing the gauge or other standard at the instant when the line is drawn. A corresponding atmospheric line is taken immediately after obtaining the line at the given pressure, so as to eliminate any difference in the temperature of the parts of the indicator. This appears to be a better method (although less readily carried on and requiring more care) than the one heretofore more commonly used, where the indicator cock is kept continually open and the pressure is gradually rising or falling through the range of comparison.

The calibration should be made for at least five points, two of these being for the pressures corresponding as near as may be to the initial and back pressures, and three for intermediate points equally distant.

For pressures above the atmosphere the proper standard recommended is the dead-weight testing apparatus or a reliable mercury column, or an accurate steam gauge proved correct, or of known error, by either of these standards. For pressures below the atmosphere the best standard to use is a mercury column.

The correct scale of spring to be used for working out the mean effective pressure of the diagrams should be the average based on the calibration.

XV. **Brake Horse-power.**—This term applies to the power delivered from the fly-wheel shaft of the engine. It is the power absorbed by a friction brake applied to the rim of the wheel or to the shaft. A form of brake is preferred that is self-adjusting to a certain extent, so that it will, of itself, tend to maintain a constant resistance at the rim of the wheel. One of the simplest brakes for comparatively small engines which may be made to embody this principle consists of a cotton or hemp rope, or a number of ropes, encircling the wheel, arranged with weighing scales or other means for showing the strain. An ordinary band brake may also be constructed so as to embody the principle. The wheel should be provided with interior flanges for holding water used for keeping the rim cool.

The water-friction brake is considered most satisfactory, not only for small powers but for large powers. It is especially adapted for high speeds, and has the advantage of being self-cooling.

XVI. **Quality of Steam.**—When ordinary saturated steam is used its quality should be obtained by the use of a throttling calorimeter attached to the main steam pipe near the throttle valve. When the steam is superheated the amount of superheating should be found by the use of a thermometer placed in a thermometer-well filled with mercury, inserted in the pipe. The sampling pipe for the calorimeter should, if possible, be attached to a section of the main pipe having a vertical direction, with the steam preferably passing upward, and the sampling nozzle should be made of a half-inch pipe having at least twenty ⅛-inch holes in its perforated surface. The readings of the calorimeter should be corrected for radiation of the instrument, or they should be referred to a normal reading. If the steam is superheated, the amount of superheating should be obtained by referring the reading of the thermometer to that of the same thermometer when the steam within the pipe is saturated, and not by taking the difference between the reading of the thermometer and the temperature of saturated steam at the observed pressure as given in a steam table.

XVII. **Speed.**—There are several reliable methods of ascertaining the speed, or the number of revolutions of the engine crank-shaft per minute. The simplest is the familiar method of counting the number of turns for a

period of one minute, with the eye fixed on the second-hand of a time-piece. Another is the use of a counter held for a minute or a number of minutes against the end of the main shaft. Another is the use of a reliable calibrated tachometer held likewise against the end of the shaft. The most reliable method, and the one we recommend, is the use of a continuous recording engine register or counter, taking the total reading each time that the general test data are recorded, and computing the revolutions per minute corresponding to the difference in the readings of the instrument. When the speed is above 250 revolutions per minute it is almost impossible to make a satisfactory counting of the revolutions without the use of some form of mechanical counter.

The determination of variation of speed during a single revolution, or the effect of the fluctuation due to sudden changes of the load, is also desirable, especially in engines driving electric generators used for lighting purposes. There is at present no recognized standard method of making such determinations, and, if such are desired, the method employed may be devised by the person making the test and described in detail in the report.

XVIII. **Recording the Data.**—Take note of every event connected with the progress of the trial, whether it seems at the time to be important or unimportant. Record the time of every event and time of taking every weight and every observation. Observe the pressures, temperatures, water heights, speeds, etc., every twenty or thirty minutes when the conditions are practically uniform, and at much more frequent intervals if the conditions vary. Observations which concern the feed-water measurement should be made with special care at the expiration of each hour of the trial, so as to divide the tests into hourly periods and show the uniformity of the conditions and results as the test goes forward. Where the water discharged from a surface condenser is weighed, it may be advisable to divide the test by this means into periods of less than one hour.

The data and observations of the test should be kept on properly-prepared blanks or in note-books containing columns suitably arranged for a clear record. As different observers have their own individual ideas as to how such records should be kept, no special form of log sheet is given as a necessary part of the code.

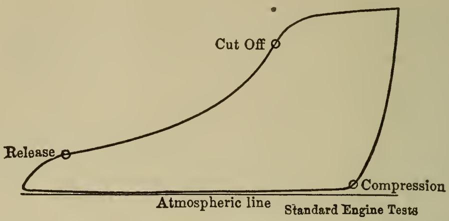
XIX. **Uniformity of Conditions.**—In a test having for an object the determination of the maximum economy obtainable from an engine, or where it is desired to ascertain with special accuracy the effect of predetermined conditions of operation, it is important that all the conditions under which the engine is operated should be maintained uniformly constant. This requirement applies especially to the pressure, the speed, the load, the rate of feeding the various supplies of water, the height of water in the gauge glasses, and the depth of water in the feed-water reservoir.

XX. **Analysis of Indicator Diagrams.**—*(a) Steam Accounted for by the Indicator:* The simplest method of computing the steam accounted for by the indicator is the use of the formula,

$$M = \frac{13750}{\text{M. E. P.}}[(C + E) \times Wc - (H + E) \times Wh],$$

which gives the weight, in pounds, per indicated horse-power per hour. In this formula the symbol "M. E. P." refers to the mean effective pressure. In multiple-expansion engines this is the combined mean effective pressure referred to the cylinder in question. The symbol C refers to the proportion of the stroke completed at points on the expansion line of the diagram near the actual cut-off or release, the symbol H to the proportion of compression, and the symbol E to the proportion of clearance, all of which are determined from the indicator diagram. The symbol Wc refers to the weight of 1 cubic foot of steam at the cut-off or release pressure, and the symbol Wh to the weight of 1 cubic foot of steam at the compression pressure, these weights being taken from steam tables of recognized accuracy. The points near the cut-off and release on the expansion line, and the point on the compression line, are located as shown on the sample diagram. They are the points in the case of the expansion and compression lines of the diagram which mark the complete closure of the valve. The

point near the cut-off, for example, lies where the curve of expansion begins after the rounding of the diagram due to the wire-drawing, which occurs while the valve is closing. This cut-off may be located by finding the point where the curve is tangent to a hyperbolic curve.



Showing Points where "Steam Accounted for by Indicator" is Computed.

Should the point in the compression curve be at the same height as the point in the expansion curve, then $Wc = Wh$, and the formula becomes

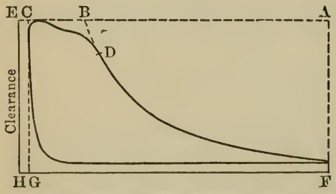
$$\frac{13750}{\text{M. E. P.}} \times (C - H) \times Wc,$$

in which $(C - H)$ represents the distance between the two points divided by the length of the diagram.

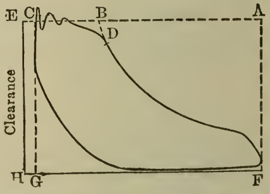
When the load and all other conditions are substantially uniform, it is unnecessary to work up the steam accounted for by the indicator from all the diagrams taken. Five or more sample diagrams may be selected and the computations based on the samples instead of on the whole.

(*b*) *Sample Indicator Diagrams:* In order that the report of a test may afford complete information regarding the conditions of the test, sample indicator diagrams should be selected from those taken and copies appended to the tables of results. In cases where the engine is of the multiple-expansion type, these sample diagrams may also be arranged in the form of a "combined" diagram.

(*c*) *The Point of Cut-off:* The term "cut-off," as applied to steam engines, although somewhat indefinite, is usually considered to be at an earlier point in the stroke than the beginning of the real expansion line. That the cut-off point may be defined in exact terms for commercial purposes, as used in steam-engine specifications and contracts, the Committee recommends that, unless otherwise specified, the *commercial cut-off*, which seems



Four-valve Engine, Slow Speed.



Single-valve Engine, High Speed.

$$\text{Commercial Cut-off} = \frac{BC}{AC}.$$

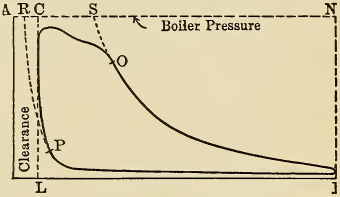
to be an appropriate expression for this term, be ascertained as follows: through a point showing the maximum pressure during admission draw a line parallel to the atmospheric line. Through the point on the expansion line, near the actual cut-off, referred to in Section XX. (*a*), draw a hyperbolic curve. The point where these two lines intersect is to be considered the *commercial cut-off* point. The percentage is then found by dividing the

length of the diagram measured to this point by the total length of the diagram, and multiplying the result by 100.

The principle involved in locating the commercial cut-off is shown in the preceding diagrams, the first of which represents a diagram from a slow-speed Corliss engine, and the second a diagram from a single-valve, high-speed engine. In the latter case, where, owing to the fling of the pencil, the steam line vibrates, the maximum pressure is found by taking a mean of the vibrations at the highest point.

The *commercial cut-off*, as thus determined, is situated at an earlier point of the stroke than the actual cut-off used in computing the "steam accounted for" by the indicator and referred to in Section XX. (*a*).

(*d*) *Ratio of Expansion:* The "commercial" ratio of expansion is the quotient obtained by dividing the volume corresponding to the piston displacement, including clearance, by the volume of the steam at the commercial cut-off, including clearance. In a multiple-expansion engine the volumes are those pertaining to the low-pressure cylinder and high-pressure cylinder, respectively.



$$\text{Ideal Ratio of Expansion} = \frac{IL}{RS}.$$

The "ideal" ratio of expansion is the quotient obtained by dividing the volume of the piston displacement by the volume of the steam at the cut-off (the latter being referred to the throttle-valve pressure), less the volume equivalent to that retained at compression. In a multiple-expansion engine the volumes to be used are those pertaining to the low-pressure cylinder and high-pressure cylinder, respectively.

(*e*) *Diagram Factor:* The diagram factor is the proportion borne by the actual mean effective pressure measured from the indicator diagram to that of a diagram in which the various operations of admission, expansion, release, and compression are carried on under assumed conditions. The factor recommended refers to an ideal diagram which represents the maximum power obtainable from the steam accounted for by the indicator diagrams at the point of cut-off, assuming, first, that the engine has no clearance; second, that there are no losses through wire-drawing the steam either during the admission or the release; third, that the expansion line is a hyperbolic curve; and fourth, that the initial pressure is that of the boiler and the back pressure that of the atmosphere for a non-condensing engine, and of the condenser for a condensing engine.

The diagram factor is useful for comparing the steam distribution losses in different engines, and is of special use to the engine designer, for by multiplying the mean effective pressure obtained from the assumed theoretical diagrams by it he will obtain the actual mean effective pressure that should be developed in an engine of the type considered. The expansion and compression curves are taken as hyperbolas, because such curves are ordinarily used by engine builders in their work, and a diagram based on such curves will be more useful to them than one where the curves are constructed according to a more exact law.

In cases where there is a considerable loss of pressure between the boiler and the engine, as where steam is transmitted from a central plant to a number of consumers, the pressure of the steam in the supply main should be used in place of the boiler pressure in constructing the diagrams.

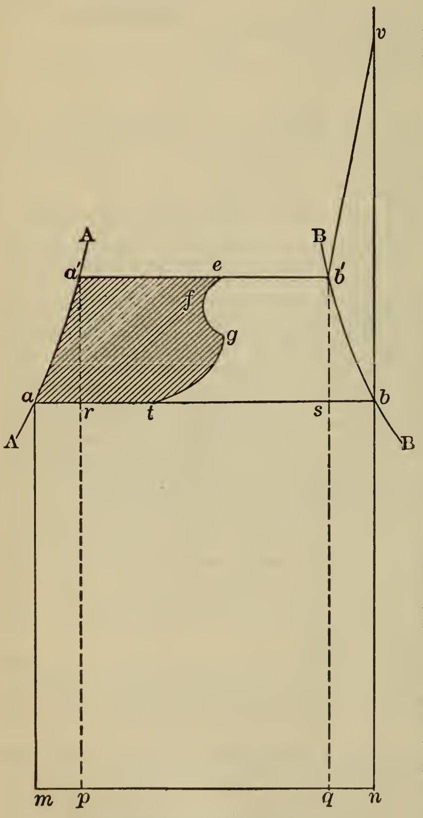
XXI. **Standards of Economy and Efficiency.**—The hourly consumption of heat, determined by employing the actual temperature of the feed water to the boiler, as pointed out in Article IX. of the Code, divided by the indicated and brake horse-power,—that is, the number of heat units consumed per indicated and per brake horse-power per hour are the standards of engine efficiency recommended by the Committee. The consumption *per hour* is chosen rather than the consumption per minute, so as to conform with the designation of time applied to the more familiar units of coal and water measurement, which have heretofore been used. The British standard, where the temperature of the feed water is taken as that corresponding to the temperature of the back-pressure steam, allowance

being made for any drips from jackets or reheaters, is also included in the tables.

It is useful in this connection to express the efficiency in its more scientific form, or what is called the "thermal efficiency ratio." The thermal efficiency ratio is the proportion which the heat equivalent of the power developed bears to the total amount of heat actually consumed, as determined by test. The heat converted into work, represented by 1 horse-power, is 1,980,000 foot-pounds per hour, and this, divided by 778, equals 2545 B. T. U. Consequently, the thermal efficiency ratio is expressed by the fraction

$$\frac{2545}{\text{B. T. U. per horse-power per hour}}.$$

XXII. Heat Analysis.—For certain scientific investigations it is useful to make a heat analysis of the diagram to show the interchange of heat from steam to cylinder walls, etc., which is going on within the cylinder. This is unnecessary for commercial tests.



Temperature-entropy Diagram.

XXIII. Temperature-entropy Diagram.—The study of the heat analysis is facilitated by the use of the temperature-entropy diagram, in which areas represent quantities of heat, the coördinates being the absolute temperature and entropy. Such a diagram is here given. When the quantity given in the steam tables is plotted, two curves, *AA* and *BB*, are obtained which may be termed the water line and the steam line, *AA* being the logarithmic curve if the specific heat of the water is taken as constant. The diagram refers to a unit weight of the agent, and the heat necessary to raise a pound of water from the temperature, *ma*, to the temperature, *pa'*, and evaporate it at that temperature, is represented by the area, *aa'b'qm*. If the steam be now expanded adiabatically, the temperature will fall to *qs*, and x per cent. $= \frac{as}{ab}$ will remain as steam, the rest being liquefied. If the steam is now rejected, it carries away with it the heat, *sqma*, the work area being *a'b'sa*, from which must be deducted the work, *w* (expressed in heat units), to pump a pound of water into the boiler. The efficiency of this cycle is evidently

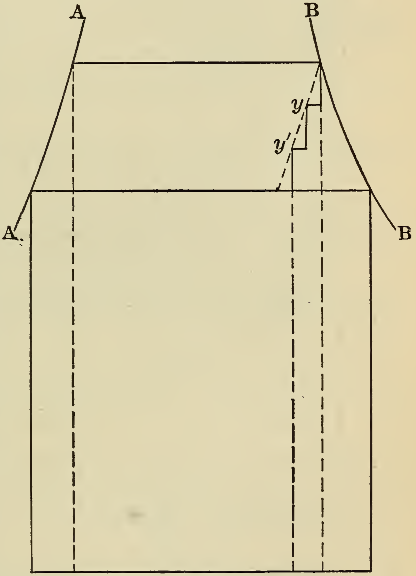
$$\frac{h + L_1 - xL_2 - w}{h + L_1}$$

in which

$$x = \frac{ar + a'b'}{ab} = \frac{\log. \frac{T_1}{T_2} + \frac{L_1}{T_1}}{\frac{L_2}{T_2}}.$$

By the action of the walls a portion of the steam is liquefied prior to the expansion, which, therefore, begins at e; and since the cooling action of the walls continues, the expansion line falls off to ef, from which point a reverse action takes place and the expansion line bends over to g. Finally, since the release takes place before the condenser temperature is reached, the heat rejection starts at g, following a line of equal volume until the exhaust-port temperature is reached at j. If enough heat is added during expansion to keep the steam theoretically saturated,—as, for example, by a water jacket,—such additional heat is represented by the area, $b'bnq$, and the additional work obtained by the triangle, $b'bs$. If the steam is superheated sufficiently to give by expansion theoretically dry steam at the end, such additional heat is represented by the area, $b'vnq$, and the additional work by $b'vbs$. Neither of these extra amounts of work are realized in practice, and it is evident from the diagram that the heat thus applied is in both cases less efficient than in the principal cycle. Nevertheless, the action in each case is to bring the point, e, nearer the point, b', and to effect a notable net economy.

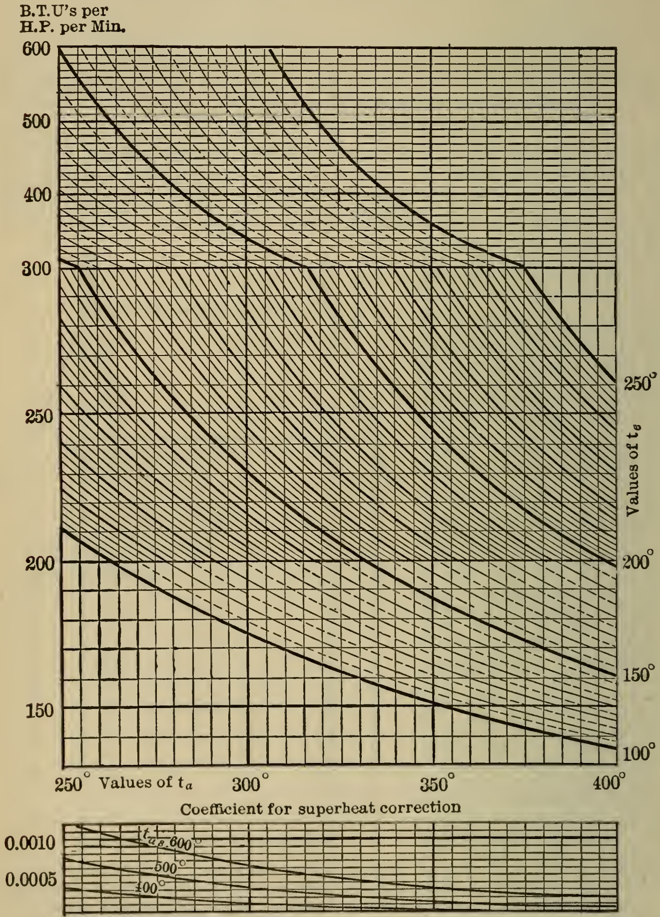
The Carnot cycle would be obtained if in the Rankine cycle the rejection of heat were stopped at r and the temperature of the mixture raised to a' by compression. This cannot be practically accomplished, but a system of feed-water heaters has been suggested and exemplified in the Nordberg engine, which is theoretically a close equivalent to it. Where steam is expanded in, say, three cylinders, the feed water may be successively heated from the receiver intermediate between each pair, the effect of which is illustrated in the above diagram. The expansion line follows the heavy line, being carried over to y by the first feed-water heater and to y' by the second feed-water heater. With an infinite number of such feed-water heaters the line, yy', would be parallel to aa', and the cycle equivalent to that of Carnot.



Temperature-entropy Diagram.

XXIV. **Ratio of Economy of an Engine to that of an Ideal Engine.**—The ideal engine recommended for obtaining this ratio is that which was adopted by the Committee appointed by the Civil Engineers of London to consider and report a standard thermal efficiency for steam engines. This engine is one which follows the Rankine cycle, where steam at a constant pressure is admitted into the cylinder with no clearance, and, after the point of cut-off, is expanded adiabatically to the back pressure. In obtaining the economy of this engine the feed water is assumed to be

returned to the boiler at the exhaust temperature. Such a cycle is preferable to the Carnot for the purpose at hand, because the Carnot cycle is theoretically impossible for an engine using superheated steam produced



Curves showing British Thermal Units Expended per Minute per Indicated Horse-power by the Ideal Steam Engine, forming part of the Rankine Cycle. (From the Minutes of Proceedings of the Civil Engineers of London.) Temperatures are expressed in degrees Fahrenheit. The upper and lower portions of the upper diagram are to different scales. This is in order that the lower and more important part may be read more easily, and accounts for the cusps in the curves.

at a constant pressure, and the gain in efficiency for superheated steam corresponding to the Carnot efficiency will be much greater than that possible for the actual cycle.

The economy of the ideal engine recommended can be readily obtained from the accompanying chart, which has been copied from the report already mentioned of the Committee appointed by the Civil Engineers of London.

In the chart, t_a represents the temperature of saturated steam at the boiler pressure, in degrees Fahrenheit; t_{as}, that of the steam furnished to the engine, should there be superheating; t_e, that of the exhaust. The British thermal units consumed per minute per indicated horse-power by the ideal engine can be read off directly from the curves given in the upper portion of the diagram. Thus, if the temperature of the exhaust, t_e, is 212° F., and the temperature of the steam at boiler pressure is 350° F., the heat consumption is 265 B. T. U. per indicated horse-power per minute. If the steam is superheated, the figure obtained as just described is corrected by employing the factor obtained from the lower part of the diagram. Opposite the temperature of saturation, corresponding to the pressure in the boiler, and on the curve corresponding to the temperature of superheated steam, t_{as}, is found a coefficient. This coefficient, multiplied by the exhaust temperature and by the heat consumption per minute obtained,—should there be no superheating,—gives the deduction to be made on account of the superheating. Thus, if the temperature of the superheated steam is 500° F. in the case already considered for saturated steam, we find, opposite 350 degrees for t_a and on the curve for $t_{as} = 500$ degrees, the coefficient, 0.00015. This gives the correction, $0.00015 \times 265 = 8.5$ B. T. U; and the heat consumption of the engine, when furnished with superheated steam, will be $265 - 8.5 = 256.5$ B. T. U. per indicated horse-power per minute.

The ratio of the economy of an engine to that of the ideal engine is obtained by dividing the heat consumption per indicated horse-power per minute for the ideal engine by that of the actual engine.

XXV. **Miscellaneous.**—In the case of tests of combined engine and boiler plants, where the full data of the boiler performance is to be determined, reference should be made to the directions given by the Boiler-test Committee of the Society, Code of 1899. (See "Transactions of the American Society of Mechanical Engineers," Volume XXI., page 34.)

In tests made for scientific research, and in those made on special forms of engines, the line of procedure must be varied according to the special objects in view; and it has been deemed unnecessary to go into particulars applying to such tests.

In testing steam pumping engines and locomotives, in accordance with the standard methods of conducting such tests recommended by the committees of the Society, reference should be made to the reports of those committees in the "Transactions," Volume XII., page 530, and in Volume XIV., page 1312.

XXVI. **Report of Test.**—The data and results of the test should be reported in the manner and in the order outlined in one of the following tables, the first of which gives, it is hoped, a complete summary of all the data and results as applied not only to the standard heat-unit test, but also to tests of combined engine and boiler for determining all questions of performance, whatever the class of service; the second refers to a short form of report giving the necessary data and results for the standard heat test; and the third to a short form of report for a feed-water test. It is the intention that the tables should be full enough to apply to any type of engine, but where not so, or where special data and results are determined, additional results may be inserted under the appropriate headings. Although these forms are arranged so as to be used for expressing the principal data and results of tests of pumping engines and locomotives, as well as for all other classes of steam engines, it is not the intention that they shall supplant the forms recommended by the committees on Duty Trials and Locomotives in cases where the full report of a test of such engines is desired.

Data and Results of Standard Heat Test of Steam Engine.

Arranged according to the Short Form advised by the Engine Test Committee of the American Society of Mechanical Engineers. Code of 1902.

1. Made by ....of....................................
   on engine located at...
   to determine ....
   ....
2. Date of trial ....
3. Type and class of engine; also of condenser................................
   ....

	1st Cyl.	2d Cyl.	3d Cyl.
4. Dimensions of main engine:			
(*a*) Diameter of cylinder, in inches....			
(*b*) Stroke of piston, in feet............			
(*c*) Diameter of piston-rod, in inches..			
(*d*) Average clearance, in per cent			
(*e*) Ratio of volume of cylinder to high-pressure cylinder			
(*f*) Horse-power constant for 1 pound mean effective pressure and 1 revolution per minute...........			

5. Dimensions and type of auxiliaries ....
   ....

Total Quantities, Time, Etc.

6. Duration of test.. hours.
7. Total water fed to boilers from main source of supply. lbs.
8. Total water fed from auxiliary supplies:
   (*a*)... lbs.
   (*b*)... lbs.
   (*c*)... lbs.
9. Total water fed to boilers from all sources............. lbs.
10. Moisture in steam or superheating near throttle ...... per cent. or deg.
11. Factor of correction for quality of steam..............
12. Total dry steam consumed for all purposes .... lbs.

Hourly Quantities.

13. Water fed from main source of supply................. lbs.
14. Water fed from auxiliary supplies:
    (*a*).. lbs.
    (*b*).. lbs.
    (*c*).. lbs.
15. Total water fed to boilers per hour .... lbs.
16. Total dry steam consumed per hour .... lbs.
17. Loss of steam and water per hour due to drips from main steam pipes and to leakage of plant.... lbs.
18. Net dry steam consumed per hour by engine and auxiliaries.. lbs.

Pressures and Temperatures (Corrected).

19. Pressure in steam pipe near throttle, by gauge .... lbs. per sq. in.
20. Barometric pressure of atmosphere, in inches of mercury .... ins.
21. Pressure in receivers, by gauge........................ lbs. per sq. in.
22. Vacuum in condenser, in inches of mercury .... ins.
23. Pressure in jackets and reheaters, by gauge .... lbs. per sq. in.
24. Temperature of main supply of feed water............. deg. Fahr.
25. Temperature of auxiliary supplies of feed water:
    (*a*).. deg. Fahr.
    (*b*).. deg. Fahr.
    (*c*).. deg. Fahr.

26. Ideal feed-water temperature, corresponding to pressure of steam in the exhaust pipe, allowance being made for heat derived from jacket or reheater drips .... deg. Fahr.

Data Relating to Heat Measurement.

27. Heat units per pound of feed water, main supply ..... B. T. U.
28. Heat units per pound of feed water, auxiliary supplies:
    (*a*).. B. T. U.
    (*b*).. B. T. U.
    (*c*).. B. T. U.
29. Heat units consumed per hour, main supply .... B. T. U.
30. Heat units consumed per hour, auxiliary supplies:
    (*a*).. B. T. U.
    (*b*).. B. T. U.
    (*c*).. B. T. U.
31. Total heat units consumed per hour for all purposes .. B. T. U.
32. Loss of heat per hour due to leakage of plant, drips, etc.................................... B. T. U.
33. Net heat units consumed per hour:
    (*a*) By engine alone.............................. B. T. U.
    (*b*) By auxiliaries.............................. B. T. U.
34. Heat units consumed per hour by engine alone, reckoned from temperature given in line 26 ...... B. T. U.

Indicator Diagrams.

	1st Cyl.	2d Cyl.	3d Cyl.
35. Commercial cut-off, in per cent. of stroke.			
36. Initial pressure, in pounds, per square inch above atmosphere			
37. Back pressure at mid-stroke, above or below atmosphere, in pounds, per square inch			
38. Mean effective pressure, in pounds, per square inch			
39. Equivalent mean effective pressure, in pounds, per square inch:			
(*a*) Referred to first cylinder...........			
(*b*) Referred to second cylinder........			
(*c*) Referred to third cylinder..........			
40. Pressures and percentages used in computing the steam accounted for by the indicator diagrams, measured to points on the expansion and compression curves..........			
Pressure above zero, in pounds, per square inch:			
(*a*) Near cut-off			
(*b*) Near release.......................			
(*c*) Near beginning of compression....			
Percentage of stroke at points where pressures are measured:			
(*a*) Near cut-off			
(*b*) Near release.......................			
(*c*) Near beginning of compression....			
41. Steam accounted for by indicator, in pounds, per indicated horse-power per hour:			
(*a*) Near cut-off			
(*b*) Near release.......................			
42. Ratio of expansion:			
(*a*) Commercial.......................			
(*b*) Ideal			

Speed.

43. Revolutions per minute .... rev.

Power.

44. Indicated horse-power developed by main engine cylinders:
    First cylinder .... H. P.
    Second cylinder .... H. P.
    Third cylinder .... H. P.
    Total .... H. P.
45. Brake horse-power developed by engine .... H. P.

Standard Efficiency and other Results.*

46. Heat units consumed by engine and auxiliaries per hour:
    (*a*) Per indicated horse-power .... B.T.U.
    (*b*) Per brake horse-power .... B.T.U.
47. Equivalent standard coal, in pounds, per hour:
    (*a*) Per indicated horse-power .... lbs.
    (*b*) Per brake horse-power .... lbs.
48. Heat units consumed by main engine per hour, corresponding to ideal maximum temperature of feed water given in line 26:
    (*a*) Per indicated horse-power .... B.T.U.
    (*b*) Per brake horse-power .... B.T.U.
49. Dry steam consumed per indicated horse-power per hour:
    (*a*) Main cylinders, including jackets .... lbs.
    (*b*) Auxiliary cylinders .... lbs.
    (*c*) Engine and auxiliaries .... lbs.
50. Dry steam consumed per brake horse-power per hour:
    (*a*) Main cylinders, including jackets .... lbs.
    (*b*) Auxiliary cylinders .... lbs.
    (*c*) Engine and auxiliaries .... lbs.
51. Percentage of steam used by main engine cylinders accounted for by indicator diagrams, near cut-off of high-pressure cylinder .... per cent.

Additional Data.

Add any additional data bearing on the particular objects of the test or relating to the special class of service for which the engine is used. Also give copies of indicator diagrams nearest the mean and the corresponding scales.

Data and Results of Feed-water Test of Steam Engine.

Arranged according to the Short Form advised by the Engine Test Committee of the American Society of Mechanical Engineers. Code of 1902.

1. Made by .... of ....
   on engine located at ....
   to determine ....
   ....
2. Date of trial ....
3. Type of engine (simple, compound, or other multiple-expansion; condensing or non-condensing) ....
   ....
4. Class of engine (mill, marine, locomotive, pumping, electric, or other).
5. Rated power of engine ....
6. Name of builders ....
7. Number and arrangement of cylinders of engine; how lagged; type of valves and of condensers ....

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* The horse-power referred to above (items 46–50) is that of the main engine, exclusive of auxiliaries.

	1st Cyl.	2d Cyl.	3d Cyl.
8. Dimensions of engine....................			
(a) Single or double acting............			
(b) Cylinder dimensions:			
Bore, in inches..................			
Stroke, in feet..................			
Diameter of piston-rod, in inches.			
Diameter of tail-rod, in inches...			
(c) Clearance, in per cent. of volume, displaced by piston per stroke:			
Head end..........................			
Crank end.........................			
Average...........................			
(d) Ratio of volume of each cylinder to volume of high-pressure cylinder...........................			
(e) Horse-power constant for 1 pound mean effective pressure and 1 revolution per minute..........			

Total Quantities, Time, Etc.

9. Duration of test .... hours.
10. Water fed to boilers from main source of supply ...... lbs.
11. Water fed from auxiliary supplies:
    (a).. lbs.
    (b).. lbs.
    (c).. lbs.
12. Total water fed from all sources .... lbs.
13. Moisture in steam or superheating near throttle*...... per cent. or deg.
14. Factor of correction for quality of steam..............
15. Total dry steam consumed for all purposes .... lbs.

Hourly Quantities.

16. Water fed from main source of supply................ lbs.
17. Water fed from auxiliary supplies:
    (a).. lbs.
    (b).. lbs.
    (c).. lbs.
18. Total water fed to boilers per hour .... lbs.
19. Total dry steam consumed per hour .... lbs.
20. Loss of steam and water per hour due to leakage of plant, drips, etc.................................. lbs.
21. Net dry steam consumed per hour by engine and auxiliaries.. lbs.
22. Dry steam consumed per hour:
    (a) Main cylinders.................................. lbs.
    (b) Jackets and reheaters .... lbs.

Pressures and Temperatures (Corrected).

23. Steam-pipe pressure near throttle, by gauge........... lbs. per sq. in.
24. Barometric pressure of atmosphere, in inches of mercury .... ins.
25. Pressure in first receiver, by gauge .... lbs. per sq. in.
26. Pressure in second receiver, by gauge .... lbs. per sq. in.
27. Vacuum in condenser:
    (a) In inches of mercury............................ ins.
    (b) Corresponding total pressure................... lbs. per sq. in.
28. Pressure in steam jackets, by gauge .... lbs. per sq. in.
29. Pressure in reheater, by gauge........................ lbs. per sq. in.
30. Superheating of steam in first receiver .... deg. Fahr.
31. Superheating of steam in second receiver .... deg. Fahr.

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* In case of superheated steam engines determine, if practicable, the temperature of the steam in each cylinder.

Indicator Diagrams.

	1st Cyl.	2d Cyl.	3d Cyl.
32. Commercial cut-off, in per cent., of stroke.			
33. Initial pressure, in pounds, per square inch above atmosphere................			
34. Back pressure at mid-stroke above or below atmosphere, in pounds, per square inch			
35. Mean effective pressure, in pounds, per square inch......................			
36. Equivalent mean effective pressure, in pounds, per square inch per indicated horse-power................			
(a) Referred to first cylinder.			
(b) Referred to second cylinder.			
(c) Referred to third cylinder.			
37. Pressures and percentages used in computing the steam accounted for by the indicator diagrams, measured to points on the expansion and compression curves..........			
Pressures above zero, in pounds, per square inch:			
(a) Near cut-off			
(b) Near release.......................			
(c) Near beginning of compression....			
Percentage of stroke at points where pressures are measured:			
(a) Near cut-off			
(b) Near release.......................			
(c) Near beginning of compression....			
38. Aggregate mean effective pressure, in pounds, per square inch referred to each cylinder given in heading			
39. Mean back pressure above zero............			
40. Steam accounted for, in pounds, per indicated horse-power per hour:			
(a) Near cut-off			
(b) Near release.......................			
41. Ratio of expansion:			
(a) Commercial			
(b) Ideal			

Speed.

42. Revolutions per minute.................................... rev.
43. Piston speed per minute .... ft.

Power.

44. Indicated horse-power developed by main engine cylinders:
First cylinder .... H. P.
Second cylinder .... H. P.
Third cylinder .... H. P.
Total .... H. P.

Efficiency Results.

45. Dry steam consumed per indicated horse-power per hour:
(a) Main cylinder, including jackets .... lbs.
(b) Auxiliary cylinders, etc. .... lbs.
(c) Engine and auxiliaries................................. lbs.
46. Percentage of steam used by main engine cylinders accounted for by indicator diagrams:

	1st Cyl.	2d Cyl.	3d Cyl.
(a) Near cut-off			
(b) Near release.......................			

Sample Diagrams.

Copies of indicator diagrams nearest the mean, with corresponding scales, should be given in connection with table.

Practical Engine Performances.

(J. B. Stanwood.)

NON-CONDENSING ENGINES.

Slide-valve Engine.—75 to 80 pounds boiler pressure; stroke, long; mean effective pressure, 33 to 38 pounds per square inch; 25 to 100 horse-power; cut-off, ⅝ stroke; performance, about 40 pounds of steam per indicated horse-power per hour. When valves and piston are tight this has been reduced to 33 pounds of dry steam per indicated horse-power per hour by careful test.

Automatic High-speed Engines with Single Valves.—75 to 80 pounds boiler pressure; stroke, about equal to piston diameter; mean effective pressure, 40 pounds per square inch; 50 to 150 horse-power; cut-off, ¼ stroke; performance, about 40 pounds of steam per horse-power per hour. When valves and piston are tight this has been reduced to 32 pounds of dry steam per indicated horse-power per hour. Valves difficult to keep tight.

Automatic High-speed Engines with Double Valves.—75 to 80 pounds boiler pressure; stroke, 1½ to 2 times piston diameter; mean effective pressure, 40 pounds per square inch; 50 to 150 horse-power; cut-off, ¼ stroke; performance, about 35 pounds of steam per indicated horse-power per hour. When valves and piston are tight this has been reduced to 30 pounds of dry steam per indicated horse-power per hour by careful test.

Automatic Cut-off Engines of the Corliss Type.—Stroke, 2 to 3 times diameter of piston; 75 to 90 pounds boiler pressure; mean effective pressure, 40 pounds per square inch; cut-off, ⅕ to ¼ stroke; performance, under 200 horse-power, 29 to 30 pounds of steam per indicated horse-power per hour, over 200 horse-power, 27 pounds of steam per indicated horse-power per hour. When valves and piston are tight this has been reduced to 23⅛ pounds of dry steam per indicated horse-power per hour.

Compound Engines.—High speed; automatic cut-off; short stroke; 110 to 120 pounds boiler pressure; mean effective pressure, 25 to 27 pounds per square inch; 6 expansions; 100 to 250 horse-power; performance, 27 pounds of steam per indicated horse-power per hour.

CONDENSING ENGINES.

Automatic Cut-off Engines of the Corliss Type.—Stroke, 2 to 3 times piston diameter; 70 to 80 pounds boiler pressure; mean effective pressure, 40 pounds per square inch; over 200 horse-power; cut-off, ⅕ stroke; about 19 to 20 pounds of steam per indicated horse-power per hour.

Compound Engines.—High speed; automatic cut-off; short stroke; 110 to 120 pounds boiler pressure; mean effective pressure, 27 to 30 pounds per square inch; 9 expansions; 200 to 500 horse-power; 17 to 19 pounds of steam per indicated horse-power per hour.

Compound Automatic Cut-off Engines of the Corliss Type.—Stroke, on high-pressure cylinder, 2 to 3 times piston diameter; 110 to 135 pounds boiler pressure; mean effective pressure, 14 to 24 pounds per square inch; over 400 horse-power; 16 to 20 expansions; 14 to 17 pounds of steam per indicated horse-power per hour. In one or two special cases, 13½ pounds of steam per indicated horse-power per hour has been obtained.

Steam-engine Proportions.

The dimensions of many of the parts of a steam engine may be determined according to the general methods given in the section on Machine Design, pages 416–481, but some additional data will be given here.

The following proportions are those recommended by James B. Stanwood, M.E., and are based on an extensive practical experience.

ENGINE PROPORTIONS.

Pressures on Wearing Surfaces.

Main bearings: 140 to 160 pounds per square inch of area, obtained by multiplying length by diameter of journal.

Crank pins: 1000 to 1200 pounds per square inch of area, obtained by multiplying length by diameter of pin.

Cross-head pins: 1200 to 1600 pounds per square inch of area, obtained by multiplying length by diameter of pin.
Cross-head surface: 35 to 40 pounds per square inch of area.

Non-condensing engines are usually designed for 100 pounds pressure per square inch of piston.

Sizes of Engine Parts, in Relation to Piston.

	Diameter of piston.
Main shaft, diameter	0.42 to 0.50
Main bearing, length	0.85 to 1.00
Crank pin, diameter	0.22 to 0.27
Crank pin, length	0.25 to 0.30
Cross-head pin, diameter	0.18 to 0.20
Cross-head pin, length	0.25 to 0.30
Piston-rod, diameter	0.14 to 0.17
Area of steam ports:	Area of piston.
Slide-valve engine	0.08 to 0.09
High-speed automatic engine	0.10 to 0.12
Corliss engine	0.07 to 0.80
Area of exhaust ports:	
Slide-valve engine	0.15 to 0.20
High-speed automatic engine	0.18 to 0.22
Corliss engine	0.10 to 0.12

Diameter of steam pipes:
- Slide-valve engine, ¼ diameter of piston to ¼ diameter of piston + ½ inch.
- Automatic high-speed engine, ⅓ diameter of piston.
- Corliss engine, $\frac{3}{10}$ diameter of piston.

Diameter of exhaust pipes:
- Slide-valve engine, ⅓ diameter of piston.
- Automatic high-speed engine, ⅜ diameter of piston.
- Corliss engine, ⅓ to ⅜ diameter of piston.

	Displacement of piston in one stroke.
Clearance spaces:	
Slide-valve engine	0.06 to 0.08
Automatic high-speed engine, single valve	0.08 to 0.15
Automatic high-speed engine, double valve	0.03 to 0.05
Automatic cut-off engine, Corliss type, long stroke	0.02 to 0.04
Weights of engines per rated horse-power:	
Slide-valve engine	125 to 135 pounds
Automatic high-speed engine	90 to 120 pounds
Corliss engine	220 to 250 pounds
Fly-wheels, weight per rated horse-power:	
Slide-valve engine	33 pounds
Automatic high-speed engine (according to size and speed)	25 to 33 pounds
Corliss engine (according to size and speed)	80 to 120 pounds

Rules for Fly-wheel Weights, Single-cylinder Engines.

Let d = diameter of cylinder, in inches;
S = stroke of cylinder, in inches;
D = diameter of fly-wheel, in feet;
R = revolutions per minute;
W = weight of fly-wheel, in pounds.

For slide-valve engines, ordinary duty, $W = 350\,000 \dfrac{d^2S}{D^2R^2}$;

For slide-valve engines, electric lighting, $W = 700\,000 \dfrac{d^2S}{D^2R^2}$;

For automatic high-speed engines, $W = 1\,000\,000 \dfrac{d^2S}{D^2R^2}$;

For Corliss engines, ordinary duty, $W = 700\,000 \dfrac{d^2S}{D^2R^2}$;

For Corliss engines, electric lighting, $W = 1\,000\,000 \dfrac{d^2S}{D^2R^2}$;

Steam Passages.

The dimensions of steam passages should be proportioned, when possible, so that the velocity of flow is not greater than 6000 feet per minute, but this is not always practicable. The following table will enable the diameters of steam pipes and the areas of steam ports to be computed for various velocities.

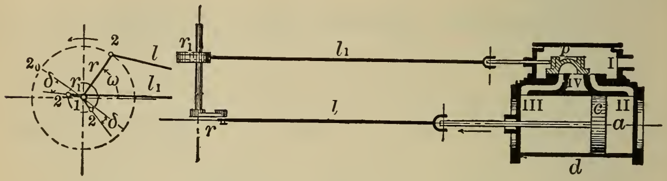
Steam-pipe Diameters and Port Areas.

Piston speed, in feet, per minute.	Velocity of steam, in feet, per minute.									
	4000		6000		8000		10000		12000	
	Steam-pipe diameter, if piston diameter = 1.	Port area, if piston area = 1.	Steam-pipe diameter, if piston diameter = 1.	Port area, if piston area = 1.	Steam-pipe diameter, if piston diameter = 1.	Port area, if piston area = 1.	Steam-pipe diameter, if piston diameter = 1.	Port area, if piston area = 1.	Steam-pipe diameter, if piston diameter = 1.	Port area, if piston area = 1.
100	.158	.025	.129	.017	.112	.013	.100	.010	.091	.008
125	.177	.031	.144	.021	.125	.016	.112	.013	.102	.010
150	.194	.037	.158	.025	.137	.019	.123	.015	.112	.013
175	.209	.044	.171	.029	.148	.022	.132	.018	.121	.015
200	.224	.050	.183	.033	.158	.025	.141	.020	.129	.017
225	.237	.056	.194	.038	.168	.028	.150	.023	.137	.019
250	.250	.063	.204	.042	.177	.031	.158	.025	.144	.021
275	.262	.069	.214	.046	.185	.034	.166	.028	.151	.023
300	.274	.075	.224	.050	.193	.038	.173	.030	.157	.025
325	.285	.081	.233	.054	.201	.041	.180	.033	.164	.027
350	.296	.088	.242	.058	.209	.044	.187	.035	.171	.029
375	.306	.094	.250	.063	.217	.047	.194	.038	.177	.031
400	.316	.100	.258	.067	.224	.050	.200	.040	.183	.033
425	.326	.106	.266	.071	.231	.053	.206	.043	.188	.035
450	.335	.113	.274	.075	.238	.056	.212	.045	.193	.038
475	.344	.119	.281	.079	.244	.059	.218	.048	.199	.040
500	.353	.125	.288	.083	.250	.063	.224	.050	.204	.042
525	.362	.131	.295	.088	.256	.066	.229	.053	.209	.044
550	.371	.138	.302	.092	.262	.069	.235	.055	.214	.046
575	.380	.144	.309	.096	.268	.072	.240	.058	.219	.048
600	.388	.150	.316	.100	.274	.075	.245	.060	.224	.050
625	.395	.156	.323	.104	.279	.078	.250	.063	.228	.052
650	.403	.163	.329	.108	.285	.081	.255	.065	.232	.054
675	.411	.169	.335	.113	.290	.084	.260	.068	.237	.056
700	.418	.175	.341	.117	.296	.088	.265	.070	.241	.058
725	.426	.181	.347	.121	.301	.091	.269	.073	.246	.060
750	.433	.188	.353	.125	.306	.094	.274	.075	.250	.063
775	.440	.194	.359	.129	.311	.097	.278	.078	.254	.065
800	.447	.200	.365	.133	.316	.100	.283	.080	.259	.067
825	.454	.206	.371	.137	.321	.103	.287	.083	.262	.069
850	.461	.213	.376	.141	.326	.106	.292	.085	.266	.071
875	.468	.219	.382	.145	.331	.109	.296	.088	.270	.073
900	.474	.225	.388	.150	.336	.113	.300	.090	.274	.075
925	.481	.231	.393	.154	.340	.116	.304	.093	.277	.077
950	.487	.238	.398	.158	.344	.119	.308	.095	.281	.079
975	.494	.244	.403	.162	.349	.122	.312	.098	.285	.081
1000	.500	.250	.408	.166	.353	.125	.316	.100	.289	.083
1025	.506	.256	.413	.170	.357	.128	.320	.103	.292	.085
1050	.512	.263	.418	.175	.361	.131	.324	.105	.295	.088
1075	.518	.269	.423	.179	.365	.134	.328	.108	.299	.090
1100	.524	.275	.428	.183	.37	.138	.332	.11	.303	.092

Valve Gears.

The admission of the steam at the proper time to the cylinder may be effected by various forms of valve gear.

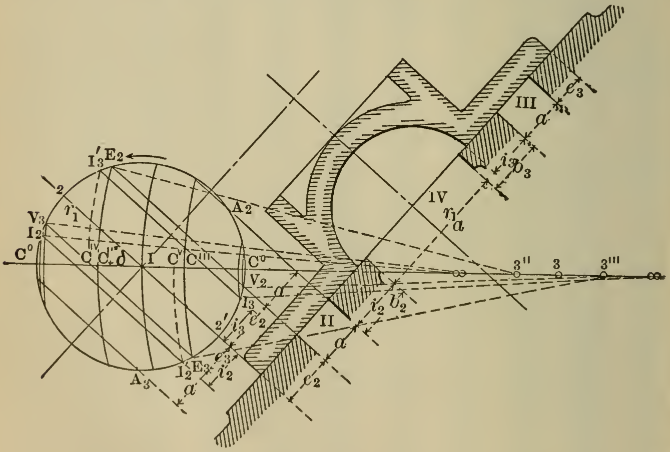
The plain slide valve, operated by a single eccentric, is shown diagrammatically in the accompanying illustration.



When the valve is so made that it just covers both ports when in the mid position, and the eccentric travel is just equal to twice the width of the port, there will be no expansion, the eccentric being placed exactly at right angles with the crank. It was soon found, however, that by making the valve with increased lap and by giving the eccentric more throw a certain degree of expansion could be obtained, together with an earlier release and compression, this resulting in better steam economy and smoother running.

In order to accomplish this result without impeding the exhaust of the steam, the eccentric, r_1, must be given the so-called angle of advance, $2^0\,1\,.\,2'$, beyond the mid position. The direction of rotation of the crank is then governed by this angle, the arrangement above giving rotation to the left, and the position, $1\,.\,2''$ for r_1, giving right-hand rotation.

The action of the slide valve may readily be represented graphically by use of Reuleaux's diagram. The angle of advance and lap being given, the point of cut-off can be determined by the following method:

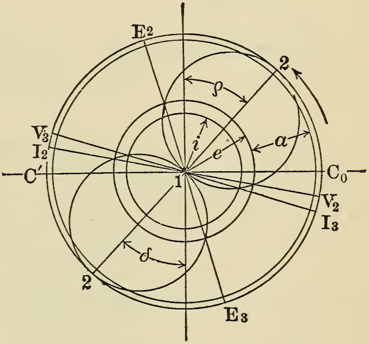


The circle, $1C^0$, represents the circle of the eccentric, and may also be taken as the crank circle on a reduced scale. C'' and C''' are two symmetrically-placed positions of the piston at which it is desired that the cut-off shall take place. Through these points, with a radius $1\,.\,3 = l$,

describe arcs from centres, $3''$ and $3'''$. Their intersections, E_2 and E_3, with the circle give the angles at which the expansion, C^0C'' and $C'C'''$, occurs,—in this instance $\frac{7}{10}$ of the stroke. We now select the point, v_2, of the crank circle at which the admission shall begin, join V_2E_2, and draw the equator, 2 . 1 . 2′, parallel to it, and the angle, 2 . 1 . C', will be the angle of advance, S, and the distance of 2 . 1 from E_2V_2, the outside lap, e_2, for the port II. The width of port, a, must also be chosen, and must be so taken that it is less than $r_1 - e_2$, and is represented by the parallel, A_2. When the crank reaches I_2,—in this instance at $\frac{98}{100}$ of the stroke,—the exhaust begins, and the distance, i_2i_2, of the parallel, I_2I_2, from the equator is the inside lap.

The construction is similar for the other half of the stroke. The angle, δ, is already known, and hence the parallel, E_3V_3 from E_3, can be at once drawn and the admission point, V_3, determined. The outside lap, e_3, is somewhat less than e_2, thus giving a correspondingly wider port opening. The inside lap, i_3, is made equal to i_2, and the bridges, b_3 and b_2, are made equal, thus giving a symmetrical valve seat. A certain amount of discretion is permissible in the selection of $b_2 = b_3$, care being taken that there is sufficient bearing at the extreme valve stroke to insure tightness. The points, I_2' and I_3', are also of importance, as they determine the closing of the exhaust. The corresponding piston positions, C^{IV} and C^V, are not symmetrical, because $i_3 = i_2$; but the inequality in the compression is not serious.

The above method of considering the influence of the ratio $\frac{l}{r}$ is very simple. It is easy to substitute any desired ratio $\frac{l_1}{r_1}$, but the variation is slight. It must be noted that the distance, 1 . 3, must be laid out to the actual scale of construction.



The application of Zeuner's diagram to the same case is made in the following manner: The circle, $1C_0$, represents, as before, the eccentric circle and the crank-pin path. The angle $C_0 . 1 . 2 = C' . 1 . 2 = 90 - \delta$. With 1 as a centre describe circles with radii e and i, here made alike for both ends of the valve; also, one of radius $e + a$. Upon 1 . 2 and 1 . 2 as diameters describe circles, called the valve circles.

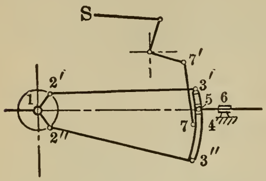
The intersection of radii from 1 with these circles gives the distance of the valve from its middle position for various crank positions. For the position $1V_2$, for instance, the admission for the left stroke begins, at $1E_2$ the expansion, at $1I$ the exhaust, etc.

The Zeuner diagram gives the valve position by means of polar coördinates, while Reuleaux's diagram is based on parallel coördinates. To be strictly correct, the valve circles, 1 . 2 and 1 . 2′, of the Zeuner diagram should fall upon each other. The arrangement shown has been adopted by Zeuner as more convenient in practice.

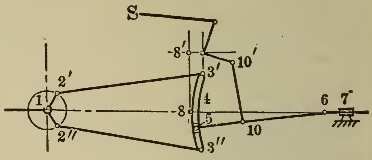
It will be seen from the preceding that the rate of expansion can be varied by altering the eccentricity and the angle of advance. This may be carried so far that the direction of rotation is changed, giving what is termed a reversing motion. A variety of reversing motions have been devised, which accomplish the desired relation of parts by shifting a reversing lever. Of these the most practical are the so-called link motions, of which a number will here be briefly shown.

No. 1 is an outline diagram of Stephenson's link motion. The link, 3′3″, of convex curvature towards the valve, is given an oscillating motion by means of the two equal eccentrics, 1 . 2′ and 1 . 2″, and is suspended from its middle point, 7, from the bell crank lever, $S7'$. The motion of the link is transmitted to the valve by means of the sliding block, 5, and rod, 6. No. 2 is Gooch's link motion. The link, 4, is driven by two eccentrics, as

before, but is curved in the opposite direction with a radius, 5 . 6, and is suspended from its middle point, 8, to a fixed pivot, 8′, while the rod, 5 . 6, is shifted by means of the lever connection, *S*10 . 10′.

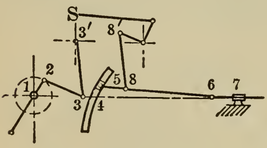


No. 1.

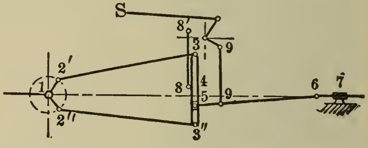


No. 2.

No. 3 is the link motion of Pius Fink. In this form the link is operated by a single eccentric instead of two, as in the previous forms. This simple mechanism is not as widely used as its merits deserve.

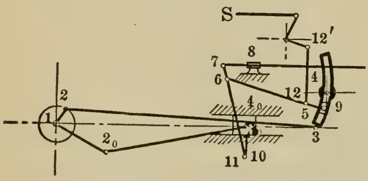


No. 3.

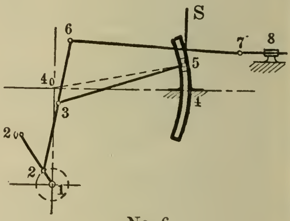


No. 4.

No. 4 is the link motion of Allen. In this design the link, 4, is straight, and both the link and the radius rod are suspended and shifted by the lever connections, 8′ . 8 and 9′ . 9.



No. 5.



No. 6.

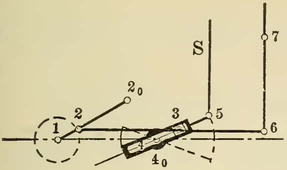
No. 5 is Walschaert's link motion. The link, 4, vibrates upon a fixed centre, 9, and is operated by an eccentric, 1 . 2. The valve rod is moved from the main cross-head by the connections, 10 . 11 . 6 . 7, and also by the radius rod, 5 . 6, which latter is suspended from the bell crank, *S* . 12′.

No. 6 is Marshall's valve gear. The curved link, 4, is rigidly secured and does not move. The eccentric, 1 . 2, moves the valve connection, 6 . 7, by means of the lever, 2 . 3 . 6, which vibrates about the point, 3, on the end of the radius rod, the other end of the rod being held by the link block, 5. Instead of the link, 4, a radius arm, 4_0 . 5, is often used, the centre, 4_0, corresponding to the centre of curvature of the link, the action being the same in both cases.

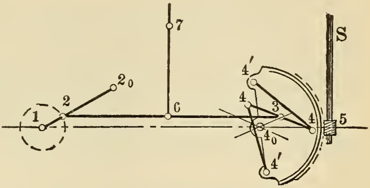
No. 7 is Brown's valve gear, which differs from the preceding by the substitution of a straight link of adjustable angle for the curved guide link.

No. 8 is Angström's valve gear. The point, 3, of the preceding gear is guided by a parallel motion, and the point, 6, is between 2 and 3, instead of beyond.

The eight preceding valve gears operate the valve approximately in the same manner as if a single eccentric of variable eccentricity and angular

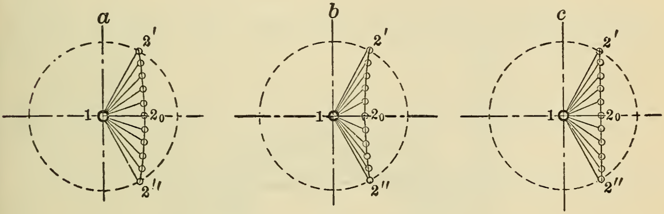


No. 7.



No. 8.

advance were used, the eccentric rod being assumed of infinite length as compared with r. The path of the successive positions of the middle point of this imaginary eccentric is called the central curve of the valve gear.



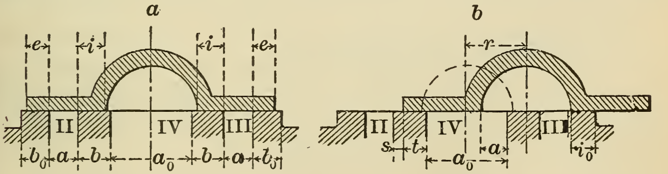
The general forms of the central curve are shown above. Form a is that for cases 1, 4, and 5; form b, for case 1, when the eccentric rods are crossed; and form c, in which the curve becomes a straight line, is for cases 2, 3, 6, 7, and 8. In the latter instance the lead is constant.

The use of the central curve is involved in the mechanism of the valve gear of the single-valve automatic cut-off engines, in which the eccentric is shifted across the shaft by the action of a centrifugal or inertia governor.

Slide Valves.

The two principal forms of slide valves in use are the plan D valve and the Allen valve, the latter being designed to give a more rapid and full port opening.

The action of the slide valve has already been discussed, and the amount of inside and outside lap may be determined for the desired steam



distribution by the use of Reuleaux's or Zeuner's diagrams. The other dimensions are determined as follows:

The width, a, of the steam ports is kept as small as is practicable, while the length at right angles to the plane of the drawing is made quite large. When a is given, the dimensions to be determined are the outside and

inside lap, e and i; the bridges, b; the width of face, b_0, beyond the ports; the width, a_0, of the exhaust port, IV; the travel, r; the length of the valve, l; and of the valve seat, l_0. The laps, e and i, are determined from the valve diagrams.

In the same manner, also, is found the greatest distance, s, in which the edge of the valves passes the edge of the port. This gives the width of bearing, t, of the valve upon the bridge, since $b = s + t$. The value of t varies greatly; the least permissible value is $t = \frac{3}{16}''$, and it is more frequently made $\frac{3}{8}''$ to $\frac{1}{2}''$. Approximately, we have, after assuming t as just given, $a_0 + t - (e + a + i) = a$. We then have

$$a_0 = 2a + l + i - t,$$

whence $$r = a + e + s,$$

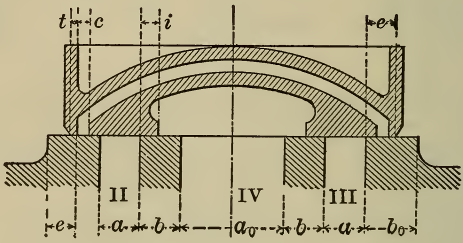
and $$l = 4a + 3l + i + 2s + t.$$

The valve face must have an inner width of bearing, t_0 (Fig. b), at least equal to t, whence for the total width of the valve face we have the value

$$a_0 + 2b + 2a + 2b_0, \text{ or}$$

$$l_0 = 4a + 3e - i + 4s + t + 2t_0.$$

The thickness of metal in the valve itself, when made of cast-iron, should be about $= \dfrac{D}{200} + 0.4''$.



The Allen Valve.

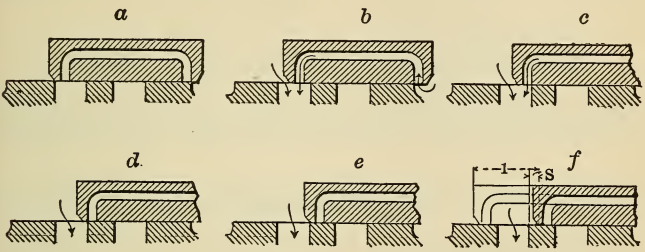
This is a double valve, and consists of one D valve over another, with a steam passage between. As before, we have $r = a + e + s$, and also make $b_0 = 2e - t$,—*i.e.*, the inner edge of the outer valve, when the valve is in mid position, is at a distance $= e$ from the edge of the valve seat. The consequence is that when the valve is moved a distance equal to e, say to the right, the passage through the valve opens to admit steam at the same instant as does the edge of the valve on the left. This gives a steam admission twice as quickly, and an opening twice as great, as would otherwise be the case.

The following positions, from a to f, will show the successive actions, the exhaust ports being omitted for simplicity.

a. The admission is just about to take place both from the edge of the valve on the left and through the passage in the valve. If we apply Zeuner's diagram, we must, from the point A, which indicates the port opening, double the width given by the Zeuner circle until the entrance to the passage in the valve is wide open, as at b. By thus doubling the opening in the diagram we obtain the curve, AB_1.

b. From this position on, the opening at the left continues to grow wider, but that through the valve on the right does not; hence, on the Zeuner diagram, from this point we return to the opening which the regular valve circle gives, to which is added the constant opening, $c = BB_1 = CC_1$, indicated by the curve, B_1C_1. This continues until the inner edge of the opening of the valve passage on the left reaches the edge of the bridge, as at c.

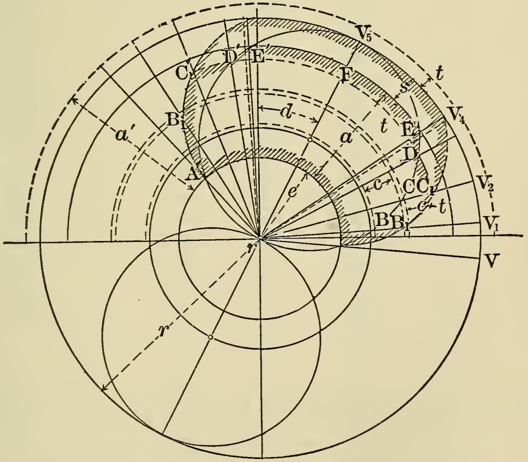
c. As the valve continues to move, the passage through it is gradually closed, but the steam port is opened to the same amount, and hence the actual port opening remains constant. This continues until the position, *d*, is reached, when the passage through the valve is entirely shut off. This is indicated in the diagram by the arc, C_1D, struck from the centre at 1.



d. The valve continues to move to the right until it is entirely upon the bridge, the corresponding portion of the diagram being the arc, *DE*, of the valve circle.

e. The valve from this position moves on the bridge beyond the port until it has travelled a distance equal to *s*, as shown at *f*, during which time the port opening remains constant, as indicated in the diagram by the arc, *EE'*, struck from the centre, 1. From this point the same actions take place successively in the reversed order.

It will be seen that Allen's valve gives a much quicker opening and also a much longer duration of the full opening than does the plain slide valve. It remains to be seen how these features can be used to the best



advantage. This is best done by making the value of *s* negative, and also $\geq t$. This makes the port opening from C_1 to C_1' in the diagram constant, as shown in the diagram.

In order that the apparent contraction of the ports by the change in the sign of *s* shall not occur, the value of *a* is made greater than would other-

wise be the case. Under these conditions we have for the exhaust port, a_0, the equation:

$$a_0 + t - e_1 - a - i = a - s,$$

in which s is given the magnitude equal to the distance which the edge of the valve is moved beyond the edge of the bridges, as in Fig. *f*. We then have

For the exhaust port, $a_0 = 2a + e_1 + i - s - t$;
For the bridge, $b = e - e_1 + s - t$;
For the passage through valve, $c = e - t - e_1$;
For the total valve, $l = 4a + 4e - e_1 + i - 3s + t$.

For a complete discussion of valves and valve gears see Zeuner's "Treatise on Valve Gears" and Auchincloss's "Link and Valve Motions;" also, compare Reuleaux's "Constructor" and Unwin's "Machine Design."

CONDENSERS.

The gain in power by use of a condenser may be estimated upon the basis of an increase of 12 pounds per square inch to the mean effective pressure in the cylinder. Upon this basis the following table shows the gain in horse-power for cylinders of various diameters for every 100 feet piston speed. For any other speed, multiply by the speed, in feet, per minute and divide by 100 to obtain the gain in horse-power.

Diameter of piston.	Horse-power gained for every 100 feet of piston speed per minute.	Diameter of piston.	Horse-power gained for every 100 feet of piston speed per minute.
5	.71	32	29.24
6	1.03	34	33.01
7	1.40	36	37.01
8	1.83	38	41.24
9	2.31	40	45.70
10	2.86	42	50.38
12	4.11	44	55.29
14	5.60	46	60.43
16	7.31	48	65.80
18	9.25	50	71.40
20	11.42	52	77.23
22	13.82	54	83.28
24	16.45	56	89.56
26	19.30	58	96.08
28	22.39	60	102.81
30	25.70		

The size of a jet condenser varies somewhat according to the speed of the engine, but is usually made from ⅓ to ½ the volume of the steam cylinder. The quantity of injection water required is from 25 to 30 times the weight of steam to be condensed, according to the pressure of the exhaust and the temperature of the water. Too much water is poor economy, since the increased burden on the air pump neutralizes the gain of the better vacuum. It is best to provide for an ample supply, in case of emergency, and cut the injection down in actual use until the minimum amount to maintain a fair vacuum is ascertained.

The temperature of the hot well is best kept at about 100° F., although it sometimes may rise to 120° F. without materially impairing the vacuum.

For surface condensers a cooling surface of 2 to 3 square feet per indicated horse-power is found satisfactory in practice, according to climate. The quantity of circulating water may be taken at about 30 times the weight of steam to be condensed.

The size of the air pump may be determined by the following formula:

$$\text{Volume of air pump} = \frac{\text{I. H. P.}}{\text{revolutions}} \times c,$$

where $c = 700$ for single-acting and jet condenser;
$= 300$ for single-acting surface condenser;
$= 470$ for double-acting horizontal pump;

$$\text{or volume of single-acting air pump} = \frac{\text{volume of low-pressure cylinder}}{23}.$$

Independent condensers are now extensively used. The following general dimensions are for standard designs, the Worthington being a jet condenser and the Wheeler a surface condenser.

Sizes of Worthington Jet Condenser.

Diameter of steam cylinders.	Diameter of water cylinders.	Length of stroke.	Diameter of steam pipe of pump.	Diameter of exhaust pipe of pump.	Diameter of engine exhaust opening.	Diameter of injection pipe.	Diameter of discharge pipe.
Inch.			Inch.	Inch.	Inch.	Inch.	Inch.
5¼ ×	4¾ ×	5	¾	1¼	4	2½	2
6 ×	5¾ ×	6	1	1½	5	3	3
7½ ×	7½ ×	6	1½	2	8	4	4
7½ ×	7 ×	10	1½	2	10	4	4
7½ ×	8½ ×	10	1½	2	12	5	5
7½ ×	10¼ ×	10	1½	2	14	7	6
9 ×	12 ×	10	2	2½	14	7	8
12 ×	14 ×	10	2½	3	16	8	10
12 ×	15 ×	10	2½	3	16	8	10
12 ×	15 ×	15	2½	3	18	10	10
12 ×	17 ×	15	2½	3	18	10	12
14 ×	17 ×	15	2½	3	18	10	12
12 ×	19 ×	15	2½	3	18	10	12
14 ×	19 ×	15	2½	3	18	10	12
17 ×	19 ×	15	3	4	18	10	12
14 ×	22 ×	15	3	4	20	10	14
17 ×	22 ×	15	3	4	20	10	14
18 ×	22 ×	18	3	4	20	10	14
18 ×	24 ×	18	3	4	20	10	14
18 ×	26 ×	18	3	4	24	12	16
18 ×	29 ×	18	3	4	24	14	18

Wheeler Surface Condenser, Mounted on Blake-Knowles Air and Circulating Pump.

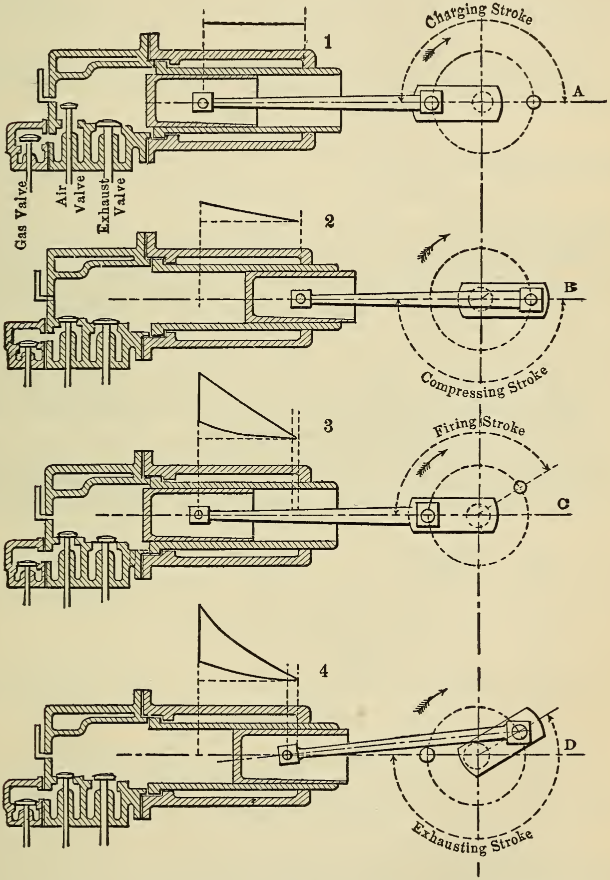
Steam per hour.	Cooling surface.	Size of cylinders: steam, air, water. Stroke.	Weight of outfit.
Lb.	Sq. ft.	Inch.	Lb.
500	80	4 × 5 × 5 × 5	1200
800	110	4 × 5 × 5 × 5	1350
1000	150	4½ × 5½ × 5½ × 6	1700
1500	180	4½ × 5½ × 5½ × 6	2000
1800	200	5½ × 6 × 6 × 7	2600
2000	210	5½ × 6 × 6 × 7	2700
2250	230	5½ × 6 × 6 × 7	2800
2500	270	6 × 8 × 8 × 7	3300
3000	310	6 × 8 × 8 × 7	3500
3500	360	6 × 8 × 8 × 7	3600
4000	430	7½ × 8 × 8 × 10	4600
4500	480	7½ × 8 × 8 × 10	4800
5000	530	7½ × 8 × 8 × 10	5600
6000	610	8 × 9 × 9 × 10	6000
7000	740	8 × 9 × 9 × 10	6300
7500	770	8 × 10 × 10 × 12	6900
8000	850	8 × 10 × 10 × 12	7200
9000	900	8 × 10 × 10 × 12	9100
10500	1000	10 × 12 × 12 × 12	9600
11000	1050	10 × 12 × 12 × 12	10700
12000	1200	10 × 12 × 12 × 12	13100
14000	1400	12 × 14 × 14 × 12	17000
16000	1600	12 × 14 × 14 × 16	19000
18000	1800	14 × 16 × 16 × 16	19800
20000	2000	14 × 16 × 16 × 16	20500
22500	2100	14 × 16 × 16 × 16	24000
25000	2360	16 × 16 × 18 × 24	....

Separate condensing plants have the advantage that they can be started before the main engines, and thus permit a vacuum to be secured at once, without blowing through. The speed of air and circulating pumps can be regulated according to the vacuum, which is not the case when they are operated by direct connection to the main engine.

In modern power plants, where there are many engines, pumps, and other steam-driven auxiliaries, it is found advantageous to provide one large central condensing plant, with independent air and water pumps, into which all the engines discharge their exhaust. When compound engines are used the exhaust steam from pumps and similar machines in which the steam is not used expansively may well be discharged into the receiver of the engine, being thus enabled to exert its expansive force upon the low-pressure piston, and then pass into the condenser. In this way much of the wastefulness of such auxiliaries may be prevented.

INTERNAL-COMBUSTION MOTORS.

Practically all of the internal-combustion motors now in active use are operated on the Beau de Rochas cycle, with a power impulse every fourth stroke. The sequence of operations is shown in the cuts, the corresponding portion of the indicator diagram being given in each case.



In the first outward stroke the mixed charge of air and gas is drawn in, and on the return stroke this is compressed. It is then ignited by an electric spark, hot tube, or similar device, and the expansion due to the explosion and combustion makes the second outward stroke,—this being the power stroke. The fourth phase in the cycle, the second inward stroke, is the exhaust.

It is advantageous to use as high a compression pressure as possible, but the limit to this is found in the heat generated by compression. If the compression is too high the charge will be ignited by this heat and an

injurious premature explosion occur. Various attempts have been made to obviate this difficulty. In the Banki engine a fine spray of water is injected into the inlet pipe with the charge, and this absorbs much of the heat of compression. The vapor of water thus produced expands with the explosion, and there is thus a combined gas and steam action. In the Diesel motor the charge drawn in is pure air, and this is compressed to about 500 pounds per square inch. A high temperature is thus produced, but there is no fuel in the cylinder to be ignited. At the end of the stroke the liquid fuel is injected and is ignited by the heat of the compressed air.

In ordinary gas engines the compression is carried from 80 to 90 pounds per square inch. The maximum pressure in such engines is about 3.5 times the compression pressure. For compressions of 100 pounds per square inch or less the mean effective pressure may be obtained from the following formula:

$$\text{M. E. P.} = 2C - 0.01C^2,$$

in which C is the compression pressure. Thus, for 50 pounds compression, this would give

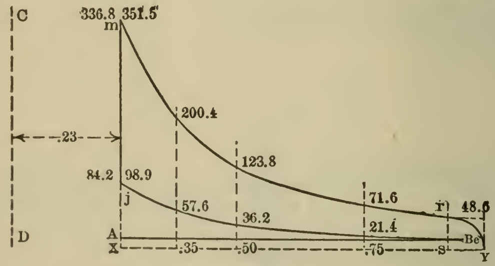
$$\text{M. E. P.} = 100 - 25 = 75 \text{ pounds.}$$

Piston speed should not exceed 700 feet per minute,—more generally 500 feet per minute is used. Maximum pressure should not be reached later than at one-tenth the stroke. The time of rise in pressure in a gas engine is: first, time taken for flame to strike back into the mixture; second, time during which pressure rises after ignition. Cylinders of large dimensions have much larger ratio of volume to surface than small ones, and are therefore more economical. The size of valves should be such that the velocity of gases calculated upon mean piston speed does not exceed 100 feet per minute.

Internal-combustion motors have a much higher thermal efficiency than steam engines, on account of the greater temperature range and, also, because of the absence of losses from cylinder condensation, owing to the fact that the working fluid is a perfect gas.

Gas engines frequently show on test thermal efficiencies of 22 to 25 per cent., while the Diesel motor has given a thermal efficiency of 38 per cent.

The general proportions of gas-engine parts may be determined according to the general principles of machine design. There are, however,



Gas-engine Diagram.

certain parts which may be given special consideration. Since gas engines may be used with fuels of various calorific values, it is necessary to assume some standard upon which proportions may be based, and in the United States it is often assumed that natural gas is the standard fuel, its calorific value being about 1000 B. T. U. per cubic foot. For gas of any other calorific value, a general rule is to make the compression ratio *inversely* as the *calorific value* of the gas. Thus for a lean gas a higher degree of compression will be required, and, although less power will be developed than with a richer gas, the thermal efficiency may be as high or even higher.

For natural gas the compression space is made about 30 per cent. of the piston displacement, so that the total volume of cylinder and clearance is

1.30 of the piston displacement, and the ratio of the clearance to the total volume is $\frac{0.30}{1.30} = 0.2308$.

Upon this assumption a typical gas-engine indicator diagram may be constructed, from which the action in the cylinder may be seen.

The following discussion is condensed from Roberts's "Gas-engine Hand-book."

The compression curve has been found experimentally to be represented by the relation

$$PV^{1.3} = K,$$

in which P is the absolute pressure at any point; V, the corresponding volume; and K, a constant. If the volume of the cylinder is taken as unity, K is the absolute pressure of the atmosphere, or 14.7 pounds per square inch.

With natural gas the pressure of explosion is about 4 times the compression pressure, both compression and explosion pressures being considered above atmospheric.

For the expansion curve the relation of pressure to volume is

$$PV^{1.35} = C,$$

in which C is a constant depending upon the maximum pressure of explosion.

To find the compression pressure with a clearance ratio of 0.2308, as determined above, we have

$$PV^{1.3} = K = 14.7,$$

$$P = \frac{14.7}{V^{1.3}} = \frac{14.7}{(0.2308)^{1.3}} = 98.88 \text{ pounds.}$$

This is absolute pressure, and the pressure above atmospheric will be

$$98.88 - 14.7 = 84.2 \text{ pounds per square inch.}$$

The explosion pressure will then be $84.2 \times 4 = 336.8$ pounds above atmosphere, or 351.5 pounds absolute. Other points in the compression curve may then be computed by the formula.

To apply the formula for the expansion curve,

$$PV^{1.35} = C,$$

the value of C must be found. This is the pressure at the end of the stroke when the volume is equal to 1; hence, we have

$$\begin{aligned} PV^{1.35} &= 351.5 \times (0.2308)^{1.35} = C, \\ &= 48.56 \text{ pounds absolute,} \end{aligned}$$

as the terminal pressure.

Intermediate points in the expansion curve may then be found, as shown in the diagram, from

$$PV^{1.35} = 48.56.$$

The mean effective pressure may then be measured from the diagram,—preferably by the use of the planimeter.

The power of the gas engine is generally determined by means of the brake, and the dimensions of parts are based on brake horse-power (B. H. P.).

The brake horse-power may be expressed in general by the formula

$$\text{B.H.P.} = \frac{D^2 \times L \times R}{C},$$

in which

D = diameter of cylinder, in inches;
L = stroke, in inches;
R = revolutions per minute;
C = constant, depending upon the fuel.

For a four-cycle engine C may be taken as 19,000 for natural gas and 18,000 for gasoline. The value of C may be determined from any engine in

which the brake horse-power has been found, and then this value can be used for subsequent computations with the same fuel.

The stroke is usually made equal to $1.5D$, and the piston speed about 600 feet per minute.

For the inlet and the exhaust passages we have

S = piston speed, in feet, per minute;
A = piston area, in square inches;
a = inlet area;
a' = exhaust area.

$$a \doteq \frac{AS}{6000};$$

$$a' = \frac{AS}{5100}.$$

The flow of water through the cylinder jacket is made 4 to 5 gallons per horse-power per hour.

The 1902 Code of the American Society of Mechanical Engineers includes the following:

Rules for Conducting Tests of Gas and Oil Engines.

Code of 1901.

I. **Objects of the Tests.**—At the outset the specific object of the test should be ascertained, whether it be to determine the fulfilment of a contract guarantee, to ascertain the highest economy obtainable, to find the working economy and the defects as they exist, to ascertain the performance under special conditions, or to determine the effect of changes in the conditions; and the test should be arranged accordingly.

II. **General Condition of the Engine.**—Examine the engine, and make notes of its general condition and any points of design, construction, or operation which bear on the objects in view. Make a special examination of all the valves by inspecting the seats and bearing surfaces, and note their condition, and see if the piston rings are gas-tight.

If the trial is made to determine the highest efficiency, and the examination shows evidence of leakage, the valves and piston rings, etc., should be made tight and all parts of the engine put in the best possible working condition before starting on the test.

III. **Dimensions, etc.**—Take the dimensions of the cylinder, or cylinders, whether already known or not. This should be done when they are hot, and in working order. If they are slightly worn, the average diameter should be determined. Measure, also, the compression space or clearance volume, which should be done, if practicable, by filling the spaces with water previously measured, the proper correction being made for the temperature. (See Section III., Steam-engine Code.)

IV. **Fuel.**—Decide upon the gas or oil to be used, and, if the trial is to be made for maximum efficiency, the fuel should be the best of its class that can readily be obtained, or one that shows the highest calorific power. (See Section IV., Steam-engine Code.)

V. **Calibration of Instruments Used in the Tests.**—All instruments and apparatus should be calibrated and their reliability and accuracy verified by comparison with recognized standards. Apparatus liable to change or to become broken during the tests, such as gauges, indicator springs, and thermometers, should be calibrated both before and after the experiments. The accuracy of all scales should be verified by standard weights. In the case of gas- or water-meters, special attention should be given to their calibration, both before and after the trial, and at the same rate of flow and pressure as exists during the trial.

VI. **Duration of Test.**—The duration of a test should depend largely upon its character and the objects in view, and in any case the test should be continued until the successive readings of the rates at which oil or gas

is consumed, taken at, say, half-hourly intervals, become uniform and thus verify each other. If the object is to determine the working economy, and the period of time during which the engine is usually in motion is some part of twenty-four hours, the duration of the test should be fixed for this number of hours. If the engine is one using coal for generating gas, the test should cover a long enough period to determine with accuracy the coal used in the gas producer; such a test should be of at least twenty-four hours' duration, and in most cases it should extend over several days.

VII. **Starting and Stopping a Test.**—In a test for determining the maximum economy of an engine, it should first be run a sufficient time to bring all the conditions to a normal and constant state. Then the regular observations of the test should begin, and continue for the allotted time.

If a test is made to determine the performance under working conditions, the test should begin as soon as the regular preparations have been made for starting the engine in practical work, and the measurements should then commence and be continued until the close of the period covered by the day's work.

VIII. **Measurement of Fuel.**—If the fuel used is coal furnished to a gas producer, the same methods apply for determining the consumption as are used in steam-boiler tests. (See Code of Rules for Conducting Boiler Tests, "Transactions of the American Society of Mechanical Engineers," Volume XXI., page 34.)

If the fuel used be gas, the only practical method of measurement is the use of a meter through which the gas is passed. Gas bags should be placed between the meter and the engine to diminish the variations of pressure, and these should be of a size proportionate to the quantity used. Where a meter is employed to measure the air used by an engine, a receiver with a flexible diaphragm should be placed between the engine and the meter. The temperature and pressure of the gas should be measured, as also the barometric pressure and temperature of the atmosphere, and the quantity of gas should be determined by reference to the calibration of the meter, taking into account the temperature and pressure of the gas.

If the fuel is oil, this can be drawn from a tank which is filled to the original level at the end of the test, the amount of oil required for so doing being weighed; or, for a small engine, the oil may be drawn from a calibrated vertical pipe.

In an engine using an igniting flame the gas or oil required for it should be included in that of the main supply, but the amount so used should be stated separately, if possible.

IX. **Measurement of Heat Units Consumed by the Engine.**—The number of heat units used is found by multiplying the number of pounds of coal or oil or the cubic feet of gas consumed by the total heat of combustion of the fuel, as determined by a calorimeter test. In determining the total heat of combustion no deduction is made for the latent heat of the water vapor in the products of combustion. There is a difference of opinion on the propriety of using this higher heating value, and for purposes of comparison care must be taken to note whether this or the lower value has been used. The calorimeter recommended for determining the heat of combustion is the Mahler, for solid fuels or oil, or the Junker, for gases, or some form of calorimeter known to be equally reliable. (See Poole on "The Calorific Power of Fuels.")

It is sometimes desirable, also, to have a complete chemical analysis of the oil or gas. The total heat of combustion may be computed, if desired, from the results of the analysis, and should agree well with the calorimeter values. (See Section XVII., Boiler-test Code.)

For the purpose of making the calorimeter test, if the fuel used is coal for generating gas in a producer, or oil, samples should be taken at the time of the engine trial and carefully preserved for subsequent determination. If gas is used, it is better to have a gas calorimeter on the spot, samples taken, and the calorimeter test made while the trial is going on.

X. **Measurement of Jacket Water to Cylinder or Cylinders.**—The jacket water may be measured by passing it through a water-meter or allowing it to flow from a measuring tank before entering the jacket, or by collecting it in tanks on its discharge. If measuring tanks are used, the

same system of arrangement is recommended as that employed for feed-water measurements in boiler and steam-engine tests. (See Section XI., Steam-engine Code.)

XI. **Indicated Horse-power.**—The directions given for determining the indicated horse-power for steam engines apply in all respects to internal-combustion engines. (See Section XIII., Steam-engine Code.)

XII. **Brake Horse-power.**—The determination of the brake horse-power, which is very desirable, is the same for internal combustion as for steam engines. (See directions given in Section XV., Steam-engine Code.)

XIII. **Speed.**—The same directions apply to internal-combustion engines as to steam engines for the determination of speed, and reference is made to Section XVII., Steam-engine Code, for suggestions on this subject.

In an engine which is governed by varying the number of explosions or working cycles, a record should be kept of the number of explosions per minute; or if the engine is running at nearly maximum load, by counting the number of times the governor causes a miss in the explosions.

XIV. **Recording the Data.**—The time of taking weights and every observation should be recorded, and note made of every event, however unimportant it may seem to be. The pressures, temperatures, meter readings, speeds, and other measurements should be observed every 20 or 30 minutes when the conditions are practically uniform, and at more frequent intervals if they are variable. Observations of the gas or oil measurements should be taken with special care at the expiration of each hour, so as to divide the test into hourly periods and reveal the uniformity, or otherwise, of the conditions and results as the test goes forward.

All data and observations should be kept on suitably-prepared blank sheets or in note-books.

XV. **Uniformity of Conditions.**—When the object of the test is to determine the maximum economy, all the conditions relating to the operation of the engine should be maintained as constant as possible during the trial.

XVI. **Indicator Diagrams and Their Analysis.**—(*a*) *Sample Diagrams:* Sample diagrams nearest to the mean should be selected from those taken during the trial and appended to the tables of the results. If there are separate compression or feed cylinders, the indicator diagrams from these should be taken and the power deducted from that of the main cylinder.

XVII. **Standards of Economy and Efficiency.**—The hourly consumption of heat, determined as pointed out in Article IX., divided by the indicated or the brake horse-power, is the standard expression of engine economy recommended.

In making comparisons between the standard for internal-combustion engines and that for steam engines, it must be borne in mind that the former relates to energy concerned in the *generation* of the force employed, whereas in the steam engine it does not relate to the entire energy expended during the process of combustion in the steam boiler. The steam engine standard does not cover the losses due to combustion, while the internal-combustion engine standard, in cases where a crude fuel such as oil is burned in the cylinder, does cover these losses. To make a direct comparison between the two classes of engines considered as complete plants for the production of power, the losses in generating the working agent must be taken into account in both cases, and the comparison must be on the basis of the fuel used; and not only this, but on the basis of the same or equivalent fuel used in each case. In such a comparison, where producer gas is used and the producer is included in the plant, the fuel consumption, which will be the weight of coal in both cases, may be directly compared.

The thermal efficiency ratio per indicated horse-power or per brake horse-power for internal-combustion engines is obtained in the same

manner as for steam engines referred to in Section XXI., Steam-engine Code, and is expressed by the fraction

$$\frac{2545}{\text{B. T. U. per horse-power per hour}}.$$

XVIII. **Heat Balance.**—For purposes of scientific research, a heat balance should be drawn which shows the manner in which the total heat of combustion is expended in the various processes concerned in the working of the engine. It may be divided into three parts: first, the heat which is converted into the indicated or brake work; second, the heat rejected in the cooling water of the jackets; and third, the heat rejected in the exhaust gases, together with that lost through incomplete combustion and radiation.

To determine the first item, the number of foot-pounds of work performed by, say, 1 pound or 1 cubic foot of the fuel is determined; and this quantity, divided by 778, which is the mechanical equivalent of 1 B. T. U., gives the number of heat units desired. The second item is determined by measuring the amount of cooling water passed through the jackets, equivalent to 1 pound or 1 cubic foot of fuel consumed, and calculating the amount of heat rejected, by multiplying this quantity by the difference in the sensible heat of the water leaving the jacket and that entering. The third item is obtained by the method of differences,—that is, by subtracting the sum of the first two items from the total heat supplied. The third item can be subdivided by computing the heat rejected in the exhaust gases as a separate quantity. The data for this computation are found by analyzing the fuel and the exhaust gases, or by measuring the quantity of air admitted to the cylinder in addition to that of the gas or oil.

XIX. **Report of Test.**—The data and results of a test should be reported in the manner outlined in one of the following tables, the first of which gives a complete summary when all the data are determined, and the second is a shorter form of report, in which some of the minor items are omitted.

XX. **Temperatures Computed at Various Points of the Indicator Diagram.**—The computation of temperatures corresponding to various points in the indicator diagram is, at best, approximate. It is possible only where the temperature of one point is known or assumed, or where the amount of air entering the cylinder along with the charge of gas or oil and the temperature of the exhaust gases is determined.

Data and Results of Test of Gas or Oil Engine.

Arranged according to the Complete Form advised by the Engine Test Committee of the American Society of Mechanical Engineers. Code of 1902.

1. Made by ....of..............................
   on engine located at...
   to determine...
   ....
2. Date of trial ....
3. Type of engine (whether oil or gas)..............................
   ....
4. Class of engine (mill, marine, motor for vehicle, pumping, or other) ..
   ....
5. Number of revolutions for one cycle, and class of cycle..........
   ....
6. Method of ignition ....
   ....
7. Name of builders...
8. Gas or oil used..
   (*a*) Specific gravity .... deg. Fahr.
   (*b*) Burning-point.................................. deg. Fahr.
   (*c*) Flashing-point .... deg. Fahr.

	1st Cyl.	2d Cyl.
9. Dimensions of engine:		
(*a*) Class of cylinder (working or for compressing the charge)		
(*b*) Vertical or horizontal		
(*c*) Single- or double-acting........................		
(*d*) Cylinder dimensions		
Bore, in inches		
Stroke, in feet........................		
Diameter of piston-rod, in inches........		
Diameter of tail-rod, in inches...........		
(*e*) Compression space or clearance, in per cent., of volume displaced by piston per stroke		
Head end........................		
Crank end........................		
Average		
(*f*) Surface, in square feet (average)		
Barrel of cylinders		
Cylinder heads........................		
Clearance and ports		
Ends of piston........................		
Piston-rod		
(*g*) Jacket surfaces or internal surfaces of cylinder heated by jackets, in square feet.		
Barrel of cylinder........................		
Cylinder heads		
Clearance and ports		
(*h*) Horse-power constant for 1 pound mean effective pressure and 1 revolution per minute		

10. Give description of main features of engine and plant, and illustrate with drawings of same given on an appended sheet. Describe method of governing. State whether the conditions were constant throughout the test.

Total Quantities.

11. Duration of test..	hours.
12. Gas or oil consumed..	cu. ft. or lbs.
13. Air supplied, in cubic feet	cu. ft.
14. Cooling water supplied to jackets	cu. ft.
15. Calorific value of gas or oil by calorimeter test, determined by...............calorimeter..................	B. T. U.

Hourly Quantities.

16. Gas or oil consumed per hour..............................	lbs.
17. Cooling water supplied per hour..........................	lbs.

Pressures and Temperatures.

18. Pressure at meter (for gas engine), in inches, of water....	ins.
19. Barometric pressure of atmosphere:	
(*a*) Reading of height of barometer	ins.
(*b*) Reading of temperature of barometer	deg. Fahr.
(*c*) Reading of barometer corrected to 32° F.	ins.
20. Temperature of cooling water:	
(*a*) Inlet	deg. Fahr.
(*b*) Outlet	deg. Fahr.
21. Temperature of gas at meter (for gas engine)	deg. Fahr.
22. Temperature of atmosphere:	
(*a*) Dry-bulb thermometer	deg. Fahr.
(*b*) Wet-bulb thermometer..............................	deg. Fahr.
(*c*) Degree of humidity	per cent.
23. Temperature of exhaust gases............................	deg. Fahr.

How determined ....

Data Relating to Heat Measurement.

24. Heat units consumed per hour (pounds of oil or cubic feet of gas per hour multiplied by the total heat of combustion) .... B. T. U.
25. Heat rejected in cooling water:
    (*a*) Total per hour .... B. T. U.
    (*b*) In per cent. of heat of combustion of the gas or oil consumed .... per cent.
26. Sensible heat rejected in exhaust gases above temperature of inlet air:
    (*a*) Total per hour .... B. T. U.
    (*b*) In per cent. of heat of combustion of the gas or oil consumed .... per cent.
27. Heat lost through incomplete combustion and radiation per hour:
    (*a*) Total per hour .... B. T. U.
    (*b*) In per cent. of heat of combustion of the gas or oil consumed .... per cent.

Speed, Etc.

28. Revolutions per minute .... rev.
29. Average number of explosions per minute ....
    How determined ....
30. Variation of speed between no load and full load .... rev.
31. Fluctuation of speed on changing from no load to full load, measured by the increase in the revolutions due to the change ....

Indicator Diagrams.

	1st Cyl.	2d Cyl.
32. Pressure, in pounds, per square inch above atmosphere:		
(*a*) Maximum pressure		
(*b*) Pressure just before ignition		
(*c*) Pressure at end of expansion		
(*d*) Exhaust pressure		
33. Temperatures, in degrees Fahr., computed from diagrams:		
(*a*) Maximum temperature (not necessarily at maximum pressure)		
(*b*) Just before ignition		
(*c*) At end of expansion		
(*d*) During exhaust		
34. Mean effective pressure, in pounds, per square inch		

Power.

35. Power, as rated by builders:
    (*a*) Indicated horse-power .... H. P.
    (*b*) Brake horse-power .... H. P.
36. Indicated horse-power actually developed:
    First cylinder .... H. P.
    Second cylinder .... H. P.
    Total .... H. P.
37. Brake horse-power, electric horse-power, or pump horse-power, according to the class of engine .... H. P.
38. Friction indicated horse-power from diagrams, with no load on engine and computed for average speed H. P.
39. Percentage of indicated horse-power lost in friction ...... per cent.

Standard Efficiency Results.

40. Heat units consumed by the engine per hour:
    (*a*) Per indicated horse-power .... B. T. U.
    (*b*) Per brake horse-power .... B. T. U.

41. Heat units consumed by the engine per minute:
(a) Per indicated horse-power .... B. T. U.
(b) Per brake horse-power .... B. T. U.
42. Thermal efficiency ratio:
(a) Per indicated horse-power.......................... per cent.
(b) Per brake horse-power .... per cent.

Miscellaneous Efficiency Results.

43. Cubic feet of gas or pounds of oil consumed per horse-power per hour:
(a) Per indicated horse-power ....
(b) Per brake horse-power ....

Heat Balance.

44. Quantities given, in per cents., of the total heat of combustion of the fuel:
(a) Heat equivalent of indicated horse-power......... per cent.
(b) Heat rejected in cooling water..................... per cent.
(c) Heat rejected in exhaust gases and lost through radiation and incomplete combustion.......... per cent.
Sum = 100 per cent.

Subdivisions of Item (c):
(c1) Heat rejected in exhaust gases .... per cent.
(c2) Lost through incomplete combustion .... per cent.
(c3) Lost through radiation, and unaccounted for..... per cent.
Sum = Item (c)....................

Additional Data.

Add any additional data bearing on the particular objects of the test or relating to the special class of service for which the engine is to be used. Also give copies of indicator diagrams nearest the mean and the corresponding scales. Where analyses are made of the gas or oil used as fuel, or of the exhaust gases, the results may be given in a separate table.

Data and Results of Standard Heat Test of Gas or Oil Engine.

Arranged according to the Short Form advised by the Engine Test Committee of the American Society of Mechanical Engineers. Code of 1902.

1. Made by............................of....................................
on engine located at...
to determine...
....
2. Date of trial ....
3. Type and class of engine ....
....
4. Kind of fuel used..
(a) Specific gravity.................................. deg. Fahr.
(b) Burning-point.................................. deg. Fahr.
(c) Flashing-point .... deg. Fahr.

5. Dimensions of engine: 1st Cyl. 2d Cyl.
(a) Class of cylinder (working or for compressing the charge).......................
(b) Single- or double-acting....................
(c) Cylinder dimensions:
Bore, in inches ....
Stroke, in feet ....
Diameter of piston-rod, in inches ....
(d) Average compression space or clearance, in per cent..............................
(e) Horse-power constant for 1 pound mean effective pressure and 1 revolution per minute ....

Total Quantities.

6. Duration of test.. hours.
7. Gas or oil consumed .... cu. ft. or lbs.
8. Cooling water supplied to jackets .... cu. ft. or lbs.
9. Calorific value of fuel by calorimeter test, determined by ....calorimeter.................. B. T. U.

Pressures and Temperatures.

10. Pressure at meter (for gas engine), in inches, of water.... ins.
11. Barometric pressure of atmosphere:
    (*a*) Reading of barometer.................................. ins.
    (*b*) Reading corrected to 32° F. .... ins.
12. Temperature of cooling water:
    (*a*) Inlet .... deg. Fahr.
    (*b*) Outlet .... deg. Fahr.
    (*c*) Degree of humidity .... deg. Fahr.
13. Temperature of gas at meter (for gas engine)............ deg. Fahr.
14. Temperature of atmosphere:
    (*a*) Dry-bulb thermometer................................ deg. Fahr.
    (*b*) Wet-bulb thermometer................................ deg. Fahr.
15. Temperature of exhaust gases.................................. deg. Fahr.

Data Relating to Heat Measurement.

16. Heat units consumed per hour (pounds of oil or cubic feet of gas per hour multiplied by the total heat of combustion).. B. T. U.
17. Heat rejected in cooling water per hour.................. B. T. U.

Speed, Etc.

18. Revolutions per minute .... rev.
19. Average number of explosions per minute................

Indicator Diagrams.

1st Cyl. 2d Cyl.

20. Pressure, in pounds, per square inch above atmosphere:
    (*a*) Maximum pressure..........................
    (*b*) Pressure just before ignition...............
    (*c*) Pressure at end of expansion ....
    (*d*) Exhaust pressure ....
    (*e*) Mean effective pressure....................

Power.

21. Indicated horse-power:
    First cylinder .... H. P.
    Second cylinder.. H. P.
    Total .... H. P.
22. Brake horse-power .... H. P.
23. Friction horse-power by friction diagrams................ H. P.
24. Percentage of indicated horse-power lost in friction...... per cent.

Standard Efficiency and Other Results.

25. Heat units consumed by the engine per hour:
    (*a*) Per indicated horse-power .... B. T. U.
    (*b*) Per brake horse-power .... B. T. U.
26. Pounds of oil or cubic feet of gas consumed per hour:
    (*a*) Per indicated horse-power .... lbs. or cu. ft.
    (*b*) Per brake horse-power............................... lbs. or cu. ft

Additional Data.

Add any additional data bearing on the particular objects of the test or relating to the special class of service for which the engine is to be used. Also give copies of indicator diagrams nearest the mean, and the corresponding scales.

NOTE.—The volume of gas measured at any temperature should be reduced to the equivalent at a standard temperature and atmospheric pressure, corrected for the effect of moisture in the gas, which is ordinarily at the saturation-point or nearly so. It is recommended that a standard be adopted for gas-engine work, the same as that used in photometry,—namely, the equivalent volume of the gas when saturated with moisture at the normal atmospheric pressure at a temperature of 60° F. In order to reduce the reading of the volume containing moist gas at any other temperature to this standard, multiply by the factor

$$\frac{459.4 + 60}{459.4 + t} \times \frac{b - (29.92 - s)}{29.4},$$

in which b is the height of the barometer, in inches, at 32° F.; t, the temperature of the gas at the meter, in degrees Fahrenheit; and s, the vacuum, in inches, of mercury corresponding to the temperature of t obtained from steam tables.

ELECTRIC POWER TRANSMISSION.

The principal applications of electricity in mechanical engineering are in the transmission of power and the independent driving of machines by electric motors. It is yet a question for debate as to whether the actual transmission losses are materially reduced by the substitution of electric driving for shafting, belting, and pulleys, but there is no doubt as to the great advantages of electricity so far as the convenient arrangement of machinery and the utilization of floor-space are concerned.

Some general data concerning electricity will here be given, and for special and fuller treatment the reader is referred to Foster's "Electrical Engineer's Pocket-book," Bell's "Electric Power Transmission," and the standard works of reference on electrical engineering.

Equivalents and Expressions of Electrical and Mechanical Units.

Name of unit.	Symbol.	What it measures.	Definition.	Formulas of standard units in absolute system (C. G. S.).	Equation.	Equivalent.	Multiplier to get number of C. G. S. corresponding units.
Ampere.	C	Electric current.	The ampere is the constant electric current which, when passed through a particular solution of nitrate of silver in water, deposits 0.001118 gramme per second. Or the flow of 1 coulomb per second.	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	$C=\frac{E}{R}$ (Ohm's Law)	Flow of 1 coulomb per second.	10^{-1}
Volt.	E	Difference of potential or electric pressure or electromotive force.	The volt is the electrical pressure, which, if applied to a conductor where resistance is 1 ohm, will produce a current of 1 ampere, and which is represented by 0.6974 of the pressure between the poles of a Clark's voltaic cell at 15° C.	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	$E=C\times R$	0.926 standard Daniel cell.	10^{-8}
Ohm.	R	Electric resistance.	The ohm is the resistance offered by a column of mercury 14.4521 grammes in mass, of constant cross-section, and 106.3 centimetres long, at the temperature of melting ice.	LT^{-1}	$R=\frac{E}{C}$	....	10^{-9}
Watt.	P	Electric power.	Is the rate of doing work when a current of 1 ampere flows between two points, having a difference of potential of 1 volt.	L^2MT^{-3}	$P=E\times C$	$\frac{1}{746}$ horse-power.	10^{-7}
Coulomb.	Q	Electric quantity.	The quantity of electricity that flows per second past a given point in a conductor which is carrying a current of 1 ampere.	$L^{\frac{1}{2}}M^{\frac{1}{2}}$	$Q=C\times T$	1.118 m. of silver deposited per second.	10^{-1}
Farad.	K	Electric capacity.	Is capacity of condenser which would require a charge of 1 coulomb to produce difference of potential of 1 volt between the two conductors forming the condenser.	$L^{-1}T^2$	$K=\frac{Q}{E}$	....	10^{-9}
Henry.	L	Electrical inductance, or self-induction.	Is the inductance in a circuit when current is changing at the rate of 1 ampere per second, and producing in that circuit a difference of potential of 1 volt. It corresponds to a rate of change of magnetic field strength through the circuit.	L	$L=\frac{E+T}{C}$	....	10^{-9}
Joule.	W	Electric energy or work.	Is the work done in one second when 1 ampere flows in conductor between two points, having a difference of potential of 1 volt.	L^2MT^{-2}	$W=\frac{1}{2}E\times Q$	0.7373 foot-pound per second.	10

NOTE.—In above C. G. S. (centimetre, gramme, second), L is the unit of length; M, of mass; and T, of time.

Analogies Between the Flow of Water and Electricity.

Water.	Electricity.
Head, difference of level, in feet. Difference of pressure per square inch, in pounds.	Volts; electro-motive force; difference of potential or of pressure; *E.* or *E. M. F.*
Resistance of pipes, apertures, etc., increases with length of pipe, with contractions, roughness, etc.; decreases with increase of sectional area. The law of increase and decrease is expressed by complex formulæ.	Ohms, resistance, *R*. The resistance increases directly as the length or the conductor or wire, and inversely as its sectional area. It varies with the nature or quality of the conductor. Conductivity is the reciprocal of specific resistance.
Rate of flow, as cubic feet per second, gallons per minute, etc., or volume divided by the time. In the mining regions sometimes expressed in "miner's inches."	Amperes; current; current strength; intensity of current; rate of flow; 1 ampere = 1 coulomb per second. Amperes $= \frac{\text{volts}}{\text{ohms}}$; $C = \frac{E}{R}$; $E = CR$.
Quantity, usually measured in cubic feet or gallons, but is also equivalent to rate of flow × time, as cubic feet per second for so many hours.	Coulomb, unit of quantity, Q = rate of flow × time, as ampere seconds; 1 ampere hour = 3600 coulombs.
Work, or energy, measured in foot-pounds: product of weight of falling water into height of fall; in pumping, product of quantity, in cubic feet, into the pressure, in pounds, per square foot against which the water is pumped.	Joule, volt-coulomb, *W*, the unit of work = product of quantity by the electro-motive force = volt-ampere. If *C* (amperes) = rate of flow, and *E* (volts) = difference of pressure between two points in a circuit, energy expended = CEt, = C^2Rt, since $E = CR$.
Power, rate of work. Horse-power, foot-pounds of work done in 1 minute ÷ 30,000. In falling water, pounds falling in 1 second ÷ 150. In water flowing in pipes, rate of flow, in cubic feet, per second × pressure resisting the flow, in pounds, per square foot ÷ 550.	Watt, unit of power, P = volts × amperes, = current or rate of flow × difference of potential. 1 watt = 0.7373 foot-pound per second = $\frac{1}{746}$ of a horse-power.

In the mechanical applications of electricity it must always be remembered that the volt corresponds to pressure and the ampere to flow, and the product—the volt-ampere—is the watt, the unit of power, 746 of which are equal to a horse-power. The kilowatt = 1000 watts is equal to

$$\frac{1000}{746} = 1.34 \text{ horse-power.}$$

The British Board of Trade Unit is equal to 1 kilowatt hour

Strands of Copper Wire.

(Roeblings.)

Copper wires are twisted into concentric strands or into ropes of 7 strands. A rope of 7 strands, each composed of 7 wires, is called a seven-by-seven rope, and is usually written 7 × 7. The number of wires that can be made into a strand is limited by the capacity of the stranding machinery,—200 wires is the usual limit of a concentric strand, and 133 wires of a rope.

In a strand of circular milage, CM, composed of n wires of diameter, d, with a weight per 1000 feet, w, we have

$$CM = d^2 \times n,$$

$$n = \frac{CM}{d^2},$$

$$d = \sqrt{\frac{CM}{n}},$$

$$w = 0.00305 \times CM.$$

The weights of strands are calculated about 1 per cent. heavier than a solid wire of the same circular milage, while the resistance is calculated for the solid wire.

The diameter of a strand may be calculated by multiplying the diameter of one wire by the factors given in the table, according to the number of wires composing the strand.

Number of Wires and Diameter in Strand Required to Equal a Given Circular Milage.

Diameter of Wires in Decimal Parts of an Inch.

Number of wires.	Area, in circular mils.						
	50 000	100 000	150 000	200 000	250 000	300 000	350 000
7	.0845	.1203	.1463	.1690	.1890	.2070	.2236
19	.0513	.0725	.0889	.1025	.1147	.1256	.1357
37	.0367	.0519	.0636	.0735	.0821	.0900	.0972
61	.0286	.0405	.0496	.0572	.0640	.0701	.0757
127	.0199	.0280	.0343	.0396	.0443	.0486	.0526
169	.0172	.0243	.0297	.0344	.0384	.0421	.0455
217	.0151	.0214	.0262	.0304	.0339	.0371	.0401
	400 000	450 000	500 000	550 000	600 000	650 000	700 000
7	.2390	.2535	.2672	.2803	.2927	.3047	.3163
19	.1450	.1538	.1622	.1701	.1776	.1849	.1919
37	.1039	.1103	.1162	.1219	.1273	.1325	.1375
61	.0809	.0858	.0905	.0949	.0991	.1032	.1071
127	.0561	.0595	.0627	.0658	.0687	.0715	.0742
169	.0486	.0516	.0543	.0571	.0595	.0620	.0643
217	.0429	.0455	.0480	.0503	.0525	.0547	.0567
	750 000	800 000	850 000	900 000	950 000	1 000 000	
7	.3273	.3380	.3484	.3585	.3684	.3770	
19	.1986	.2050	.2115	.2176	.2236	.2294	
37	.1423	.1470	.1515	.1559	.1602	.1644	
61	.1108	.1145	.1180	.1214	.1247	.1280	
127	.0768	.0793	.0818	.0841	.0864	.0887	
169	.0666	.0687	.0709	.0729	.0749	.0769	
217	.0588	.0607	.0625	.0644	.0661	.0678	

Copper Wire Table of American Institute of Electrical Engineers.

Giving Weights and Lengths of Cool, Warm, and Hot Wires, of Matthiessen's Standard of Conductivity, for Brown & Sharpe Gauge.

Gauges to the nearest fourth significant digit.				Weight.				Length.			
B. & S. or A. W. G.	Diameter, in inches.	Area.		Pounds per foot.	Pounds per ohm.			Feet per pound.	Feet per ohm.		
		Circular mils.	Sq. inch. sq. mils.		At 20° C.	At 50° C.	At 80° C.		At 20° C.	At 50° C.	At 80° C.
0000	.460 000	211 600.0	166 190.0	.640 500	13090.000	11720.000	10570.000	1.561	20440.0	18290.0	16510.0
000	.409 600	167 800.0	131 790.0	.508 000	8232.000	7369.000	6647.000	1.969	16210.0	14510.0	13090.0
00	.364 800	133 100.0	104 518.0	.402 800	5177.000	4634.000	4182.000	2.482	12850.0	11500.0	10380.0
0	.324 900	105 500.0	82 887.0	.319 500	3256.000	2914.000	2630.000	3.130	10190.0	9123.0	8232.0
1	.289 300	83 690.0	65 732.0	.253 300	2048.000	1833.000	1654.000	3.947	8083.0	7235.0	6528.0
2	.257 600	66 370.0	52 128.0	.200 900	1288.000	1153.000	1040.000	4.977	6410.0	5738.0	5177.0
3	.229 400	52 630.0	41 339.0	.159 300	810.000	725.000	654.200	6.276	5084.0	4550.0	4106.0
4	.204 300	41 740.0	32 784.0	.126 400	509.400	455.900	411.400	7.914	4031.0	3608.0	3256.0
5	.181 900	33 100.0	25 999.0	.100 200	320.400	286.700	258.700	9.98	3197.0	2862.0	2582.0
6	.162 000	26 250.0	20 618.0	.079 460	201.500	180.300	162.700	12.58	2535.0	2269.0	2048.0
7	.144 300	20 820.0	16 351.0	.063 020	126.700	113.400	102.300	15.87	2011.0	1800.0	1624.0
8	.128 500	16 510.0	12 967.0	.049 980	79.690	71.330	64.360	20.01	1595.0	1427.0	1288.0
9	.114 400	13 090.0	10 283.0	.039 630	50.120	44.860	40.480	25.23	1265.0	1132.0	1021.0
10	.101 900	10 380.0	8 155.0	.031 430	31.520	28.210	25.460	31.82	1003.0	897.6	809.9
11	.090 740	8 234.0	6 467.0	.024 930	19.820	17.740	16.010	40.12	795.3	711.8	642.3
12	.080 810	6 530.0	5 129.0	.019 770	12.470	11.160	10.070	50.59	630.7	564.5	509.4
13	.071 960	5 178.0	4 067.0	.015 680	7.840	7.017	6.332	63.79	500.1	447.7	404.0
14	.064 080	4 107.0	3 225.0	.012 430	4.931	4.413	3.982	80.44	396.6	355.0	320.3
15	.057 070	3 257.0	2 558.0	.009 858	3.101	2.776	2.504	101.4	314.5	281.5	254.0
16	.050 820	2 583.0	2 029.0	.007 818	1.950	1.746	1.575	127.9	249.4	225.3	201.5
17	.045 260	2 048.0	1 609.0	.006 200	1.226	1.098	.990 600	161.3	197.8	177.1	159.8
18	.040 300	1 624.0	1 276.0	.004 917	.771 300	.690 400	.623 000	203.4	156.9	140.4	126.7
19	.035 890	1 288.0	1 012.0	.003 899	.485 100	.434 200	.391 800	256.5	124.4	111.4	100.5

20	.031 960	1 022.000	802.00	.003 092	.305 100	.273 100	.246 400	323.4	98.660	88.310	79.68
21	.028 460	810.100	636.30	.002 452	.191 900	.171 700	.155 000	407.8	78.240	70.030	63.19
22	.025 350	642.400	504.60	.001 945	.120 700	.108 000	.097 460	514.2	62.050	55.540	50.11
23	.022 570	509.500	400.20	.001 542	.075 890	.067 930	.061 290	648.4	49.210	44.040	39.74
24	.020 100	404.000	317.30	.001 223	.047 730	.042 720	.038 550	817.6	39.020	34.930	31.52
25	.017 900	320.400	251.70	.000 9699	.030 020	.026 870	.024 240	1031.0	30.950	27.700	24.99
26	.015 940	254.100	199.60	.000 7692	.018 880	.016 900	.015 250	1300.0	24.540	21.970	19.82
27	.014 200	201.500	158.30	.000 6100	.011 870	.010 630	.009 588	1639.0	19.460	17.420	15.72
28	.012 640	159.800	125.50	.000 4837	.007 466	.006 683	.006 030	2067.0	15.430	13.820	12.47
29	.011 260	126.700	99.53	.000 3836	.004 696	.004 203	.003 792	2607.0	12.240	10.960	9.886
30	.010 030	100.500	78.94	.000 3042	.002 953	.002 643	.002 385	3287.0	9.707	8.688	7.840
31	.008 928	79.700	62.60	.000 2413	.001 857	.001 662	.001 500	4145.0	7.698	6.890	6.217
32	.007 950	63.210	49.64	.000 1913	.001 168	.001 045	.000 9436	5227.0	6.105	5.464	4.930
33	.007 080	50.130	39.37	.000 1517	.000 7346	.000 6575	.000 5933	6591.0	4.841	4.333	3.910
34	.006 305	39.750	31.22	.000 1203	.000 4620	.000 4135	.000 3731	8311.0	3.839	3.436	3.101
35	.005 615	31.520	24.76	.000 09543	.000 2905	.000 2601	.000 2347	10480.0	3.045	2.725	2.459
36	.005 000	25.000	19.64	.000 07568	.000 1827	.000 1636	.000 1476	13210.0	2.414	2.161	1.950
37	.004 453	19.830	15.57	.000 06001	.000 1149	.000 1029	.000 09281	16660.0	1.915	1.714	1.547
38	.003 965	15.720	12.35	.000 04759	.000 07210	.000 06454	.000 05824	21010.0	1.519	1.359	1.226
39	.003 531	12.470	9.79	.000 03774	.000 04545	.000 04068	.000 03671	26500.0	1.204	1.078	.9726
40	.003 145	9.888	7.77	.000 02993	.000 02858	.000 02559	.000 02309	33410.0	.955	.8548	.7713

The data from which this table has been computed are as follows: Matthiessen's standard resistivity, Matthiessen's temperature coefficient, specific gravity of copper = 8.89. Resistance in terms of the international ohm.

Matthiessen's standard 1 metre-gramme of hard drawn copper = 0.1469 B. A. U. at 0° C. Ratio of resistivity, hard or soft copper, 1.0226.

Matthiessen's standard 1 metre-gramme of soft drawn copper = 0.14365 B. A. U. at 0° C. 1 B. A. U. = 0.9866 international ohm.

Matthiessen's standard 1 metre-gramme of soft drawn copper = 9.141729 international ohms at 0° C.

Temperature coefficients of resistance for 20° C., 50° C., and 80° C., 1.07968, 1.20625, and 1.33681, respectively. 1 foot = 0.3048028 metre, 1 pound = 453.59256 grammes.

Although the entries in the table are carried to the fourth significant digit, the computations have been carried to at least five figures. The last digit is therefore correct to within half a unit, representing an arithmetical degree of accuracy of at least one part in two thousand. The diameters of the B. & S. or A. W. G. wires are obtained from the geometrical series, in which No. 0000 = 0.46 inch and No. 36 = 0.005 inch, the nearest fourth significant digit being retained in the areas and diameters so deduced.

It is to be observed that while Matthiessen's standard of resistivity may be permanently recognized, the temperature coefficient of its variation which he introduced, and which is here used, may in future undergo slight revision.

F. B. CROCKER, W. E. GEYER, G. A. HAMILTON, A. E. KENNELLY, *Chairman*,

Committee on "Units and Standards."

Table for the Conversion of Mils. ($\frac{1}{1000}$ inch) into Centimetres.

Mils.	Centi-metres.	Mils.	Centi-metres.	Mils.	Centi-metres.	Mils.	Centi-metres.
1	.00254	26	.06602	51	.1295	76	.1931
2	.00508	27	.06856	52	.1321	77	.1956
3	.00762	28	.07110	53	.1346	78	.1981
4	.01016	29	.07364	54	.1372	79	.2006
5	.01270	30	.07618	55	.1397	80	.2032
6	.01524	31	.07872	56	.1422	81	.2057
7	.01778	32	.08126	57	.1448	82	.2083
8	.02032	33	.08380	58	.1473	83	.2108
9	.02286	34	.08634	59	.1499	84	.2133
10	.02540	35	.08888	60	.1524	85	.2159
11	.02793	36	.09142	61	.1549	86	.2184
12	.03047	37	.09396	62	.1575	87	.2209
13	.03301	38	.09650	63	.1600	88	.2235
14	.03555	39	.09904	64	.1626	89	.2260
15	.03809	40	.1016	65	.1651	90	.2286
16	.04063	41	.1041	66	.1676	91	.2311
17	.04317	42	.1067	67	.1702	92	.2336
18	.04571	43	.1092	68	.1727	93	.2362
19	.04825	44	.1118	69	.1752	94	.2387
20	.05079	45	.1143	70	.1778	95	.2413
21	.05333	46	.1168	71	.1803	96	.2438
22	.05587	47	.1194	72	.1829	97	.2465
23	.05841	48	.1219	73	.1854	98	.2489
24	.06095	49	.1245	74	.1879	99	.2514
25	.06348	50	.1270	75	.1905	100	.2540

"National Electrical Code."

Rules and Requirements of the National Board of Fire Underwriters for the Installation of Wiring and Apparatus for Electric Light, Heat, and Power as Recommended by the Underwriters' National Electric Association.

Edition of 1901.

The National Electrical Code is the result of the united efforts of the various electrical, insurance, architectural, and allied interests which have, through the National Conference on Standard Electrical Rules, composed of delegates from various national associations, unanimously voted to recommend it to their respective associations for approval or adoption.

The following is a list of the associations represented in the conference, all of which have approved of the Code:

American Institute of Architects,
American Institute of Electrical Engineers,
American Society of Mechanical Engineers,
American Street Railway Association,
Factory Mutual Fire Insurance Companies,
National Association of Fire Engineers,
National Board of Fire Underwriters,
National Electric Light Association,
Underwriters' National Electric Association.

GENERAL PLAN GOVERNING THE ARRANGEMENT OF RULES.

Class A.—**Central Stations, Dynamo, Motor, and Storage=battery Rooms, Transformer Sub=stations, etc.** Rules 1 to 11.

Class B.—**Outside Work,** all systems and voltages. Rules 12 and 13.

Class C.—**Inside Work.** Rules 14 to 39. Subdivided as follows:

General Rules, applying to all systems and voltages. Rules 14 to 17.

Constant=current Systems. Rules 18 to 20.

Constant=potential Systems.

All voltages. Rules 21 to 23.

Voltage not over 550. Rules 24 to 31.

Voltage between 550 and 3500. Rules 32 to 37.

Voltage over 3500. Rules 38 and 39.

Class D.—**Specifications for Wires and Fittings.** Rules 40 to 63.

Class E.—**Miscellaneous.** Rules 64 to 67.

Class F.—**Marine Wiring.** Rules 68 to 80.

CLASS A.—STATIONS AND DYNAMO ROOMS.

Includes Central Stations, Dynamo, Motor, and Storage-battery Rooms, Transformer Sub-stations, etc.

1. Generators.

(*a*) Must be located in a dry place.

(*b*) Must never be placed in a room where any hazardous process is carried on, nor in places where they would be exposed to inflammable gases or flyings of combustible materials.

(*c*) Must be insulated on floors or base frames, which must be kept filled to prevent absorption of moisture, and also kept clean and dry. Where frame insulation is impracticable, the Inspection Department having jurisdiction may, in writing, permit its omission, in which case the frame must be permanently and effectively grounded.

A high-potential machine which, on account of great weight or for other reasons, cannot have its frame insulated from the ground, should be surrounded with an insulated platform. This may be made of wood mounted on insulating supports, and so arranged that a man must always stand upon it in order to touch any part of the machine.

In case of a machine having an insulated frame, if there is trouble from static electricity due to belt friction, it should be overcome by placing near the belt a metallic comb connected with the earth, or by grounding the frame through a very high resistance of not less than 200 ohms per volt generated by the machine.

(*d*) Every constant-potential generator must be protected from excessive current by a safety fuse, or equivalent device, of approved design in each lead wire.

These devices should be placed on the machine or as near it as possible.

Where the needs of the service make these devices impracticable, the Inspection Department having jurisdiction may, in writing, modify the requirements.

(*e*) Must each be provided with a water-proof cover.

(*f*) Must each be provided with a name-plate, giving the maker's name, the capacity in volts and amperes, and the normal speed, in revolutions, per minute.

2. Conductors.

From generators to switchboards, rheostats, or other instruments, and thence to outside lines.

(*a*) Must be in plain sight or readily accessible.

(*b*) Must have an *approved* insulating covering, as called for by rules in Class C for similar work, except that in central stations, on exposed circuits, the wire which is used must have a heavy-braided, non-combustible outer covering.

Bus bars may be made of bare metal.

(*c*) Must be kept so rigidly in place that they cannot come in contact.

(*d*) Must in all other respects be installed under the same precautions as required by rules in Class C for wires carrying a current of the same volume and potential.

3. Switchboards.

(*a*) Must be so placed as to reduce to a minimum the danger of communicating fire to adjacent combustible material.

Special attention is called to the fact that switchboards should not be built down to the floor nor up to the ceiling, but a space of at least 10 or 12 inches should be left between the floor and the board, and from 18 to 24 inches between the ceiling and the board, in order to prevent fire from communicating from the switchboard to the floor or ceiling, and also to prevent the forming of a partially-concealed space very liable to be used for storage of rubbish and oily waste.

(*b*) Must be made of non-combustible material or of hard wood, in skeleton form, filled to prevent absorption of moisture.

(*c*) Must be accessible from all sides when the connections are on the back, but may be placed against a brick or stone wall when the wiring is entirely on the face.

(*d*) Must be kept free from moisture.

(*e*) Bus bars must be equipped in accordance with rules for placing conductors.

4. Resistance Boxes and Equalizers.

(For Construction Rules, see No. 60.)

(*a*) Must be placed on a switchboard, or, if not thereon, at a distance of a foot from combustible material, or separated therefrom by a non-inflammable, non-absorptive insulating material.

5. Lightning Arresters.

(For Construction Rules, see No. 63.)

(*a*) Must be attached to each side of every overhead circuit connected with the station.

It is recommended to all electric light and power companies that arresters be connected at intervals over systems in such numbers and so located as to prevent ordinary discharges entering (over the wires) buildings connected to the lines.

(*b*) Must be located in readily-accessible places away from combustible materials, and as near as practicable to the point where the wires enter the building.

Station arresters should generally be placed in plain sight on the switchboard.

In all cases, kinks, coils, and sharp bends in the wires between the arresters and the outdoor lines must be avoided, as far as possible.

(*c*) Must be connected with a thoroughly good and permanent ground connection by metallic strips or wires having a conductivity not less than that of a No. 6 B. & S. copper wire, which must be run as nearly in a straight line as possible from the arresters to the earth connection.

Ground wires for lightning arresters must not be attached to gas-pipes within the buildings.

It is often desirable to introduce a choke coil in circuit between the arresters and the dynamo. In no case should the ground wire from a lightning arrester be put into iron pipes, as these would tend to impede the discharge.

6. Care and Attendance.

(*a*) A competent man must be kept on duty where generators are operating.

(*b*) Oily waste must be kept in approved metal cans and removed daily.

Approved waste cans shall be made of metal, with legs raising can 3 inches from the floor, and with self-closing covers.

7. Testing of Insulation Resistance.

(*a*) All circuits, except such as are permanently grounded in accordance with Rule 13A, must be provided with reliable ground detectors. Detectors

which indicate continuously, and give an instant and permanent indication of a ground, are preferable. Ground wires from detectors must not be attached to gas-pipes within the building.

(*b*) Where continuously-indicating detectors are not feasible, the circuits should be tested at least once per day, and preferably oftener.

(*c*) Data obtained from all tests must be preserved for examination by the Inspection Department having jurisdiction.

These rules on testing to be applied at such places as may be designated by the Inspection Department having jurisdiction.

8. Motors.

(*a*) Must be insulated on floors or base frames, which must be kept filled to prevent absorption of moisture; and must be kept clean and dry. Where frame insulation is impracticable, the Inspection Department having jurisdiction may, in writing, permit its omission, in which case the frame must be permanently and effectively grounded.

A high-potential machine, which on account of great weight or for other reasons cannot have its frame insulated, should be surrounded with an insulated platform. This may be made of wood mounted on insulating supports, and so arranged that a man must stand upon it in order to touch any part of the machine.

In case of a machine having an insulated frame, if there is trouble from static electricity due to belt friction it should be overcome by placing near the belt a metallic comb connected to the earth, or by grounding the frame through a very high resistance of not less than 200 ohms per volt generated by the machine.

(*b*) Must be wired under the same precautions as required by rules in Class C for wires carrying a current of the same volume and potential.

The leads or branch circuits should be designed to carry a current at least 50 per cent. greater than that required by the rated capacity of the motor, to provide for the inevitable overloading of the motor at times without overfusing the wires.

(*c*) The motor and resistance box must be protected by a cutout and controlled by a switch (see No. 17, *a*), said switch plainly indicating whether "on" or "off." Where one-fourth horse-power or less is used on low-tension circuits a single-pole switch will be accepted. The switch and rheostat must be located within sight of the motor, except in such cases where special permission to locate them elsewhere is given in writing by the Inspection Department having jurisdiction.

(*d*) Must have their rheostats or starting-boxes located as to conform to the requirements of No. 4.

In connection with motors, the use of circuit-breakers, automatic starting-boxes, and automatic under-load switches is recommended, and they *must* be used when required.

(*e*) Must not be run in series-multiple or multiple-series, except on constant-potential systems, and then only by special permission of the Inspection Department having jurisdiction.

(*f*) Must be covered with a water-proof cover when not in use, and, if deemed necessary by the Inspection Department having jurisdiction, must be inclosed in an approved case.

From the nature of the question, the decision as to what is an approved case must be left to the Inspection Department having jurisdiction to determine in each instance.

(*g*) Must, when combined with ceiling fans, be hung from insulated hooks, or else there must be an insulator interposed between the motor and its support.

(*h*) Must each be provided with a name-plate, giving the maker's name, the capacity in volts and amperes, and the normal speed, in revolutions, per minute.

9. Railway Power Plants.

(*a*) Must be equipped in each feed wire before it leaves the station with an *approved* automatic circuit-breaker (see No. 52), or other device, which will immediately cut off the current in case of an accidental ground. This device must be mounted on a fire-proof base, and in full view and reach of the attendant.

10. Storage or Primary Batteries.

(*a*) When current for light and power is taken from primary or secondary batteries, the same general regulations must be observed as applied to similar apparatus fed from dynamo generators developing the same difference of potential.

(*b*) Storage-battery rooms must be thoroughly ventilated.

(*c*) Special attention is directed to the rules for rooms where acid fumes exist. (See No. 24, *j* and *k*.)

(*d*) All secondary batteries must be mounted on non-absorptive, non-combustible insulators, such as glass or thoroughly vitrified and glazed porcelain.

(*e*) The use of any metal liable to corrosion must be avoided in cell connections of secondary batteries.

11. Transformers.

(For Construction Rules, see No. 62.)

(*a*) In central or sub-stations the transformers must be so placed that smoke from the burning out of the coils or the boiling over of the oil (where oil-filled cases are used) could do no harm.

CLASS B.—OUTSIDE WORK.

All Systems and Voltages.

12. Wires.

(*a*) Service wires must have an *approved* rubber insulator covering. (See No. 41.) Line wires, other than services, must have an *approved* weather-proof or rubber insulating covering (Nos. 41 and 44). All the wires must have an insulation equal to that of the conductors they confine.

(*b*) Must be so placed that moisture cannot form a cross connection between them, not less than a foot apart, and not in contact with any substance other than their insulating supports. Service blocks must be covered over their entire surface with at least two coats of water-proof paint.

(*c*) Must be at least 7 feet above the highest point of flat roofs, and at least 1 foot above the ridge of pitched roofs over which they pass or to which they are attached.

(*d*) Must be protected by dead insulated guard iron or wires from possibility of contact with other conducting wires or substances to which current may leak. Special precautions of this kind must be taken where sharp angles occur, or where any wires might possibly come in contact with electric light or power wires.

(*e*) Must be provided with petticoat insulators of glass or porcelain. Porcelain knobs or cleats and rubber hooks will not be approved.

(*f*) Must be so spliced or joined as to be both mechanically and electrically secure without solder. The joints must then be soldered, to insure preservation, and covered with an insulation equal to that on the conductors.

All joints must be soldered, even if made with some form of patent splicing device. This ruling applies to joints and splices in all classes of wiring covered by these rules.

(*g*) Must, where they enter buildings, have drip loops outside, and the holes through which the conductors pass must be bushed with non-combustible, non-absorptive insulating tubes slanting upward towards the inside.

(*h*) Telegraph, telephone, and similar wires must not be placed on the same cross-arm with electric light or power wires; and when placed on the same pole with such wires the distance between the two inside pins of each cross-arm must not be less than 26 inches.

(*i*) The metallic sheaths to cables must be permanently and effectively connected to "earth."

Trolley Wires.

(*j*) Must not be smaller than No. 0 B. & S. copper, or No. 4 B. & S. silicon bronze, and must readily stand the strain put upon them when in use.

(*k*) Must have a double insulation from the ground. In wooden-pole construction the pole will be considered as one insulation.

(*l*) Must be capable of being disconnected at the power plant, or of being divided into sections, so that, in case of fire on the railway route, the current may be shut off from the particular section and not interfere with the work of the firemen. This rule also applies to feeders.

(*m*) Must be safely protected against accidental contact where crossed by other conductors.

Guard wires should be insulated from the ground, and should be electrically disconnected in sections of not more than 300 feet in length.

Ground Return Wires.

(*n*) For the diminution of electrolytic corrosion of underground metal work, ground return wires must be so arranged that the difference of potential between the grounded dynamo terminal and any point on the return circuit will not exceed 25 volts.

It is suggested that the positive pole of the dynamo be connected to the trolley line, and that whenever pipes or other underground metal work are found to be electrically positive to the rails or surrounding earth that they be connected by conductors arranged so as to prevent, as far as possible, current flow from the pipes into the ground.

13. Transformers.

(For Construction Rules, see No. 62.)

(*a*) Must not be placed inside of any building, excepting central stations, unless by special permission of the Inspection Department having jurisdiction.

(*b*) Must not be attached to the outside walls of buildings, unless separated therefrom by substantial supports.

13A. Grounding Low-potential Circuits.

The grounding of low-potential circuits under the following regulations is only allowed when so arranged that under normal conditions there will be no flow of current through the ground wire.

Direct-current 3-Wire Systems.

(*a*) Neutral wire may be grounded, and when grounded the following rules must be complied with:

1. Must be grounded at the central station on a metal plate buried in coke beneath permanent moisture level, and also through all available underground water- and gas-pipe systems.

2. In underground systems the neutral wire must also be grounded at each distributing box through the box.

3. In overhead systems the neutral wire must be grounded every 500 feet, as provided in Sections *c*, *e*, and *f*.

The Inspection Department having jurisdiction may require grounding, if they deem it necessary.

2-wire direct-current systems having no accessible neutral point are not to be grounded.

Alternating Current Secondary Systems.

(*b*) The neutral point of transformers or the neutral wire of distributing systems may be grounded, and when grounded the following rules must be complied with:

1. Transformers feeding 2-wire systems must be grounded at the centre of the secondary coils.

2. Transformers feeding systems with a neutral wire must have the neutral wire grounded at the transformer and at least every 250 feet beyond.

Inspection Department having jurisdiction may *require* grounding, if they deem it necessary.

Ground Connections.

(*c*) The ground wire in D. C. 3-wire systems must not at central stations be smaller than the neutral wire, and not smaller than No. 6 B. & S. elsewhere.

(*d*) The ground wire in A. C. systems must never be less than No. 6 B. & S., and must always have equal carrying capacity to the secondary lead of the transformer, or the combined leads where transformers are banked.

(*e*) The ground wire must be kept outside of buildings, but may be directly attached to the building or pole. The wire must be carried in as nearly a straight line as possible, and kinks, coils, and sharp bends must be avoided.

(*f*) The ground connections for central stations, transformer sub-stations, and banks of transformers must be made through metal plates buried in coke below permanent moisture level, and connections should also be made to all available underground piping systems. For individual transformers and building services the ground connection may be made as above, or may be made to water or other piping systems running into the buildings. This connection may be made by carrying the ground wire into the cellar and connecting on the street side of meters, main clocks, etc.

In connecting ground wires to piping systems, where possible, the wires should be soldered into one or more brass plugs, and the plugs forcibly screwed into a pipe-fitting, or, where the plugs are cast-iron, into a hole tapped to the pipe itself. For large stations, where connecting to underground pipes with bell and spigot joints, it is well to connect to several lengths, as the pipe joints may be of rather high resistance. Where such plugs cannot be used the surface of the pipe may be filed or scraped bright, the wire wound around it, and a strong clamp put over the wire and firmly bolted together.

Where ground plates are used a No. 16 copper plate, about 3 × 6 feet in size, with about 2 feet of crushed coke or charcoal about pea size both under and over it, would make a ground of sufficient capacity for a moderate-size station, and would probably answer for the ordinary sub-station or bank of transformers. For a large central station considerable more area might be necessary, depending upon the other unground connections available. The ground wire should be riveted to such a plate in a number of places, and soldered for its whole length. Perhaps even better than a copper plate is a cast-iron plate with projecting forks, the idea of the fork being to distribute the connection to the ground over a fairly broad area, and to give a large surface contact. The ground wire can probably best be connected to such a cast-iron plate by brass plugs screwed into the plate to which the wire is soldered. In all cases the joint between the plate and the ground wire should be thoroughly protected against corrosion by suitable painting with water-proof paint or some equivalent.

CLASS C.—INSIDE WORK.

All Systems and Voltages.

GENERAL RULES.—ALL SYSTEMS AND VOLTAGES.

14. Wires.

(For Special Rules, see Nos. 18, 24, 32, 38, and 39.)

(*a*) Must not be of smaller size than No. 14 B. & S., except as allowed under Rules 24, *t*, and 45, *b*.

(*b*) Tie wires must have an insulation equal to that of the conductors they confine.

(*c*) Must be so spliced or joined as to both mechanically and electrically secure without solder; they must be then soldered to insure preservation, and the joint covered with an insulation equal to that on the conductors.

Standard wires must be soldered before being fastened under clamps or binding screws; and, when they have a conductivity greater than No. 10 B. & S. copper wire, they will be soldered into lugs.

All joints must be soldered, even if made with some form of patent splicing device. This ruling applies to joints and splices in all classes of wiring covered by these rules.

(*d*) Must be separated from contact with walls, floors, timbers, or partitions through which they may pass by non-combustible, non-absorptive insulating tubes, such as glass or porcelain.

Bushings must be long enough to bush the entire length of the hole in one continuous piece, or else the hole must first be bushed by a continuous

water-proof tube, which may be a conductor, such as iron pipe; the tube then is to have a non-conducting bushing pushed in at each end, so as to keep the wire absolutely out of contact with the conducting pipe.

(*e*) Must be kept free from contact with gas-, water-, or other metallic piping, or any other conductors or conducting material which they may cross, by some continuous and firmly-fixed non-conductor, creating a separation of at least 1 inch. Deviations from this rule may sometimes be allowed by special permission.

(*f*) Must be so placed in wet places that an air space will be left between conductors and pipes in crossing, and the former must be run in such a way that they cannot come in contact with the pipe accidentally. Wires should be run over, rather than under, pipes upon which moisture is likely to gather, or which, by leaking, might cause trouble on a circuit.

15. Underground Conductors.

(*a*) Must be protected, when brought into a building, against moisture and mechanical injury, and all combustible material must be kept removed from the immediate vicinity.

(*b*) Must not be so arranged as to shunt the current through a building around any catch-box.

16. Carrying Capacity of Wires.

Below is a table which must be followed in placing interior conductors, showing the allowable carrying capacity of wires and cables of 98 per cent. conductivity, according to the standard adopted by the American Institute of Electrical Engineers.

B. & S. G.	Table A, rubber-covered wires. (See No. 41.) Amperes.	Table B, weather-proof wires. (See Nos. 42–44.) Amperes.	Circular mils.	Circular mils.	Table A, rubber-covered wires. (See No. 41.) Amperes.	Table B, weather-proof wires. (See Nos. 42–44.) Amperes.
18	3	5	1 624	200 000	200	300
16	6	8	2 583	300 000	270	400
14	12	16	4 107	400 000	330	500
12	17	23	6 530	500 000	390	590
10	24	32	10 380	600 000	450	680
8	33	46	16 510	700 000	500	760
6	46	65	26 250	800 000	550	840
5	54	77	33 100	900 000	600	920
4	65	92	41 740	1 000 000	650	1000
3	76	110	52 630	1 100 000	690	1080
2	90	131	66 370	1 200 000	730	1150
1	107	156	83 690	1 300 000	770	1220
0	127	185	105 500	1 400 000	810	1290
00	150	220	133 100	1 500 000	850	1360
000	177	262	167 800	1 600 000	890	1430
0000	210	312	211 600	1 700 000	930	1490
				1 800 000	970	1550
				1 900 000	1010	1610
				2 000 000	1050	1670

The lower limit is specified for rubber-covered wires to prevent gradual deterioration of the high insulations by the heat of the wires, but not from

fear of igniting the insulation. The question of drop is not taken into consideration in the tables on page 711.

The carrying capacity of 16- and 18-wire is given, but no smaller than 14 is to be used, except as allowed under Rules 24, *t*, and 45, *b*.

17. Switches, Cutouts, Circuit-breakers, etc.

(For Construction Rules, see Nos. 51, 52, and 53.)

(*a*) Must, whenever called for, unless otherwise provided (for exceptions, see No. 8, *c*, and No. 22, *c*), be so arranged that the cutouts will protect, and the opening of the switch or circuit-breaker will disconnect, all of the wires,—that is, in a 2-wire system the 2 wires, and in a 3-wire system the 3 wires, must be protected by the cutout, and disconnected by the operation of the switch or circuit-breaker.

(*b*) Must not be placed in the immediate vicinity of easily-ignitible stuff or where exposed to inflammable gases or dust or to flyings of combustible material.

(*c*) Must, when exposed to dampness, either be inclosed in a water-proof box or mounted on porcelain knobs.

CONSTANT-CURRENT SYSTEMS.

Principally Series Arc Lighting.

18. Wires.

(See, also, Nos. 14, 15, and 16.)

(*a*) Must have an *approved* rubber insulating covering. (See No. 41.)

(*b*) Must be arranged to enter and leave the building through an *approved* double-contact service switch (see No. 51), mounted in a non-combustible case, kept free from moisture, and easy of access to police or firemen. So-called "snap switches" must not be used on high-potential circuits.

(*c*) Must always be in plain sight, and never incased, except when *required* by the Inspection Department having jurisdiction.

(*d*) Must be supported on glass or porcelain insulators, which separate the wire at least 1 inch from the surface wired over, and must be kept *rigidly* at least 8 inches from each other, except within the structure of lamps, on hanger-boards, in cutout boxes, or like places, where a less distance is necessary.

(*e*) Must, on side walls, be protected from mechanical injury by a substantial boxing, retaining an air space of 1 inch around the conductors, closed at the top (the wires passing through bushed holes), and extending not less than 7 feet from the floor. When crossing floor-timbers in cellars or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than ½ inch in thickness.

19. Arc Lamps.

(For Construction Rules, see No. 57.)

(*a*) Must be carefully isolated from inflammable material.

(*b*) Must be provided at all times with a glass globe surrounding the arc, securely fastened upon a closed base. No broken or cracked globes to be used.

(*c*) Must be provided with a wire netting (having a mesh not exceeding 1¼ inches) around the globe and an *approved* spark arrester (see No. 58), when readily-inflammable material is in the vicinity of the lamps, to prevent escape of sparks, melted copper, or carbon. It is recommended that plain carbons, not copper-plated, be used for lamps in such places.

Arc lamps, when used in places where they are exposed to flyings of easily-inflammable material, should have the carbons inclosed completely in a globe in such manner as to avoid the necessity for spark arresters.

For the present, globe and spark arresters will not be required on so-called "inverted arc" lamps, but this type of lamp must not be used where exposed to flyings of easily-inflammable materials.

(*d*) Where hanger-boards (see No. 56) are not used lamps must be hung from insulating supports other than their conductors.

20. Incandescent Lamps in Series Circuits.

(*a*) Must have the conductors installed as provided in No. 18, and each lamp must be provided with an automatic cutout.
(*b*) Must have each lamp suspended from a hanger-board by means of rigid tube.
(*c*) No electro-magnetic device for switches and no system of multiple-series or series-multiple lighting will be approved.
(*d*) Under no circumstances can they be attached to gas fixtures.

CONSTANT-POTENTIAL SYSTEMS.

General Rules, all Voltages.

21. Automatic Cutouts (Fuses and Circuit-breakers).

(See No. 17, and for Construction, Nos. 52 and 53.)

(*a*) Must be placed on all service wires, either overhead or underground, as near as possible to the point where they enter the building and inside the walls, and arranged to cut off the entire current from the building.
Where the switch required by Rule No. 22 is inside the building, the cutout required by this section must be placed so as to protect it.
(*b*) Must be placed at every point where a change is made in the size of wire (unless the cutout in the larger wire will protect the smaller). (See No. 16.)
(*c*) Must be in plain sight or inclosed in an *approved* box (see No. 54) and readily accessible. They must not be placed in the canopies or shells of fixtures.
(*d*) Must be so placed that no set of incandescent lamps, whether grouped on one fixture or several fixtures or pendants, requiring more than 660 watts shall be dependent upon one cutout. Special permission may be given in writing by the Inspection Department having jurisdiction for departure from this rule in case of large chandeliers, stage borders, and illuminated signs.
(*e*) Must be provided with fuses, the rated capacity of which does not exceed the allowable carrying capacity of the wire; and, when circuit-breakers are used, they must not be set more than about 30 per cent. above the allowable carrying capacity of the wire, unless a fusible cutout is also installed in the circuit. (See No. 16.)

22. Switches.

(See No. 17, and for Construction, No. 51.)

(*a*) Must be placed on all service wires, either overhead or underground, in a readily-accessible place, as near as possible to the point where the wires enter the building, and arranged to cut off the entire current.
(*b*) Must always be placed in dry, accessible places, and be grouped, as far as possible. Knife switches must be so placed that gravity will tend to open rather than close the switch.
(*c*) Must not be single-pole, except when the circuits which they control supply not more than six 16-candle-power lamps or their equivalent.
(*d*) Where flush switches are used, whether with conduit systems or not, the switches must be inclosed in boxes constructed of or lined with fire-resisting material. No push-buttons for bells, gas-lighting circuits, or the like shall be placed in the same wall-plate with switches controlling electric light or power wiring.

23. Electric Heaters.

(*a*) Must, if stationary, be placed in a safe situation, isolated from inflammable materials, and be treated as sources of heat.
(*b*) Must each have a cutout and *indicating* switch. (See No. 17, *a*.)
(*c*) Must have the attachments of feed wires to the heaters in plain sight, easily accessible, and protected from interference, accidental or otherwise.

(*d*) The flexible conductors for portable apparatus, such as irons, etc., must have an *approved* insulating covering. (See No. 45, *h*.)
(*e*) Must each be provided with name-plate, giving the maker's name and the normal capacity in volts and amperes.

LOW-POTENTIAL SYSTEMS.

550 Volts or Less.

Any circuit attached to any machine, or combination of machines, which develops a difference of potential between any two wires of over 10 volts and less than 550 volts, shall be considered as a low-potential circuit, and as coming under this class, unless an approved transforming device is used, which cuts the difference of potential down to 10 volts or less. The primary circuit not to exceed a potential of 3500 volts.

24. Wires.

General Rules.

(See, also, Nos. 14, 15, and 16.)

(*a*) Must not be laid in plaster, cement, or similar finish.
(*b*) Must never be fastened with staples.
(*c*) Must not be fished for any great distance, and only in places where the inspector can satisfy himself that the rules have been complied with.
(*d*) Twin wires must never be used, except in conduits or where flexible conductors are necessary.
(*e*) Must be protected on side walls from mechanical injury. When crossing floor-timbers in cellars or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than ½ inch in thickness and not less than 3 inches in width.
Suitable protection on side walls may be secured by a substantial boxing, retaining an air space of 1 inch around the conductor, closed at the top (the wires passing through bushed holes), and extending not less than 5 feet from the floor; or by an iron-armored or metal-sheathed insulating conduit sufficiently strong to withstand the strain it will be subjected to; or plain metal pipe lined with insulating tubing, which must extend ½ inch beyond the end of the metal tube.
The pipe must extend not less than 5 feet above the floor, and may extend through the floor in place of a floor bushing.
If iron pipes are used with alternating currents, the two or more wires of a circuit *must* be placed in the same conduit. In this case the insulation of each wire must be reinforced by a tough conduit tubing projecting beyond the ends of the iron pipe at least 2 inches.
(*f*) When run immediately under roofs, or in proximity to water tanks or pipes, will be considered as exposed to moisture.

Special Rules.

For Open Work:

In dry places:

(*g*) Must have an *approved* rubber or "slow-burning," water-proof insulation. (See Nos. 41 and 42.)
(*h*) Must be rigidly supported on non-combustible, non-absorptive insulators, which separate the wires from each other and from the surface wired over in accordance with following table:

Voltage.	Difference from surface.	Distance between wires.
0 to 225	½ inch	2½ inches
225 to 550	1 inch	4 inches

Rigid supporting requires, under ordinary conditions, where wiring along flat surfaces, supports at least every 4½ feet. If the wires are liable to be disturbed, the distance between supports should be shortened. In buildings of mill construction, mains of No. 8 B. & S. wire or over, where not liable to be disturbed, may be separated about 4 inches and run from

timber to timber,—not breaking around,—and may be supported at each timber only.

This rule will not be interpreted to forbid the placing of the neutral of a 3-wire system in the centre of a 3-wire cleat, provided the outside wires are separated in accordance with table on page 714.

In damp places, such as breweries, sugar-houses, packing-houses, stables, dye-houses, paper or pulp mills, or buildings specially liable to moisture or acid or other fumes liable to injure the wires or their insulation, except where used for pendants:

(*i*) Must have an *approved* rubber insulating covering. (See No. 41.)

(*j*) Must be rigidly supported on non-combustible, non-absorptive insulators, which separate the wire at least 1 inch from the surface wired over, and they must be kept apart at least 2½ inches.

Rigid supporting requires, under ordinary conditions, where wiring over flat surfaces, supports at least every 4½ feet. If the wires are liable to be disturbed, the distance between supports should be shortened. In buildings of mill construction, mains of No. 8 B. & S. wire or over, where not liable to be disturbed, may be separated about 4 inches and run from timber to timber,—not breaking around,—and may be supported at each timber only.

(*k*) Must have no joints or splices.

For Molding Work:

(*l*) Must have *approved* rubber insulating covering. (See No. 41.)

(*m*) Must never be placed in molding in concealed or damp places.

For Conduit Work:

(*n*) Must have an *approved* rubber insulating covering. (See No. 41.)

(*o*) Must not be drawn in until all mechanical work on the building has been, as far as possible, completed.

(*p*) Must, for alternating systems, have the two or more wires of a circuit drawn in the same conduit.

It is advised that this be done for direct-current systems also, so that they may be changed to alternating systems at any time, induction troubles preventing such a change unless this construction is followed.

For Concealed "Knob and Tube" Work:

(*q*) Must have an *approved* rubber insulating covering. (See No. 41.)

(*r*) Must be rigidly supported on non-combustible, non-absorptive insulators, which separate the wire at least 1 inch from the surface wired over, and must be kept at least 10 inches apart, and, when possible, should be run singly on separate timbers or studding.

Rigid supporting requires, under ordinary conditions, where wiring along flat surfaces, supports at least every 4½ feet. If the wires are liable to be disturbed, the distance between supports should be shortened.

(*s*) When, from the nature of the case, it is impossible to place concealed wiring on non-combustible insulating supports of glass or porcelain, an *approved* armored cable with single or twin conductors (see No. 48) may be used where the difference of potential between wires is not over 300 volts, provided it is installed without joints between outlets, and the cable armor properly enters all fittings and is rigidly secured in place; or, if the difference of potential between wires is not over 300 volts, and if wires are not exposed to moisture, they may be fished on the loop system if separately incased throughout in approved flexible tubing or conduits.

For Fixture Work:

(*t*) Must have an *approved* rubber insulating covering (see No. 46), and shall not be less in size than No. 18 B. & S.

(*u*) Supply conductors, and especially the splices to fixture wires, must be kept clear of the grounded part of gas-pipes; and, where shells are used, the latter must be constructed in a manner affording sufficient area to allow this requirement.

(*v*) Must, when fixtures are wired outside, be so secured as not to be cut or abraded by the pressure of the fastenings or motion of the fixture.

25. Interior Conduits.

(See, also, Nos. 24, *n* to *p*, and 49.)

The object of a tube or conduit is to facilitate the insertion or extraction of the conductors, to protect them from mechanical injury, and, as far as possible, from moisture. Tubes or conduits are to be considered merely as raceways, and are not to be relied upon for insulation between wire and wire, or between the wire and the ground.

(*a*) No conduit tube having an internal diameter of less than ⅝ inch shall be used. (If conduit is lined, measurement to be taken inside of lining.)

(*b*) Must be continuous from one junction box to another or to fixtures, and the conduit tube must properly enter all fittings.

(*c*) Must be first installed as a complete conduit system, without the conductors.

(*d*) Must be equipped at every outlet with an *approved* outlet box.

(*e*) Metal conduits, where they enter junction boxes, and at all other outlets, etc., must be fitted with a capping of *approved* insulating material, fitted so as to protect wire from abrasion.

(*f*) Must have the metal of the conduit permanently and effectively grounded.

26. Fixtures.

(See, also, No. 24, *t* to *v*.)

(*a*) Must, when supported from the gas-piping of a building, be insulated from the gas-pipe system by means of *approved* insulating joints (see No. 59) placed as close as possible to the ceiling.

It is recommended that the gas outlet pipe be protected above the insulating joint by a non-combustible, non-absorptive insulating tube, having a flange at the lower end where it comes in contact with the insulating joint; and that, where outlet tubes are used, they be of sufficient length to extend below the insulating joint, and that they be so secured that they will not be pushed back when the canopy is put in place. Where iron ceilings are used care must be taken to see that the canopy is thoroughly and permanently insulated from the ceiling.

(*b*) Must have all burs, or fins, removed before the conductors are drawn into the fixture.

(*c*) The tendency to condensation within the pipes should be guarded against by sealing the upper end of the fixture.

(*d*) No combination fixture in which the conductors are concealed in a space less than ¼ inch between the inside pipe and the outside casing will be approved.

(*e*) Must be tested for "contacts" between conductors and fixture, for "short circuits," and for ground connections before it is connected to its supply conductors.

(*f*) Ceiling blocks for fixtures should be made of insulating material; if not, the wires in passing through the plate must be surrounded with non-combustible, non-absorptive insulating material, such as glass or porcelain.

(*g*) Under no conditions shall there be a difference of potential of more than 300 volts between wires contained in or attached to the same fixture.

27. Sockets.

(For Construction Rules, see No. 55.)

(*a*) In rooms where inflammable gases may exist the incandescent lamp and socket must be inclosed in a vapor-tight globe and supported on a pipe hanger, wired with *approved* rubber-covered wire (see No. 41) soldered directly to the circuit.

(*b*) In damp or wet places, or over specially-inflammable stuff, water-proof sockets must be used.

When water-proof sockets are used they should be hung by separate-stranded, rubber-covered wires, not smaller than No. 14 B. & S., which should preferably be twisted together when the drop is over 3 feet. These wires should be soldered direct to the circuit wires, but supported independently of them.

28. Flexible Cord.

(*a*) Must have an *approved* installation and covering. (See No. 45.)
(*b*) Must not be used where the difference of potential between the two wires is over 300 volts.
(*c*) Must not be used as a support for clusters.
(*d*) Must not be used except for pendants, wiring of fixtures, and portable lamps or motors.
(*e*) Must not be used in show windows.
(*f*) Must be protected by insulating bushings where the cord enters the socket.
(*g*) Must be so suspended that the entire weight of the socket and lamp will be borne by knots under the bushing in the socket, and above the point where the cord comes through the ceiling-block or rosette, in order that the strain may be taken from the joints and binding screws.

29. Arc Lights on Low-potential Circuits.

(*a*) Must have a cutout (see No. 17, *a*) for each lamp of each series of lamps.
The branch conductors should have a carrying capacity about 50 per cent. in excess of the normal current required by the lamp, to provide for heavy current required when lamp is started or when carbons become stuck, without overfusing the wires.
(*b*) Must only be furnished with such resistances or regulators as are inclosed in non-combustible material, such resistances being treated as sources of heat. Incandescent lamps must not be used for resistance devices.
(*c*) Must be supplied with globes and protected by spark arresters and wire netting around globe, as in the case of arc lights on high-potential circuits. (See Nos. 19 and 58.)

30. Economy Coils.

(*a*) Economy and compensator coils for arc lamps must be mounted on non-combustible, non-absorptive insulating supports, such as glass or porcelain, allowing an air space of at least 1 inch between frame and support, and in general to be treated like sources of heat.

31. Decorative Series Lamps.

(*a*) Incandescent lamps run in series shall not be used for decorative purposes inside of buildings, except by special permission in writing from the Inspection Department having jurisdiction.

32. Car-wiring.

(*a*) Must be always run out of reach of the passengers, and must have an *approved* rubber insulating covering. (See No. 41.)

33. Car-houses.

(*a*) Must have the trolley wires securely supported on insulating hangers.
(*b*) Must have the trolley hangers placed at such distance apart that in case of a break in the trolley wire contact cannot be made with the floor.
(*c*) Must have cutout switch located at a proper place outside of the building, so that all trolley circuits in the building can be cut out at one point; and line circuit-breakers must be installed, so that when this cutout switch is open the trolley wire will be dead at all points within 100 feet of the building. The current must be cut out of the building whenever the same is not in use or the road not in operation.
(*d*) Must have all lamps and stationary motors installed in such a way that one main switch can control the whole of each installation,—lighting or power,—independently of main feeder-switch. No portable incandescent lamps or twin wire allowed, except that portable incandescent lamps may be used in the pits, connections to be made by two *approved* rubber-

covered, flexible wires (see No. 41), properly protected against mechanical injury; the circuit to be controlled by a switch placed outside of the pit.

(*e*) Must have all wiring and apparatus installed in accordance with rules under Class C for constant-potential systems.

(*f*) Must not have any system of feeder distribution centring in the building.

(*g*) Must have the rails bonded at each joint with no less than No. 2 B. & S. annealed copper wire; also, a supplementary wire to be run for each track.

(*h*) Must not have cars left with trolley in electrical connection with the trolley wire.

34. Lighting and Power from Railway Wires.

(*a*) Must not be permitted, under any pretense, in the same circuit with trolley wires with a ground return, except in electric railway cars, electric car houses, and their power stations; nor shall the same dynamo be used for both purposes.

HIGH-POTENTIAL SYSTEMS.

550 to 3500 Volts.

Any circuit attached to any machine, or combination of machines, which develops a difference of potential between any two wires of over 300 volts and less than 3500 volts, shall be considered as a high-potential circuit, and as coming under that class, unless an approved transforming device is used which cuts the difference of potential down to 300 volts or less.

35. Wires.

(See, also, Nos. 14, 15, and 16.)

(*a*) Must have an *approved* rubber insulating covering. (See No. 41.)

(*b*) Must be always in plain sight and never incased, except where required by the Inspection Department having jurisdiction.

(*c*) Must be rigidly supported on glass or porcelain insulators, which raise the wire at least 1 inch from the surface wired over, and must be kept apart at least 4 inches for voltages up to 750, and at least 8 inches for voltages over 750.

Rigid supporting requires, under ordinary conditions, where wiring along flat surfaces, supports at least about every 4½ feet. If the wires are unusually liable to be disturbed, the distance between supports should be shortened.

In buildings of mill construction, mains of No. 8 B. & S. wire or over, where not liable to be disturbed, may be separated about 6 inches for voltages up to 750, and about 10 inches for voltages above 750, and run from timber to timber,—not breaking around,—and may be supported at each timber only.

(*d*) Must be protected on side walls from mechanical injury by a substantial boxing, retaining an air space of 1 inch around the conductors, closed at the top (the wires passing through bushed holes), and extending not less than 7 feet from the floor. When crossing floor-timbers in cellars or in rooms where they might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than ½ inch in thickness.

36. Transformers (when permitted inside buildings). (See No. 13.)

(For Construction Rules, see No. 62.)

(*a*) Must be located at a point as near as possible to that at which the primary wires enter the building.

(*b*) Must be placed in an inclosure constructed of or lined with fire-resisting material; the inclosure to be used only for this purpose and to be kept securely locked, and access to the same allowed only to responsible persons.

(c) Must be effectually insulated from the ground, and the inclosure in which they are placed must be practically air-tight, except that it shall be thoroughly ventilated to the out-door air, if possible, through a chimney or flue. There should be at least 6 inches air space on all sides of the transformer.

37. Series Lamps.

(a) No system of multiple-series or series-multiple for light or power will be approved.

(b) Under no circumstances can lamps be attached to gas fixtures.

EXTRA HIGH-POTENTIAL SYSTEMS.

Over 3500 Volts.

Any circuit attached to any machine, or combination of machines, which develops a difference of potential between any two wires of over 3500 volts, shall be considered as an extra high-potential circuit, and as coming under that class, unless an approved transforming device is used, which cuts the difference of potential down to 3500 volts or less.

38. Primary Wires.

(a) Must not be brought into or over building, except power and sub-stations.

39. Secondary Wires.

(a) Must be installed under rules for high-potential systems when their immediate primary wires carry a current of over 3500 volts, unless the primary wires are entirely underground within city and village limits.

The presence of wires carrying a current with a potential of over 3500 volts in the streets of cities, towns, and villages is considered to increase the fire hazard. Extra high-potential circuits are also objectionable in any location where telephone, telegraph, and similar circuits run in proximity to them. As the underwriters have no jurisdiction over streets and roads, they can only take this indirect way of discouraging such systems; but further, it is strongly urged that municipal authorities absolutely refuse to grant any franchise for right of way for overhead wires carrying a current of extra high-potential through streets or roads which are used to any great extent for public travel or for trunk-line, telephone, or telegraph circuits.

CLASS D.—FITTINGS, MATERIALS, AND DETAILS OF CONSTRUCTION.

All Systems and Voltages. Insulated Wires, Rules 40 to 48.

40. General Rules.

(a) Copper for insulated conductors must never vary in diameter so as to be more than $\frac{2}{1000}$ inch less than the specified size.

(b) Wires and cables of all kinds designed to meet the following specifications must be plainly tagged or marked as follows:

1. The maximum voltage at which the wire is designed to be used.
2. The words "National Electrical Code Standard."
3. Name of the manufacturing company, and, if desired, trade-name of the wire.
4. Month and year when manufactured.

41. Rubber-covered.

(a) Copper for conductors must be thoroughly tinned.

Insulation for Voltages Between 0 and 600.

(*b*) Must be of rubber or other approved substance, and be of a thickness not less than that given in the following table for B. & S. gauge sizes:

From	18 to	16, inclusive,		$\frac{1}{32}$ inch.
From	14 to	8, inclusive,		$\frac{3}{64}$ inch.
From	7 to	2, inclusive,		$\frac{1}{16}$ inch.
From	1 to	0,000, inclusive,		$\frac{5}{64}$ inch.
From	0,000 to	500,000	C. M.,	$\frac{3}{32}$ inch.
From	500,000 to	1,000,000	C. M.,	$\frac{7}{64}$ inch.
Larger than		1,000,000	C. M.,	$\frac{1}{8}$ inch.

Measurements of insulating wall are to be made at the thinnest portion of the dielectric.

(*c*) The completed coverings must show an insulation resistance of at least 100 megohms per mile during 30 days' immersion in water at 70° F.

(*d*) Each foot of the completed covering must show a dielectric strength sufficient to resist throughout 5 minutes the application of an electromotive force of 3000 volts per $\frac{1}{64}$ inch thickness of insulation under the following conditions:

The source of alternating electro-motive force shall be a transformer of at least 1 kilowatt capacity. The application of the electro-motive force shall first be made at 4000 volts for 5 minutes, and then the voltage increased by steps of not over 3000 volts, each held for 5 minutes, until the rupture of the insulation occurs. The tests for dielectric strength shall be made on a sample of wire which has been immersed for 72 hours in water, 1 foot of which is submerged in a conducting liquid held in a metal trough, one of the transformer terminals being connected to the wire and the other to the metal of the trough.

Insulation for Voltages Between 600 and 3500.

(*e*) The thickness of the insulating walls must not be less than those given in the following table for B. & S. gauge sizes:

From	14 to	1, inclusive,		$\frac{3}{32}$ inch.
From	0 to	500,000	C. M.,	$\frac{3}{32}$ inch, covered by a tape or a braid.
Larger than		500,000	C. M.,	$\frac{4}{32}$ inch, covered by a tape or a braid.

(*f*) The requirements as to insulation and break-down resistance for wires for low-potential systems shall apply, with the exception that an insulation resistance of not less than 300 megohms per mile shall be required.

(*g*) Wire for arc-light circuits exceeding 3500 volts potential shall have an insulating wall not less than $\frac{6}{32}$ inch in thickness, and shall withstand a break-down test of at least 30,000 volts, and have an insulation of at least 500 megohms per mile.

The tests on this wire to be made under the same conditions as for low-potential wires.

Specifications for insulations for alternating currents exceeding 3500 volts have been considered, but on account of the somewhat complex conditions in such work it has so far been deemed inexpedient to specify general insulations for this use.

(*h*) All of the above insulations must be protected by a substantial braided covering, properly saturated with a preservative compound, and sufficiently strong to withstand all the abrasion likely to be met with in practice, and sufficiently elastic to permit all wires smaller than No. 7 B. & S. gauge to be bent around a cylinder with twice the diameter of the wire, without injury to the braid.

42. Slow-burning, Weather-proof.

(*a*) The insulation shall consist of two coatings, the inner one to be fire-proof in character, the outer to be weather-proof. The inner fire-proof coating must comprise at least $\frac{6}{10}$ of the total thickness of the wall. The

completed covering must be of a thickness not less than that given in the following table for B. & S. gauge sizes:

From	14 to	8,	inclusive,	$\frac{3}{64}$ inch.
From	7 to	2,	inclusive,	$\frac{1}{16}$ inch.
From	2 to	0,000,	inclusive,	$\frac{5}{64}$ inch.
From	0,000 to	500,000	C. M.,	$\frac{3}{32}$ inch.
From	500,000 to	1,000,000	C. M.,	$\frac{7}{64}$ inch.
Larger than		1,000,000	C. M.,	$\frac{1}{8}$ inch.

Measurements of insulating wall are to be made at the thinnest portion of the dielectric.

(*b*) The inner fire-proof coating shall be layers of cotton or other thread, the outer one of which must be braided. All the interstices of these layers are to be filled with the fire-proofing compound. This is to be material whose solid constituent is not susceptible to moisture, and which will not burn even when ground in an oxidizable oil, making a compound which, while proof against fire and moisture, at the same time has considerable elasticity, and which, when dry, will suffer no change at a temperature of 250° F., and which will not burn at even higher temperature.

(*c*) The weather-proof coating shall be a stout braid thoroughly saturated with a dense, moisture-proof compound thoroughly slicked down, applied in such manner as to drive any atmospheric moisture from the cotton braiding, thereby securing a covering to a greater degree waterproof and of high insulating power. This compound to retain its elasticity at zero Fahrenheit, and not to drip at 160° F.

This wire is not as burnable as the old "weather-proof," nor as subject to softening under heat, but still is able to repel the ordinary amount of moisture found in-doors. It would not usually be used for outside work.

43. Slow-burning.

(*a*) The insulation shall be the same as the "slow-burning, weather-proof," except that the outer braiding shall be impregnated with a fire-proofing compound similar to that required for the interior layers, and with the outer surface finished smooth and hard.

This "slow-burning" wire shall only be used with special permission of the Inspection Department having jurisdiction.

This is practically the old "Underwriters'" insulation. It is specially useful in hot, dry places, where ordinary insulations would perish; also where wires are bunched, as on the back of a large switchboard or in a wire tower, so that the accumulation of rubber or weather-proof insulation would result in an objectionably large mass of highly-inflammable material.

Its use is restricted, as its insulating qualities are not high and are damaged by moisture.

44. Weather-proof.

(*a*) The insulating covering shall consist of at least 3 braids thoroughly impregnated with a dense moisture repellent, which will not drip at a temperature lower than 180° F. The thickness of insulation shall be not less than that of "slow-burning, weather-proof." The outer surface shall be thoroughly slicked down.

This wire is for out-door use, where moisture is certain and where fire-proof qualities are not necessary.

45. Flexible Cord.

(*a*) Must be made of stranded copper conductors, each strand to be not larger than No. 26 or smaller than No. 30 B. & S. gauge, and each stranded conductor must be covered by an approved insulation and protected from mechanical injury by a tough, braided outer covering.

For Pendent Lamps:

In this class is to be included all flexible cord which under usual conditions hangs freely in air, and which is not likely to be moved sufficiently to come in contact with surrounding objects.

(*b*) Each stranded conductor must have a carrying capacity equivalent to not less than a No. 18 B. & S. gauge wire.

(*c*) The covering of each stranded conductor must be made up as follows:

1. A tight, close wind of fine cotton.
2. The insulation proper, which shall be either water-proof or slow-burning.
3. An outer cover of silk or cotton.

The wind of cotton tends to prevent a broken strand puncturing the insulation and causing a short circuit. It also keeps the rubber from corroding the copper.

(*d*) Water-proof insulation must be solid, at least $\frac{1}{32}$ inch thick, and must show an insulation resistance of 50 megohms per mile throughout 2 weeks' immersion in water at 70° F., and stand the test prescribed for low-tension wires as far as they apply.

(*e*) Slow-burning insulation must be at least $\frac{1}{32}$ inch in thickness, and composed of substantial, elastic, slow-burning materials, which will suffer no damage at a temperature of 250° F.

(*f*) The outer protecting braiding should be so put on and sealed in place that when cut it will not fray out, and where cotton is used it should be impregnated with a flame-proof paint, which will not have an injurious effect on the insulation.

For Portables:

In this class is included all cord used on portable lamps, small portable motors, etc.

(*g*) Flexible cord for portable use must have water-proof insulation, as required in Section *d* for pendent cord, and in addition be provided with a reinforcing cover especially designed to withstand the abrasion it will be subject to in the uses to which it is to be put.

For Portable Heating Apparatus:

(*h*) Must be made up as follows:

1. A tight, close wind of fine cotton.
2. A thin layer of rubber about $\frac{1}{100}$ inch thick, or other cementing material.
3. A layer of asbestos insulation at least $\frac{3}{64}$ inch thick.
4. A stout braid of cotton.
5. An outer reinforcing cover especially designed to withstand abrasion.

This cord is in no sense water-proof, the thin layer of rubber being specified in order that it may serve merely as a seal to help hold in place the fine cotton and asbestos, and it should be so put on as to accomplish this.

46. Fixture Wire.

(*a*) Must have a solid insulation, with a slow-burning, tough, outer covering, the whole to be $\frac{1}{32}$ inch in thickness, and show an insulation resistance between conductors, and between either conductor and the ground, of at least 1 megohm per mile, after 1 week's submersion in water at 70° F., and after 3 minutes' electrification with 550 volts.

47. Conduit Wire.

Must comply with the following specifications:

(*a*) For metal conduits, having a lining of insulating material, single wires must comply with No. 41, and all duplex, twin, and concentric conductors must comply with No. 41, and must also have each conductor separately braided or taped, and a substantial braid covering the whole.

(*b*) For unlined metal conduits, conductors must conform to the specifications given for lined conduits, and in addition have a second outer fibrous covering at least $\frac{1}{32}$ inch in thickness, and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit.

The braid required around each conductor in duplex, twin, and concentric cables is to hold the rubber insulation in place and prevent jamming and flattening.

48. Armored Cable.

(*a*) The armor of such cables must be at least equal in thickness and of equal strength to resist penetration by nails, etc., as the armor of metal covering of metal conduits. (See No. 49, *b*.)

(*b*) The conductors in same—single wire or twin conductors—must have an insulating covering as required by No. 41, any filler used to secure a round exterior must be impregnated with a moisture repellent, and the whole bunch of conductors and fillers must have a separate exterior covering of insulating material at least $\frac{1}{32}$ inch in thickness, conforming to the insulation standard given in No. 41, and covered with a substantial braid.

Very reliable insulation is specified, as such cables are liable to hard usage, and in part of their length may be subject to moisture, while they may not be easily removable, so that a breakdown of insulation is likely to be expensive.

49. Interior Conduits.

(For Wiring Rules, see Nos. 24 and 25.)

(*a*) Each length of conduit, whether insulated or uninsulated, must have the maker's name or initials stamped in the metal or attached thereto in a satisfactory manner, so that the inspectors can readily see the same.

Metal Conduits with Lining of Insulating Material.

(*b*) The metal covering or pipe must be equal in strength to the ordinary commercial forms of gas-pipe of the same size, and its thickness must be not less than that of standard gas-pipe, as shown by the following table:

Size, in inches.	Thickness of wall, in inches.	Size, in inches.	Thickness of wall, in inches.
½	.109	1¼	.140
⅝	.111	1½	.145
¾	.113	2	.154
1	.134		

An allowance of $\frac{2}{100}$ inch for variation in manufacturing and loss of thickness by cleaning will be permitted.

(*c*) Must not be seriously affected externally by burning out a wire inside the tube when the iron pipe is connected to one side of the circuit.

(*d*) Must have the insulating lining firmly secured to the pipe.

(*e*) The insulating lining must not crack or break when a length of the conduit is uniformly bent at temperature of 212° F. to an angle of 90°, with a curve having a radius of 15 inches for pipes of 1 inch and less, and 15 times the diameter of pipe for larger pipes.

(*f*) The insulating lining must not soften injuriously at a temperature below 212° F., and must leave water in which it is boiled practically neutral.

(*g*) The insulating lining must be at least $\frac{1}{32}$ inch in thickness; and the materials of which it is composed must be of such a nature as will not have a deteriorating effect on the insulation of the conductor, and be sufficiently tough and tenacious to withstand the abrasion test of drawing long lengths of conductors in and out of same.

(*h*) The insulating lining must not be mechanically weak after 3 days' submersion in water, and when removed from the pipe entire must not absorb more than 10 per cent. of its weight of water during 100 hours of submersion.

(*i*) All elbows or bends must be so made that the conduit or lining of same will not be injured. The radius of the curve of the inner edge of any elbow not to be less than 3½ inches. Must have not more than the equivalent of 4 quarter-bends from outlet to outlet, the bends at the outlets not being counted.

Unlined Metal Conduits.

(*j*) Plain iron or steel pipes of equal thickness and strengths specified for lined conduits in No. 49, *b*, may be used as conduits, provided their in-

terior surfaces are smooth and free from burs; pipe to be galvanized, or the interior surfaces coated or enamelled to prevent oxidation with some substance which will not soften, so as to become sticky and prevent wire from being withdrawn from the pipe.

(*k*) All elbows or bends must be so made that the conduit will not be injured. The radius of the curve of the inner edge of any elbow not to be less than 3½ inches. Must have not more than the equivalent of 4 quarter-bends from outlet to outlet, the bends at the outlet not being counted.

50. Wooden Moldings.

(For Wiring Rules, see No. 24.)

(*a*) Must have, both outside and inside, at least two coats of water-proof paint, or be impregnated with a moisture repellent.

(*b*) Must be made of two pieces, a backing and capping, so constructed as to thoroughly incase the wire, and provide a ½-inch tongue between the conductors, and a solid backing, which, under grooves, shall not be less than ⅜ inch in thickness, and must afford suitable protection from abrasion.

It is recommended that only hardwood molding be used.

51. Switches.

(See Nos. 17 and 22.)

(*a*) Must be mounted on non-combustible, non-absorptive insulating bases, such as slate or porcelain.

(*b*) Must have carrying capacity sufficient to prevent undue heating.

(*c*) Must, when used for service switches, indicate, on inspection, whether the current be "on" or "off."

(*d*) Must be plainly marked, where it will always be visible, with the name of the maker and the current and voltage for which the switch is designed.

(*e*) Must, for constant-potential systems, operate successfully at 50 per cent. overload in amperes, with 25 per cent. excess voltage under the most severe conditions they are liable to meet with in practice.

(*f*) Must, for constant-potential systems, have a firm and secure contact; must make and break readily, and not stop when motion has once been imparted by the handle.

(*g*) Must, for constant-current systems, close the main circuit and disconnect the branch wires when turned "off;" must be so constructed that they shall be automatic in action, not stopping between points when started, and must prevent an arc between the points under all circumstances. They must indicate, upon inspection, whether the current be "on" or "off."

52. Cutouts and Circuit-breakers.

(For Installation Rules, see Nos. 17 and 21.)

(*a*) Must be supported on bases of non-combustible, non-absorptive insulating material.

(*b*) Cutouts must be provided with covers, when not arranged in approved cabinets, so as to obviate any danger of the melted fuse metal coming in contact with any substance which might be ignited thereby.

(*c*) Cutouts must operate successfully, under the most severe conditions they are liable to meet with in practice, on short circuits with fuses rated at 50 per cent. above, and with a voltage 25 per cent. above the current and voltage for which they are designed.

(*d*) Circuit-breakers must operate successfully, under the most severe conditions they are liable to meet with in practice, on short circuits when set at 50 per cent. above the current, and with a voltage 25 per cent. above that for which they are designed.

(*e*) Must be plainly marked, where it will always be visible, with the name of the maker and current and voltage for which the device is designed.

53. Fuses.

(For Installation Rules, see Nos. 17 and 21.)

(*a*) Must have contact surfaces or tips of harder metal having perfect electrical connection with the fusible part of the strip.

(*b*) Must be stamped with about 80 per cent. of the maximum current they can carry indefinitely, thus allowing about 25 per cent. overload before fuse melts.

With naked, open fuses of ordinary shapes and not over 500 amperes capacity, the *maximum* current which will melt them in about 5 minutes may be safely taken as the melting-point, as the fuse practically reaches its maximum temperature in this time. With larger fuses a longer time is necessary.

Inclosed fuses, where the fuse is often in contact with substances having good conductivity to heat and often of considerable volume, require a much longer time to reach a maximum temperature, on account of the surrounding material, which heats up slowly.

These data are given to facilitate testing.

(*c*) Fuse terminals must be stamped with the maker's name, initials, or some known trade-mark.

54. Cutout Cabinets.

(*a*) Must be so constructed, and cutouts so arranged, as to obviate any danger of the melted fuse metal coming in contact with any substance which might be ignited thereby.

A suitable box can be made of marble, slate, or wood, strongly put together, the door to close against a rabbet, so as to be perfectly dust-tight; and it should be hung on strong hinges and held closed by a strong hook or catch. If the box is wood, the inside should be lined with sheets of asbestos-board about $\frac{1}{16}$ inch in thickness, neatly put on and firmly secured in place by shellac and tacks. The wire should enter through holes bushed with porcelain bushings; the bushings tightly fitting the holes in the box, and the wires tightly fitting the bushings (using tape to build up the wire, if necessary), so as to keep out the dust.

55. Sockets.

(See No. 27.)

Sockets of all kinds, including wall receptacles, must be constructed in accordance with the following specifications:

(*a*) **Standard Sizes.**—The standard lamp socket shall be suitable for use on any voltage not exceeding 250, and with any size lamp up to 50 candle-power. For lamps larger than 50 candle-power a standard keyless socket may be used, or, if a key is required, a special socket designed for the current to be used must be made. Any special sockets must follow the general spirit of these specifications.

(*b*) **Marking.**—The standard socket must be plainly marked 50 candle-power, 250 volts, and with either the manufacturer's name or registered trade-mark. Special large sockets must be marked with the current and voltage for which they are designed.

(*c*) **Shell.**—Metal used for shells must be moderately hard, but not hard enough to be brittle or so soft as to be easily dented or knocked out of place. Brass shells must be at least 0.013 inch in thickness, and shells of any other material must be thick enough to give the same stiffness and strength of brass.

(*d*) **Lining.**—The inside of the shells must be lined with insulating material, which shall absolutely prevent the shell from becoming a part of the circuit, even though the wires inside the socket should start from their position under binding screws.

The material used for lining must be at least $\frac{1}{32}$ inch in thickness, and must be tough and tenacious. It must not be injuriously affected by the heat from the largest lamp permitted in the socket, and must leave the water in which it is boiled practically neutral. It must be so firmly secured to the shell that it will not fall out with ordinary handling of the socket. It is preferable to have the lining in one piece.

(*e*) **Cap.**—Caps, when of sheet brass, must be at least 0.013 inch in thickness, and when cast or made of other metals must be of equivalent strength. The inlet piece, except for special sockets, must be tapped and threaded for ordinary ⅛-inch pipe. It must contain sufficient metal for a full, strong thread, and, when not of the same piece as the cap, must be joined to it in a way to give the strength of a single piece.

There must be sufficient room in the cap to enable the ordinary wireman to easily and quickly make a knot in the cord, and push it into place in cap without crowding. All parts of the cap upon which the knot is likely to bear must be smooth and well insulated.

(*f*) **Frame and Screws.**—The frame holding moving parts must be sufficiently heavy to give ample strength and stiffness.

Brass pieces containing screw threads must be at least 0.06 of an inch in thickness.

Binding-post screws must not be smaller than No. 5 wire, and about 40 threads per inch.

(*g*) **Spacing.**—Points of opposite polarity must everywhere be kept not less than $\frac{3}{64}$ inch apart unless separated by a reliable insulation.

(*h*) **Connections.**—The connecting points for the flexible cord must be made to very securely grip a No. 16 or 18 B. & S. conductor. A turned-up lug, arranged so that the cord may be gripped between the screw and the lug in such a way that it cannot possibly come out, is strongly advised.

(*i*) **Lamp-holder.**—The socket must firmly hold the lamp in place, so that it cannot be easily jarred out, and must provide a contact good enough to prevent undue heating with maximum current allowed. The holding-pieces, springs, and the like, if a part of the circuit, must not be sufficiently exposed to allow them to be brought in contact with anything outside of lamp and socket.

(*j*) **Base.**—The inside parts of the socket, which are of insulating material (except the lining), must be made of porcelain.

(*k*) **Key.**—The socket key-handle must be of such a material that it will not soften from the heat of a 50 candle-power lamp hanging downward, in air at 70° F., from the socket, and must be securely, but not necessarily rigidly, attached to the metal spindle it is designed to turn.

(*l*) **Sealing.**—All screws in porcelain pieces, which can be firmly sealed in place, must be so sealed by a water-proof compound which will not melt below 200° F.

(*m*) **Putting Together.**—The socket must, as a whole, be so put together that it will not rattle to pieces. Bayonet joints or equivalent are recommended.

(*n*) **Test.**—The socket, when slowly turned "on and off" at the rate of about 2 or 3 times per minute, must "make and break" the circuit 6000 times before failing when carrying a load of 1 ampere at 220 volts.

(*o*) **Keyless Sockets.**—Keyless sockets of all kinds must comply with requirements for key sockets as far as they apply.

(*p*) **Sockets of Insulating Materials.**—Sockets made of porcelain or other insulating material must conform to the above requirements as far as they apply, and all parts must be strong enough to withstand a moderate amount of hard usage without breaking.

(*q*) **Inlet Bushing.**—When the socket is not attached to fixtures the threaded inlet must be provided with a strong insulating bushing having a *smooth* hole of at least $\frac{15}{64}$ inch in diameter. The corners of the bushing must be rounded and all inside fins removed, so that in no place will the cord be subjected to the cutting or wearing action of a sharp edge.

56. Hanger-boards.

(*a*) Hanger-boards must be so constructed that all wires and current-carrying devices thereon shall be exposed to view and thoroughly insulated by being mounted on a non-combustible, non-absorptive insulating substance. All switches attached to the same must be so constructed that they shall be automatic in their action, cutting off both poles to the lamp,

not stopping between points when started, and preventing an arc between points under all circumstances.

57. Arc Lamps.

(For Installation Rules, see No. 19.)

(*a*) Must be provided with reliable stops to prevent carbons from falling out in case the clamps become loose.

(*b*) Must be carefully insulated from the circuit in all their exposed parts.

(*c*) Must, for constant-current systems, be provided with an *approved* hand switch, also an automatic switch that will shunt the current around the carbons should they fail to feed properly.

The hand switch to be approved, if placed anywhere except on the lamp itself, must comply with requirements for switches on hanger-boards, as laid down in No. 56.

58. Spark Arresters.

(See No. 19, *c*.)

(*a*) Spark arresters must so close the upper orifice of the globe that it will be impossible for any sparks thrown off by the carbons to escape.

59. Insulating Joints.

(See No. 26, *a*.)

(*a*) Must be entirely made of material that will resist the action of illuminating gases and will not give way or soften under the heat of an ordinary gas-flame or leak under a moderate pressure. They shall be so arranged that a deposit of moisture will not destroy the insulating effect, and shall have an insulating resistance of at least 250,000 ohms between the gas-pipe attachments, and be sufficiently strong to resist the strain they will be liable to be subjected to in being installed.

(*b*) Insulating joints having soft rubber in their construction will not be approved.

60. Resistance Boxes and Equalizers.

(For Installation Rules, see No. 4.)

(*a*) Must be equipped with metal or with other non-combustible frames.

The word "frame" in this section relates to the entire case and surroundings of the rheostat, and not alone to the upholding supports.

61. Reactive Coils and Condensers.

(*a*) Reactive coils must be made of non-combustible material, mounted on non-combustible bases, and treated, in general, like sources of heat.

(*b*) Condensers must be treated like apparatus operating with equivalent voltage and currents. They must have non-combustible cases and supports, and must be isolated from all combustible materials, and, in general, treated like sources of heat.

62. Transformers.

(For Installation Rules, see Nos. 11, 13, and 33.)

(*a*) Must not be placed in any but metallic or other non-combustible cases.

(*b*) Must be constructed to comply with the following tests:

1. Shall be run for 8 consecutive hours at a full load in watts under conditions of service, and at the end of that time the rise in temperature, as measured by the increase of resistance of the primary coil, shall not exceed 135° F.

2. The insulation of transformers, when heated, shall withstand continuously for 5 minutes a difference of potential of 10,000 volts (alter-

nating) between primary and secondary coils and core, and between the primary coils and core and a no-load "run" at double voltage for 30 minutes.

63. Lightning Arresters.

(For Installation Rules, see No. 5.)

(*a*) Must be mounted on non-combustible bases, and must be so constructed as not to maintain an arc after the discharge has passed, and must have no moving parts.

CLASS E.—MISCELLANEOUS.

64. Signalling Systems (governing wiring for telephone, telegraph, district messenger, and call-bell circuits, fire and burglar alarms, and all similar systems).

(*a*) Outside wires should be run in underground ducts or strung on poles, and, as far as possible, kept off of buildings, and must not be placed on the same cross-arm with electric light or power wires.

(*b*) When outside wires are run on same pole with electric light or power wires the distance between the two inside pins of each cross-arm must not be less than 26 inches.

(*c*) All aerial conductors and underground conductors which are directly connected to aerial wires must be provided with some approved protective device, which shall be located as near their point of entrance to the building as possible, and not less than 6 inches from curtains or other inflammable material.

(*d*) If the protector is placed inside of building, wires—from outside supports to binding-posts of protector—shall comply with the following requirements:

1. Must be of copper, and not smaller than No. 16 B. & S. gauge.
2. Must have an *approved* rubber insulating covering. (See No. 41.)
3. Must have drip loops in each wire immediately outside the building.
4. Must enter buildings through separate holes sloping upward from the outside. When practicable, holes to be bushed with non-absorptive, non-combustible insulating tubes extending through their entire length. Where tubing is not practicable, the wires shall be wrapped with two layers of insulating tape.
5. Must be supported on porcelain insulators, so that they will not come in contact with anything other than their designed supports.
6. A separation between wires of at least 2½ inches must be maintained.

In case of crosses, these wires may become a part of a high-voltage circuit, so that similar care to that given high-voltage circuits is needed in placing them. Reliable porcelain bushings at the entrance holes are desirable, and are only waived under adverse conditions, because the state of the art in this type of wiring makes an absolute requirement inadvisable.

(*e*) The ground wire of the protective device shall be run in accordance with the following requirements:

1. Shall be of copper, and not smaller than No. 16 B. & S.
2. Must have an *approved* rubber insulating covering. (See No. 41.)
3. Shall run in as straight a line as possible to a good, permanent ground, to be made by connecting to water- or gas-pipe, preferably water-pipe. If gas-pipe is used, the connection, in all cases, must be made between the meter and service pipes. In the absence of other good ground, the ground shall be made by means of a metallic plate or bunch of wires buried in permanently moist earth.
4. Shall be kept at least 3 inches from all other conductors, and supported on porcelain insulators, so as not to come in contact with anything other than its designated supports.

In attaching a ground wire to a pipe it is often difficult to make a thoroughly-reliable solder joint. It is better, therefore, where possible, to carefully solder the wire to a brass plug, which may then be firmly screwed into a pipe fitting.

Where such joints are made underground they should be thoroughly painted and taped to prevent corrosion.

(*f*) The protector, to be approved, must comply with the following requirements:

1. Must be mounted on non-combustible, non-absorptive insulating bases, so designed that when the protector is in place all parts which may be alive will be thoroughly insulated from the wall holding the protector.

2. Must have the following parts:

A lightning arrester which will operate with a difference of potential between wires of not over 500 volts, and so arranged that the chance of accidental grounding is reduced to a minimum.

A fuse designed to open the circuit in case the wires become crossed with light or power circuits. The fuse must be able to open the circuit without arcing or serious flashing when crossed with any ordinary commercial light or power circuit.

A heat coil which will operate before a sneak current can damage the instrument the protector is guarding.

The heat coil is designed to warm up and melt out with a current large enough to endanger the instruments, if continued for a long time, but so small that it would not blow the fuses ordinarily found necessary for such instruments. These smaller currents are often called "sneak" currents.

3. The fuses must be so placed as to protect the arrester and heat coils, and the protector terminals must be plainly marked "line," "instrument," "ground."

(*g*) Wires beyond the protector, except where bunched, must be neatly arranged and securely fastened in place in any convenient, workmanlike manner. They must not come nearer than 6 inches to any electric light or power wire in the building, unless incased in approved tubing so secured as to prevent its slipping out of place.

The wires would ordinarily be insulated, but the kind of insulation is not specified, as the protector is relied upon to stop all dangerous currents. Porcelain tubing or circular loom conduit may be used for incasing wires where required as above.

(*h*) Wires connected with outside circuits, where bunched together within any building, or inside wires, where laid in conduits or ducts with electric light or power wires, must have fire-resisting coverings, or else must be inclosed in an air-tight tube or duct.

It is feared that if a burnable insulation were used a chance spark might ignite it and cause a serious fire, for many installations contain a large amount of very readily-burnable matter.

65. Electric Gas-lighting.

Where electric gas-lighting is to be used on the same fixture with the electric light:

(*a*) No part of the gas-piping or fixture shall be in electric connection with the gas-lighting circuit.

(*b*) The wires used with the fixtures must have a non-inflammable insulation, or, where concealed between the pipe and shell of the fixture, the insulation must be such as required for fixture wiring for the electric light.

(*c*) The whole installation must test free from "grounds."

(*d*) The two installations must test perfectly free from connection with each other.

66. Insulation Resistance.

The wiring in any building must test free from grounds,—*i.e.*, the complete installation must have an insulation between conductors and between all conductors and the ground (not including attachments, sockets, receptacles, etc.) of not less than the following:

Up to 5 amperes	4,000,000 ohms.
Up to 10 amperes	2,000,000 ohms.
Up to 25 amperes	800,000 ohms.
Up to 50 amperes	400,000 ohms.
Up to 100 amperes	200,000 ohms.
Up to 200 amperes	100,000 ohms.
Up to 400 amperes	25,000 ohms.
Up to 800 amperes	25,000 ohms.
Up to 1600 amperes	12,500 ohms.

All cutouts and safety devices in place in the preceding.

Where lamp sockets, receptacles, and electroliers, etc., are connected, one-half of the preceding insulation will be required.

67. Soldering Fluid.

(*a*) The following formula for soldering fluid is suggested:

Saturated solution of zinc chloride.................	5 parts.
Alcohol..	4 parts.
Glycerine.....................................	1 part.

CLASS F.—MARINE WORK.

68. Generators.

(*a*) Must be located in a dry place.

(*b*) Must have their frames insulated from their bed-plates.

(*c*) Must each be provided with a water-proof cover.

(*d*) Must each be provided with a name-plate, giving the maker's name, the capacity in voltage and amperes, and normal speed in revolutions per minute.

69. Wires.

(*a*) Must have an *approved* insulating covering.

The insulation for all conductors, except for portables, to be approved, must be at least ⅛ inch in thickness and be covered with a substantial water-proof and flame-proof braid. The physical characteristics shall not be affected by any change in temperature up to 200° F. After 2 weeks' submersion in salt water at 70° F. it must show an insulation resistance of 1 megohm per mile after 3 minutes' electrification with 550 volts.

(*b*) Must have no single wire larger than No. 12 B. & S. Wires to be stranded when greater carrying capacity is required. No single solid wire smaller than No. 14 B. & S., except in fixture wiring, to be used.

Stranded wires must be soldered before being fastened under clamps or binding screws, and when they have a conductivity greater than No. 10 B. & S. copper wire they must be soldered into lugs.

(*c*) Must be supported in approved molding, except at switchboards and portables,

Special permission may be given for deviation from this rule in dynamo-rooms.

(*d*) Must be bushed with hard-rubber tubing ⅛ inch in thickness when passing through beams and non-water-tight bulkheads.

(*e*) Must have, when passing through water-tight bulkheads and through all decks, a metallic stuffing-tube lined with hard rubber. In case of deck tubes they shall be boxed near deck to prevent mechanical injury.

(*f*) Splices or taps in conductors must be avoided as far as possible. Where it is necessary to make them they must be so spliced or joined as to be both mechanically and electrically secure without solder. They must then be soldered to insure preservation, covered with an insulating compound equal to the insulation of the wire, and further protected by a water-proof tape. The joint must then be coated or painted with a water-proof compound.

70. Portable Conductors.

(*a*) Must be made of two stranded conductors, each having a carrying capacity equivalent to not less than No. 14 B. & S. wire, and each covered with an approved insulation and covering.

Where not exposed to moisture or severe mechanical injury, each stranded conductor must have a solid insulation at least $\frac{1}{32}$ inch in thickness, and must show an insulation resistance between conductors, and between either conductor and the ground, of at least 1 megohm per mile after 1 week's submersion in water at 70° F. and after 3 minutes' electrification with 590 volts, and be protected by a slow-burning, tough-braided, outer covering.

Where exposed to moisture and mechanical injury—as for use on decks, holds, and fire-rooms—each stranded conductor shall have a solid insulation, to be approved, of at least $\frac{1}{32}$ inch in thickness and protected by a tough braid. The two conductors shall then be stranded together, using a jute filling. The whole shall then be covered with a layer of flax, either woven or braided, at least $\frac{1}{32}$ inch in thickness, and treated with a non-inflammable, water-proof compound. After 1 week's submersion in water at 70° F., at 550 volts and a 3 minutes' electrification, must show an insulation between the two conductors, or between either conductor and the ground, of 1 megohm per mile.

71. Bell or Other Wires.

(*a*) Shall never run in same duct with lighting or power wires

72. Table of Capacity of Wires.

B. & S. G.	Area, actual C. M.	Number of strands.	Size of strands, B. & S. G.	Amperes.
19	1 288	....	....	....
18	1 624	....	....	3
17	2 048	....	....	....
16	2 583	....	....	6
15	3 257	....	....	....
14	4 107	....	....	12
12	6 530	....	....	17
....	9 016	7	19	21
....	11 368	7	18	25
....	14 336	7	17	30
....	18 081	7	16	35
....	22 799	7	15	40
....	30 856	19	18	50
....	38 912	19	17	60
....	49 077	19	16	70
....	60 088	37	18	85
....	75 776	37	17	100
....	99 064	61	18	120
....	124 928	61	17	145
....	157 563	61	16	170
....	198 677	61	15	200
....	250 527	61	14	235
....	296 387	91	15	270
....	373 737	91	14	320
....	413 639	127	15	340

When greater conducting area than that of a single wire is required, the conductor shall be stranded in a series of 7, 19, 37, 61, 91, or 127 wires, as may be required, the strand consisting of 1 central wire, the remainder laid around it concentrically, each layer to be twisted in the opposite direction from the preceding.

73. Switchboards.

(*a*) Must be made of non-combustible, non-absorptive insulating material, such as marble or slate.

(*b*) Must be kept free from moisture, and must be located so as to be accessible from all sides.

(*c*) Must have a main switch, main cutout, and ammeter for each generator.

Must also have a voltmeter and ground detector.

(*d*) Must have a cutout and switch for each side of each circuit leading from board.

74. Resistance Boxes.

(*a*) Must be made of non-combustible material.

(*b*) Must be located on switchboard or away from combustible material. When not placed on switchboard they must be mounted on non-inflammable, non-absorptive insulating material.

(*c*) Must be so constructed as to allow sufficient ventilation for the uses to which they are put.

75. Switches.

(*a*) Must have non-combustible, non-absorptive insulating bases.

(*b*) Must operate successfully, at 50 per cent. overload in amperes with 25 per cent. excess voltage, under the most severe conditions they are liable to meet with in practice, and must be plainly marked, where they will always be visible, with the name of the maker and the current and voltage for which the switch is designed.

(*c*) Must be double pole when circuits which they control supply more than six 16-candle-power lamps or their equivalent.

(*d*) When exposed to dampness they must be inclosed in a water-tight case.

76. Cutouts.

(*a*) Must have non-combustible, non-absorptive insulating bases.

(*b*) Must operate successfully, under the most severe conditions they are liable to meet with in practice, on short circuit, with fuse rated at 50 per cent. above and with a voltage 25 per cent. above the current and voltage they are designed for, and must be plainly marked, where they will always be visible, with the name of the maker and current and voltage for which the device is designed.

(*c*) Must be placed at every point where a change is made in the size of the wire (unless the cutout in the larger wire will protect the smaller).

(*d*) In places such as upper decks, holds, cargo spaces, and fire-rooms a water-tight and fire-proof cutout may be used, connecting directly to mains when such cutout supplies circuits requiring not more than 660 watts energy.

(*e*) When placed anywhere except on switchboards and certain places, as cargo spaces, holds, fire-rooms, etc., where it is impossible to run from centre of distribution, they shall be in a cabinet lined with fire-resisting material.

(*f*) Except for motors, searchlights, and diving-lamps, shall be so placed that no group of lamps requiring a current of more than 6 amperes shall ultimately be dependent upon 1 cutout.

A single-pole, covered cutout may be placed in the molding when same contains conductor supplying circuits requiring not more than 220 watts energy.

77. Fixtures.

(*a*) Shall be mounted on blocks made from well-seasoned lumber treated with two coats of white lead or shellac.

(*b*) Where exposed to dampness the lamp must be surrounded by a vapor-proof globe.

(*c*) Where exposed to mechanical injury the lamp must be surrounded by a globe protected by a stout wire guard.

(*d*) Shall be wired with same grade of insulation as portable conductors which are not exposed to moisture or mechanical injury.

78. Sockets.

(*a*) No portion of the lamp socket or lamp base exposed to contact with outside objects shall be allowed to come into electrical contact with either of the conductors.

79. Wooden Moldings.

(*a*) Must be made of well-seasoned lumber and be treated inside and out with at least two coats of white lead or shellac.

(*b*) Must be made of two pieces, a backing and a capping, so constructed as to thoroughly incase the wire and provide a ½-inch tongue between the conductors, and a solid backing which, under grooves, shall not be less than ⅜ inch in thickness.

(*c*) Where molding is run over rivets, beams, etc., a backing strip must first be put up and the molding secured to this.

(*d*) Capping must be secured by brass screws.

80. Motors.

(*a*) Must be wired under the same precautions as with a current of same volume and potential for lighting. The motor and resistance box must be protected by a double-pole cutout and controlled by a double-pole switch, except in cases where ¼ horse-power or less is used.

The leads or branch circuits should be designed to carry a current at least 50 per cent. greater than that required by the rated capacity of the motor, to provide for the inevitable overloading of the motor at times.

(*b*) Must be thoroughly insulated. Where possible, should be set on base frames made from filled, hard, dry wood, and raised above surrounding deck. On hoists and winches they shall be insulated from bed-plates by hard rubber, fibre, or similar insulating material.

(*c*) Shall be covered with a water-proof cover when not in use.

(*d*) Must each be provided with a name-plate giving maker's name, the capacity in volts and amperes, and the normal speed, in revolutions, per minute.

GENERAL SUGGESTIONS.

In all electric work, conductors, however well insulated, should always be treated as bare, to the end that under no conditions, existing or likely to exist, can a grounding or short circuit occur, and so that all leakage from conductor to conductor, or between conductor and ground, may be reduced to the minimum.

In all wiring special attention must be paid to the mechanical execution of the work. Careful and neat running, connecting, soldering, taping of conductors, and securing and attaching of fittings are specially conducive to security and efficiency, and will be strongly insisted on.

In laying out an installation, except for constant-current systems, the work should, if possible, be started from a centre of distribution, and the switches and cutouts controlling and connected with the several branches be grouped together in a safe and easily-accessible place, where they can be readily got at for attention or repairs. The load should be divided as evenly as possible among the branches, and all complicated and unnecessary wiring avoided.

The use of wire-ways for rendering concealed wiring permanently accessible is most heartily endorsed and recommended; and this method of accessible, concealed construction is advised for general use.

Architects are urged, when drawing plans and specifications, to make provision for the channelling and pocketing of buildings for electric light or power wires, and in specifications for electric gas-lighting to require a 2-wire circuit, whether the building is to be wired for electric lighting or not, so that no part of the gas fixtures or gas-piping be allowed to be used for the gas-lighting circuit.

General Wiring Formulas.

(General Electric Company.)

The following general formulas may be used to determine the size of copper conductors, volts loss in lines, current per conductor, and the weight of copper per circuit for any system of electrical distribution:

$$\text{Area of conductor, circular mils.} = \frac{D \times W \times C}{P \times E^2};$$

$$\text{Current in main conductors} = \frac{W \times T}{E};$$

$$\text{Volts loss in line} = \frac{P \times E \times B}{100};$$

$$\text{Pounds of copper} = \frac{D^2 \times W \times C \times A}{P \times E^2 \times 1{,}000{,}000}.$$

Where

W = total watts delivered;
D = distance of transmission (one way), in feet;
P = loss in line, in per cent., of power delivered,—that is, of W,
E = voltage between main conductors at receiving or consumer's end of circuit.

For continuous current $C = 2160$, $T = 1$, $B = 1$, and $A = 6.04$.

Values of *A*, *C*, and *T*.

System.	Values of *A*.	Values of *C*. Percentage of power factor.					Values of *T*. Percentage of power factor.				
		100	95	90	85	80	100	95	90	85	80
Single phase	6.04	2160	2400	2660	3000	3380	1.00	1.05	1.11	1.17	1.25
Two-phase (4-wire) ...	12.08	1080	1200	1330	1500	1690	.50	.53	.55	.59	.62
Three-phase (3-wire) ..	9.06	1080	1200	1330	1500	1690	.58	.61	.64	.68	.72

The following formula will be found a convenient one for calculating the copper required for long-distance, three-phase transmission circuits.

$$\text{Pounds of copper} = \frac{M^2 \times Kw. \times 300{,}000{,}000}{P \times E^2}.$$

M = distance of transmission, in miles;
$Kw.$ = the power delivered, in kilowatts.

Power factor is assumed to be approximately 95 per cent.

Application of Formulas.

The value of C for any particular power factor is obtained by dividing 2160, the value for continuous current, by the square of that power factor for single-phase, and by twice the square of that power factor for 3-wire three-phase, or 4-wire two-phase.

The value of B depends upon the size of wire, frequency, and power factor. It is equal to 1 for continuous current and for alternating current with 100 per cent. power factor, and sizes of wire given in the preceding table of wiring constants.

The figures given are for wires 18 inches apart, and are sufficiently accurate for all practical purposes, provided the displacement in phase between current and electro-motive force at the receiving end is not very much greater than that at the generator; in other words, provided the reactance of the line is not excessive or the line loss unusually high. For example, the constants should not be applied at 125 cycles if the largest conductors are used and the loss is 20 per cent. or more of the power delivered.

Values of B.

Number of wire, B. & S. gauge.	Area of wire, in circular mils.	Weight of bare wire per 1000 feet.	Resistance of wire per 1000 feet, at 20° C.	25 cycles. Percentage of power factor.				40 cycles. Percentage of power factor.				60 cycles. Percentage of power factor.				125 cycles. Percentage of power factor.			
				95	90	85	80	95	90	85	80	95	90	85	80	95	90	85	80
		Lb.	Ohms.																
4/0	211 600	640.73	.04879	1.23	1.29	1.33	1.34	1.52	1.53	1.61	1.67	1.62	1.84	1.99	2.09	2.35	2.86	3.24	3.49
3/0	167 805	508.12	.06154	1.18	1.22	1.24	1.24	1.40	1.41	1.48	1.51	1.49	1.66	1.77	1.95	2.08	2.48	2.77	2.94
2/0	133 079	402.97	.07758	1.14	1.16	1.16	1.16	1.25	1.32	1.35	1.37	1.34	1.52	1.60	1.66	1.86	2.18	2.40	2.57
1/0	105 560	319.00	.09775	1.10	1.11	1.10	1.09	1.19	1.24	1.26	1.26	1.31	1.40	1.46	1.49	1.71	1.96	2.13	2.25
1	83 694	253.43	.1234	1.07	1.07	1.05	1.03	1.14	1.17	1.18	1.17	1.24	1.30	1.34	1.36	1.56	1.75	1.88	1.97
2	66 373	200.98	.1556	1.05	1.04	1.02	1.0	1.11	1.12	1.12	1.10	1.18	1.23	1.25	1.26	1.45	1.60	1.70	1.77
3	52 633	159.38	.1962	1.03	1.02	1.0	1.0	1.07	1.08	1.07	1.05	1.14	1.17	1.18	1.17	1.35	1.46	1.53	1.57
4	41 742	126.40	.2473	1.02	1.0	1.0	1.0	1.05	1.06	1.03	1.0	1.11	1.12	1.11	1.10	1.27	1.35	1.40	1.43
5	33 102	100.23	.3120	1.0	1.0	1.0	1.0	1.03	1.01	1.0	1.0	1.08	1.08	1.06	1.04	1.21	1.27	1.30	1.31
6	26 250	79.49	.3934	1.0	1.0	1.0	1.0	1.02	1.0	1.0	1.0	1.05	1.04	1.02	1.0	1.16	1.20	1.21	1.21
7	20 816	63.03	.4958	1.0	1.0	1.0	1.0	1.01	1.0	1.0	1.0	1.03	1.02	1.0	1.0	1.12	1.14	1.14	1.13
8	16 509	49.99	.6250	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.02	1.0	1.0	1.0	1.09	1.10	1.09	1.07
9	13 090	39.6	.7886	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.06	1.06	1.04	1.02
10	10 382	31.4	.994	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.04	1.03	1.0	1.0

At lower frequencies, however, the constants are reasonably correct, even under such extreme conditions. They represent about the true values at 10 per cent. line loss, are close enough at all losses less than 10 per cent., and often, at least for frequencies up to 40 cycles, close enough for even much larger losses. Where the conductors of a circuit are much nearer each other than 18 inches the volts loss will be less than that given by the formula, and if close together, as with a multiple-conductor cable, the loss will be only that due to the resistance.

The value of T depends on the system and power factor. It is equal to 1 for continuous current and for single-phase current of 100 per cent. power factor.

The value of A and the weights of wires in the tables are based on 0.00000302 of a pound as the weight of a foot of copper wire of 1 circular mil area.

In using the formulas and constants on page 734, it should particularly be observed that P stands for the per cent. loss in the line of the *delivered* power, not for the per cent. loss in the line of the power at the generator, and that E is the power at the end of the line and not at the generator.

When the power factor cannot be more accurately determined, it may be assumed to be as follows for any alternating system operating under average conditions: Incandescent lighting and synchronous motors, 95 per cent.; lighting and induction motors together, 85 per cent.; induction motors alone, 80 per cent.

In continuous-current 3-wire systems the neutral wire for feeders should be made of one-third the section obtained by formula for either of the outside wires. In both continuous- and alternating-current systems the neutral conductor for secondary mains and house-wiring should be taken as large as the other conductors.

The 3 wires of a three-phase circuit and the 4 wires of a two-phase circuit should all be of the same size, and each conductor should be of the cross-section given by the first formula.

Report of the Committee on Standardization.

American Institute of Electrical Engineers.
1898.

GENERAL PLAN.

Electrical apparatus will be treated under the following heads:

I. **Commutating Machines,** which comprise a constant magnetic field, a closed-coil armature, and a multi-segmental commutator connected thereto.

Under this head may be classed the following: Direct-current generators, direct-current motors, direct-current boosters, motor-generators, dynamotors, converters, and closed-coil arc machines.

A booster is a machine inserted in series in a circuit to change its voltage, and may be driven either by an electric motor or otherwise. In the former case it is a motor-booster.

A motor-generator is a transforming device consisting of two machines, a motor and a generator, mechanically connected together.

A dynamotor is a transforming device combining both motor and generator action in one magnetic field, with two armatures, or with an armature having two separate windings.

For converters, see III.

II. **Synchronous Machines,** which comprise a constant magnetic field and an armature receiving or delivering alternating currents in synchronism with the motion of the machine,—*i.e.*, having a frequency equal to the product of the number of pairs of poles and the speed of the machine, in revolutions, per second.

III. **Synchronous Commutating Machines.**—These include: 1, synchronous converters,—*i.e.*, converters from alternating to direct, or from direct to alternating current; and 2, double-current generators,—*i.e.*, generators producing both direct and alternating currents.

A converter is a rotary device transforming electric energy from one form into another without passing it through the intermediary form of mechanical energy.

A converter may be either:

(*a*) A direct-current converter, converting from a direct current to a direct current, or

(*b*) A synchronous converter, formerly called a rotary converter, converting from an alternating to a direct current, or *vice versa*. Phase converters are converters from an alternating-current system to an alternating-current system of the same frequency but different phase.

Frequency converters are converters from an alternating-current system of one frequency to an alternating-current system of another frequency, with or without changes of phase.

IV. **Rectifying Machines, or Pulsating-current Generators,** which produce a unidirectional current of periodically varying strength.

V. **Stationary Induction Apparatus,**—*i.e.*, stationary apparatus changing electric energy from one form into another without passing it through an intermediary form of energy. These comprise:

(*a*) Transformers, or stationary induction apparatus, in which the primary and secondary windings are electrically insulated from each other.

(*b*) Auto-transformers, formerly called compensators,—*i.e.*, stationary induction apparatus, in which part of the primary winding is used as a secondary winding, or conversely.

(*c*) Potential regulators, or stationary induction apparatus having a coil in shunt and a coil in series with the circuit, so arranged that the ratio of transformation between them is variable at will.

These may be divided into:

1. Compensator potential regulators, in which the number of turns of one of the coils is changed.

2. Induction potential regulators, in which the relative positions of primary and secondary coils is changed.

3. Magneto-potential regulators, in which the direction of the magnetic flux with respect to the coils is changed.

(*d*) Reactive coils, or reactance coils, formerly called choking coils,—*i.e.*, stationary induction apparatus, *used to produce impedance or phase displacement.*

VI. **Rotary Induction Apparatus,** which consists of primary and secondary windings, rotating with respect to each other. They comprise:

(*a*) Induction motors.
(*b*) Induction generators.
(*c*) Frequency changers.
(*d*) Rotary phase converters.

EFFICIENCY.

1. The "efficiency" of an apparatus is the ratio of its net power output to its gross power input.*

2. Electric power should be measured at the terminals of the apparatus.

3. In determining the efficiency of alternating-current apparatus the electric power should be measured when the current is in phase with the E. M. F., unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, etc.

4. Mechanical power in machines should be measured at the pulley, gearing, coupling, etc., thus excluding the loss of power in said pulley, gearing, or coupling, but including the bearing friction and windage. The magnitude of bearing friction and windage may be considered as independent of the load. The loss of power in the belt and the increase of bearing friction due to belt tension should be excluded. Where, however, a machine is mounted upon the shaft of a prime mover in such a manner that it cannot be separated therefrom, the frictional losses in bearings and in windage, which ought by definition to be included in determining the efficiency, should be excluded, owing to the practical impossibility of determining them satisfactorily. The brush friction, however, should be included.

(*a*) Where a machine has auxiliary apparatus, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the machine, but to the plant, consisting of machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

5. The efficiency may be determined by measuring all the losses individually and adding their sum to the output to derive the input, or subtracting their sum from the input to derive the output. All losses should be measured at, or reduced to, the temperature assumed in continuous operation, or in operation under conditions specified. (See Sections 25 to 31.)

In order to consider the application of the foregoing rules to various machines in general use, the latter may be conveniently divided into classes, as follows:

I. Commutating Machines.

6. In commutating machines the losses are:

(*a*) Bearing friction and windage. (See Section 4.)

(*b*) Molecular magnetic friction and eddy currents in iron and copper. These losses should be determined with the machine on open circuit, and at a voltage equal to the rated voltage $+ Ir$ in a generator and $- Ir$ in a motor, where I denotes the current strength and r denotes the internal resistance of the machine. They should be measured at the correct speed and voltage, since they do not usually vary in proportion to the speed or to any definite power of the voltage.

(*c*) Armature resistance losses, I^2r', where I is the current strength in the armature and r' is the resistance between armature brushes, excluding the resistance of brushes and brush contacts.

(*d*) Commutator brush friction.

(*e*) Commutator brush-contact resistance. It is desirable to point out that with carbon brushes the losses, (*d*) and (*e*), are usually considerable in low-voltage machines.

(*f*) Field excitation. With separately-excited fields the loss of power in the resistance of the field coils alone should be considered. With shunt fields or series fields, however, the loss of power in the accompanying rheostat should also be included, the said rheostat being considered as an essential part of the machine, and not as separate auxiliary apparatus.

(*b*) and (*c*) are losses in the armature, or "armature losses;" (*d*) and (*e*), "commutator losses;" (*f*), "field losses."

7. The difference between the total losses under load and the sum of the losses above specified should be considered as "load losses," and are usually trivial in commutating machines of small field distortion. When

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* An exception should be noted in the case of storage batteries or apparatus for storing energy, in which the efficiency, unless otherwise qualified, should be understood as the ratio of the energy output to the energy intake in a normal cycle.

the field distortion is large, as is shown by the necessity for shifting the brushes between no load and full load, or with variations of load, these load losses may be considerable, and should be taken into account. In this case the efficiency may be determined either by input and output measurements or the load losses may be estimated by the method of Section II.

8. Boosters should be considered and treated like other direct-current machines in regard to losses.

9. In motor-generators, dynamotors, or converters the efficiency is the $\frac{\text{electric output}}{\text{electric input}}$.

II. Synchronous Machines.

10. In synchronous machines the output or input should be measured with the current in phase with the terminal E. M. F., except when otherwise expressly specified.

Owing to the uncertainty necessarily involved in the approximation of load losses, it is preferable, whenever possible, to determine the efficiency of synchronous machines by input and output tests.

11. The losses in synchronous machines are:

(*a*) Bearing friction and windage. (See Section 4.)

(*b*) Molecular magnetic friction and eddy currents in iron, copper, and other metallic parts. These losses should be determined at open circuit of the machine at the rated speed and at the rated voltage, $+ Ir$ in a synchronous generator, $- Ir$ in a synchronous motor, where $I =$ current in armature, $r =$ armature resistance. It is undesirable to compute these losses from observations made at other speeds or voltages.

These losses may be determined either by driving the machine by a motor, or by running it as a synchronous motor and adjusting its fields so as to get minimum current input, and measuring the input by wattmeter. The former is the preferable method; and in polyphase machines the latter method is liable to give erroneous results in consequence of unequal distribution of currents in the different circuits, caused by inequalities of the impedance of connecting leads, etc.

(*c*) Armature-resistance loss, which may be expressed by pI^2r, where $r =$ resistance of one armature circuit or branch, $I =$ the current in such armature circuit or branch, and $p =$ the number of armature circuits or branches.

(*d*) Load losses as defined in Section 7. While these losses cannot well be determined individually, they may be considerable, and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short circuit and at full-load current,—that is, by determining what may be called the "short-circuit core loss." With the low field intensity and great lag of current existing in this case the load losses are usually greatly exaggerated.

One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

(*e*) Collector-ring friction and contact resistance. These are generally negligible, except in machines of extremely low voltage.

(*f*) Field excitation. In separately-excited machines the I^2r of the field-coils proper should be used. In self-exciting machines, however, the loss in the field rheostat should be included. (See Section 6, *f*.)

III. Synchronous Commutating Machines.

12. In synchronous converters the power on the alternating-current side is to be measured with the current in phase with the terminal E. M. F., unless otherwise specified.

13. In double-current generators the efficiency of the machine should be determined as a direct-current generator, in accordance with Section 6, and as an alternating-current generator, in accordance with Section 11. The two values of efficiency may be different, and should be clearly distinguished.

14. In synchronous converters the losses should be determined when driving the machine by a motor. These losses are:

(*a*) Bearing friction and windage. (See Section 4.)

(*b*) Molecular magnetic friction and eddy currents in iron, copper, and metallic parts. These losses should be determined at open circuit and at the rated terminal voltage, no allowance being made for the armature

resistance, since the alternating and the direct currents flow in opposite directions.

(*c*) Armature resistance. The loss in the armature is qI^2r, where I = direct current in armature, r = armature resistance, and q, a factor which is equal to 1.37 in single-phasers, 0.56 in three-phasers, 0.37 in quarter-phasers, and 0.26 in six-phasers.

(*d*) Load losses. The load losses should be determined in the same manner as described in Section 11, *d*, with reference to the direct-current side.

(*e*) and (*f*) Losses in commutator and collector-friction and brush-contact resistance. (See Sections 6 and 11.)

(*g*) Field excitation. In separately-excited fields the I^2r loss in the field-coils proper should be taken, while in shunt and series fields the rheostat loss should be included, except where fields and rheostats are intentionally modified to produce effects outside of the conversion of electric power, as for producing phase displacement for voltage control. In this case 25 per cent. of the I^2r loss in the field proper at non-inductive alternating circuit should be added as proper estimated allowance for normal rheostat losses. (See Section 6, *f*.)

15. Where two similar synchronous machines are available their efficiency can be determined by operating one machine as a converter from direct to alternating, and the other as a converter from alternating to direct, connecting the alternating sides together and measuring the difference between the direct-current input and the direct-current output. This process may be modified by returning the output of the second machine through two boosters into the first machine and measuring the losses. Another modification might be to supply the losses by an alternator between the two machines, using potential regulators.

IV. Rectifying Machines, or Pulsating-current Generators.

16. These include open-coil arc machines, constant-current rectifiers, constant-potential rectifiers.

The losses in open-coil arc machines are essentially the same as in Sections 6 to 9 (closed-coil commutating machines). In alternating-current rectifiers, however, the output must be measured by wattmeter and not by voltmeter and ammeter, since owing to the pulsation of current and E. M. F. a considerable discrepancy may exist between watts and volt-amperes, amounting to as much as 10 or 15 per cent.

17. In constant-current rectifiers, transforming from constant-potential alternating to constant direct current by means of constant-current transformers and rectifying commutators, the losses in the transformers are to be included in the efficiency, and have to be measured when operating the rectifier, since in this case the losses are generally greater than when feeding an alternating secondary circuit. In constant-current transformers the load losses are usually larger than in constant-potential transformers, and thus should not be neglected.

The most satisfactory method of determining the efficiency in rectifiers is to measure electric input and electric output by wattmeter. The input is usually not non-inductive, owing to a considerable phase displacement and to wave distortion. For this reason the apparent efficiency should also be considered, since it is usually much lower than the true efficiency. The power consumed by the synchronous motor or other source driving the rectifier should be included in the electric input.

V. Stationary Induction Apparatus.

18. Since the efficiency of induction apparatus depends upon the wave shape of E. M. F., it should be referred to a sine wave of E. M. F., except where expressly specified otherwise. The efficiency should be measured with non-inductive load and at rated frequency, except where expressly specified otherwise. The losses are:

(*a*) Molecular magnetic friction and eddy currents measured at open circuit and at rated voltage — Ir, where I = rated current, r = resistance of primary circuit.

(*b*) Resistance losses. The sum of the I^2r of primary and of secondary in a transformer, or of the two sections of the coil in the compensator or auto-transformer, where I = current in the coil or section of coil, r = resistance.

(*c*) Load losses,—*i.e.*, eddy currents in the iron, and especially in the copper conductors, caused by the current. They should be measured by short-circuiting the secondary of the transformer and impressing upon the primary an E. M. F. sufficient to send full-load current through the transformer. The loss in the transformer under these conditions, measured by wattmeter, gives the load losses $= I^2r$ losses in both primary and secondary coils.

(*d*) Losses due to the methods of cooling, as power consumed by the blower in air-blast transformers and power consumed by the motor driving-pumps in oil- or water-cooled transformers. Where the same cooling apparatus supplies a number of transformers, or is installed to supply future additions, allowance should be made therefor.

19. In potential regulators the efficiency should be taken at the maximum voltage for which the apparatus is designed, and with non-inductive load, unless otherwise specified.

VI. Rotary Induction Apparatus.

20. Owing to the existence of load losses, and since the magnetic density in the induction motor under load changes in a complex manner, the efficiency should be determined by measuring the electric input by wattmeter and the mechanical output at the pulley, gear, coupling, etc.

21. The efficiency should be determined at the rated frequency, and the input measured with sine waves of impressed E. M. F.

22. The efficiency may be calculated from the apparent input, the power factor, and the power output. The same applies to induction generators. Since phase displacement is inherent in induction machines, their apparent efficiency is also important.

23. In frequency changers,—*i.e.*, apparatus transforming from a polyphase system to an alternating system of different frequency, with or without a change in the number of phases and phase converters,—*i.e.*, apparatus converting from an alternating system, usually single-phase, to another alternating system, usually polyphase, of the same frequency, the efficiency should also be determined by measuring both output and input.

VII. Transmission Lines.

24. The efficiency of transmission lines should be measured with non-inductive load at the receiving end, with the rated receiving pressure and frequency, also with sinusoidal impressed E. M. F.'s, except where expressly specified otherwise, and with the exclusion of transformers or other apparatus at the ends of the line.

RISE OF TEMPERATURE.

General Principles.

25. Under regular service conditions the temperature of electrical machinery should never be allowed to remain at a point at which permanent deterioration of its insulating material takes place.

26. The rise of temperature should be referred to the standard conditions of a room-temperature of 25° C., a barometric pressure of 760 millimetres, and normal conditions of ventilation,—that is, the apparatus under test should neither be exposed to draught nor inclosed, except where expressly specified.

27. If the room-temperature during the test differs from 25° C., the observed rise of temperature should be corrected by ½ per cent. for each degree C.* Thus, with a room-temperature of 35° C. the observed rise of temperature has to be decreased by 5 per cent., and with a room-temperature of 15° C. the observed rise of temperature has to be increased by 5 per cent. The thermometer indicating the room-temperature should be screened from thermal radiation emitted by heated bodies or from draughts of air. When it is impracticable to secure normal conditions of ventilation

* This correction is also intended to compensate, as nearly as is at present practicable, for the error involved in the assumption of a constant temperature coefficient of resistivity,—*i.e.*, 0.4 per cent. per degree C., taken with varying initial temperatures.

on account of an adjacent engine or other sources of heat, the thermometer for measuring the air-temperature should be placed so as fairly to indicate the temperature which the machine would have if it were idle, in order that the rise of temperature determined shall be that caused by the operation of the machine.

28. The temperature should be measured after a run of sufficient duration to reach practical constancy. This is usually from 6 to 18 hours, according to the size and construction of the apparatus. It is permissible, however, to shorten the time of the test by running a lesser time on an overload in current and voltage, then reducing the load to normal, and maintaining it thus until the temperature has become constant.

In apparatus intended for intermittent service, as railway motors, starting rheostats, etc., the rise of temperature should be measured after a shorter time, depending upon the nature of the service, and should be specified.

In apparatus which, by the nature of their service, may be exposed to overload, as railway converters, and in very high voltage circuits a smaller rise of temperature should be specified than in apparatus not liable to overloads or in low-voltage apparatus. In apparatus built for conditions of limited space, as railway motors, a higher rise of temperature must be allowed.

29. In electrical conductors the rise of temperature should be determined by their increase of resistance. For this purpose the resistance may be measured either by galvanometer test or by drop-of-potential method. A temperature coefficient of 0.4 per cent. per degree C. may be assumed for copper.* Temperature elevations measured in this way are usually in excess of temperature elevations measured by thermometers.

30. It is recommended that the following maximum values of temperature elevation should not be exceeded:

Commutating machines, rectifying machines, and synchronous machines:

Field and armature, by resistance, 50° C.
Commutator and collector rings and brushes, by thermometer, 55° C.
Bearings and other parts of machine, by thermometer, 40° C.

Rotary induction apparatus:

Electric circuits, 50° C., by resistance.
Bearings and other parts of the machine, 40° C., by thermometer.

In squirrel-cage or short-circuited armatures, 55° C., by thermometer, may be allowed.

Transformers for continuous service,—electric circuits, by resistance, 50° C.; other parts, by thermometer, 40° C., under conditions of normal ventilation.

Reactive coils, induction and magneto regulators and transformers of 15 kilowatts or less,—electric circuits, by resistance, 55° C.; other parts, by thermometer, 45° C.

Where a thermometer, applied to a coil or winding, indicates a higher temperature elevation than that shown by resistance measurement, the thermometer indication should be accepted. In using the thermometer care should be taken so to protect its bulb as to prevent radiation from it, and, at the same time, not to interfere seriously with the normal radiation from the part to which it is applied.

31. In the case of apparatus intended for intermittent service, the temperature elevation which is attained at the end of the period corresponding to the term of full load should not exceed 50° C., by resistance, in electric circuits. In the case of transformers intended for intermittent service or not operating continuously at full load, but continuously in circuit, as in the ordinary case of lighting transformers, the temperature elevation above the surrounding air-temperature should not exceed 50° C., by resistance, in electric circuits, and 40° C., by thermometer, in other parts, after the period corresponding to the term of full load. In this instance the best load should not be applied until the transformer has been in circuit for a sufficient time to attain the temperature elevation due to core loss. With transformers for commercial lighting the duration of the full-load test

* By the formula $R_T = R_t\,(1 + 0.004\theta)$. Where R_t is the resistance at room-temperature, R_T the resistance when heated, and θ the temperature elevation (T-t) in degrees centigrade.

may be taken as 3 hours, unless otherwise specified. In the case of railway, crane, and elevator motors the conditions of service are necessarily so varied that no specific period corresponding to the full load term can be stated.

INSULATION.

32. The ohmic resistance of the insulation is of secondary importance only, as compared with the dielectric strength or resistance to rupture by high voltage.

Since the ohmic resistance of the insulation can be very greatly increased by baking,—but the dielectric strength is liable to be weakened thereby,—it is preferable to specify a high dielectric strength rather than a high insulation resistance. The high-voltage test for dielectric strength should always be applied.

Insulation Resistance.

33. Insulation resistance tests should, if possible, be made at the pressure for which the apparatus is designed.

The insulation resistance of the complete apparatus must be such that the rated voltage of the apparatus will not send more than $\frac{1}{1,000,000}$ of the full load current, at the rated terminal voltage, through the insulation. Where the value found in this way exceeds 1 megohm, 1 megohm is sufficient.

Dielectric Strength.

34. The dielectric strength or resistance to rupture should be determined by a continued application of an alternating E. M. F. for one minute. The source of alternating E. M. F. should be a transformer of such size that the charging current of the apparatus as a condenser does not exceed 25 per cent. of the rated capacity of the transformer.

35. The high-voltage tests should not be applied when the insulation is low owing to dirt and moisture, and should be applied before the machine is put into commercial service.

36. It should be pointed out that tests at high voltages considerably in excess of the normal voltages are admissible on new machines, to determine whether they fulfil their specifications, but should not be made subsequently at a voltage much exceeding the normal, as the actual insulation of the machine may be weakened by such tests.

37. The test for dielectric strength should be made with the completely-assembled apparatus, and not with its individual parts, and the voltage should be applied as follows:

1. Between electric circuits and surrounding conducting material, and,
2. Between adjacent electric circuits, where such exist, as in transformers.

The tests should be made with a sine wave of E. M. F., or, where this is not available, at a voltage giving the same striking distance between needle-points in air as a sine wave of the specified E. M. F., except where expressly specified otherwise. As needles, new sewing-needles should be used. It is recommended to shunt the apparatus during the test by a spark gap of needle-points set for a voltage exceeding the required voltage by 10 per cent.

38. The following voltages are recommended for apparatus, not including transmission lines or switchboards:

Rated terminal voltage.	Capacity.	Testing voltage.
Not exceeding 400 volts.	Under 10 kilowatts.	1000 volts.
Not exceeding 400 volts.	10 kilowatts and over.	1500 volts.
400 and over, but less than 800 volts.	Under 10 kilowatts.	1500 volts.
400 and over, but less than 800 volts.	10 kilowatts and over.	2000 volts.
800 and over, but less than 1200 volts.	Any.	3500 volts.
1200 and over, but less than 2500 volts.	Any.	5000 volts.
2500 and over.	Any.	Double the normal rated voltages.

Synchronous motor fields and fields of converters started from the alternating current side should be tested at 5000 volts.

Synchronous motors and synchronous converter field-coils should be tested at 5000 volts, since in the starting of such machines a high voltage is induced in their field-coils.

Alternator field circuits should be tested under a breakdown test voltage corresponding to the rated voltage of the exciter referred to an output equal to the output of the alternator,—*i.e.*, the exciter should be rated for this test as having an output equal to that of the machine it excites.

Condensers should be tested at twice their rated voltage and at their rated frequency.

The above values are effective values, or square roots of mean square reduced to a sine wave of E. M. F.

39. In testing insulation between different electric circuits, as between primary and secondary of transformers, the testing voltage must be chosen corresponding to the high-voltage circuit.

40. In transformers of from 10,000 volts to 20,000 volts it should be considered as sufficient to operate the transformer at twice its rated voltage by connecting first the one and then the other terminal of the high-voltage winding to the core and to the low-voltage winding. The test of dielectric resistance between the low-voltage winding and the core should be in accordance with the recommendation in Section 39 for similar voltages and capacities.

41. When machines or apparatus are to be operated in series, so as to employ the sum of their separate E. M. F.'s, the voltage should be referred to this sum, except where the frames of the machine are separately insulated, both from ground and from each other.

REGULATION.

42. The term "regulation" should have the same meaning as the term "inherent regulation," at present frequently used.

43. The regulation of an apparatus intended for the generation of constant potential, constant current, constant speed, etc., is to be measured by the maximum variation of potential current, speed, etc., occurring within the range from full load to no load under such constant conditions of operation as give the required full-load values, the conditions of full load being considered in all cases as the normal condition of operation.

44. The regulation of an apparatus intended for the generation of a potential, current, speed, etc., varying in a definite manner between full load and no load, is to be measured by the maximum variation of potential, current, speed, etc., from the satisfied condition, under such constant conditions of operation as give the required full-load values.

If the manner in which the variation in potential, current, speed, etc., between full load and no load is not specified, it should be assumed to be a simple linear relation.

The regulation of an apparatus may, therefore, differ according to its qualification for use. Thus, the regulation of a compound-wound generator specified as a constant-potential generator will be different from that it possesses when specified as an over-compounded generator.

45. The regulation is given in percentage of the full-load value of potential, current, speed, etc., and the apparatus should be steadily operated during the test under the same conditions as at full load.

46. The regulation of generators is to be determined at constant speed, of alternating apparatus at constant impressed frequency.

47. The regulation of a generator unit, consisting of a generator united with a prime mover, should be determined at constant conditions of the prime mover,—*i.e.*, constant steam pressure, head, etc. It would include the inherent speed variations of the prime mover. For this reason the regulation of a generator unit is to be distinguished from the regulation of either the prime mover or of the generator contained in it and taken separately.

48. In apparatus generating, transforming, or transmitting alternating currents, regulation should be understood to refer to non-inductive load,—that is, to a load in which the current is in phase with the E. M. F. at the output side of the apparatus, except where expressly specified otherwise.

49. In alternating apparatus receiving electric power, regulation should refer to a sine wave of E. M. F., except where expressly specified otherwise.

50. In commutating machines, rectifying machines, and synchronous machines, as direct-current generators and motors, alternating-current and polyphase generators, the regulation is to be determined under the following conditions:

(*a*) At constant excitation in separately-excited fields,
(*b*) With constant resistance in shunt-field circuits, and
(*c*) With constant resistance shunting series fields,—*i.e.*, the field adjustment should remain constant, and should be so chosen as to give the required full-load voltage at full-load current.

51. In constant-potential machines the regulation is the ratio of the maximum difference of terminal voltage from the rated full-load value (occurring within the range from full load to open circuit) to the full-load terminal voltage.

52. In constant-current machines the regulation is the ratio of the maximum difference of current from the rated full-load value (occurring within the range from full load to short circuit) to the full-load current.

53. In constant-power machines the regulation is the ratio of maximum difference of power from the rated full-load value (occurring within the range of operation specified) to the rated power.

54. In over-compounded machines the regulation is the ratio of the maximum difference in voltage from a straight line connecting the no-load and full-load values of terminal voltage as function of the current to the full-load terminal voltage.

55. In constant-speed, continuous-current motors the regulation is the ratio of the maximum variation of speed from its full-load value (occurring within the range from full load to no load) to the full-load speed.

56. In transformers the regulation is the ratio of the rise of secondary terminal voltage from full load to no load (at constant primary impressed terminal voltage) to the secondary terminal voltage.

57. In induction motors the regulation is the ratio of the rise of speed from full load to no load (at constant impressed voltage) to the full-load speed.

The regulation of an induction motor is, therefore, not identical with the slip of the motor, which is the ratio of the drop in speed from synchronism to synchronous speed.

58. In converters, dynamotors, motor-generators, and frequency-changers the regulation is the ratio of the maximum difference of terminal voltage at the output side from the rated full-load voltage (at constant impressed voltage and at constant frequency) to the full-load voltage on the output side.

59. In transmission lines, feeders, etc., the regulation is the ratio of maximum voltage difference at the receiving end between no load and full non-inductive load to the full-load voltage at the receiving end, with constant voltage impressed upon the sending end.

60. In steam engines the regulation is the ratio of the maximum variation of speed in passing from full load to no load (at constant steam pressure at the throttle) to the full-load speed.

61. In a turbine or other water motor the regulation is the ratio of the maximum variation of speed from full load to no load (at constant head of water,—*i.e.*, at constant difference of level between tail-race and head-race) to the full-load speed.

VARIATION AND PULSATION.

62. In prime movers which do not give an absolutely uniform rate of rotation or speed, as in steam engines, the "variation" is the maximum angular displacement in position of the revolving member from the position it would occupy at uniform rotation, expressed in degrees,—that is, with one revolution at 300°; and the pulsation is the ratio of the maximum change of speed in an engine cycle to the average speed.

63. In alternators, or alternating-current circuits in general, the variation is the maximum difference in phase of the generated wave of E. M. F. from a wave of absolutely constant frequency, expressed in degrees, and is due to the variation of the prime mover. The pulsation is the ratio of the maximum change of frequency during an engine cycle to the average frequency.

64. If n = number of poles, the variation of an alternator is $\frac{n}{2}$ times the variation of its prime mover if direct connected, and $\frac{n}{2}p$ times the variation of the prime mover if rigidly connected thereto in the velocity ratio, p.

65. The pulsation of an alternating-current circuit is the same as the pulsation of the prime mover of its alternator.

RATING.

66. Both electrical and mechanical power should be expressed in kilowatts, except when otherwise specified. Alternating-current apparatus should be rated in kilowatts on the basis of non-inductive condition,—*i.e.*, with the current in phase with the terminal voltage.

67. Thus, the electric power generated by an alternating-current apparatus equals its rating only at non-inductive load,—that is, when the current is in phase with the terminal voltage.

68. Apparent power should be expressed in kilovolt-amperes, as distinguished from real power in kilowatts.

69. If a power factor other than 100 per cent. is specified, the rating should be expressed in kilovolt-amperes and power factor at full load.

70. The full-load current of an electric generator is that current which, with the rated full-load terminal voltage, gives the rated kilowatts; but in alternating-current apparatus, only at non-inductive load.

71. Thus, in machines in which the full-load voltage differs from the no-load voltage, the full-load current should refer to the former.

If P = rating of an electric generator and E = full-load terminal voltage, the full-load current is:

$I = \frac{P}{E}$ in a continuous-current machine or single-phase alternator;

$I = \frac{P}{E\sqrt{3}}$ in a three-phase alternator;

$I = \frac{P}{2E}$ in a quarter-phase alternator.

72. Constant-current machines, such as series arc-light generators, should be rated in kilowatts based on terminal volts and amperes at full load.

73. The rating of a fuse or circuit-breaker should be the current strength at which it will open the circuit, and not the working-current strength.

CLASSIFICATION OF VOLTAGES AND FREQUENCIES.

74. In direct-current, low-tension generators the following average terminal voltages are in general use, and are recommended:

125 volts. 250 volts. 550 volts.

75. In direct-current and alternating-current, low-pressure circuits the following average terminal voltages are in general use, and are recommended:

110 volts. 220 volts.

In direct-current power circuits, for railway and other service, 500 volts may be considered as standard.

76. In alternating-current, high-pressure circuits at the receiving end the following pressures are in general use, and are recommended:

1000 volts.	3000 volts.	10,000 volts.	20,000 volts.
2000 volts.	6000 volts.	15,000 volts.	

77. In alternating-current, high-pressure generators or generating systems the following terminal voltages are in general use, and are recommended:

1150 volts. 2300 volts. 3450 volts.

These pressures allow of a maximum drop in transmission of 15 per cent. of the pressure at the receiving end. If the drop required is greater than 15 per cent., the generator should be considered as special.

78. In alternating-current circuits the following approximate frequencies are recommended as desirable:

25 ~ or 30 ~. 40 ~. 60 ~. 120 ~.*

These frequencies are already in extensive use, and it is deemed advisable to adhere to them as closely as possible.

OVERLOAD CAPACITIES.

79. All guaranties on heating, regulation, sparking, etc., should apply to the rated load, except where expressly specified otherwise, and in alternating-current apparatus to the current in phase with the terminal E. M. F., except where a phase displacement is inherent in the apparatus.

80. All apparatus should be able to carry a reasonable overload without self-destruction by heating, sparking, mechanical weakness, etc., and with an increase of temperature elevation not exceeding 15° C. above those specified for full loads. (See Sections 25 to 31.)

81. Overload guaranties should refer to normal conditions of operation regarding speed, frequency, voltage, etc., and to non-inductive conditions in alternating apparatus, except where a phase displacement is inherent in the apparatus.

82. The following overload capacities are recommended:

1. In direct-current generators and alternating-current generators, 25 per cent. for ½ hour.
2. In direct-current motors and synchronous motors, 25 per cent. for ½ hour, 50 per cent. for 1 minute, except in railway motors and other apparatus intended for intermittent service.
3. Induction motors, 25 per cent. for ½ hour, 50 per cent. for 1 minute.
4. Synchronous converters, 50 per cent. for ½ hour.
5. Transformers, 25 per cent. for ½ hour, except in transformers connected to apparatus for which a different overload is guaranteed, in which case the same guaranties shall apply for the transformers as for the apparatus connected thereto.
6. Exciters of alternators and other synchronous machines, 10 per cent. more overload than is required for the excitation of the synchronous machine at its guaranteed overload and for the same period of time.

APPENDIX I.

EFFICIENCY.

Efficiency of Phase-displacing Apparatus.

In apparatus producing phase displacement, as, for example, synchronous compensators, exciters of induction generators, reactive coils, condensers, polarization cells, etc., the efficiency should be understood to be the ratio of the volt-ampere activity to the volt-ampere activity plus power loss.

The efficiency may be calculated by determining the losses individually, adding to them the volt-ampere activity, and then dividing the volt-ampere activity by the sum.

1. In synchronous compensators and exciters of induction generators the determination of losses is the same as in other synchronous machines under Sections 10 and 11.
2. In reactive coils the losses are molecular friction, eddy losses, and I^2r loss. They should be measured by wattmeter. The efficiency of re-

* The frequency of 120 ~ may be considered as covering the already-existing commercial frequencies between 120 ~ and 140 , and the frequency of 60 ~ as covering the already-existing commercial frequencies between 60 ~ and 70 ~.

active coils should be determined with a sine wave of impressed E. M. F., except where expressly specified otherwise.

3. In condensers the losses are due to dielectric hysteresis and leakage, and should be determined by wattmeter with a sine wave of E. M. F.

4. In polarization cells the losses are those due to electric resistivity and a loss in the electrolyte of the nature of chemical hysteresis, and are usually very considerable. They depend upon the frequency, voltage, and temperature, and should be determined with a sine wave of impressed E. M. F., except where expressly specified otherwise.

APPENDIX II.

APPARENT EFFICIENCY.

In apparatus in which a phase displacement is inherent to their operation, apparent efficiency should be understood as the ratio of net power output to volt-ampere input.

Such apparatus comprise induction motors, reactive synchronous converters, synchronous converters controlling the voltage of an alternating-current system, self-exciting synchronous motors, potential regulators, and open magnetic circuit transformers, etc.

Since the apparent efficiency of apparatus generating electric power depends upon the power factor of the load, the apparent efficiency, unless otherwise specified, should be referred to a load power factor of unity.

APPENDIX III.

POWER FACTOR AND INDUCTANCE FACTOR.

The power factor in alternating circuits or apparatus may be defined as the ratio of the electric power in watts to volt-amperes.

The inductance factor is to be considered as the ratio of wattless volt-amperes to total volt-amperes.

Thus, if p = power factor, q = inductance factor; then

$$p^2 + q^2 = 1.$$

The power factor is the

$$\frac{(\text{energy component of current or E. M. F.})}{\text{total current or E. M. F.}},$$

and the inductance factor is the

$$\frac{(\text{wattless component of current or E. M. F.})}{(\text{total current of E. M. F.})} = \frac{\text{true power}}{\text{volt-amperes}}.$$

Since the power factor of apparatus supplying electric power depends upon the power factor of the load, the power factor of the load should be considered as Unity, unless otherwise specified.

APPENDIX IV.

The following notation is recommended:

E, e = voltage, E. M. F., potential difference;
I, i = current;
P = power;
ϕ = magnetic flux;
β = magnetic density;
R, r = resistance;
X, x = reactance;
Z, z = impedance;
L, l = inductance;
C, c = capacity.

Vector quantities, when used, should be denoted by capital italics.

APPENDIX V.

Table of sparking distances in air between opposed sharp needle-points, for various effective sinusoidal voltages, in inches and in centimetres.

Kilovolts. Square root of mean square.	Distance.		Kilovolts. Square root of mean square.	Distance.	
	Inches.	Centimetres.		Inches.	Centimetres.
5	.225	.57	60	4.65	11.8
10	.470	1.19	70	5.85	14.9
15	.725	1.84	80	7.10	18.0
20	1.000	2.54	90	8.35	21.2
25	1.300	3.3	100	9.60	24.4
30	1.625	4.1	110	10.75	27.3
35	2.00	5.1	120	11.85	30.1
40	2.45	6.2	130	12.95	32.9
45	2.95	7.5	140	13.95	35.4
50	3.55	9.0	150	15.0	38.1

CARY T. HUTCHINSON, CHARLES P. STEINMETZ,
A. E. KENNELLY, LEWIS B. STILWELL,
JOHN LIEB, JR., ELIHU THOMSON,
F. B. CROCKER, *Chairman.*

ELECTRIC DRIVING.

The general opinion is in favor of independent driving, each tool having its own motor attached. In some cases a group of small machines may be operated to advantage from a short line-shaft driven by an electric motor, but in the great majority of cases the independent driving is to be preferred.

The advantages of independent driving are well set forth in a paper by F. B. Duncan before the Engineers' Society of Western Pennsylvania.

1. Greater output per machine due to positive nature of drive; in many cases this is at least 50 per cent.

2. Ability to accurately determine—by means of recording instruments centrally located, with a multi-point switch—whether tools are being kept at work in proper manner, thereby affording a graphic record of the time each machine is in operation and its consumption of power. This will also enable the detection of tools that are in bad condition due to abnormal friction of bearings or moving parts.

3. The flexibility of placement of machine tools to suit the passage of the work through the shop.

4. Better light and absence of dirt due to belts, shafting, pulley hangers, etc., and less first cost of building owing to the lighter overhead construction permissible when no shafting, pulleys, hangers, or belt tension have to be taken care of.

5. Free head room for crane service.

6. Ability to shut down or start up any one machine independently of all others.

Mr. Duncan also gives the following data sheet of power required by a number of different machine tools. These represent average practice, using ordinary tool steels, but for the modern high-speed tool steels the cutting speeds may be increased to 80 to 100 feet per minute for cast- or wrought-iron, in which case the power required will be about **three times that** given on pages 750–753.

For planers the maximum power is that required for reversing the platen, as will be seen.

Data Sheet of Motor Power on Standard Machine Tools.

No. 1.

Description of machine, Planer.
Make of machine, Niles Tool Company.
Size of machine, 10′ × 10′ × 20′.
Number of cutting tools, 3.
Size of cut, ¾″ × ⅛″, each tool.
Cutting speed, 18 feet per minute.
Material machined, cast-iron.
Weight on platen, 40 tons.
Power for cut, 26.54 H. P.
Power for reverse, 42.93 H. P.
Power for return, 23.56 H. P.
Ratio of return, 3 to 1.
Method of drive, motor belted to counter-shaft.
Kind of motor, Direct-current Compound-wound.

Remarks.—Not enough fly-wheel effect on counter-shaft to equalize load at moment of reversal. A 30 H. P. motor was used for above drive with good results.

No. 2.

Description of machine, Planer.
Make of machine, Pond Machine Company.
Size of machine, 8′ × 8′ × 20′.
Number of cutting tools, 3.
Size of cut, ⅝″ × ⅛″, each tool.
Cutting speed, 18 feet per minute.
Material machined, cast-iron.
Weight on platen, 32 tons.
Power for cut, 16 H. P.
Power for reverse, 28.15 H. P.
Power for return, 14.80 H. P.
Ratio of return, 3 to 1.
Method of drive, motor belted to counter-shaft.
Kind of motor, Direct-current Compound-wound.

Remarks.—Not enough fly-wheel effect on counter-shaft to equalize load at moment of reversal. A 25 H. P. motor was used on this machine with good results.

No. 3.

Description of machine, Planer.
Make of machine, Pond Machine Company.
Size of machine, 66′ × 60′ × 12′.
Number of cutting tools, 2.
Size of cut, ½″ × $\frac{1}{16}$″.
Cutting speed, 21 feet per minute.
Material machined, open-hearth steel castings.
Weight on platen, 4 tons.
Power for cut, 10 H. P.
Power for reverse, 16 H. P.
Power for return, 14 H. P.
Ratio of return, 3½ to 1.
Method of drive, Direct-current Compound-wound Motor, mounted on housing of planer with 42-inch, 1500-pound fly-wheel, running at 400 revolutions per minute, mounted on motor-shaft. Fly-wheel used as driving pulley for return of platen.

Remarks.—A series of recording ammeter cards taken on this planer showed it was idle an average of 2½ hours per day, showing a saving of power by use of individual motor drive. The above 2½ hours was generally made up of short periods for setting work, taking measurements, etc.

No. 4.

Description of machine, Planer.
Make of machine, Gray.
Size of machine, 28″ × 32″ × 6′.
Number of cutting tools, 1.
Size of cut, $\frac{3}{4}$″ × $\frac{1}{8}$″.
Cutting speed, 22 feet per minute.
Material machined, cast-iron.
Weight on platen, 3 tons.
Power for cut, 3.1 H. P.
Power for reverse, 4.4 H. P.
Power for return, 3.8 H. P.
Ratio of return, 4 to 1.
Method of drive, Direct-current Compound-wound Motor, mounted on platen housings, with fly-wheel 30 inches in diameter, 496 pounds, 800 revolutions per minute, mounted on motor-shaft and used as pulley for return of platen.

Remarks.—Average load on motor, 2.48. A 3 H. P. motor at 800 revolutions per minute gave first-class service. Rheostat used in series with shunt field to raise cutting speed on light work to 30 feet per minute.

No. 5.

Description of machine, Turret Lathe.
Make of machine, Gisholt Machine Company.
Size of machine, 28 inches swing.
Number of cutting tools, 5.
Size of cut, $\frac{3}{4}$″ × $\frac{5}{16}$″, 1 tool; $\frac{1}{2}$″ × $\frac{5}{64}$″, 4 tool.
Cutting speed, 25 feet.
Material machined, Tropenas cast-steel.
Power for cut, 3.9 H. P.
Weight of casting, 400 pounds.
Method of drive, Direct-current Compound-wound Motor, 600 revolutions per minute, geared to headstock gear in place of cone pulley. Speed variations on motor 100 per cent. in all,—25 per cent. by armature control below normal, and 75 per cent. increase above normal by resistance in shunt field. Eleven points in controller, giving, with the three gear speeds, 33 changes of speed in all. An increase in output of 100 per cent. was obtained on this machine by changing from belt to geared motor drive.

No. 6.

Description of machine, Drill-press.
Make of machine, W. F. & John Barnes.
Size of machine, 21 inches.
Motor power required, 1 H. P.
Method of drive, Direct-current Compound Motor, mounted on frame of press and belted down to driving pulley. Starter and reversing switch mounted on frame of press within reach of operator seated at table.

No. 7.

Description of machine, Radial Drill-press.
Make of machine, Niles Tool Works.
Size of machine, No. 1, 5-foot arm from centre of column.
Motor power required (maximum), 2.03 H. P.
Size of motor used, 2 H. P., 600 revolutions per minute.
Method of drive, Vertical Direct-current Compound-wound Motor, mounted on top of column and geared to driving-shaft. Raw-hide pinion used on motor-shaft.

No. 8.

Description of machine, Double-end Emery-wheel Stand.
Size of wheel, 18″ × 2″.
Speed of wheels, 950 revolutions per minute.
Kind of work, 2 laborers grinding castings.
Maximum horse-power, 6 H. P. momentarily.
Average horse-power, 3.5 H. P.
Horse-power motor required, 5 H. P. open, with dust-proof covers.
Method of drive, Direct-current Compound-wound Motor, mounted on grinder-shaft between the wheels.

No. 9.

Description of machine, Vertical Boring Mill.
Make of machine, Pond Machine Company.
Size of machine, 10-foot table.
Number of cutting tools, 2.
Size of cut, $\frac{3}{4}$″ × $\frac{1}{16}$″
Cutting speed, 20 feet per minute.
Material machined, cast-iron.
Weight on table, 3.5 tons.
Motor power required, 8.58 H. P.
Method of drive, Direct-current Compound-wound Motor, belted to counter-shaft. 12 H. P. motor gave good results on heaviest cuts and weights of castings.

No. 10.

Description of machine, Slotter.
Make of machine, Bement & Miles.
Number of cutting tools, 1.
Size of cut, $\frac{3}{8}$″ × $\frac{1}{16}$″.
Speed of tool, 20 feet per minute.
Material machined, open-hearth steel castings.
Motor power required, 6.98 H. P.
Method of drive, Direct-current Compound-wound Motor, belted to counter-shaft.

No. 11.

Description of machine, Flat Turret Lathe.
Make of machine, Jones & Lamson.
Size of machine, 2″ × 24″, their standard.
Motor power required, 1½ H. P. for satisfactory service.

No. 12.

Description of machine, Tool Grinder.
Make of machine, Gisholt Machine Company.
Size of wheel, their standard cup wheel.
Speed of wheel, 16 to 18 revolutions per minute.
Maximum horse-power required, 7 for short periods.
Average horse-power required, 4.
Method of drive, Direct-current Compound-wound Inclosed Motor, mounted on grinder-shaft, with field rheostat in series with shunt coils to increase speed from 1600 to 1800. A 5 H. P. open motor with inclosing covers gave good satisfaction on this grinder.

No. 13.

Description of machine, Engine Lathe.
Make of machine, Hendey Norton.
Size of machine, 16 inches.
Motor power required, approximate, 2 H. P. at maximum.
Method of drive, Direct-current Compound-wound Motor, mounted on support, bolted to bed of lathe, and equipped with clutch and cone pulley, with belt to headstock cone.

No. 14.

Description of machine, Engine Lathe.
Make of machine, Putnam.
Size of machine, 18″ × 6′ between centres.
Motor power required, 2.1 H. P.
Method of drive, Direct-current Compound-wound Motor, geared to counter-shaft.

No. 15.

Description of machine, Engine Lathe.
Make of machine, Pond Machine Company.
Size of tool, 36″ × 10′ between centres.
Motor power required, 10 H. P.
Method of drive, Direct-current Compound-wound Motor, direct-geared to counter-shaft.

On all the preceding machines, where motors are geared, raw-hide pinions were used on motor-shaft.

Electric Cranes.

In discussing electric driving before the Engineer's Society of Western Pennsylvania, Mr. S. S. Wales gives data as to the power required for electric cranes.

As in a general crane specification the actual weights of material and gear reduction, etc., are not known, some arbitrary assumptions will have to be made and some empirical formulæ will be used, but as both are founded on facts and experience some reliance may be placed in them.

An electric crane is divided into three general parts,—bridge, trolley, and hoist,—each of which has its own motor and controlling system, and each subjected to different conditions of work.

For the bridge, where the ratio of axle-bearings to diameter of wheel is between 1 to 5 and 1 to 6, the following table will answer our purpose for weights and traction for different spans.

Let

L = working load of crane, in tons;
W = weight of bridge alone, in tons;
w = weight of trolley alone, in tons;
S = speed, in feet, per minute;
P = pounds per ton required.

Span.	*W.*	*P.*
25 feet.	.3 *L.*	30 pounds.
50 feet.	.6 *L.*	35 pounds.
75 feet.	1.0 *L.*	40 pounds.
100 feet.	1.5 *L.*	45 pounds.

For the trolley we would assume the weight and traction as shown in the following table:

L.	*W.*	*P.*
1 to 25 tons.	.3 *L.*	30 pounds.
25 to 75 tons.	.4 *L.*	35 pounds.
75 to 150 tons.	.5 *L.*	40 pounds.

Now the power required for bridge will be

$$\frac{(L + W + w) \times P \times S}{33000} = \text{horse-power},$$

which result will be used in connection with the motor characteristic to determine the gear reduction from motor to track wheel. As the nominal horse-power rating of a series motor is based on an hour's run, with a rise of 75° C. above the surrounding air, and as conditions of bad track, bad bearings, or poor alignment of track wheels may be met with, in factory operation 1½ times the above result should be taken as the proper size motor for the bridge.

For the trolley the power required would be

$$\frac{(L + w) \times P \times S}{33000} = \text{horse-power},$$

which will be used for speed and gear reductions; but $1\frac{1}{4}$ times this should be used for size of motor.

For hoist work we cannot have so large a margin of power, as the variation from full load to no load may imply a possible dangerous increase of speed, and unless the crane is to be subjected to its maximum load continuously, or is to be worked where the temperature of the surrounding air will be high, it is safe to use the size by assuming 1 horse-power per 10 foot-tons per minute of hoisting. This is nearly equal to assuming the useful work done as 60 per cent. of the power consumed.

As an illustration, let us take a crane of 50-ton capacity; lifting speed of hoist, 15 feet per minute; bridge to be 70 feet span and to run 200 feet per minute with load; trolley to travel 100 feet per minute with full load. On the foregoing assumption the bridge would weigh 50 tons and require 40 pounds per ton for traction, and the trolley would weigh 20 tons and require 35 pounds per ton for traction.

Mr. Wales also gives formulas for the power required for driving the rollers in rolling-mill tables.

The power required by roller tables in mill work varies greatly, as they are subjected to tight bearings and lack of oil to a greater extent than electric cranes; and as there will be from $2\frac{1}{2}$ to 3 bearings to each roller, and many rollers per table, the chances for trouble are greatly multiplied.

For the average conditions of mill tables, where each roller is driven by a mitre gear from a common line-shaft and with usual mill lubrication, the following empirical formulæ, derived from the test of 20 tables, represent about the power required:

$$\frac{W \times D \times S \times N}{950\,000} = \text{horse-power},$$

where w = weight of roller, in pounds, the load to be carried on table being considered as uniformly distributed over all rollers, $1\text{–}N$ to each.

D = diameter of bearings, in inches;
S = speed of table, in revolutions per minute, of rollers;
N = number of rollers in table.

The same $1\frac{1}{2}$ times power required for size of motors should be taken as for crane bridges.

This takes no account of diameter of roller used, which would of course have some effect on the power required to move the load to be handled, and would also show some fly-wheel effect when starting, but still it will check fairly well with tables now in use under existing conditions, two examples of which are given here:

$N = 18$;
$W = 1000$ pounds;
$D = 4\frac{1}{2}$ inches;
$S = 200$ revolutions per minute.

Diameter of roller, 10 inches.

$$\frac{1600 \times 4\frac{1}{2} \times 200 \times 18}{950\,000} = 27.2 \text{ horse-power}.$$

From actual test under working conditions, this table required 28.8 horse-power, or the nearest Westinghouse motor being No. 38,—50 horse-power,—this type should be used. As a matter of fact, this table is equipped with a 30 horse-power motor, and is the source of continual annoyance from over-load.

$N = 16$;
$W = 1000$ pounds;
$D = 3$ inches;
$S = 110$ revolutions per minute.

$$\frac{1000 \times 3 \times 115 \times 16}{950\,000} = 5.8 \text{ horse-power}.$$

By actual test 5.5 horse-power was required.

Choice of Motors and System.

In discussing the selection of electric motors for driving machinery, Mr. P. R. Moses, writing in the *Engineering Magazine* for September, 1901, says:

"The best system in general will be that which will be free from breakdown, able to stand hard usage and frequent sudden overloads, simple and safe to handle, with parts standard and available. It should be uniform and applicable to all the requirements liable to arise in the work contemplated, the speed of the motors should be variable at will of the operator, and in some cases, like hoisting, should vary inversely with the load to prevent undue use of power. The motors should start with small currents and should have high efficiencies at average loads. The first cost should be as low as possible, and the number of parts a minimum.

"The alternating two- or three-phase system at low pressure (500 to 220 volts) meets the first few conditions slightly better than the direct-current system of the same voltages. This system consists of a polyphase generator composed of a stationary and a revolving part, an exciter—sometimes revolving on shaft, sometimes belted to shaft—for delivering the current required to magnetize the fields, a system of distributing wires and motors frequently built without brushes, but sometimes, where adjustable speed and good dynamo regulation are required, with brushes and collector rings. The system is simpler than the direct-current system, in that no current has to be delivered to any moving part of the motors. In the direct-current system, current must be delivered to the rotating armatures of the motors through brushes of carbon and commutators made up of copper bars held firmly, clamped by a collar, with mica between the bars and between the bars and collar. This commutator is the chief difference between the motors of the polyphase alternating systems and the motors of the direct-current systems. The connections to the commutator and the commutator itself are the only parts of the motors in which trouble is liable to arise, with careful construction; and although probably a hundred thousand are in use daily, and the manufacture has been carefully studied, trouble does arise,—generally on account of accumulation of grease or dirt, allowing the current to jump from the copper bars to the iron frame of the machines, or from breaking of connections between winding and commutator lugs, caused by frequent stopping and starting, combined with overheating and slow cooling. This alternation of heating and cooling causes the copper to become brittle and—unless the connections are made flexible—to break off.

"The advantages of simplicity, durability, and freedom from breakdown, therefore, are with the alternating polyphase motors,—more especially of the brushless type; but, unfortunately for the polyphase system at the present day, all the other requirements are much more easily and better met by the direct-current motor.

"The alternating-current system is not yet fully standardized, but is constantly being perfected and broadened in its scope. Its parts are obtainable from but two or three first-class companies; it is not applicable yet to charging storage batteries, to railroad work, or to hoisting, although it has been used for both the last; the speed of motors is not adjustable unless the brush and collector-ring type is used; the starting currents under load are large, causing more or less fluctuation in lights; and the first cost of dynamos and motors is between 25 per cent. and 35 per cent. higher than that of direct-current apparatus of the same capacity. Therefore, unless the value of adjustable speeds, 25 per cent. less first cost, higher average efficiency, etc., are balanced by the possibility of commutator troubles, the direct-current system is at present preferable and advisable for ordinary cases of factory transmission. In such cases the polyphase alternating current's value is confined to the transmission of power from distant sources at higher pressures. In special cases the alternating-current system may prove advisable even for medium distances. One of these instances is in hat, candy, or similar factories, where electricity is largely used for heating, as well as for power and for light; the ease with which an alternating current can be transformed—*i.e.*, small quantity at high pressure changed to large quantity at low pressure, or *vice versa*—gives this system the preference, as quantity and not pressure is the essential feature of electric heating. Works where naphtha or other explosive gas is used require a sparkless motor, and the alternating motor is the only one fitted for this use. Machine shops with a number of small, scattered machine

tools may be better suited by one system or the other, depending on the question as to the advisability of direct connection of tools, the amount of probable overload, and the question of speed variation. In mills, large machine shops, and factories of all kinds, where the power to transform is not valuable, there is no benefit to be derived from the polyphase current sufficient to offset the disadvantages mentioned.

"As to the best pressure to be used for transmitting direct current, the answer cannot be so definite. The advantages of a high transmitting pressure, such as 440 to 500 volts, are low first cost and small bulk of wiring, switches, and controlling apparatus. The disadvantages are increased liability to ground, danger of shock, increased number and decreased size of field armature wires and of commutator bars. The liability to ground may be guarded against in the construction, but the other difficulties are inherent, and are sufficiently objectionable to make the 500-volt motor and system inadvisable, except for such purposes as electric railroads, where power has to be transmitted a long distance. The 220 to 250-volt motor has only half as many commutator segments, armature conductors, and connections as the 500-volt motor; and the 110 to 125-volt motor but one-quarter the number. The sizes of these parts increase in proportion to the decrease in number; hence, the lower the voltage of a motor, the more substantial and the stronger mechanically. The lower voltage offers the advantage, too, of decreased tendency of the current to jump to the frame or from one wire to another. The winding of the fields of the 120-volt motor is made up of less than one-half the number of turns used in the 240-volt motor, and while the wire has twice the area, it does not occupy twice the space; on this account the field-coils of the low-voltage motor do not heat as much as the higher-voltage motor. On the other hand, the resistances used to control the speed and starting current, the switches, fuses, and the wiring are much larger and heavier for the 120-volt motor than for the 240-volt motor, and for motors of large size this is a decided disadvantage, as it interferes with ease of regulation and control, and increases the first cost.

"For small motors, 5 to 50 horse-power, for transmission over short distances (from 200 to 400 feet), and for fluctuating work, such as elevators, presses, cranes, punches, etc., the 125-volt 2-wire system seems preferable. For large motors, transmission over comparatively long distances, or for steady work, where mechanical strength is not essential and where the motor will receive attention, as in silk mills, carpet works, etc., or for work where minimum weight and size are important, the 240-volt system, or one of higher pressure, is usually the best fitted for the purpose.

"It is my opinion that at present prices the 125-volt direct-current 2-wire system is the preferable one in most instances, and little or no trouble with the motors or dynamos is experienced at this voltage. Where the distance through which the power is to be transmitted is such as to make the cost of wires for carrying the current too great, the 240-volt direct-current 3-wire system should be used. Where there are special features, such as those heretofore mentioned, or where the power is transmitted a distance too great for the 240-volt direct-current system, the three-phase alternating-current system, 60 to 70 cycles per minute, becomes necessary and gives thoroughly satisfactory results.

"The greatest drawback to the alternating-current system to-day is its excessive cost, which brings no equivalent advantage. In the course of a few years this difference will disappear, and the system will probably be used for all situations demanding a higher pressure than 125 volts."

Speed Variation.

The question of the control of speed with electric driving of machine tools is an important one, especially as the use of modern high-speed tool steels involves the correct speeding for each diameter and material in the lathe, boring mill, or other machine. This subject was thoroughly discussed at a meeting of the Engineer's Club of St. Louis, January 7, 1903, and some abstract of the points there made represent the latest opinions.

Mr. W. A. Layman says:

"The individual drive system may be generally classified under three headings:

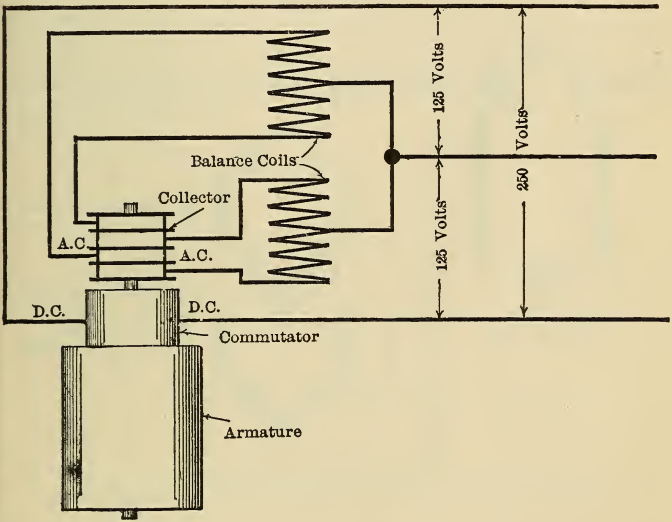
"Rheostatic control systems,
"Multi-voltage control,
"Special systems for special tools.

"In the rheostatic control system the motor is of the well-known shunt type, supplied from a constant-potential system of distribution. Speed variation above the normal speed of the motor is secured by the introduction of resistance into the motor shunt-field circuit; speed variation below normal is secured by the introduction of resistance into the armature circuit.

"The disadvantages of the system are its inefficiency when armature resistance is made use of for speed reduction, and variation of speed on a given armature resistance with variation of load. To overcome both disadvantages, motors have been designed capable of a very wide variation in speed by variation of field resistance.

"The multiple-voltage control is regarded with favor by many.

"The Westinghouse and the General Electric Companies use a 3-wire system, as shown in the illustration.



Westinghouse 3-wire System for Variable Speed Control.

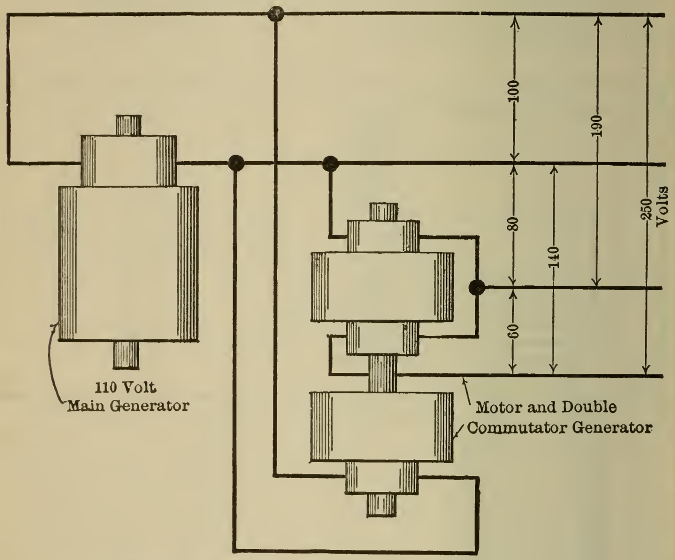
"The usual direct-current generator is provided with a set of collector rings, these collector rings being connected to the armature winding in such a way as to establish an exact two-phase relation between the potentials of the two pairs of collector rings. By means of choking coils, connected as shown, the neutral wire of the 3-wire system is exactly and constantly maintained, irrespective of load, at zero potential relative to the outside wires.

"In connection with this 3-wire system the individual tool is equipped with a standard 250-volt shunt motor, and speed variation is secured in two ways: first, by running the armature either on 250 volts (normal speed condition); and second, by running it on 125 volts (half normal speed condition). For any speed desired between normal and half normal, shunt field resistance is introduced.

"The shunt motor is capable of 100 per cent. speed variation by variation of shunt resistance when the armature is on half voltage (and correspondingly at half load). If speed above normal full speed is required, shunt resistance is again introduced.

"The Bullock Electric Manufacturing Company advocates a system as

shown in the illustration. A generator, standard in every respect, is supplemented by a small motor-generator set, the design of which is such that a 4-wire system of distribution is established, providing for six different voltages upon which the motor armature may be operated without the use of armature resistance. The form of motor used is the standard shunt-wound type. Without the use of field resistance six speeds may be secured, corresponding in ratio to the ratio of the voltages supplied by the 4-wire distribution system. By means of shunt resistance any speed intermediate to that possible with the several armature voltages may be secured. The motor-generator set is so proportioned as to take care of the



Bullock Electric Manufacturing Company's Multiple-voltage System.

unbalanced load. This system is also adaptable to 3-wire distribution, where less speed variation is required; and in the event of 3-wire distribution an increased amount of field regulation is introduced. This 3-wire distribution differs from the Westinghouse and General Electric systems, in that the voltages on the two sides of the intermediate wire differ, thus giving three pressures instead of two."

In the same discussion Mr. W. Cooper said, in considering the fact that many tools are not designed to stand the high speeds that modern tool steels will permit, "that even if the machines will not stand the maximum speed of the tool, they may yet be operated at the highest speeds of which they are capable.

"There is no reason why a machine tool that is adapted to do a certain work should not do this work at two or three times the speed. The reason for this seems obvious, in the fact that the strains on a machine are due entirely to the torque required to make a given cut. With this given cut the speed may be increased three or four times without producing any greater strains on the machine itself, because the torque remains constant. However, the horse-power will increase directly in proportion as the increase in speed; and right here we have a factor that limits the ordinary belted machine tool,—the belts will not pull the load. For instance, suppose that we have a given machine running with a belt on the largest

step of the cone pulley on the machine, and taking a certain cut; assume the cutting speed to be 20 feet a minute. If it is desired to increase this cutting speed to, say, 80 feet per minute, it is found necessary to put the belt on the smallest step on the cone on the machine. We at once encounter the difficulty that the belt will not begin to pull the cut. This is also true of the various mechanical speed-changing devices that have been introduced. Thus it will be seen that machine tools that were designed on the lines of the cutting speed of 20 feet per minute are not adapted at all to cutting speed of 80 feet per minute. However, they are not limited by the strength of the tool, but by the pulling power. Under these conditions it is only necessary to increase the pulling power of this machine to make it do four times the actual work that it formerly did.

"This can readily be accomplished by the use of the electric motor, so that the limit lies in the stiffness of the bed or frame of the machine to carry the increased load without springing or chatter."

In a paper read before the Iron and Steel Institute, in 1903, by Mr. D. Selby-Bigge, the power required to operate electrically a great variety of machine tools was tabulated. Although in many instances the depth of cut and the speed of cutting surfaces is not given, yet it is understood that these records represent average practice with ordinary tool steels, and that for lathes, boring mills, etc., the power given in the tables would have to be increased in proportion to the increased capacity given by use of the modern high-speed steels.

Motor Tests.

Designation of machine.	Work done.		E. H. P. absorbed by motor with machine running light.	E. H. P. absorbed with load on.	Size of motor used. B. H. P.	Remarks.
	Maximum.	Average.				
Large plate-edge planer.	35′ × 1″.	7/8″ or 1″ plate.	14.75	24.50	30	Cut 0.05″ thick on 1″ plate 4′ long at time of test. This machine took 29.5 horse-power to reverse.
Small plate-edge planer.	15′ × 3/4″.	5/8″ to 3/4″ plate.	6.33	20.10	25	Cutting 3/4″ plate 22″ long. 21 horse-power to reverse.
Plate-straightening rolls.	4′ 6″ × 4′ 6″ × 1 1/8″.	10″ × 1/2″ to 15″ × 3/4″.	4.50	6.10	......	10″ × 1/2″ plate.
				16.10	......	Plate 4′ square by 1 3/16″ thick.
				14.00	15	Same plate on slow speed. This machine took 19.1 E. H. P. to reverse.
Cold saw	16″ × 6″ girder.	About 12″ × 6″.	1.45	2.60	7	Cutting 12″ × 3 1/2″ × 1/2″ channel. Saw fitted with friction gear for feed, which prevents a heavy load.
Punch shears and angle cutter.	12″ shear, 7/8″ plate.	....	3.85	6.00	10	3/4″ hole, 3/8″ plate. 25 per minute.
	15/16″ hole, 7/8″ plate.	3/4″ hole, 1/2″ plate.	....	8.85	......	15/16″ hole, 3/4″ plate. 20 per minute.
				5.15	......	Shearing 10″ × 1/2″ plate.
				4.90	......	Cutting 3″ × 3″ × 3/8″ angles.
Large ending machine.	Knives 22 1/2″ long.	....	1.88	15.42	15	Ending 20″ × 7 1/2″ girder, 1/2″ web, 1″ flange.
Straightener and angle cutter.	Angles and small bars.	....	1.53	11.55	10	Cutting 3 1/2″ × 3 1/2″ × 1/2″ angle (momentary load).
				6.15	......	Straightening 6″ × 6″ angle-piece.

Shafting driving 6 radial drills.	1″ holes.	¾″.	3.70	7.5	8	All six drills at work. ¾″ holes in girder steel.
Large straightener	20″ × 7½″ girders.	12″ × 6″ girders.	3.96	6.9	8	Straightening 20″ × 7½″ girder.
Milling-tool notching machine.	....	....	2.42	8.05, maximum load.	8	4″ × ½″ notches in flanges of 8″ × 4″ girders.
				5.8, average load.	......	Two notches cut at once.
Joggling machine	1¼″ plate.	¾″ plate.	9.85	16.00	......	Joggling $\frac{11}{16}$″ plate.
Scarphing winch	2 tons at 40 feet per minute.	....	5.65	12.10	......	....
				10.75	......	....
Shafting	Driving lathes, nibblers, planes, machines, etc.	Slotting machines, drills, milling,—about 70 in all.	21 E. H. P.	40–48	50	No. 1 shaft drives about 70 machines.
			First motor, 23 E. H. P.; second motor, 22 E. H. P.	40–50	50	No. 2 shaft drives about 40 machines on first motor.
			Second motor, 24 E. H. P.	42–50	50	No. 3 shaft drives about 25 machines,—slots and planers.
Grindstones	....	....	20.5 E. H. P.	53–55	60	7 grindstones, varying from 5 to 10 feet in diameter; each has fast and loose belt. Also 1 Tusca grinding machine on same shaft.
Cranes	....	....	Driving-ropes doing no work, 23 E. H. P.	48	50	All rope-driven cranes. 2–30 ton cranes, 1–15 ton cranes, 2–10 ton cranes; all cranes on nearly full load at once.
Cold rolls (locomotive work).	Plates 40′ × 4′ 6″ × 1½″.	1″ plates.	Driving countershafts and belts only, 5 E. H. P.	30	30	Straightening a plate 36′ × 4′ 6″ × 1¼″.
				7	......	Straightening a plate 9′ × 17″ × 1″. Machine is driven by cross and open belts, and has 4 top rolls and 3 under rolls.

Motor Tests.—*Continued.*

Designation of machine.	Work done.		E. H. P. absorbed by motor with machine running light.	E. H. P. absorbed with load on.	Size of motor used. B. H. P.	Remarks.
	Maximum.	Average.				
Joiners' shop machines	....	....	Driving shafting, 5.5 E. H. P.	10	20	During a week this motor was not seen to rise above 10 horse-power. When busy it absorbs about 17 to 18 E. H. P. It drives 12 tools, including 2 circular saws.
Vertical ship-yard drills and countersinks.	1¼″ holes and 1¾″ countersinks.	....	Driving counter-shaft, off which 2 per cent. drills are worked, 3.5 E. H. P.	7.5	7	1″ hole and 1½″ countersink; 2 machines doing 280 holes in 8 minutes.
Punch and shears	1¼″ holes in 1.	Plate.	2.7 E. H. P.	6.0	12	⅞″ holes, ⅝″ plate, 36 strokes per minute.
	Shearing 1″.	Plate.	....	15.0	......	Shearing ⅝″ plate 16 feet per minute.
Punch and shears	Shearing 1″.	Plate.	2.75 E. H. P.	3.5	12	⅝″ holes, ½″ plate, 34 strokes per minute.
				11.0	......	Shearing ½″ plate 11 feet per minute.
Mangle rolls..........	1⅛″ plates.	¾″ × ⅝″.	4.5 E. H. P.	13.5	15	Motor is a series motor, with a tramway controller; belt drives direct to machine, and motor is reversed when mangling. This arrangement has given most excellent results.

Wood-working Tools.						
Hatch-boring machine	1″ holes.	⅝″ holes.	2.05 E. H. P.	3.0	3.5	Boring ⅝″ hole in hatch 22″ broad, in 2 minutes.
Wood-planing machine.	....	....	4.4 E. H. P.	8.8	8	Cutting ⅛″ off plank 10″ broad × 17′ long, in 1.5 minutes.
Wood-planing machine (heavy).	....	....	4.5 E. H. P.	16.0	16	⅛″ off plank 7″ broad × 13′ 6″ long, in 30 seconds.
32″ circular saw	....	....	0.9 E. H. P.	14.8	15	Cutting 3″ teak—hand-fed—14′ long, in 20 seconds.
Heaviest class of shearing machinery.	Steel tires of locomotive wheels.	General scrap.	2 E. H. P.	29.0	30	Machine cut about 35 pieces 11″ × 1⅛″ section, in 1 minute.
Punching machine....	2″ diameter of holes in 1¼″ plates.	1¼″ holes in 1″.	3 E. H. P.	10.5	12	Punching 19 holes per minute, 1⅞″ in diameter, through 1″ plate. Machine driven by belt.
Large ship-yard rolls..	26′ rolls to bend 1¼″ plates.	⅝″ and ¾″ plates.	8 E. H. P.	26.0	30	Bending plates 20′ × 4′ × ¾″ (load at reverse 26 E. H. P.). Rolls driven by cross and open belts.
18″ dock pump	....	....	....	74.0	70	Pump discharges about 500,000 gallons per hour.
5″ dock pump	....	....	....	11.0	10	Leakage pump.
Plate-edge planer	⅝″, ½″, and ⅜″ plates.	Planing about 24′ per minute.	5½ E. H. P.	⅜″ plates, 10.0	20	This machine took 23 E. H. P. to reverse; motor has cross and open belts on; special wide pulley, thus dispensing with the use of a counter-shaft.
Punch and shears	1¼″ × 1¼″.	1″ holes in ⅝″ plates.	3.5 E. H. P.	7.5	10	1″ hole, ⅝″ plate. 31 strokes per minute.

Motor Tests.

Works department or machine.	Work done by motor.		E. H. P. absorbed.				Size and type of motor.	Remarks.
	Total of machines on motor.	Usual and average machines on load.	Shafting and all machines light.	Average machines light.	Average machine loaded.	Maximum load on motor.		
Fitting-shop motor.	2–9″ centre lathes. 1–12″ centre lathes. 1–3′ centre chuck lathe. 1 planing machine, 9-foot stroke. 1 slotting machine. 1 milling machine. 1 shaping machine. 1 punch and shears machine. 2–2½″ vertical drills. 1 emery wheel. 1 cold saw (hack). 1 fan, supplying 9 smiths' hearths. Shafting: 20′ × 3″ C shaft; five others to 10 feet long.	2–9″ centre lathes. 1–12″ centre lathes. 1–3′ chuck lathe. 1 slotting machine. 1 drilling machine. 1 emery wheel. Fan.	Shafting alone, 1.75 E. H. P. All machines light. 62 E. H. P.	3.7 E. H. P.	9.5 E. H. P.	14.9 E. H. P., with fluctuations to 19 E. H. P.	20 B. H. P.	
No. 1. Galvanizing-house motor.	4 large galvanizing pots, each for 20 tons of metal. 2 drying machines, attached. 1 large sheet-stretching machine. 1 large corrugating machine (press). 2 circular shearing machines, counter-shafting, and belts.	All machines, pots, and drying machine continuous, and others intermittent.	Shafting alone, 1.5 E. H. P.	....	Average full load, 11.2 E. H. P.	Fluctuation to about 18 E. H. P.	20 B. H. P.	

No. 2. Galvanizing-house motor.	1 large galvanizing pot, for 20 tons of metal. 2 small galvanizing pots, each for 10 tons of metal. 1 drying machine, attached. 1 rolling machine. 1 stretching machine.	All machines, pots, and drying machine continuous, and others intermittent.	Shafting alone, 1 E. H. P.	....	Average full load, 6.9 E. H. P.	Fluctuation to about 12 E. H. P.	10 B. H. P.	
"Dolomite" crusher.	1–9′ in diameter pan mill, short counter-shaft.	....	2.6 E. H. P.	....	8.2 E. H. P.	11.5 E. H. P., occasionally.	10 B. H. P.	
Stamping-house motor.	20 stamping machines, of various sizes and types, for armature and field core plates.	....	6.5 E. H. P.	....	9.8 to 10.3 E. H. P.	11 to 12.5 E. H. P.	10 B. H. P.	
Steel charging machine, in steel mill.	1 electrically-driven charging machine, charging and discharging,—*i.e.*, heating furnaces and supplying live rolls with ingots up to 10 cwt. each.	....	Motor and first, second, and third motion shafts and gearing, 1.55 E. H. P.	Lifting, 3.2 to 4 E. H. P.; travelling, 7.8 E. H. P.; slewing, 4.9 E. H. P.; racking, 5 E. H. P.	11, 11 to 12.8, 6.3 to 8, and 6.5 to 8.5 E. H. P.	Maximum, when working 3 motions simultaneously, about 22 E. H. P., under worst conditions.	20 B. H. P.	Weight of standard billet lifted, 10 cwts. (maximum, 12 cwts.). Travelling rails, 6′ gauge; wheels, 2′ diameter. Collector gear = D. P. overhead, bare trolley cable, with tramway collecting gear. Travel of peel = 12′ 6″. When peel is racked in towards machine, the distance between centre of bogie and steel billet is approximately 20′, taken radially. When peel is racked out it is approximately 32′ 6″ from centre of support. Total lift of peel = 3′.

Motor

Tests on Cranes. Quick-motion,

Taken August

Works department.	Size and type of travelling crane.	Work done by crane.		Actual tests on crane.		
		Average.	Maximum.	Motion.	E. H. P. absorbed. Crane light.	
					Starting effort.	Running Power.
No. 1 crane. Plate mills loading.	5-ton 3-motor crane. Works in exposed position, both in and out of shop.	1 to 3 tons.	$4\frac{1}{2}$ tons.	Lifting.	13.4	.50
				Traversing.	14.8	7.70
				Travelling.	23.6	11.80
No. 2 crane. Plate mills floor.	5-ton 3-motor crane. Works under cover.	1 to 2 tons.	4 tons.	Lifting.	18.0	9.60
				Traversing.	20.6	9.35
				Travelling.	28.5	11.3
No. 3 crane (new). Plate mills loading.	6-ton 3-motor crane. Built and erected by Dowlais Cardiff Works. Works in exposed position, both in and out of shop.	1 to 3 tons.	$4\frac{1}{2}$ to 5 tons.	Lifting.	22.3	9.5
				Traversing.	11.3	6.2
				Travelling.	29.5	12.0

NOTE.—The starting efforts given above can be regarded only as approximate, being merely momentary, and volts drop being disregarded. The cranes above are on 220 volts circuit. One longitudinal trolley wire

Tests.

Overhead Travelling.

18, 1902.

Figures taken August 18, 1902.				Size of motor. H. P., brake or electrical.	Remarks.
E. H. P. absorbed. Crane loaded.		Speeds and loads during test.			
Starting effort.	Running power.	Actual load.	Approximate speeds.		
29.1	18.3	3 tons 6 cwt.	43 feet per minute.	22 E. H. P.	This crane has the motors fixed on the main girders above end carriage. Drives by square shaft and gear. Water starting and regulating switch, and metallic lever controlling switch.
16.8	9.6		125 feet per minute.	15 E. H. P.	
29.4	12.6		150 feet per minute.	22 E. H. P.	
27.8	12.8	3 tons.	Speeds approximately as above.	22 E. H. P.	As above.
24.5	10.3			15 E. H. P.	
34.0	12.4			22 E. H. P.	
29.4	19.2	3 tons 6 cwt.	60 feet per minute.	20 B. H. P.	This crane has travel motor on main girders and lift and traverse motors on the bogie. Gear driven. Water starting and regulating switch, and metallic lever controlling switch.
14.9	7.3		150 feet per minute.	20 B. H. P.	
32.8	13.8		165 feet per minute.	20 B. H. P.	

only employed, the return being to "earth." The above working-load tests present a fair and good heavy average load, and are seldom exceeded under actual working conditions in these works.

Motor Tests.

Taken August 18, 1902.

Description of machine.	Work done by machine.	E. H. P. absorbed.		Type and size of motor.	Remarks.
		Light.	Loaded.		
3-ton skull-breaking winch.	Lifts ball weighing 3 tons 8 cwt. to height of 50 feet at speed of 60 feet per minute (timed).	8.5	17.8	2-pole open type armature at bottom, 18 E. H. P., series-wound.	This winch is of ordinary band pattern, driven through works and spur gear, with brakes and clutch. Water starting and regulating switch.

A table of machine tests in a somewhat different form is appended:

Condensing Plant.

One 10-horse-power motor of 220 volts, driving direct-coupled 3-inch centrifugal pump, driving also with belt air pump 9½ inches in diameter by 9-inch stroke, and feed pump 2 inches in diameter by 9-inch stroke. Boiler pressure, 200 pounds.

Operation.	Revolutions.	Amperes.	Volts.	Vacuum, in inches.	E. H. P.	
					Total.	Actual per operation.
Centrifugal pump	1100	6	240	....	1.9	1.9
Centrifugal with air and feed pump.............	160	12	240	27	3.8	1.9

Brass-shop Motor, 5 Horse-power, 240 Volts.

Operation.	Revolutions.	Volts.	Amperes.	E. H. P.	
				Total.	Actual per operation.
Motor and shaft	220	250	7.00	2.3	2.3
Disc grinder, 18-inch emery discs running light...................	1800	246	8.50	2.8	.5
Facing 6½-inch brass valves......	1800	246	24.00	7.9	5.6
6-inch capstan lathe (light)	....	248	9.75	3.2	.9
Turning and screwing 1⅛-inch brass bars for ¾-inch tap bolts..	....	248	12.0	4.0	1.7
Parting 1⅛-inch brass bars for ¾-inch tap bolts................	....	248	10.0	3.3	1.0

No. 1 Foundry.—Roots Blower, Acme No. M.

Operation.	Revolutions.	Volts.	Amperes.	E. H. P. Total.	E. H. P. Average.	Time, in hours.
Motor and shafting, light running	....	245	14	4.6	....	......
Blowing cupola:						
Maximum	360	246	104	32.7	28.2	$4\frac{1}{2}$
Minimum	350	233	66	21.7		

Total weight of iron melted, 22 tons 10 hundredweights; total weight of iron melted per hour, 5 tons.

No. 2 Foundry.—Roots Blower, Acme No. K.

Operation.	Revolutions.	Volts.	Amperes.	E. H. P. Total.	E. H. P. Average.	Time, in hours.
Motor and shafting, light running	....	232	9.5	2.96	....	......
Blowing cupola:						
Maximum	430	237	57.0	17.1	15.94	3
Minimum	394	225	50.0	15.2		

Total weight of iron melted, 12 tons; total weight of iron melted per hour, 4 tons.

REMARKS.—No. 1 cupola is capable of melting on an average 7 tons per hour.

Boiler Shop.—Vertical Plate-bending Rolls.

Length of rolls, 11 feet 7 inches; diameter of rolls, 1 foot 11 inches; mean size of plates rolled, 20′ × 10′ 6″ × 1″; maximum size of plates rolled, 16′ × 11′ 5″ × $1\frac{1}{2}$″.

Operation.	Amperes.	Volts.	E. H. P. Total.	E. H. P. Actual per operation.	Time, in minutes.
Motor and shafting, light running	20	242	6.40	6.4	......
Rolls, light running	24	242	7.70	1.3	......
Rolling plate, 23′ × 11′ 2″ × $1\frac{1}{4}$″........................	90–68	233	Average. 19.2	12.8	3
Putting squeeze on plates......	30–60	233	9–18	6–12	......
Reversing the rolls	50	233	15.6	9.2	......

Air Compressor, Belt Driving from Motor.

Operation.	Revolutions.	Volts.	Amperes.	E. H. P.	Remarks.
Motor, shafting, and pumps.	175	230	22	6.7	Air compressor, 9 inches in diameter, 10-inch stroke.
Pumping up to maximum pressure.	170	230	70	21.5	Maximum pressure, 80 pounds.

Pattern-shop Motor, 15 Horse-power, 220 Volts.

Operation.	Revolutions.	Amperes.	Volts.	E. H. P. Total.	E. H. P. Actual per operation.
Motor and shafting	170	9.5	233	2.9	2.9
Circular saw, 2 feet 8 inches in diameter, running light	800	10.5	233	3.2	.3
Cutting yellow pine 11 inches deep, 7 feet per minute	Maximum.	40.0	233	12.4	9.2
	Minimum.	18.0	233	5.6	2.4
	Average.	29.0	233	9.0	5.8
Thickness of machine, 2 feet 6 inch bed, running light	3800	12.0	230	3.7	.8
Surfacing yellow pine 11 inches wide, 13 feet per minute	....	17.9	232	5.5	1.8

37 Horse-power Motor.

Operation.	Revolutions.	Amperes.	Volts.	E. H. P. Total.	E. H. P. Actual per operation.
Motor and shafting	....	12	230	3.7	3.7
Circular saw, 3 feet in diameter, running light	1200	21	230	6.4	2.7
Sawing yellow pine 11 inches deep, 20 feet per minute	....	71	230	21.5	15.1
Circular saw, 33 inches in diameter, running light	....	26	230	8.0	4.3
Cross-cut lignum-vitæ, 9¼ inches deep by 18 inches long	....	41	230	12.6	4.6

30-Ton Cranes. Boiler Shop, 25 Horse-power Motor, 220 Volts.

Operation.	Amperes.	Volts.	E. H. P. Total.	E. H. P. Actual per operation.	Time, in minutes.	Feet.
Motor and shafting and belts (light)	12.0	230	4.3	4.30	....	......
Heaving (light)	16.0	230	4.9	.60	....	......
Cross travel (light)	15.0	230	4.8	.50	....	......
Longitudinal travel (light)	16.0	230	4.9	.60	....	......
Longitudinal and cross travel (light)	18.0	230	5.5	1.20	....	......
Travelling long	16.2	230	4.9	.68	1	90
Weight, 16 tons, heaving	36.0	230	11.1	6.2	1	5
Cross travel	30.0	230	9.2	4.4	....	......
Longitudinal travel	18.0	230	5.5	.6	1	90

THE COST OF POWER.

In a series of articles in the Journal of the Franklin Institute, October, November, December, 1901, Mr. Clyde D. Gray gives an extended discussion of the cost of power under various conditions, and from these papers the following abstract is made:

Water Power.

The costs of water-power plants are widely different, depending upon the location, size, and extent of the hydraulic works needed, length of penstock and flume, and many other things that differ in the various localities. Below are given some figures in regard to the costs of plants. These are low-head plants fitted with turbine wheels, and are used principally for mill or factory purposes. The costs do not include costs of dam unless so specified, but include everything else in the plant. The horse-power basis upon which they are figured is the horse-power delivered at the wheel shaft.

WATER-PLANT COSTS.

Place.	Cost per D. H. P.	Authority.
Lawrence, Massachusetts	$68.67	Manning, A. S. M. E., Vol. X., p. 499.
Manchester, New Hampshire	66.00	
Lowell, Massachusetts, 13 feet head	110.00	C. T. Main, A. S. M. E., Vol. XI., p. 108.
Lowell, Massachusetts, 18 feet head	57.00	
Lawrence, Massachusetts	63.00	
Lawrence, Massachusetts, 1000 horse-power	67.50	
Concord, New Hampshire (with dam)	57.75	Webber, A. S. M. E., Vol. XVII., p. 41.
Augusta, Georgia	34.20	
Columbia, South Carolina	37.50	
Caratonk Falls, Maine (with dam)	24.00	
Omaha, Nebraska (estimate)	67.33	*Eng. Mag.*, Vol. VII., p. 409.
Zurich (with dam)	100.00	*Eng. Mag.*, February, 1900.
Paderna, Italy (with dam)	120.00	
Big Cottonwood, 3000 horse-power	108.25	*Eng. News*, October 1, 1896.
Average without dam (excluding Lowell, $110.00)	$53.41	
Average with dam	79.55	

It is probable that the cost of such plants will be from $40.00 to $60.00, excluding the cost of dam, but including all other parts; and when the dam is included that it will be from $60.00 to $100.00. Webber, in *Iron Age*, February and March, 1893, says that water-power plants can be put in for $100.00 per horse-power; and Stilwell, in A. I. E. E., Vol. X., p. 484, says that the cost may be as low as $65.00.

The cost of water power per horse-power year is variable, depending, as it does, upon the first cost of plant; and hence no very good average can be found. The following table may serve to show the costs in some cases that have been reported.

COST OF WATER POWER.

Place.	Cost per H. P. year.	Authority.
Lawrence, Massachusetts	$13.70	C. T. Main, A. S. M. E., Vol. XIII., p. 140.
Canada (lowest)	6.25	Meyer, *Sci. Am.*, February 9, 1882.
Cottonwood	16.10	*Eng. News*, October 1, 1896.
Lawrence, Massachusetts, 1000 horse-power	22.62	Manning, A. S. M. E., Vol. X., p. 48.
Lawrence, Massachusetts, 500 horse-power	19.13	Main, A. S. M. E., Vol. XIII., p. 140.
Concord, New Hampshire.....	8.64	Webber, A. S. M. E., Vol. XVII., p. 41.
Augusta, Georgia..............	11.05	
Columbia, South Carolina.....	9.50	
Omaha, Nebraska (estimate)..	8.08	*Eng. Mag.*, Vol. VII., p. 409.
Norway (electrolytic work)...	11.25	*Chem. Ind.*, Vol. XXIII., p. 121.
Niagara (sold for).............	13.00	Emery, A. I. E. E., Vol. XII., p. 358.
Estimate on plant.............	5.42	Webber, W. O., *Eng. Mag.*, Vol. XV., p. 926.
Average of the above	$10.72	

From the above table it may be seen that the cost per horse-power year is $10.72. Webber gives it as $10.00 to $12.00 (*Iron Age*, February and March, 1893); and Conant, in an article in the *Street Railway Journal* for October, 1898, gives the cost as ranging from $10.40 to $22.40. A fair average may be taken as varying from $10.00 to $15.00.

Steam Power.

SUMMARY OF BOILER TESTS.

Water Evaporation per Pound of Fuel.

Authority.	No. of tests.	Water evaporated, in pounds.	Kind of fuel.
Kent (Christie), A. S. M. E., Vol. XVIII., p. 365	95	11.11	All kinds.
Barrus, horizontal tubular	16	10.76	Anthracite.
Barrus, horizontal tubular, low flue temperature	6	10.40	
Barrus, horizontal tubular, high flue temperature	5	11.00	
Barrus, horizontal tubular	10	11.59	Cumberland.
Average from Gray's Tables, W. T.....	37	10.80	All kinds.
Average from Gray's Tables, tubular..	23	10.40	
Average of all the above..............	192	10.86	

SUMMARY OF ENGINE TESTS.

Pounds of Water per Horse-power Hour.

Authority.	Automatic.	Corliss.
Simple, Non-condensing.		
Carpenter (Sibley theses, *Sib. Jour.*, Vol. XIV., p. 228)	34.3	28.3
L. Bell, "Electrical Transmission of Power"	33.0	28.0
Hutton, "Mechanical Engineering of Power-plants"	33.0	29.0
Thurston & Carpenter, A. I. E. E., Vol. X., p. 297	31.5	28.6
Davis, C. H., *Eng. Mag.*, Vol. XII., p. 942	36.0	31.5
Average of Gray's Tables	33.4	28.9
Average of all the above	33.8	29.0
Compound, Non-condensing.		
Carpenter	32.3	....
Bell	24.0	22.0
Hutton	26.0	24.0
Thurston & Carpenter	24.5	....
Davis	27.0	26.0
Gray's Tables	23.6	....
Average	26.2	24.0
Simple, Condensing.		
Bell	25.0	21.0
Hutton	22.0	20.0
Thurston & Carpenter	....	23.0
Thurston, *Eng. Mag.*, Vol. VII., p. 844	....	25.0
Davis	31.0	26.5
Gray's Tables	22.2	20.2
Average	23.1	22.6
Compound, Condensing.		
Carpenter	22.7	18.3
Bell	20.8	18.0
Hutton	20.0	18.0
Thurston & Carpenter	18.8	17.2
Davis	22.5	23.0
Thurston	....	18.0
Gray's Tables	19.8	15.7
Average	20.6	17.7
Triple, Condensing.		
Bell	17.0	13.0
Hutton	17.0	16.0
Thurston & Carpenter	....	14.6
Thurston	....	14.0
Gray's Tables	....	13.3
Average	17.0	14.2

STEAM-PLANT COSTS.

The cost of steam plants varies greatly with the locality, size, kind of machinery, boilers, and many other items. The table given herewith shows good approximations to the costs of various constructions.

TABLE OF PLANT COSTS.

Authority.	Cost per H. P.	Remarks.
Manning, A. S. M. E., Vol. X., p. 48	$68.26	Total—engine, boilers, stack, 500 horse-power plant.
	52.50	Total—engine, boilers, stack, 1000 horse-power plant.
Field, C. J., A. S. M. E., Vol. XVI., p. 504	50.00	Steam plant complete.
Webber, A. S. M. E., Vol. XVII., p. 41	65.00	Plant complete.
	54.71	High speed, condensing, from Emery's tables for 550.
	59.51	Low speed, from Emery.
	60.35	Compound low, from Emery.
	70.00	Triple compound, from Emery.
Dean, A. S. M. E., Vol. XIX., p. 301	70.00	Simple Corliss, condensing, best, 1000 horse-power.
	57.00	Compound, condensing, best, 1000 horse-power.
	60.50	Actual cost of a yarn mill, 1132 horse-power.
Rathwell, A. I. M. E., Vol. XVII., p. 555	40.00	Engines and boilers.
Western Elec., March 16, 1901	60.00	Complete plant.
Carpenter, *Sib. Jour.*, Vol. XIV., p. 298	28.60	Simple slide valve, non-condensing.*
	30.20	Corliss, non-condensing.
	30.00	Compound slide valve, non-condensing.
	30.00	Compound, non-condensing.
	33.25	Compound Corliss, condensing.
Elec. World, February 2, 1901, p. 214	28.50	Estimates on plant for South Africa.
Thurston, *Eng. Mag.*, Vol. VII., p. 841	38.00	Engines, boilers, and piping, simple, condensing.
	45.00	Compound, condensing.
	53.00	Triple, condensing.
	62.00	Quadruple, condensing.

Field, in A. S. M. E., Vol. XVI., p. 504, gives the average cost of steam plants as ranging from $50.00 to $55.00 per horse-power, and Professor Ryan, in an article in the *Engineering Magazine*, Vol. VII., p. 733, says that the cost of steam plants with high-speed engines is about $50.00, and that for slow-speed Corliss engines ranges from $65.00 to $75.00. This is exclusive of the cost of the buildings.

The costs of electric plants are dependent upon the cost of engines and boilers, and their cost is usually a constant quantity, for the cost of dynamos is nearly constant per kilowatt plus the cost of engine plant. The cost of dynamos and other electrical apparatus may be assumed as ranging from $20.00 to $35.00, including switchboard; hence, the cost of complete plants for electric lighting and power may be assumed to be $75.00 to $100.00, according to circumstances.

* The costs under this are for engines, boilers, and piping alone, exclusive of cost of building.

COST OF STEAM POWER.

The following table is a condensation of a more detailed one in Mr. Gray's paper, and gives practically the same result. The costs are for the total of fixed and operative charges, in cents, per horse-power hour.

Authority.	Cost per H. P. hour, cents.
Emery, A. I. E. E., for 3080 hours per annum	.784
Emery, A. I. E. E., for 7090 hours per annum	.617
Emery, *Eng. Mag.*, for 3080 hours per annum	.856
Webber, 650 horse-power, for 3080 hours per annum	.720
Webber, 1050 horse-power, for 3080 hours per annum	.646
Hale, for 2985 hours per annum	.557
Main, for 3080 hours per annum	.637
Foster, for 3080 hours per annum	.824
Gray's Table	.720
Average of all	.707

Dr. Louis Bell, in his book, "Electrical Transmission of Power," gives as the cost for 10-hour day, full load, with large compound-condensing engines, 0.8 to 1.0 cent per horse-power hour, and for simple engines, 1.5 to 2.5 cents; while, if the load is partial and intermittent, these figures become 1.0 to 1.5 and 3.0 to 4.0 cents, respectively.

The cost of engines varies considerably with the class, but the following table is a very good approximation for the different kinds:

Simple slide-valve engine	$7.00 to $10.00
Simple Corliss or low-speed type	11.00 to 13.00
Compound slide-valve	12.00 to 15.00
Compound Corliss	18.00 to 23.00
High-speed automatic	10.00 to 13.00
Low-speed automatic	15.00 to 17.00

In addition to this is the price of boilers, which is approximately $10.00 to $12.00 for the plain tubular and about $15.00 for the water-tube type, and the cost of pumps, which is about $2.00 for a non-condensing and $4.00 for a condensing plant, including heaters.

Gas Power.

The following tables contain some tests of different gas engines, using various kinds of gas. Some of these are small and others large, although there are but few tests of the larger sizes, from the fact that they have not been on the market until recently. The amount of gas used per horse-power is in some cases based upon the indicated, and in others upon the developed or brake horse-power. This is indicated by an (I.) or (B.) placed after the column.

Using Natural Gas.

Kind.	H. P.	Cubic feet per H. P.	Authority.
Westinghouse	621	9.3 (I.)	Miller & Gladden, *Sib. Jour.*, June, 1900.
	67	10.4 (B.)	*Lond. Eng.*, January 4, 1901.

The Westinghouse Company will guarantee a gas consumption of 12 cubic feet of gas per B. H. P. on their small engines, and as low as 10 cubic feet of gas per B. H. P. for the larger sizes. The Standard Automatic Gas Engine Company guarantees less than 15 cubic feet per B. H. P.

Using Coal Gas.

Kind.	H. P.	Cubic feet per H. P.	Authority.
Westinghouse	9	14.5 (I.)	Budd & Moody, Sibley thesis.
Springfield	12	16.5 (I.)	Spier & Keely, Sibley thesis.
	10	15.5 (I.)	Perry.
		16.6 (B.)	*Lond. Eng.*, January 4, 1901.
Campbell	......	15.4 (B.)	Donkin, *Eng. Mag.*, December, 1900.
Otto-Crossley	17	24.1 (B.)	London *Elec. Eng.*, January 25, 1901.
Clerk	9	30.4 (B.)	
Atkinson, differential	......	25.7	
Atkinson, cycle	6	22.5	
Griffin	16	28.5	
Forward	6	22 5	
Simplex	......	20.4	
Wells Bros.	12	27.8	
	18	21.0	
Premier	61	19.7	
	50	17.0 (I.)	
Railway plant	31	21.0 (B.)	Hill & Brocksmit thesis.
Average on B. H. P.	......	22.9	

This average is rather higher than can be expected of the best modern engines, for these are all rather small units. About 17 or 18 cubic feet may be expected of the average modern engine of moderate size.

Using Producer Gas.

Kind.	H. P.	Cubic feet per H. P.	Coal, in pounds.	Authority.
Crossley	142	65.7 (I.)	.92	*Lond. Eng.*, January 4, 1901.
Koerting	349	83.2 (B.)	......	Mond. gas, January 4, 1901.
	377	60.1 (I.)	......	
Diesel	7	....	1.31	
	170	....	.88	Dowson gas, January 4, 1901.
Simplex	220	86.8 (B.)	1.30	Witz, Dowson gas.
	59	77.7 (B.)	1.30	
Otto	....	....	Less than 2.00	Dowson gas.
Simplex	220	....	.80	*Lond. Eng.*, November 30, 1894.
Crossley	280	....	1.03	Richmond, *Eng. Mag.*, Vol. X., p. 853.
Otto	....	(I.)	.95	Spangler, *Cass. Mag.*, Vol. IX., p. 47.
Crossley	....	(I.)	.76	
Le Tombe	....	(B.)	1.60	*Power Quarterly*, 1901.
	100	(I.)	1.03	Adams, *St. Ry. Jour.*, June, 1900.
	320	(I.)	.81	
Crossley-Otto	28	(I.)	1.41	Trans. I. C. E., Vol. V., p. 73.
	118	(I.)	.76	
Atkinson	22	(I.)	1.06	
Stockport	76	(I.)	.86	
	....	109.0 (B.)	1.25	Adams, *Eng. Mag.*, Vol. XVI., p. 513.
Average of those on I. H. P.			1.04	

Average cubic feet on B. H. P., 82, or on I. H. P., 62.9. These values are only approximate, as the data are not very complete.

Using Blast-furnace Gas.

Kind.	H. P.	Cubic feet per H. P.	Authority.
Cockerill	182.0	116.5 (B.)	Donkin, *Eng. Mag.*, December, 1900.
	650.0	135.7 (B.)	
	725.0	101.0 (I.)	
Otto type	79.5	79.4 (I.)	
	175.0	145.2 (B.)	Booth, *Cass. Mag.*, March 9, 1901.
Wishaw, England	....	140.0	
Cockerill	661.0	112.9 (B.)	Hubert, test at factory.
	715.0	102.3 (B.)	
Acme	15.0	90.0 (B.)	"Colliery Guardian," quoted in *Eng. Mag.*, Vol. XV., p. 140.
Cockerill	575.0	111.4 (B.)	*Eng. Mag.*, Vol. XIX., p. 587.
Cockerill, with blower	....	101.0 (B.)	
Frieden	177.0	133.0 (B.)	Schutte.
	90.0	231.0 (B.)	
	100.0	91.0 (I.)	
Average	....	115.1	

COST OF GAS POWER.

The following table gives some figures on the cost of power produced by different-sized engines using various fuels. The kind of gas used is indicated by the capital letters in parentheses, as well as whether the horse-power is based upon the indicated or the brake.

Kind. Place.	Cubic feet of gas.	Coal per H. P.	Cost, cents.	Authority.
	......	1.00	1.00 (B. P.)	*Elec. Eng.*, January 25, 1901.
	17	....	1.02 (B. C.)	
Ordinary	20	....	.87 (B. P.)	Guy, "Electric Light and Power."
	23	....	1.50 (B. P.)	
Schwabing	....	1.50	1.50 (B. P.)	
	....	....	3.00 (B. C.)	
Otto	......	1.20	.16 (I. P.)	Cost of fuel alone, Robinson, "Gas and Petroleum Engine."
Ordinary	200	....	.56	Eberle, *Eng. Mag.*, Vol. XIV., p. 687.
	250	....	2.00 (B. P.)	*Elec. World*, 1897, p. 822.
Clausthal	......	1.73	(B. P.)	*Elec. World*, Vol. XXXVI., p. 457.
	......	.93	.90 (B. P.)	Cassier's, 9.
Average engine	......	....	.50 (B. N.)	Kerr, *Cass. Mag.*, Vol. XVIII., p. 425.
	20	....	2.00 (B. C.)	
Glasgow	......	1.25	(B. B.)	Fuel alone, *Cass. Mag.*, Vol. XVIII., p. 425.
Ordinary	......	....	2.00 (B. C.)	Bolton, A. S. M. E., Vol. XX., p. 873.
Average	......	....	2.40 (I. C.)	Krone, Dg., *Elec. World*, 1900, p. 443.
Crossley	......	2.05 per K. W.	....	*Eng. Mag.*, Vol. XV., p. 295.
Le Tombe	......	1.60	....	*Power Quarterly*, October, 1900.
Otto	......	1.00	(B. P.)	
Charon	......	1.06	(B. P.)	
Crossley	......	1.23	(B. P.)	

Cost of Gas Power.—*Continued.*

Kind. Place.	Cubic feet of gas.	Cost, cents.	Authority.
Oil engine	......	1.74 (B.)	Guy, "Electric Light and Power."
Gasoline engine .	......	1.50 (B.)	Kerr, *Cass. Mag.*, Vol. XVIII., p. 425.
Diesel	......	2.00 (B.)	
Ordinary	....	.66 (B. B.)	Meyer, *Sci. Am.*, February 9, 1901.
	....	.83 (B. P.)	
	25	3.10 (B. C.)	*West. Elec.*, February 23, 1901.
Blast furnace....	80	....	Dg. *Elec. World*, January 19, 1901.

The letters in the parentheses are read as follows: The first one refers to brake or indicated horse-power, and the second to the kind of gas used, either natural, producer, coal, or blast-furnace.

The costs of gas-engine plants are not very different from those of steam plants. Mr. N. W. Perry, in A.I. E. E., 1894, says that the cost of producers or generators is about $11.00 per horse-power, which is less than that of steam boilers. He also gives an estimate by Dawson on a plant to have an output of 400 kilowatts, occupying a floor-space 27 × 54 feet on one level, and costing, complete, about $10.38 per horse-power.

Electric Power.

While electricity can be considered only as a secondary source of power, requiring itself to be generated from some prime mover, some data as to costs will be found acceptable.

The following table gives some figures on the cost of generating electric power. The costs are based upon the kilowatt-hour, which is the practical unit used to designate power, it being equal to 1.34 horse-power hours. The cost is based, in most cases, upon the power delivered to the feeders that carry the current to the point of application. The cost for lighting and for power is usually different, as the power used for motors does not need such careful regulation of the pressure, and again the amount is usually large as compared with the amount sold for lighting, so that the cost to produce is less per unit, and the amount is not so variable; hence, the machines can be run at better efficiency.

The cost also depends upon the load factor of the plant,—that is, upon the ratio of the average to the maximum output of the station, and the cost increases as this factor decreases.

COST OF ELECTRIC POWER.

Place and use	Cost per K. W. hour, cents.	Authority.
Cheltenham, England, lighting...	5.06	London, *E. Rev.*, January 4, 1901.
Dundee, England, lighting........	4.90	
London, price sold, lighting	5.00	London, *E. Eng.*, January 4, 1901.
London, price sold, motors	5.00	
Estimated operating expense......	.58	Humphrey, *Lond. Eng.*, January 4, 1901. These seven are for the operating expenses only.
Lighting, 0.5 load factor...........	1.02	
Manufacturing works, large	.92	
Manufacturing works, fairly constant	1.23	
Manufacturing works, small, constant	2.60	
Ordinary works, varying load	1.92	
Small works, varying load	3.60	

Cost of Electric Power.—*Continued.*

Place and use.	Cost per K. W. hour, cents.	Authority.
Dudley, England, selling price....	4.00	*Elec. World*, February 2, 1901, for tramways.
	10.00	Other uses.
Average for United Kingdom	5.34	Garcke, *Elec. World*, January 26, 1901.
American, practice, range 3.00 to..	7.50	Bolton, A. S. M. E., Vol. XX., p. 873.
Met. Electric Railway, Chicago, operating expenses............	.88	*E. R.*, February 15, 1901.
Glasgow, Scotland, operating expenses........................	2.56	B. I. E. E.
Lighting, to get to customer	3.56	Field, *Cass. Mag.*, March, 1896.
Railway, operating expenses......	.90	
Railway, large, at bus bars........	1.00	Kennelly, *Cass. Mag.*, Vol. XIII., p. 531.
Railway, large, operating expenses	.50	
Niagara power in Buffalo	2.00	
Railway, estimated, 33 per cent. load factor	1.00	Conant, *St. Ry. Jour.*, Vol. XIV., p. 621.
Kansas City, operating expenses..	.40	Conant, *St. Ry. Jour.*, Vol. XIV., p. 71.
Average at board.................	8.00	Editorial, *St. Ry. Jour.*, Vol. XIV., p. 92.
Brooklyn Heights, operating expenses........................	.62	....
Denver Railway..................	1.10	*Elec. World*, February 26, 1901.
Denver, motor work	4.00	
Kansas City Railway, operating expenses, 1899	.43	*W. E.*, October 20, 1900.
Kansas City Railway, operating expenses, 1900	.41	
Met. Street Railway, New York, 1898............................	1.57	*St. Ry. Jour.*, November, 1898.
Estimate at bus bars	.50	Bell, "El. Trans. of Power."

From the above table it may be seen that the cost of generating power is extremely variable in the different cases, depending upon the purpose for which it is used, the load factor, the cost of fuel, and the size of plant. For the case of large plants run by compound condensing engines, with generators directly connected, operating under fairly good load factors, it may be assumed that the cost of power per kilowatt at the bus-bars is not far from 1 cent, and it may be less with careful attendance. For water-power plants this figure may be lowered. The cost of distribution is so variable that no attempt has been made to estimate it, and it can only be approximated for specific cases.

The cost of electrical machinery depends upon the price of steel and copper to a large extent, and so is variable; but it may be assumed to range from $15.00 to $25.00 per kilowatt output for generators, motors, or rotaries of the medium or large sizes. The price per unit increases as the size decreases, as they are less efficient and require more material and more labor in manufacture.

WORKS MANAGEMENT.

The operative management of engineering establishments is necessarily governed largely by the character of the product, but there are certain basic principles which may be stated in a general form.

The object of good management is the production of good work at a minimum cost. Good work involves good tools and skilful mechanics; but good tools are costly, and skilful mechanics demand high wages. At the same time, it is fully established that poor and antiquated machinery and cheap labor are both unprofitable things. The problems of good management, therefore, may be divided into the successful use of tools and the successful handling of men.

There are two ways to reduce the cost of a manufactured product,—one being to cut down wages and capital charge, and the other to increase the output. In other words, the value of a fraction may be diminished either by diminishing the numerator or by increasing the denominator. The latter is recognized as being the best and most satisfactory method.

In order to increase the output of the workmen two things are necessary,—one, to systematize the operations; the other, to provide an incentive to the men.

Systematization of shop methods involves the principle of employing a limited number of highly-skilled and highly-paid mechanics to keep the tools in order, to maintain them at a high efficiency, and to devise improved methods, while the tools themselves are attended by a much lower grade of labor, costing less, and at the same time competent to perform the limited duties assigned to them. This also includes the use of the most efficient handling appliances and the proper arrangement of machines, so that machines are kept supplied with rough parts, while finished parts are promptly transferred to the next machines in orderly sequence.

The incentive to the men involves the use of some system of wage adjustment by which the man's earnings depend to a greater or less extent upon his output. The daily or hourly wage system, in which the pay depends wholly upon the time, has been found satisfactory in the moderate-sized shop, in which comparatively few men are employed, and where the owner or superintendent can keep his eyes upon everything and establish personal relations with all the men. The great difficulty in the extension of the system to large establishments lies in the fact that the day's output is determined by the men themselves, and naturally tends to a minimum.

In order to provide an incentive to the workman, **piece-work** has long been used, but, except in rare instances, it has not proved satisfactory. The reason for this is readily seen. After a piece price has been put upon a certain article there is undoubtedly a direct incentive to the workman to do as many pieces as he can, since the more he does the more he makes. As soon as he has thus established a new rate for himself the employer compares the wages the man is receiving with his former pay, and comes to the conclusion that the man is making too much. The piece rate is accordingly cut, and soon the establishment reaches a sort of equilibrium in which the man does only about enough to make his piece wages equal the current day rate. The incentive is thus only temporary, and the method becomes unsatisfactory to employer and employee alike.

The **Premium Plan** is intended to obviate this defect in piece-work. Instead of fixing a price upon the piece, a **time** is fixed within which it can be completed. If the work is finished just in this time, or takes a longer time, the workman is paid the regular hourly wages. If, however, he finishes the job in less than the allotted time he is credited with a certain portion of the time saved, usually receiving one-half the time. Thus, if the time fixed for a piece of work is six hours, and the man does it in four hours, there are two hours saved, and he receives one hour's additional wages. The essential element in the premium system lies in the fact that the time for a piece, when once fixed, is never changed unless some alteration is made in the piece affecting the work upon it, or unless some new tool or method is furnished by the employer to aid in accelerating the work. The premium system, with various modifications, is in successful use in many large establishments.

The **Bonus System** differs from the premium system, in that a definite sum of money is allotted as an extra payment for completing a job within

an allotted time limit. The job is analyzed into its elementary operations and the actual time in which they can be performed, and a time-card containing this information is given to the man with the job. If he gets the work out in less time he gets a bonus, the amount of which is fixed according to the value of the job, and if not, he gets his time wages, anyhow.

It will be seen that all these methods involve the determination *by some one else* of the time in which the job should be done, and this is one of the essentials of successful shop management. If the time is left to the judgment or choice of the man, the employer is relinquishing one of the most essential elements of his control, and success may be endangered. In all methods it is important to be liberal with the men. Any attempt to squeeze them to a minimum wage rate is to invite failure. The object of every improved system should be not to reduce wages but to increase output.

The importance of this will be seen when it is understood that the fixed charges of an establishment must be paid out of the product, regardless of the wage cost, and hence it is important that a maximum output be attained to bear the establishment charges.

Cost Keeping.

The subject of cost keeping is closely allied to that of general works management, and, while details must differ in various establishments, there are certain fundamental principles underlying all successful systems.

The following general outline of a cost-keeping system is condensed from a chapter by Mr. J. Newton Gunn, forming a portion of "The Complete Cost Keeper," by H. L. Arnold, published by the *Engineering Magazine*, New York and London:

"The first consideration is to see that the plant is properly divided as to its various departments,—that is to say, that each foreman has as many men as he can profitably handle and no more, that no one foreman has charge of two or more classes of work which are in no way related, and that the disposition of the work in the building is not such as to preclude the possibility of the foreman in charge giving to the work the best supervision and direction possible.

"Having determined so much, the next step is to obtain a complete statement of all the operations performed throughout the establishment.

"These are then divided into two main classes,—first, operations which are performed so directly upon the work that it is possible to charge the labor and material expended to a given production-order; and second, those operations which are so general in their character as to be necessarily treated as indirect expense, together with the cost of operations which are of the nature of expenses incidental to the operation and maintenance of the plant rather than to any particular class of work.

"In the particular plant, which is the example selected for presentation here, the work is of such a character and the performances are of such duration that in the majority of cases the direct operations are chargeable to specific production-orders, and in every case it is possible for the workmen to record their own time and performances. Where one workman performs several minor operations on a single production-order these operations are recorded on his time-card, but he is not required to individualize the actual time spent on each of the minor operations. The rule which governs in determining whether he shall indicate the time down to the least operation, or record the time on a group of operations, is that the time shall be separately recorded for all operations which could not be performed under one contract if the work were on a piece-work basis. It may be noted here that prior to the installation of the system taken as an example no piece-work was performed in the factory. Under the system adopted the indirect operations are classified wholly by the results which are obtained; thus, sweeping belongs to the group of operations incidental to the care of the plant, while oiling of shafts and care of belting pertains to the production and use of power. For all these indirect operations fixed or standing order-numbers are provided, and all the indirect work performed is charged to one or another of these standing order-numbers, unless a specific production- or plant-order has previously been given therefor.

"In order to reduce the mechanical effort in the making of the time-card, specially devised time-cards are prepared for each department, the margin of the time-card containing the principal operations performed in that department, so that the record to be made on the card consists of the date, the workman's number, the workman's name, quantity and description of the work in hand, a check-mark or cross opposite the operation performed by the workman, a check-mark indicating the time the work was commenced, and another showing the time the work was finished. The record of the work performed as indicated on these time-cards is checked at the end of each day by the foreman or sub-foreman in charge, and the cards are then immediately transferred to the office of the paymaster.

"The same methods are used in securing the original records of material disbursed in producing an order. Forms termed Material Cards serve to record the entire history of all material from the time it enters the factory until it becomes finished product, and has all of its costs properly recorded upon the cost-cards. These material cards are differentiated in color to unmistakably indicate to the workman and clerks the different classes of records. Thus, a white material-received card is used for recording all material when it enters the factory, no matter whether this is raw material or parts purchased ready to assemble into a complete machine. A buff material-delivered card shows that material has been transferred from one of the storerooms to some department, or from one department to another. A salmon-colored materials-returned card shows that material is to be credited to the order-number or account which appears thereon, as the material is returned, either from a department to the storeroom or from one department to another department from which it may have been received. A blue material-requisition card indicates a requisition made by a foreman or a storekeeper for the purchase of material by the purchasing department.

"The schedule of machine details records those parts of any one completed unit of factory product which are carried in stock in the storeroom ready for use by the assembling department, each schedule being individually numbered, and a single schedule representing all the parts necessary to complete a machine of a certain size and type. Duplicates of each of these schedules are in possession of each of the following officials: superintendent, cost clerk, stores-ledger clerk, storekeeper, and foreman of assembling department. These schedules enable the foreman of—for instance—the assembling department to use a single materials-delivered card calling for all parts on, say, schedule number seventeen, which he sends to the storekeeper, thus saving the writing and possible errors which would be incidental to the reproduction in detail of the individual items covered by the entire schedule every time a machine was to be assembled or two or more were to be assembled. These material cards always bear either the production-order number or plant-order number to which they are related, or, if the material is solely chargeable to expense, must bear the standing-order number which indicates the division of the expense account for which they are disbursed. It may be said here that one of the most important precedents to accurate factory accounting is absolute uniformity of nomenclature throughout the factory. Any variation in naming leads to doubt, and opens the way to inaccurate returns.

"As an example of organization the following, representing a successful modern plant, is given:

"The chief of the entire office and factory staff is the President and Treasurer, who is Acting General Manager, and is responsible only to the Directors of the Company. The next in authority is the General Factory Manager, who bears the entire responsibility of production, and who is subordinate to the President alone. The General Factory Manager's staff consists of a Cost Clerk, who is also Paymaster, and a Purchasing Clerk, who is also Production-order Clerk. The Production-order Clerk originates production-orders on the factory under requisitions from the Superintendent.

"The general factory manager also has a stores-ledger clerk, who is responsible for all records of material up to the time these records are turned over to the cost clerk; a stenographer; a superintendent, who supplements the general factory manager in all his work, but is more especially responsible for the direct supervision of the foreman. The drawing-room is in charge of the constructing engineer and chief draughts-

man, who has charge of the subordinate draughtsmen, and who undertakes the designing of new machinery and the remodelling of old, under the immediate direction of the superintendent. The chief draughtsman is also responsible for the revision of the schedules of machine details, which of course must be made to conform to the drawings in his possession.

"Each department has a foreman in charge, who supplies the workmen with pads of time-cards and pads of material-order cards. All stores are kept by a storekeeper, who, with one assistant, has in charge all raw material, or rough stores, and also all semi-finished material or parts, as well as finished parts and machines. Heavy materials, such as rough castings and finished machines, which from their bulk and the expense attendant upon handling them cannot economically go into the storeroom, are delivered to, and kept in, the part of the factory where they are to be used; the nearest foreman is made responsible for their care and disbursement.

"The president has as his executive force a stenographer, a filing clerk, a bookkeeper, and an order clerk, who handles the shipping orders and does the billing, as well as renders general assistance to the bookkeeper. Directly responsible to the president is the general manager of the Sales Department, who has a number of salesmen responsible in turn to him. A branch office, which is purely a department of the selling organization, is responsible, in so far as its selling functions go, to the general manager of the sales department, but for its finances it is directly responsible to the President-treasurer.

"A complete shop telephone system gives communication from the superintendent's office to all the foremen and to the stores department.

"The only bound book kept is a ledger, which contains the commercial accounts, and from which the general statement of the business is made up. This book is simply a general ledger containing personal accounts and customers' accounts. There are no other books employed in the accounting organization, every other record being made either on cards or on loose sheets. These cards and sheets are provided for as to their storage either in card-cabinets or in binders, or in filing boxes. The same record at different stages of its work may be in any one of the three forms of receptacle specified.

"Origination of Production-order.

"A production-order originates either from a specific shipping order, which demands a specific machine, or else from the depletion of the stock of a given machine or machine detail; stock shortage is immediately reported by the stores-ledger clerk, as he has a schedule showing the maximum and minimum limits not only of the parts of machines but of all complete machines. The schedule limits of stock are assumed, at first, and revised as experience may dictate.

"The moderate size of the concern under consideration precludes any necessity for having a formal record of the instructions that the superintendent gives to the production-order clerk for the creation of a production-order; these instructions are in every case verbal, the superintendent vitalizing the production-order as soon as made, and assuming the responsibility by affixing his signature thereto.

"Other orders, termed 'plant-orders,' are issued by the production-order clerk. These cover all labor and material expended for the betterment of the plant or for experimental purposes; in the case of the production-order it is expected that a direct profit will be realized from all work ordered to be performed, while plant-orders are expected to be indirectly profitable. Both production-orders and plant-orders are carbon-sheet copies in duplicate, the original going to the foreman who is to do the initial work, the duplicate remaining at the desk of the production-order clerk, where it is filed by family,—that is, by class of machine and number of machine, or class of work,—until the return of the original order with the foreman's signature gives notice that the order has been filled.

"Both production- and plant-orders, before being sent to the foreman of the proper production department, go to the chief draughtsman, who supplements each order with the drawings needed for its production, and the drawings and order then go together to the department foreman. The foreman, on receipt of this order with its drawings, starts the work, mak-

ing immediate provision for identification of the work throughout his department, either by tags showing the production-order number in case of large pieces, by painting the production-order number on the piece, or, in case of a large number of small pieces, by adding to the receptacle containing the parts a memorandum-slip bearing the production-order number.

"The material-delivered card, signed by any foreman, obtains from the storekeeper the stores needful for the completion of an order. At any time during the progress of the work, or at its completion, any excess of material may be returned, recorded on a 'material-returned' card. All these material-delivered and material-returned cards are identified by having the production- or plant-order number upon them.

"There is no limitation as to the amount of material which a foreman may draw for the completion of any one order, but as often as four times each day it is the duty of the assistant storekeeper to return all material cards to the stores-ledger clerk, on whom the responsibility is placed of seeing that the proper limits of material required for the completion of any order are not exceeded.

"The Sub-production-order.

"The sub-production-order is produced in duplicate by manifolding by any foreman who may require the product of any other department. The original goes to the department making the required product, and the duplicate to the office of the superintendent. Both original and duplicate bear the production-order number under which the foreman using the sub-production-order number is operating. On the completion of the required details by the department receiving the sub-production-order, the order is returned with the work to the originating foreman, who checks the sub-production-order, certifies its completion, and forwards it at once to the office of the superintendent.

"A plant-order is issued by the superintendent at the request of any foreman, or as his own judgment may dictate. Plant-orders cover all work in the nature of repairs or betterments to the plant or building, all special tools and special experiments, and, in fact, any work to be performed which will eventually become an addition to the plant or an expense-charge to the business. These orders are made in duplicate by manifolding, the original going to the foreman of the department required to produce the work, while the duplicate remains in the office of the superintendent, plant-orders being handled precisely as production-orders are handled.

"There is one case in which it is necessary to make an exception to the rule requiring all production-orders to be issued by the superintendent. This occurs in the general machine shop, where it sometimes happens that certain machines are left idle. The foreman of that department is supplied with a pad of production-orders, and has authority to issue to himself an order for the undertaking of such work as will keep his machines busy, sending the duplicate order immediately to the office of the superintendent, on whom rests the responsibility of approving the order or of communicating with the foreman and stopping the work, if for any reason that particular work is not justifiable in the judgment of the superintendent. In no case, however, is work stopped until the particular operation in progress is completed.

"The production- and sub-production-orders, with the plant-orders, account for all direct labor and material and a portion of the expense-labor, but it is necessary that the general factory manager, superintendent, draughting department, the foremen, the general laborers, and men employed in the production of power, heat, and light shall have some definite means by which to indicate the services which they have rendered to the company. To this end a communication is addressed to all the heads of departments, and through them brought to the notice of the employees, which communication is a standing order under which all employees of the company are to work. The standing order directs that whenever workmen perform any of the operations or render any of the services enumerated on the list accompanying the order, the labor-time, together with any material that may be drawn from the stores department necessary to complete the service, shall be charged to some one of the various numbers set

opposite the items enumerated. These numbers are called standing-order numbers, and are specific orders, to be treated in the accounting as are the production- or plant-orders.

"For convenience, and as an aid to all persons concerned in becoming familiar with these numbers, the numbers from 1 to 499 relate to the plant and expenses incidental to production. The numbers from 500 to 999 specify a general classification of the product, and relate to such operations upon the product as testing, which operation, though it may be performed—as in the case of a dynamo—upon a specific machine, gives information so general in its character as to be of service to the entire class to which this machine belongs, and therefore is an item of cost which may more justly be spread over the entire related product than charged to the individual machine under test.

"The plant-orders commence with 1000 and continue in numerical sequence, each number being preceded by the letter 'P.' Production-orders commence at 1000 and run in sequence, but have no distinguishing letter.

"The numbers from 1 to 499 are subdivided into five general groups, defined as follows:

"From 1 to 99, additions to or betterments of the 'permanent investment,'—buildings and plant; 100 to 199 report depreciations of the permanent investment and cost of up-keep of plant; 200 to 299 carry the time-cost of foremen, superintendents, draughtsmen, and all other indirect labor, excepting 'engineers' (American) or 'engine drivers' (English); 300 to 399 cover all labor and material used in the generation and distribution of power, heat, and light; 400 to 499 include all alterations, errors, and changes, also fixed charges, such as taxes and insurances."

General Expense.

In addition to the determination of the material and direct wage charges upon a job there is the item of indirect charges, usually called *general expense*. This includes interest and depreciation on plant, cost of motive power, lighting, insurance, taxes, etc., as well as the wages and salary expense of men whose work is not chargeable to any direct job, but must be borne by the entire output. This must be included in the actual shop cost; and while the total of such charges can usually be determined with a fair degree of accuracy, there are various methods of distributing it over the different jobs.

The commonest method of charging general expense is to make it a *percentage of the wage charge.*

Thus, if the grand total of all the expenditures which cannot be properly charged directly to shop orders be summed up for the year, it will be found to bear a certain relation to the total of the direct wages for the same period. In many cases the indirect charges will be found to be as much as the direct wages; in other instances they will reach 80 per cent. of the wages, etc. This relation having been determined, it is only necessary to add to the cost of materials and labor an amount equal to the expense percentage of the labor to have the entire shop cost. Thus, if the material on a job cost \$12.00, and the direct labor charges amounted to \$80.00, and the expense percentage had been determined to be 100 per cent., the amount added for general expense would be $80 \times .70 = \$56.00$, and the total shop cost would be $12 + 80 + 56 = \$148.00$.

This method has the advantage of simplicity, but it is not always satisfactory, since it assumes that the expense charge bears a direct relation to the labor charge. But it is evident that the expense goes on just as heavily for the time of a cheap workman as it does for a highly-paid one, and so this method saddles the jobs upon which skilled mechanics are working with more than their fair share of expense, and does not put enough upon those done by cheaper men.

Another and better method of distributing general expense is the so-called "hourly burden."

In this system the expense is made to depend upon *time*, instead of wages. Thus, if the total expense for a year be divided by the total number of hours during which all the workmen were occupied during that year, we shall obtain an expense charge per hour for every man's time.

A shop containing 100 men, each man working 3000 hours per year, will

The Effects at Different Rates of Depreciation for Terms of Years.

Years.	1 per cent.	2 per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	10 per cent.	12 per cent.	15 per cent.	20 per cent.
1	.990 000	.980 000	.970 000	.960 000	.950 000	.940 000	.930 000	.920 000	.900 000	.880 000	.850 000	.800 000
2	.980 100	.960 400	.940 900	.921 600	.902 500	.883 600	.864 900	.846 400	.810 000	.774 400	.722 500	.640 000
3	.970 299	.941 192	.912 673	.884 736	.857 375	.830 584	.804 357	.778 688	.729 000	.681 472	.614 125	.512 000
4	.960 596	.922 368	.885 292	.849 346	.814 506	.780 749	.748 052	.716 392	.656 100	.599 695	.522 006	.409 600
5	.950 990	.903 921	.858 734	.815 372	.773 781	.733 904	.695 688	.659 081	.590 490	.527 731	.443 705	.327 680
6	.941 480	.885 843	.832 972	.782 757	.735 092	.689 870	.646 990	.606 355	.531 441	.464 404	.377 149	.262 144
7	.932 066	.868 126	.807 982	.751 477	.698 337	.648 478	.601 700	.557 846	.478 297	.408 675	.320 577	.209 715
8	.922 745	.850 763	.783 743	.721 389	.663 420	.609 569	.559 581	.513 218	.430 467	.359 634	.272 490	.167 772
9	.913 517	.833 748	.760 231	.692 534	.630 249	.572 995	.520 411	.472 161	.387 420	.316 478	.231 617	.134 218
10	.904 382	.817 073	.737 424	.664 832	.598 737	.538 616	.483 982	.434 388	.348 678	.278 500	.196 874	.107 372
11	.895 338	.800 732	.715 301	.638 239	.568 800	.506 299	.450 103	.399 637	.313 811	.245 080	.167 343	.085 899
12	.886 385	.784 717	.693 842	.612 709	.540 360	.475 921	.418 596	.367 666	.282 429	.215 671	.142 232	.068 720
13	.877 522	.769 023	.673 026	.588 201	.513 342	.447 366	.389 294	.338 253	.254 186	.189 790	.120 905	.054 976
14	.868 746	.753 643	.652 836	.564 673	.487 675	.420 524	.362 043	.311 192	.228 768	.167 015	.102 770	.043 981
15	.860 059	.738 570	.633 250	.542 086	.463 291	.395 292	.336 700	.286 297	.205 891	.146 973	.087 354	.035 184
16	.851 458	.723 798	.614 253	.520 402	.440 127	.371 575	.313 131	.263 393	.185 302	.129 336	.074 251	.028 148
17	.842 943	.709 323	.595 825	.499 586	.418 200	.349 281	.291 212	.242 322	.166 772	.113 816	.063 113	.022 518
18	.834 514	.695 136	.577 950	.479 603	.397 214	.328 324	.270 827	.222 936	.150 095	.100 158	.053 646	.018 014
19	.826 169	.681 233	.560 612	.460 419	.377 354	.308 624	.251 869	.205 101	.135 085	.088 139	.045 599	.014 412
20	.817 907	.667 609	.543 794	.442 002	.358 486	.290 107	.234 238	.188 693	.121 577	.077 562	.038 760	.011 529
21	.809 728	.654 257	.527 300	.424 322	.340 562	.272 701	.217 842	.173 597	.109 419	.068 255	.032 946	.009 223
22	.801 631	.641 171	.511 655	.407 349	.323 533	.256 358	.202 593	.159 709	.098 477	.060 064	.028 004	.007 379
23	.793 615	.628 348	.496 306	.391 055	.307 357	.240 958	.188 411	.146 933	.088 629	.052 856	.023 803	.005 903
24	.785 678	.615 781	.481 416	.375 413	.291 989	.226 501	.175 222	.135 178	.079 766	.046 513	.020 233	.004 722
25	.777 822	.603 466	.466 974	.360 396	.277 390	.212 911	.162 957	.124 364	.071 790	.040 921	.017 198	.003 778
26	.770 043	.591 396	.452 965	.345 980	.263 520	.200 136	.151 550	.114 415	.064 611	.036 010	.014 618	.003 022
27	.762 343	.579 568	.439 376	.332 141	.250 344	.188 128	.140 941	.105 261	.058 150	.027 289	.012 425	.002 418
28	.754 720	.567 977	.426 194	.318 855	.237 827	.176 840	.131 075	.096 840	.052 335	.024 014	.010 562	.001 934
29	.747 172	.556 618	.413 408	.306 101	.225 935	.166 230	.121 900	.089 093	.047 101	.021 132	.008 977	.001 547
30	.739 701	.545 485	.401 006	.293 857	.214 639	.156 256	.110 076	.081 966	.042 391	.018 596	.007 631	.001 238

have $100 \times 3000 = 300{,}000$ hours among which to divide the general expense. If the total expense charges for the year amount to $75,000, the *burden* per hour will be $\frac{7{,}500{,}000}{300{,}000} = 25$ cents.

To find the expense charge against any job by this method the number of hours' time put on it will be multiplied by 0.25, regardless of the rate of wages paid to the men. This plan is very satisfactory in many cases, but it has been criticised in one respect. The expense properly chargeable to some important and expensive tools, it is maintained, should be greater than that charged against work done with tools which cost less to buy and to operate; hence, in many shops there is what is termed a *machine rate* for various machines, this rate being computed—more or less arbitrarily—from the importance and cost of the machine. It is impossible to provide for all the general expense by machine rates, because the various tools are not in continual operation, but it has been proposed by Mr. Hamilton Church to combine the two methods, using machine rates for the important tools, and having a supplementary hourly burden to take care of the balance of the expense charges.

The method to be selected depends largely upon the character of the work. When there is much uniformity in the methods and products the hourly burden is satisfactory, and when there is a diversified product the establishment may often be divided into departments, each with a fairly uniform product, and each with its own hourly burden. The choice of method must therefore be made by the works manager according to the conditions of operation.

The question of depreciation is one which demands attention, and it is now considered upon an entirely different basis from that which formerly obtained. At one time the amount charged off from the inventory value of a machine tool was based upon its durability, and the original cost was divided by the length of time such a tool might reasonably be supposed to last.

It is now well understood, however, that a machine may become superannuated in a few years, and while it is still in perfect condition, simply by the appearance of some improved machine or method of doing the same work, the improved method rendering a cheaper product possible and causing the old machine to be unsuited for competition.

Since the time which may elapse before a machine becomes superannuated is a very uncertain quantity, it is most necessary that the maximum output of all new machines be gotten from them as soon as practicable after they have been put into operation; and it is good management to place as high a depreciation rate upon a machine as it can reasonably stand, and drive the tool as hard as possible, so that it may pay for itself in a few years. If no improved device or method appears the machine will still be available, while if a new and improved tool comes out the old machine may, and indeed must, be promptly scrapped to make way for the new one.

If this method were more closely followed there would be fewer superannuated establishments; and a shop which uses its tools up instead of nursing and coddling them is dealing more fairly by its stockholders than is the old-time plant.

The deduction for depreciation may be made in the form of a percentage of the cost of the machine or tool, and in such case the table opposite will be found convenient. This method is defective, however, in that the depreciation never equals the full original cost, and hence there is always some value left to the tool. Another plan is to charge off one-tenth, one-fifth, or one-third of the cost of the machine, as the case may be, and so have the entire inventory value wiped out at the expiration of ten, five, or three years, after which it may at any time be scrapped without compunction.

Valuable works upon the subject of works management and cost keeping are the following: J. Slater Lewis's "Commercial Organization of Factories," Arnold's "Complete Cost Keeper," Arnold's "Factory Manager and Accountant," Garcke & Fells's "Factory Accounts," Matheson's "Depreciation of Factories," and Metcalfe's "Cost of Manufactures."

APPENDIX

ALUMINUM.

In various modern structures aluminum or some of its alloys are used when lightness is of importance, and the following information, furnished by the Pittsburg Reduction Company, will be found useful:

The low specific gravity of aluminum is one of its most striking properties, being 2.56 in ordinary castings of pure aluminum, and 2.68 in the compressed and worked.

Specific Gravity at 62° F. of Aluminum and Aluminum Alloys.

Aluminum commercially pure, cast	2.56
Nickel-aluminum alloy ingots, for rolling	2.72
Nickel-aluminum casting alloy	2.85
Special casting alloy, cast	3.00
Aluminum commercially pure, as rolled, sheets, and wire	2.68
Aluminum commercially pure, annealed	2.66
Nickel-aluminum alloy, as rolled, sheets, and wire	2.76
Nickel-aluminum alloy, sheets annealed	2.74

Weight.

Using these specific gravities, assuming water at 62° F. and at standard barometric height as 62.355 pounds per cubic foot.

Sheet of cast-aluminum, 12 inches square and 1 inch thick, weighs	13.3024 pounds.
Sheet of rolled aluminum, 12 inches square and 1 inch thick, weighs	13.9259 pounds.
Bar of cast-aluminum, 1 inch square and 12 inches long, weighs	1.1085 pounds.
Bar of rolled aluminum, 1 inch square and 12 inches long, weighs	1.1605 pounds.
Bar of aluminum, cast, 1 inch round and 12 inches long, weighs	.8706 pound.
Bar of rolled aluminum, 1 inch round and 12 inches long, weighs	.9114 pound.
The weight per cubic inch of pure cast-aluminum is	.0920 pound.
The weight per cubic inch of pure rolled aluminum, annealed, is	.0970 pound.
The weight per cubic foot of pure cast-aluminum is	159.6288 pounds.
The weight per cubic foot of pure rolled aluminum is	167.1114 pounds.

Strength of Aluminum.

The tensile, crushing, and transverse tests of aluminum vary considerably with different conditions of hardness, due to cold working; also by the amount of work that has been put upon the metal, the character of the section, amount of hardening ingredients, etc. Cast-aluminum has about an equal strength to cast-iron in tension, but under compression it is comparatively weak. The following is a table giving the average results of many tests of aluminum of 99 per cent. purity:

Elastic limit per square inch in tension	castings	8,500 pounds.
	sheet	12,500 to 25,000 pounds.
	wire	16,000 to 33,000 pounds.
	bars	14,000 to 23,000 pounds.
Ultimate strength per square inch in tension	castings	18,000 pounds.
	sheet	24,000 to 40,000 pounds.
	wire	30,000 to 55,000 pounds.
	bars	28,000 to 40,000 pounds.
Per cent. of reduction of area in tension	castings	15 per cent.
	sheet	20 to 30 per cent.
	wire	40 to 60 per cent.
	bars	30 to 40 per cent.

Elastic limit per square inch under compression in cast cylindrical short columns, with length twice the diameter .... 3,500 pounds.
Ultimate strength per square inch under compression in cast cylindrical short columns, with length twice the diameter .... 12,000 pounds.
The modulus of elasticity of cast-aluminum is about 11,500,000.

Aluminum in castings can readily be strained to the unit stress of 1500 pounds per square inch in compression, and to 5000 pounds per square inch in tension. It is rather an open metal in its texture; and for cylinders, to stand pressure, an increase in thickness over the ordinary formulæ should be given to allow for its porosity.

Nickel-aluminum.

In order to obtain a greater strength than is possessed by pure aluminum, and at the same time to retain as much as possible the advantage of the low specific gravity, an alloy containing from 2 to 5 per cent. of nickel and copper is made, this having a specific gravity of about 2.85, as compared with 2.56 for pure aluminum.

The following table gives the average results of many tests of nickel-aluminum:

Elastic limit per square inch in tension	castings	8,500 to 12,000 pounds.
	sheet	21,000 to 30,000 pounds.
	bars	18,500 to 25,000 pounds.
Ultimate strength per square inch in tension	castings	18,000 to 28 000 pounds.
	sheet	35,000 to 50,000 pounds.
	bars	30,000 to 45,000 pounds.
Per cent. of reduction of area	castings	6 to 8 per cent.
	sheet	12 to 20 per cent.
	bars	12 to 15 per cent.
Elastic limit per square inch under compression in short columns, with length twice the diameter		6,000 to 10,000 pounds.
Ultimate strength per square inch under compression in short columns, with length twice the diameter		1,600 to 24,000 pounds.

Aluminum for Structural Purposes.

In the use of aluminum for structural purposes, a great deal depends upon the specific purpose to which it is desired to apply the metal, so as to know just what is the proper grade that should be used; but, generally speaking, for purposes where aluminum is brought into tension,—such as in sheets or in rolled shapes, as angles, beams, etc.,—an ultimate tensile strength of from 32,000 to 40,000 pounds per square inch may be reckoned upon, and using a safety factor of 4 gives an allowable working strain of from 8000 to 10,000 pounds. This, of course, is not for pure metal, but for the stronger alloys.

The ultimate tensile strength of pure metal in plates and shapes may be taken at from 24,000 to 28,000 pounds. With the same safety factor of 4 it gives an allowable working strain of from 6000 to 7000 pounds.

For the alloys of cast-aluminum in tension the ultimate strength may be taken at from 18,000 to 28,000 pounds per square inch. Using a safety factor here of 5, as aluminum castings are quite uniform and solid, a working strain is obtained of from 3600 to 5600 pounds per square inch.

It is difficult to give a value for the ultimate strength of pure cast-aluminum in tension, for the reason that while the ordinary pure aluminum will run about 16,000 pounds per square inch, this can be increased very considerably by cold working, and in some cases to as much as 24,000 pounds per square inch. Using a safety factor of 4 gives an allowable working strain of from 3200 to 4800 pounds.

In compression, the alloys of aluminum in rolled plates and structural shapes—such as struts, columns, etc.—have an ultimate tensile strength of from 26,000 to 34,000 pounds per square inch, which, using a safety factor of 4, gives an allowable working strain of from 6500 to 8500 pounds per square inch.

Pure aluminum sheets and structural shapes in compression have an ultimate tensile strength of from 20,000 to 24,000 pounds per square inch, which, with a safety factor of 4, gives an allowable working strain of from 5000 to 6000 pounds per square inch.

Castings of aluminum in compression can be taken at 16,000 pounds per square inch for pure aluminum, and from this to 24,000 pounds per square inch for the alloys. Using again a safety factor of 5, an allowable working strain is given of from 3200 to 4600 pounds per square inch. But the pure metal should not be used in castings, except for electrical purposes, as it is similar to pure copper in being difficult to cast, and is soft, comparatively weak, and has a large shrinkage. In its stead, alloys with from 5 to 20 per cent. of copper, nickel, or other hardeners should be used.

The alloys of aluminum in rivets and similar shapes in shear have an ultimate shearing strength of from 24,000 to 27,000 pounds, which, using here a safety factor of 6, gives an allowable working strain of from 4000 to 4500 pounds per square inch.

The ratios of the ultimate shearing strength to the ultimate tensile strength for double-riveted joints is about 60 per cent., and for single-riveted joints the ratio is about 70 per cent. The ratio for steel is about 75 per cent.

In bearing, the ultimate value of the alloys of aluminum is from 32,000 to 40,000 pounds per square inch, which, using a safety factor of 4, gives an allowable working strain of from 8000 to 10,000 pounds.

Electrical Properties of Aluminum.

The electrical conductivity of silver being taken as 100, that of pure aluminum is about 63.

Aluminum is practically non-magnetic, and may therefore be used for many purposes in electrical work, where a magnetic metal would be useless. At the same time, its electrical conductivity is excellent, as the following electrical conductivities of various metals will show. Aluminum may therefore in the future be largely used in the windings of field magnets on dynamos, where weight is an object, and, in general, for switches, brushes, brush-holders, and apparatus where its non-tarnishing and non-corrosive qualities render it specially valuable.

As is the case with other metals of good electrical conductivity, the conducting power of aluminum is greatly decreased as the result of the presence of alloying metals. Pure aluminum has a much higher relative conductivity to pure copper than is ordinarily given in the books, occasioned by the considerable impurities in the aluminum that has been in the past tested for its relative electrical conductivity.

The following tests, made by Mr. Charles F. Scott and Professor J. W. Richards, will serve to show the relative conductivity of various samples:

Sample.	Ohms per 1000 feet, .01 inch diameter, at 15° C.	Percentage of conductivity at 15° C.	Percentage of variation between 15° C. and 80° C.
Pure copper wire	101.83	100.00	.388
No. 1, 99½ per cent. pure aluminum.	161.40	63.09	.385
No. 2, 99 per cent. pure aluminum	163.80	62.17	.385
No. 3, 98 per cent. pure aluminum	181.30	56.17	.360
No. 4, XB, nickel-aluminum alloy	174.10	58.48	.361
No. 5, X C W C, copper-zinc-aluminum alloy	185.10	55.01	.359
Result of Professor J. W. Richards on the 99½ per cent. pure aluminum.	....	64.50	.300

Taking into account the relative specific gravity as well as the relative conductivity, it has been computed that when the price of aluminum per pound is double that of copper their values for electrical conductors are equal.

LOCOMOTIVE DATA.

The following formulas are those of the Baldwin Locomotive Works, Philadelphia, and have shown themselves reliable in the practice of that well-known establishment.

Speed Resistance, Locomotive and Train.

$$R = 3 + \frac{V}{6}.$$

R = resistance, in pounds, per ton of 2000 pounds;
V = speed, in miles, per hour.

This formula represents the resistance for sustained speed, and the element of acceleration is not taken into consideration. It is deduced from the results obtained by comparison of a large number of indicator cards taken at various speeds.

Grade Resistance.

The resistance for a straight grade of 1 foot per mile is 0.3788 pound per ton.

If

G = grade, in feet, per mile;
T = weight of train, in tons (2000 pounds);
R = resistance, in pounds.

$$R = 0.3788\ GT.$$

Curve Resistance.

Taking the curve as expressed in degrees of deflection from a tangent measured from stations 100 feet apart, the resistance of curves may be expressed as in proportion to the number of degrees in the curve. The resistance naturally varies with the construction of the road-bed, speed of train, and other conditions of service, so that no general rule can be expected to apply to all cases. Approximately, with moderate speed and under ordinary conditions, the resistance may be computed on the basis that each degree of curvature is equal to a straight grade of 1½ feet per mile.

The following formula corresponds to this allowance:

Let

A = angle of curve, in degrees;
T = weight of train, in tons;
R = resistance, in pounds.

$$R = 0.5682\ AT.$$

Acceleration Resistance.

The resistance opposed to the acceleration of a train from any speed to any higher speed may be computed by the following formula:

Let

R = the resistance, in pounds;
T = weight of train, in tons (2000 pounds);
V = initial speed, in miles, per hour;
V' = accelerated speed.

$$R = 0.0132\,(V'^2 - V^2)\,T.$$

Thus, for a weight of 1 ton and an acceleration from 30 miles per hour to 50 miles, we have

$$\begin{aligned} &0.0132\,(50^2 - 30^2) &&= \\ &0.0132\,(2500 - 900) &&= \\ &0.0132 \times 1600 &&= 21 \text{ pounds,} \end{aligned}$$

and this, multiplied by the weight of the train, in tons, will give the total resistance due to the acceleration.

Tractive Power.

Let

d = diameter of cylinder;
l = length of stroke;
D = diameter of driving-wheels, in inches;
T = tractive power, in pounds, per pound of mean effective pressure in cylinder.

$$T = \frac{d^2 l}{D}.$$

The mean effective pressure may be taken as equal to 85 per cent. of the boiler pressure.

The tractive power of a locomotive multiplied by the speed, in miles, per hour, divided by 375, gives the horse-power.

THE POWERING OF STEAMSHIPS.

The most reliable method of determining the power required to propel a vessel at a given speed is to use a model of the hull in a testing tank, and this should be done in all important designs.

The following tables (pages 796–799), originally prepared by Nystrom, will be found to agree closely with the results attained by modern steamships in actual service, and may be used when experimental data are lacking.

The average powering of modern steamships is about 1 horse-power per ton of displacement, while for the fast liners it reaches 2 horse-power per ton and over.

Steamship Performance.

Displacement, in tons.	Knots, or nautical miles per hour.									
	1	2	3	4	5	6	7	8	9	10
T	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*
1	.004	.035	.118	.280	.55	.949	1.50	2.24	3.20	4.38
2	.007	.055	.190	.444	.87	1.51	2.40	3.55	5.08	6.96
3	.009	.075	.248	.598	1.14	1.98	3.12	4.79	6.91	9.12
4	.010	.084	.300	.673	1.40	2.40	3.80	5.39	8.06	11.1
5	.012	.102	.348	.818	1.52	2.78	4.40	6.55	9.36	12.2
6	.014	.115	.390	.924	1.81	3.12	4.96	7.39	10.6	14.5
7	.016	.128	.435	1.025	2.01	3.48	5.50	8.20	11.7	16.1
8	.017	.138	.479	1.125	2.20	3.80	6.01	8.96	12.8	17.5
9	.019	.151	.501	1.211	2.38	4.12	6.51	9.69	13.8	19.0
10	.020	.161	.552	1.30	2.54	4.42	6.98	10.4	14.9	20.3
11	.022	.175	.590	1.40	2.72	4.70	7.46	11.1	15.9	21.8
12	.023	.185	.624	1.48	2.88	4.99	7.90	11.8	16.8	23.0
13	.024	.195	.654	1.56	3.04	5.25	8.33	12.5	17.7	24.3
14	.024	.198	.690	1.62	3.18	5.52	8.75	13.0	18.6	25.4
15	.026	.213	.725	1.70	3.32	5.80	9.20	13.6	19.5	26.6
16	.028	.223	.780	1.78	3.49	6.04	9.55	14.2	20.4	27.9
17	.029	.236	.785	1.89	3.64	6.28	9.95	15.0	21.2	29.1
18	.030	.242	.815	1.94	3.78	6.52	10.3	15.5	22.0	30.2
19	.031	.250	.850	2.00	3.90	6.80	10.7	16.0	22.8	31.2
20	.032	.258	.875	2.06	4.02	7.00	11.1	16.5	23.0	32.2
25	.038	.300	1.015	2.40	4.14	8.12	12.9	19.2	24.2	33.1
30	.042	.338	1.14	2.70	5.30	9.18	14.6	21.6	31.0	42.4
35	.047	.375	1.26	3.00	5.89	10.1	16.2	24.0	34.2	47.1
40	.050	.409	1.39	3.27	6.41	11.1	17.6	26.2	37.5	51.3
45	.056	.445	1.50	3.56	6.95	12.0	19.0	28.5	40.5	55.6
50	.056	.474	1.61	3.79	7.44	12.9	20.5	30.3	43.2	59.5
55	.062	.501	1.72	4.06	7.95	13.8	21.8	32.5	46.2	63.6
60	.067	.538	1.80	4.30	8.41	14.4	23.1	34.4	49.1	67.3
65	.071	.570	1.90	4.56	8.88	15.1	24.4	36.5	51.8	71.0
70	.074	.597	2.02	4.77	9.36	16.2	25.5	38.2	54.4	74.9
75	.078	.625	2.12	5.00	9.77	16.9	26.8	40.0	56.8	78.0
80	.081	.650	2.20	5.20	10.2	17.6	28.0	41.6	58.1	81.6
85	.085	.680	2.30	5.44	10.6	18.4	29.2	43.5	62.0	85.0
90	.088	.705	2.38	5.64	11.0	19.1	30.5	45.2	64.5	88.4
95	.088	.710	2.49	5.68	11.4	19.9	31.3	47.0	66.6	91.5
100	.094	.755	2.56	6.04	11.8	20.5	32.4	48.4	68.5	94.5
110	.101	.810	2.73	6.48	12.6	21.9	34.6	51.8	73.2	101.0
125	.109	.877	2.98	7.02	13.7	23.8	37.5	56.2	80.0	110.0
150	.124	.99	3.38	7.72	15.5	27.0	42.8	61.7	90.5	124.0
175	.138	1.10	3.72	8.81	17.2	29.8	47.2	70.5	100.0	138.0
200	.150	1.20	4.06	9.6	18.8	32.5	51.5	76.9	110.0	150.0
225	.162	1.30	4.39	10.4	20.2	35.1	56.0	83.3	118.0	162.0
250	.175	1.40	4.70	11.2	21.9	37.6	59.8	89.2	127.0	175.0
275	.188	1.50	5.04	11.9	23.2	40.3	63.8	95.2	136.0	186.0
300	.196	1.57	5.31	12.6	24.5	42.5	67.5	100.0	142.0	196.0
325	.201	1.66	5.63	13.3	26.0	45.0	71.2	106.0	152.0	208.0
350	.220	1.75	5.91	14.0	27.4	47.3	75.0	112.0	159.0	219.0
375	.228	1.82	6.12	14.6	28.6	49.0	78.4	117.0	166.0	229.0
400	.240	1.91	6.42	15.3	29.8	51.4	81.7	122.0	172.0	238.0
450	.250	2.06	6.98	16.5	32.2	55.8	88.5	132.0	188.0	258.0
500	.276	2.21	7.45	17.7	34.6	59.6	94.3	141.0	200.0	276.0
550	.295	2.36	7.98	18.9	36.9	63.8	101.0	151.0	215.0	295.0
600	.312	2.50	8.40	20.0	39.0	67.2	107.0	160.0	226.0	313.0
650	.330	2.64	8.90	21.1	41.2	71.2	113.0	169.0	240.0	329.0
700	.348	2.78	9.32	22.2	43.3	74.6	119.0	177.0	250.0	337.0
750	.362	2.90	9.80	23.2	45.2	78.4	124.0	186.0	264.0	352.0
800	.380	3.03	10.2	24.2	47.3	81.5	130.0	194.0	274.0	378.0
850	.394	3.15	10.6	25.2	49.2	85.0	135.0	202.0	288.0	394.0
900	.410	3.28	11.0	26.2	51.1	88.1	140.0	210.0	296.0	409.0
950	.422	3.41	11.4	27.3	53.1	91.8	146.0	218.0	310.0	445.0

Steamship Performance.

Knots, or nautical miles per hour.										Displacement, in tons.
11	12	13	14	15	16	17	18	19	20	
H	H	H	H	H	H	H	H	H	H	T
5.85	7.59	9.63	12.0	14.8	17.9	21.6	25.6	30.1	35.1	**1**
9.28	12.0	15.3	19.1	23.5	28.4	34.2	40.6	47.8	54.7	**2**
12.2	15.8	20.0	25.0	30.8	38.3	44.8	53.3	62.6	73.0	**3**
14.8	19.2	24.4	30.3	37.4	43.1	54.3	64.5	75.9	88.4	**4**
17.2	22.2	28.3	35.2	43.4	52.4	63.0	74.9	88.0	97.8	**5**
19.4	25.1	31.9	39.7	49.0	59.1	71.1	84.5	99.2	116.0	**6**
21.4	27.8	35.3	44.0	54.0	65.5	79.0	93.7	110.0	128.0	**7**
23.4	30.4	38.6	48.1	59.3	68.7	86.2	102.0	121.0	140.0	**8**
25.3	32.9	41.8	52.1	64.0	77.5	93.2	110.0	130.0	152.0	**9**
27.2	35.3	44.8	55.8	68.8	83.2	100.0	119.0	140.0	163.0	**10**
29.0	37.6	47.8	59.7	73.5	89.0	107.0	127.0	150.0	174.0	**11**
30.7	39.9	50.6	63.2	77.7	94.4	113.0	134.0	158.0	184.0	**12**
32.4	42.0	53.3	66.6	82.0	99.6	120.0	142.0	167.0	194.0	**13**
34.0	44.2	56.0	70.0	86.0	105.0	126.0	149.0	176.0	203.0	**14**
35.6	46.3	58.7	73.5	90.0	109.0	131.0	156.0	183.0	213.0	**15**
37.2	48.3	61.3	76.5	94.0	114.0	137.0	163.0	192.0	223.0	**16**
38.7	50.2	63.8	79.6	98.0	120.0	143.0	170.0	200.0	233.0	**17**
40.2	52.2	66.2	82.7	102.0	124.0	148.0	176.0	207.0	242.0	**18**
41.7	54.0	68.7	85.8	106.0	128.0	154.0	182.0	215.0	250.0	**19**
43.2	56.0	71.0	88.9	111.0	132.0	159.0	189.0	222.0	258.0	**20**
50.0	65.0	82.5	103.0	127.0	154.0	184.0	194.0	258.0	265.0	**25**
56.5	73.4	93.2	117.0	143.0	173.0	208.0	248.0	291.0	339.0	**30**
62.6	81.3	103.0	130.0	159.0	192.0	230.0	274.0	322.0	377.0	**35**
68.4	88.8	113.0	141.0	173.0	209.0	252.0	300.0	350.0	410.0	**40**
74.0	96.2	122.0	152.0	188.0	228.0	273.0	324.0	382.0	445.0	**45**
79.4	103.0	131.0	164.0	201.0	242.0	293.0	346.0	410.0	476.0	**50**
84.6	110.0	140.0	174.0	215.0	260.0	312.0	370.0	437.0	509.0	**55**
90.0	117.0	149.0	185.0	226.0	285.0	330.0	393.0	464.0	538.0	**60**
94.7	123.0	156.0	195.0	240.0	292.0	349.0	414.0	488.0	568.0	**65**
99.6	130.0	164.0	206.0	252.0	306.0	367.0	437.0	512.0	599.0	**70**
104.0	135.0	171.0	214.0	264.0	320.0	383.0	455.0	536.0	624.0	**75**
109.0	141.0	180.0	224.0	276.0	333.0	400.0	467.0	561.0	653.0	**80**
113.0	147.0	187.0	234.0	287.0	348.0	417.0	496.0	584.0	680.0	**85**
118.0	153.0	194.0	243.0	298.0	362.0	433.0	516.0	607.0	707.0	**90**
122.0	158.0	201.0	251.0	309.0	376.0	448.0	533.0	629.0	732.0	**95**
126.0	164.0	207.0	259.0	318.0	387.0	464.0	551.0	648.0	756.0	**100**
135.0	175.0	222.0	277.0	340.0	414.0	495.0	588.0	693.0	807.0	**110**
146.0	190.0	241.0	300.0	370.0	450.0	539.0	640.0	753.0	878.0	**125**
165.0	215.0	273.0	342.0	420.0	494.0	609.0	724.0	852.0	992.0	**150**
183.0	238.0	302.0	378.0	464.0	564.0	675.0	802.0	946.0	1100.0	**175**
200.0	260.0	330.0	412.0	506.0	615.0	737.0	875.0	1027.0	1201.0	**200**
217.0	281.0	358.0	447.0	548.0	666.0	800.0	947.0	1118.0	1300.0	**225**
232.0	301.0	384.0	478.0	588.0	714.0	855.0	1016.0	1200.0	1400.0	**250**
248.0	322.0	409.0	510.0	627.0	762.0	912.0	1087.0	1286.0	1490.0	**275**
262.0	340.0	432.0	540.0	662.0	806.0	966.0	1146.0	1347.0	1573.0	**300**
277.0	360.0	457.0	570.0	700.0	852.0	1010.0	1213.0	1428.0	1665.0	**325**
290.0	378.0	480.0	600.0	737.0	896.0	1073.0	1276.0	1500.0	1750.0	**350**
305.0	395.0	502.0	627.0	770.0	936.0	1122.0	1332.0	1570.0	1830.0	**375**
317.0	412.0	522.0	654.0	803.0	976.0	1170.0	1402.0	1632.0	1907.0	**400**
343.0	446.0	567.0	708.0	870.0	1060.0	1265.0	1500.0	1770.0	2065.0	**450**
368.0	478.0	607.0	759.0	932.0	1131.0	1358.0	1611.0	1896.0	2213.0	**500**
393.0	510.0	648.0	810.0	995.0	1210.0	1450.0	1720.0	2025.0	2362.0	**550**
415.0	540.0	684.0	856.0	1036.0	1280.0	1532.0	1820.0	2140.0	2500.0	**600**
440.0	570.0	724.0	905.0	1111.0	1350.0	1618.0	1923.0	2265.0	2636.0	**650**
460.0	599.0	759.0	938.0	1166.0	1417.0	1700.0	2016.0	2373.0	2770.0	**700**
483.0	627.0	797.0	995.0	1220.0	1485.0	1780.0	2113.0	2490.0	2900.0	**750**
503.0	654.0	830.0	1038.0	1274.0	1548.0	1857.0	2206.0	2593.0	3026.0	**800**
525.0	680.0	866.0	1080.0	1330.0	1620.0	1935.0	2300.0	2710.0	3152.0	**850**
545.0	708.0	898.0	1123.0	1380.0	1675.0	2009.0	2385.0	2803.0	3274.0	**900**
565.0	734.0	933.0	1170.0	1430.0	1740.0	2080.0	2478.0	2920.0	3400.0	**950**

Steamship Performance.

Displacement, in tons.	Knots, or nautical miles per hour.									
	1	2	3	4	5	6	7	8	9	10
T	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*
1000	.438	3.50	11.8	28.0	54.9	94.6	150	225	318	439
1100	.456	3.75	12.5	30.0	58.4	100	160	239	338	467
1200	.500	4.00	13.4	32.0	62.0	107	170	254	359	495
1300	.515	4.12	14.0	33.0	65.3	112	179	267	378	523
1400	.548	4.38	14.9	35.0	68.7	119	189	281	398	549
1500	.562	4.50	15.5	36.0	71.9	124	197	295	417	575
1600	.578	4.62	16.2	37.0	75.0	130	206	307	435	600
1700	.594	4.75	16.9	38.0	78.1	135	215	320	453	625
1800	.625	5.00	17.5	40.0	81.2	140	224	332	470	649
1900	.634	5.25	18.1	42.0	84.2	145	231	345	488	673
2000	.700	5.60	18.8	44.0	87.0	150	239	356	504	696
2100	.719	5.75	19.4	46.0	90.0	155	247	369	521	720
2200	.735	5.88	20.0	47.0	92.7	160	255	380	537	741
2300	.765	6.12	20.6	49.0	95.6	165	262	391	554	764
2400	.788	6.28	21.1	50.2	98.4	170	270	402	569	786
2500	.805	6.44	21.8	51.5	101.0	174	277	414	585	808
2600	.828	6.62	22.4	53.0	104.0	179	285	424	600	826
2700	.851	6.81	23.0	54.5	106.0	184	292	436	616	850
2800	.872	6.98	23.5	55.8	109.0	188	299	446	631	871
2900	.876	7.12	24.0	57.1	111.0	192	306	457	646	893
3000	.909	7.35	24.6	58.8	114.0	197	313	467	660	913
3100	.931	7.45	25.1	59.8	117.0	201	320	478	676	933
3200	.952	7.62	25.6	61.0	119.0	205	327	488	690	952
3300	.972	7.78	26.1	62.2	121.0	209	334	498	704	972
3400	.992	7.94	26.8	63.5	124.0	214	340	508	718	992
3500	1.01	8.10	27.2	64.8	127.0	218	347	518	733	1010
3600	1.03	8.25	27.8	66.0	129.0	222	354	528	746	1025
3700	1.05	8.39	28.2	67.1	131.0	226	360	538	759	1049
3800	1.08	8.60	28.7	68.5	133.0	230	367	548	774	1070
3900	1.09	8.70	28.9	69.6	135.0	234	373	558	787	1087
4000	1.11	8.85	29.9	70.8	138.0	238	380	567	801	1105
4100	1.13	9.01	30.4	71.1	140.0	242	386	577	814	1122
4200	1.14	9.14	30.9	73.1	142.0	246	392	586	827	1141
4300	1.16	9.30	31.4	74.4	145.0	250	398	595	840	1160
4400	1.18	9.42	31.9	75.5	147.0	254	404	604	853	1179
4500	1.19	9.56	32.4	76.5	150.0	258	410	613	866	1198
4600	1.22	9.72	32.8	77.7	152.0	261	416	622	879	1216
4700	1.23	9.86	33.4	78.9	154.0	266	422	631	891	1232
4800	1.25	10.0	33.9	80.0	156.0	270	428	640	904	1248
4900	1.28	10.1	34.4	81.1	158.0	274	434	649	916	1265
5000	1.30	10.3	34.8	82.7	160.0	277	440	658	929	1282
5250	1.32	10.6	35.6	85.0	165.0	283	455	670	959	1324
5500	1.36	10.9	36.4	87.5	171.0	290	469	700	990	1367
5750	1.40	11.2	37.5	90.0	176.0	298	483	721	1024	1408
6000	1.42	11.4	38.0	92.8	181.0	303	497	742	1050	1448
6250	1.47	11.9	40.2	95.2	188.0	322	512	762	1065	1488
6500	1.52	12.2	41.2	97.8	191.0	330	526	782	1078	1526
6750	1.56	12.5	42.4	100.0	196.0	339	540	802	1123	1567
7000	1.60	12.9	43.2	103.0	202.0	346	554	822	1174	1616
7250	1.64	13.1	44.4	105.0	205.0	355	566	842	1198	1644
7500	1.68	13.5	45.5	108.0	210.0	364	579	861	1226	1682
7750	1.72	13.8	46.5	110.0	215.0	372	599	879	1253	1719
8000	1.75	14.0	47.4	112.0	220.0	379	603	899	1280	1757
8250	1.78	14.2	48.4	115.0	224.0	387	615	918	1306	1793
8500	1.81	14.5	49.4	116.0	229.0	395	628	929	1333	1829
8750	1.84	14.9	50.0	119.0	233.0	403	640	955	1354	1865
9000	1.88	15.2	51.1	122.0	238.0	411	653	973	1385	1902
9250	1.92	15.4	52.2	124.0	242.0	418	668	991	1411	1937
9500	1.95	15.6	53.2	126.0	246.0	426	683	1008	1437	1972
10000	2.05	16.4	55.1	131.0	256.0	441	714	1044	1488	2042

Steamship Performance.

Knots, or nautical miles per hour.										Displacement, in tons.
11	12	13	14	15	16	17	18	19	20	
H	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*H*	*T*
585	759	963	1206	1480	1798	2157	2560	3008	3514	**1000**
622	806	1024	1284	1574	1913	2295	2723	3203	3736	**1100**
660	858	1090	1360	1670	2030	2435	2890	3400	3907	**1200**
696	903	1147	1432	1758	2136	2564	3043	3576	4178	**1300**
732	950	1204	1508	1850	2248	2697	3200	3762	4394	**1400**
766	995	1264	1580	1938	2355	2825	3252	3943	4605	**1500**
800	1038	1317	1648	2020	2458	2948	3500	4113	4803	**1600**
833	1083	1374	1718	2107	2561	3072	3646	4286	5006	**1700**
864	1123	1422	1784	2188	2660	3140	3785	4448	5195	**1800**
897	1166	1479	1850	2270	2760	3310	3928	4615	5390	**1900**
927	1205	1527	1913	2345	2854	3420	4060	4770	5570	**2000**
958	1247	1582	1979	2382	2948	3535	4195	4935	5762	**2100**
988	1284	1628	2037	2500	3038	3642	4325	5084	5935	**2200**
1017	1324	1680	2102	2578	3134	3755	4460	5241	6120	**2300**
1047	1360	1723	2160	2646	3220	3860	4580	5386	6290	**2400**
1077	1400	1777	2222	2725	3313	3970	4715	5542	6470	**2500**
1102	1435	1820	2280	2796	3400	4075	4835	5655	6637	**2600**
1131	1473	1870	2338	2868	3486	4180	4960	5832	6813	**2700**
1160	1508	1911	2395	2935	3568	4280	5076	5970	6970	**2800**
1189	1545	1960	2452	3010	3655	4385	5200	6115	7142	**2900**
1215	1582	2000	2508	3075	3740	4485	5318	6255	7300	**3000**
1242	1614	2048	2565	3145	3822	4585	5440	6394	7470	**3100**
1268	1648	2092	2616	3210	3905	4680	5550	6525	7622	**3200**
1296	1683	2134	2671	3280	3985	4775	5670	6666	7781	**3300**
1320	1717	2178	2725	3343	4063	4870	5784	6784	7936	**3400**
1347	1750	2220	2779	3408	4143	4965	5893	6936	8090	**3500**
1373	1783	2264	2830	3475	4222	5060	6010	7061	8250	**3600**
1398	1815	2303	2881	3534	4300	5155	6115	7184	8400	**3700**
1422	1848	2348	2941	3606	4385	5250	6238	7333	8563	**3800**
1446	1880	2385	2986	3660	4453	5340	6336	7444	8696	**3900**
1473	1912	2427	3038	3725	4530	5430	6444	7580	8847	**4000**
1497	1944	2468	3086	3785	4610	5520	6550	7700	8988	**4100**
1520	1975	2507	3137	3850	4680	5610	6655	7830	9141	**4200**
1545	2008	2546	3186	3910	4750	5700	6761	7950	9285	**4300**
1568	2037	2585	3238	3970	4825	5790	6865	8072	9432	**4400**
1593	2070	2624	3286	4025	4900	5875	6970	8195	9572	**4500**
1614	2100	2664	3333	4087	4975	5960	7070	8320	9710	**4600**
1639	2130	2702	3382	4145	5040	6045	7172	8437	9850	**4700**
1663	2160	2740	3431	4202	5112	6130	7275	8555	9990	**4800**
1686	2190	2779	3478	4260	5193	6215	7375	8673	10120	**4900**
1708	2220	2817	3525	4321	5253	6300	7475	8792	10250	**5000**
1760	2293	2909	3640	4414	5426	6507	7723	9081	10601	**5250**
1822	2365	3000	3755	4608	5600	6715	7972	9370	10953	**5500**
1876	2436	3090	3868	4744	5767	6917	8204	9652	11269	**5750**
1930	2507	3180	3981	4880	5935	7120	8436	9935	11586	**6000**
1982	2574	3261	4094	5013	6096	7313	8519	10203	11902	**6250**
2035	2642	3352	4207	5146	6258	7505	8603	10472	12218	**6500**
2088	2710	3438	4320	5281	6419	7698	8986	10741	12534	**6750**
2141	2778	3524	4434	5417	6580	7892	9370	11010	12851	**7000**
2191	2842	3606	4531	5542	6733	8076	9587	11265	13152	**7250**
2241	2907	3688	4629	5668	6886	8260	9805	11521	13453	**7500**
2290	2971	3770	4726	5794	7039	8445	10022	11776	13754	**7750**
2340	3036	3852	4824	5920	7192	8628	10240	12032	14056	**8000**
2488	3098	3931	4923	6042	7340	8806	10451	12280	14345	**8250**
2636	3161	4011	5023	6164	7488	8984	10662	12528	14634	**8500**
2784	3223	4095	5123	6286	7637	9162	10823	12776	14922	**8750**
2933	3286	4170	5222	6408	7785	9340	11084	13024	15211	**9000**
3080	3346	4247	5343	6516	7926	9512	11289	13364	15493	**9250**
3222	3407	4324	5465	6645	8068	9685	11494	13505	15775	**9500**
3370	3529	4478	5708	6882	8351	10030	11904	13987	16340	**10000**

Conversion of Horse-power into Kilowatts and Kilowatts into Horse-power.

No.	Kilowatts to horse-power.	Horse-power to kilowatts.	No.	Kilowatts to horse-power.	Horse-power to kilowatts.	No.	Kilowatts to horse-power.	Horse-power to kilowatts.
1	1.341	.746	38	50.943	28.3	75	100.545	55.9
2	2.681	1.49	39	52.283	29.1	76	101.886	56.7
3	4.022	2.24	40	53.624	29.8	77	103.226	57.4
4	5.363	2.98	41	54.965	30.6	78	104.567	58.2
5	6.703	3.73	42	56.305	31.3	79	105.907	58.9
6	8.044	4.48	43	57.646	32.1	80	107.248	59.7
7	9.384	5.22	44	58.986	32.8	81	108.588	60.4
8	10.725	5.97	45	60.327	33.6	82	109.929	61.2
9	12.065	6.71	46	61.667	34.3	83	111.269	61.9
10	13.406	7.46	47	63.008	35.1	84	112.610	62.7
11	14.747	8.21	48	64.349	35.8	85	113.951	63.4
12	16.087	8.95	49	65.689	36.5	86	115.292	64.2
13	17.428	9.7	50	67.030	37.3	87	116.632	64.9
14	18.768	10.4	51	68.371	38.0	88	117.973	65.6
15	20.109	11.2	52	69.711	38.8	89	119.313	66.4
16	21.450	11.9	53	71.052	39.5	90	120.654	67.1
17	22.790	12.7	54	72.392	40.3	91	121.995	67.9
18	24.131	13.4	55	73.733	41.0	92	123.335	68.6
19	25.471	14.2	56	75.074	41.8	93	124.676	69.4
20	26.812	14.9	57	76.414	42.5	94	126.016	70.1
21	28.153	15.7	58	77.755	43.3	95	127.357	70.9
22	29.493	16.4	59	79.095	44.0	96	128.698	71.6
23	30.834	17.2	60	80.436	44.8	97	130.038	72.4
24	32.174	17.9	61	81.777	45.5	98	131.379	73.1
25	33.515	18.6	62	83.117	46.2	99	132.719	73.8
26	34.856	19.4	63	84.458	47.0	100	134.06	74.6
27	36.196	20.1	64	85.798	47.7	200	268.12	149.0
28	37.537	20.9	65	87.139	48.5	300	402.18	224.0
29	38.877	21.6	66	88.480	49.2	400	536.24	298.0
30	40.218	22.4	67	89.820	50.0	500	670.30	373.0
31	41.559	23.1	68	91.161	50.7	600	804.36	448.0
32	42.899	23.9	69	92.501	51.5	700	938.42	522.0
33	44.240	24.6	70	93.842	52.2	800	1072.48	597.0
34	45.580	25.4	71	95.183	53.0	900	1206.54	671.0
35	46.921	26.1	72	96.523	53.7	1000	1340.60	746.0
36	48.261	26.9	73	97.864	54.5	....	....	....
37	49.602	27.6	74	99.204	55.2	....	....	....

Unit Equivalents for Electric-heating Problems.

1 kilowatt hour =
- 1000 watt hours.
- 1.34 horse-power hours.
- 2,656,400 foot-pounds.
- 3,600,000 joulés.
- 3440 heat units.
- 366,848 kilogrammetres.
- 0.229 pound of coal oxidized with perfect efficiency.
- 3 pounds of water evaporated at 212° F.
- 22.9 pounds of water raised from 62° to 212° F.
- 8 cents at usual rates for electric heating.

1 horse-power hour =
- 0.746 kilowatt hour.
- 1,980,000 foot-pounds.
- 2580 heat units.
- 273,740 kilogrammetres.
- 0.172 pound of coal oxidized with perfect efficiency.
- 225 pounds of water evaporated at 212° F.
- 17.2 pounds of water raised from 62° to 212° F.
- 6 cents at usual rates for electric heating.

1 kilowatt =
- 1000 watts.
- 1.34 horse-power.
- 2,656,400 foot-pounds per hour.
- 4424 foot-pounds per minute.
- 73.73 foot-pounds per second.
- 3440 heat units per hour.
- 573 heat units per minute.
- 9.55 heat units per second.
- 0.229 pound of coal oxidized per hour.
- 3 pounds of water evaporated per hour at 212° F.

1 horse-power =
- 746 watts.
- 0.746 kilowatts.
- 33,000 foot-pounds per minute.
- 550 foot-pounds per second.
- 2580 heat units per hour.
- 43 heat units per minute.
- 0.71 heat unit per second.
- 0.172 pound of coal oxidized per hour.
- 2.25 pounds of water evaporated per hour at 212° F.

1 joulé =
- 1 watt second.
- 0.00000278 kilowatt hour.
- 0.102 kilogrammetre.
- 0.00094 heat unit.
- 0.73 foot-pound.

1 foot-pound =
- 1.36 joulés.
- 0.1383 kilogrammetre.
- 0.000000377 kilowatt hour.
- 0.000291 heat unit.
- 0.0000005 horse-power hour.

1 watt =
- 1 joulé per second.
- 0 00134 horse-power.
- 0.001 kilowatt.
- 3.44 heat units per hour.
- 0.73 foot-pound per second.
- 0.003 pound of water evaporated per hour.
- 44.24 foot-pounds per minute.

1 watt per square inch =
- 8.26 thermal units per square foot per minute.
- 120° F. above surrounding air (japanned cast-iron surface).
- 66° C. above surrounding air (japanned cast-iron surface).

1 heat unit =
- 1.048 watt seconds.
- 778 foot-pounds.
- 0.252 caloric (kg. d.).
- 108 kilogrammetres.
- 0.000291 kilowatt hour.
- 0.000388 horse-power hour.
- 0.0000667 pound of coal, oxidized.
- 0.00087 pound of water evaporated at 212° F.

1 heat unit per square foot per minute =
- 0.021 watt per square inch.
- 0.0174 kilowatt.
- 0.0232 horse-power.

1 kilogrammetre =
- 7.23 foot-pounds.
- 0.00000366 horse-power hour.
- 0.00000272 kilowatt hour.
- 0.0092 heat unit.

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THE END.

A SYSTEM OF ELECTRIC DRIVE FOR MACHINE TOOLS, WITH METHODS OF VARIABLE SPEED CONTROL.

(As furnished by the Crocker-Wheeler Company.)

Besides eliminating the disadvantages of line shafting, belting, and the inflexibility of location, the individual drive of machine tools by electric motors increases the efficiency and output of a machine shop. The ordinary belt-driven tool usually has a speed range obtained by mechanical means of from 20 : 1 to 50 : 1, with increasing speed steps of about 30 to 50 per cent. The Crocker-Wheeler system for the multiple-voltage operation of machine shops not only extends the speed range, but also reduces the speed increment per step to about 10 per cent., which has been found by experience to be as small an amount as would be desirable to use. This system is a method of electric-power distribution at different voltages, which enables standard motors to be operated at variable speed by changing the potential of the current at their terminals. The generating plant supplies the highest voltage of the system. This voltage may be termed the primary, and is divided by a 3-unit balancing transformer into three unvarying voltages of unequal value, which are maintained between the wires of a 4-wire circuit, various connections of which afford six different and distinct voltages.

The principle on which this system of speed control is based is that in a separately-excited shunt motor the speed of the armature is proportional to the voltage supplied to its terminals. If this voltage remains constant, the speed will remain constant even with varying load.

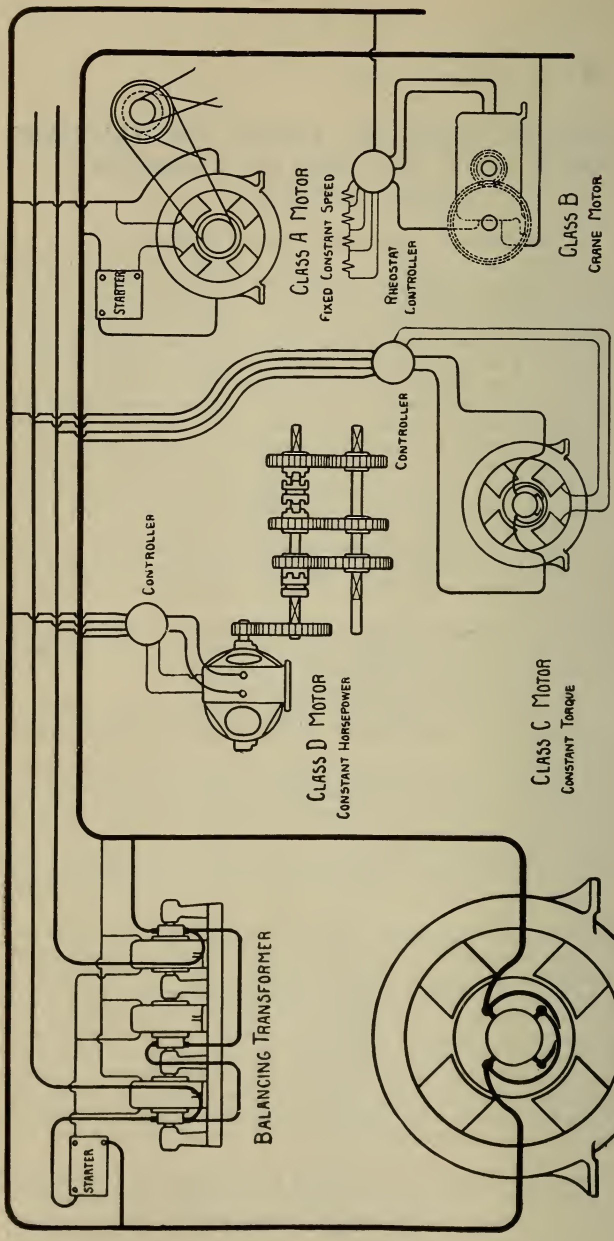
It is the function of the balancer to maintain these voltages constant and to accommodate the unbalance of currents between the four wires of the distribution circuit.

As the conditions of machine-tool operation will result in the various motors of the system being nearly equally distributed on the various circuits, the unbalanced current will be but a small percentage of the total current taken by all the motors.

The intermediate wires of the system are extended to the variable speed motors only, the constant-speed and crane motors and the lighting being supplied in the usual manner from the outside wires at the generator voltage.

Those motors requiring variable speed are connected to the 4-wire circuit by means of a controller of the drum type adapted for mounting on the tool in a place convenient for the operator. The action of this controller is such that, as the drum revolves, the armature terminals of the motor are connected to the six circuits—afforded by this system—in the proper sequence, and the travel of the drum from one position to the next is so quickened by the action of a spring that contacts are made and broken at a high rate of speed, preventing the formation of arcs and eliminating the possibility of the drum stopping between contacts. This gives six fundamental motor speeds, which are subject to a further refinement by varying the motor's field strength sufficiently to cover the gaps between them.

The speed range obtained on the voltage points alone is 6 : 1, being proportional to the ratio of maximum to minimum voltages. The addition of field-resistance points above the highest voltage points extends the total range in the controller to a value of 10 : 1. For exceptional cases the range


Class A Motor
Fixed Constant Speed
Starter
Rheostat
Controller
Class B
Crane Motor
Controller
Controller
Class D Motor
Constant Horsepower
Class C Motor
Constant Torque
Balancing Transformer
Starter
Generator 240 Volts.

may be increased to a maximum of 12:1, the proper range in any case being determined by the character of the machine tool and the work which it performs.

The Crocker-Wheeler system, as outlined, has certain positive advantages, of which the most important are the following:

1. Variable speed, under instant control, over any range.
2. Every speed constant, regardless of the load.
3. Controllers simple and convenient of attachment.
4. The horse-power of the motor but slightly in excess of that required by the tool.
5. Output of machine tools much greater than when they are belt-driven.
6. Easy of adaptation to existing shops with 2-wire system of electric-power distribution.
7. Employment of standard motors.
8. Ability to maintain high cutting speeds due to superior facilities for manipulation.

Motors used in an ordinary shop equipment may be divided into classes A, B, C, or D, according to the nature of their duty.

Class A being constant-speed motors, such as drive groups of small tools by shafting.

Class B, controllable-speed motors, generally of the series-wound type, as used on cranes.

The duty which the motors in both these classes have to perform is such that their demand for current is intermittent and often excessive, consequently they are best suited for connection to the outside mains, and such speed regulation as they may require can be obtained by rheostatic control.

The other two classes, C and D, are controllable-speed motors for the drive of individual tools, where the speed should be maintained constant at any one of a number of fixed values.

Class C is formed of motors driving pressure blowers, punch presses, planers, etc., which demand approximately constant torque at all speeds, the horse-power diminishing with the speed. This characteristic of the tool being identical with the power characteristic of the motor on this system, the normal horse-power of the motor need not be greater than the maximum demanded by the tool.

Class D covers those motors operating lathes, boring mills, etc., where the torque increases as the speed diminishes. If the range required by these tools is to be obtained by using a motor through its maximum range, the motor would be very large and unnecessarily expensive. For this class a speed range of approximately 3:1 has been selected as a basis for the determination of the most suitable sizes of motors, with respect to the duty which they have to perform. A motor, therefore, to give a constant horse-power throughout this range, must have a normal rating of about twice the horse-power required by the tool. This range, however, may be extended to cover the entire range required by the tool by using one or more additional gear runs. The method is an advantageous compromise between the use of an excessively large motor with no gears and the constant-speed motor with many gears.

The extreme facility of manipulation which this system affords enables the machinist to push his tool to the highest limit of cutting speed and gives large increases in output. Results show that as much as 20 per cent. increase in output over a belt-driven tool may be obtained by this system of motor drive. As by actual test in commercial plants it has been shown that 2¼ per cent. increase is sufficient to warrant the outlay necessary for individual drive, the possibility of large saving in operating expense is at once apparent.

FOR HEAVY LIFTING

(Try This)

a compact powerful Triplex Block enables a man to handle a load that five men cannot handle with the old slow and dangerous hoisting methods.

Compare the Triplex with any other block and it gives double speed at the same pull.

TO COMPARE SPEEDS: start both hooks together at the floor and pull equally on both hand chains; note the rise of the hooks.

TO DETERMINE EASE: hoist the same load with one block then with the other; note ease of pulling with the Triplex.

We make three types of Blocks, each the best for its purpose. Write us and we will send you a book containing some valuable hoisting hints.

We send Blocks for trial use, and pay the freight if not kept. ½ to 20 tons, any height of hoist.

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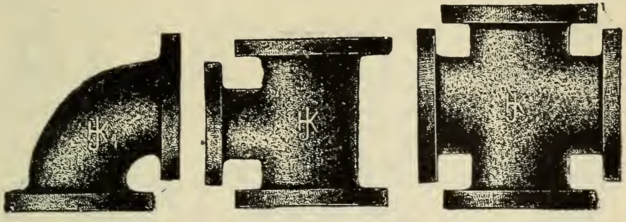

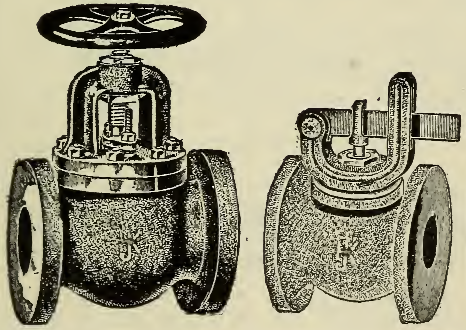

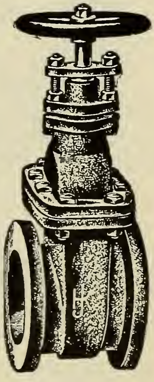

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